Resonant enhancement of macroscopic quantum tunneling in Josephson junctions: Influence of coherent two-level systems

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We report a theoretical study of the macroscopic quantum tunneling (MQT) in small Josephson junctions containing randomly distributed two-level systems. We focus on a Josephson phase escape for switching from the superconducting (the zero-voltage) state to a resistive one. Above the crossover temperature T_{cr} the thermal fluctuations of the Josephson phase induce such a switching, and as $T < T_{cr}$ the regime of the MQT occurs. In the absence of two-level systems (TLSs) a magnetic field applied parallel to the junction plane results in a smooth reduction of $T_{cr}(\Phi)$, where Φ is an applied magnetic flux. As the TLSs are present in Josephson junctions we obtain a resonant enhancement of the MQT. This phenomenon manifests itself by a narrow peak in the dependence of $T_{cr}(\Phi)$ occurring in the intermediate range of Φ, i.e., $0 < \Phi < \phi_0$ (ϕ_0 is the magnetic flux quantum). We explain this effect quantitatively by a strong resonant suppression of the potential barrier for the Josephson phase escape that is due to the *coherent quantum Rabi oscillations* in two-level systems present in the junction.

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I. INTRODUCTION

Currently there is a great interest in experimental and theoretical studies of macroscopic quantum phenomena in diverse Josephson systems [1–9]. It is well known that at low temperatures switching from the superconducting (the zero-voltage) state to a resistive one occurs in the form of macroscopic quantum tunneling (MQT) of a Josephson phase [1,3,4,6–9]. At high temperatures the so-called Josephson phase escape phenomenon is determined by thermal fluctuations.

In a simplest case of a single degree of freedom, i.e., the Josephson phase φ, the crossover temperature T_{cr} between these two regimes is determined by the frequency of small oscillations ω_0 of the Josephson phase on the bottom of potential well. Since the Josephson plasma frequency ω_p is determined by the critical current I_c as $\omega_p \propto I_c^{1/2}$, one can expect that ω_0 and the crossover temperature T_{cr} vary with an applied magnetic field. Indeed, the crossover temperature is written as [1] $k_B T_{cr} = \frac{\hbar\omega_0}{2\pi} = \frac{\hbar\omega_p}{2\pi}(1-j)^{1/4}$, where $j = I/I_c$ is the normalized external current I. The Josephson phase escape in the MQT regime occurs as the potential barrier $U_0 \simeq \frac{\hbar I_c}{2e}(1-j)^{3/2}$ becomes comparable with the energy of small oscillations $\hbar\omega_0$. Therefore, the typical values of the external current I allowing the Josephson phase escape are $(1-j) \propto I_c^{-2/5}$. The crossover temperature T_{cr} is written as

$$k_B T_{cr} \propto I_c^{2/5}. \tag{1}$$

A magnetic field applied parallel to the junction plane results in the reduction of the critical current I_c and one can expect a smooth decrease of the crossover temperature T_{cr} with the external flux Φ in the region $0 < \Phi < \phi_0$. Here, $\phi_0 = hc/2e$ is the magnetic flux quantum.

A crucial condition allowing one to obtain the dependence (1) is the absence of interactions of the Josephson phase with other degrees of freedom. E.g., one can expect in the correspondence with a generic analysis [3] that the Josephson phase interaction with a large amount of linear oscillators (such an interaction has been used as a model of dissipative environment) results in a suppression of both the MQT and the crossover temperature. However, a careful preparation of experimental setup has allowed one to reduce these undesirable effects.

The interaction of a Josephson phase with other degrees of freedom can result in the reduction of the potential barrier and, therefore, lead to an enhancement of the MQT [8–13]. E.g., in the presence of a magnetic field the intrinsic cavity modes are excited, and an enhancement of the MQT has been obtained [10,11]. However, for small Josephson junctions the probability of the cavity modes excitation becomes rather small [10], and an enhancement of the MQT is also small. Moreover, such an enhancement of the MQT has to lead to a smooth dependence of the crossover temperature T_{cr} on an applied magnetic field. In all these cases the equilibrium dynamics of the Josephson phase interacting with a set of linear oscillators has been considered. Notice here that a strong back influence of the Josephson phase on the oscillators dynamics suppresses the resonant effects, and therefore, the interaction of the Josephson phase with a set of linear oscillators cannot lead to the resonant enhancement of the MQT.

On the other hand, we recall that in the nonequilibrium case as the Josephson junction is subject to an externally applied microwave radiation the extremely pronounced resonant effects have been observed in the MQT phenomena [13,14]. Such a resonant interaction of the Josephson phase with an applied microwave radiation results in a strong suppression of the potential barrier, and a resonant enhancement of the MQT occurs as $\omega_0 \simeq \omega$, where ω is the frequency of applied microwave radiation.

Therefore, an interesting question arises in this field: Is it possible to observe a *resonant enhancement of the MQT* in the equilibrium state of the Josephson junction?

In this paper we show that such a resonant enhancement of the MQT naturally occurs in the Josephson junctions containing a large amount of microscopic *coherent two-level* *systems* (TLSs). Indeed, it has been established that the TLSs are intrinsically present in an amorphous interlayer of "typical" Josephson junctions [15–17]. These defects can exist in the two quantum states with an energy separation between them, Δ_0, and the tunneling splitting Δ. At high temperatures $k_B T \geqslant \Delta$ or as $\Delta \leqslant \Delta_0$ the thermal fluctuations induce incoherent random switchings between these states, i.e., the Poisson noise occurs. However, in the opposite regime ($k_B T < \Delta$ and $\Delta > \Delta_0$) the TLSs establish the *coherent quantum oscillations* (Rabi oscillations) of the frequency $\Omega = \Delta/\hbar$. These coherent TLSs resonantly excite the oscillations of the Josephson phase as the resonant condition, i.e., $\Omega \simeq \omega_0$, is satisfied. Since the frequency of Josephson phase oscillations ω_0 strongly depends on magnetic field ($\omega_0 \propto I_c^{1/2}$) the variation of applied magnetic field allows one to fulfill the resonant condition. Moreover, the back influence of the Josephson phase dynamics on the dynamics of coherent TLSs is small as the tunneling splitting between the two low-lying states Δ is much smaller than other characteristic energies, e.g., the energy of small oscillations around equilibrium positions of fluctuators, $\hbar\tilde{\omega}$. Therefore, similarly to a nonequilibrium case these coherent quantum oscillations [13] induce a strong reduction of the potential barrier and an enhancement of the MQT in a narrow region of applied magnetic field.

The paper is organized as follows: In Sec. II we present a model and describe the Josephson phase dynamics in the presence of coherent two-level defects. In Sec. III the escape rate of the Josephson phase in the MQT regime and the magnetic field dependence of the crossover temperature T_{cr} are obtained. Section IV provides conclusions.

II. THE JOSEPHSON JUNCTION CONTAINING TLSs IN THE PRESENCE OF MAGNETIC FIELD: THE JOSEPHSON PHASE DYNAMICS

Next, we quantitatively analyze the MQT regime of the Josephson phase escape in small (the junction size $L < \lambda_J$, where λ_J is the Josephson penetration length) Josephson junctions in the presence of magnetic field H applied parallel to the junction plane along the y direction. We also take into account TLSs randomly distributed in an amorphous interlayer of the Josephson junction. The schematic of such a system is presented in Fig. 1(a).

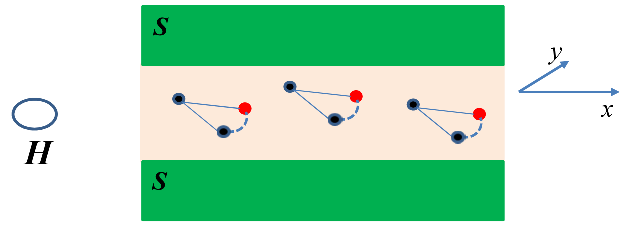
The dynamics of a small Josephson junction is characterized by time t and coordinate x dependent Josephson phase $\varphi(t,x)$. Moreover, since the magnetic field penetrates small Josephson junctions homogeneously, and $H \propto d\varphi/dx$, the Josephson phase is written as

$$\varphi(t,x) = \frac{2\pi\Phi x}{\phi_0 L} + \chi(t,x), \tag{2}$$

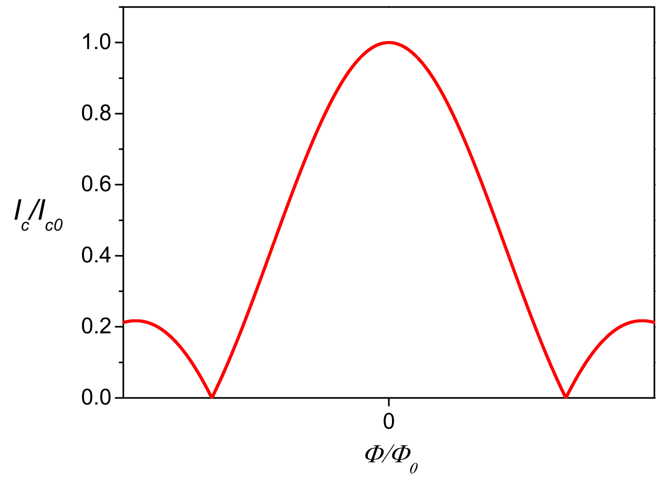
where $\Phi = HS$ is the external magnetic flux; S is the junction area.

For small Josephson junctions we can also neglect excitation of cavity modes, and the Lagrangian L_J depends on spatially averaged time-dependent Josephson phase $\chi(t)$ as

$$L_J = E_{J0}\left[\frac{1}{2\omega_{p0}^2}\dot{\chi}^2 - \gamma(\Phi)[1-\cos(\chi)] - j\chi\right], \tag{3}$$



(a)



(b)

FIG. 1. (Color online) (a) The schematic of a small Josephson junction in the presence of magnetic field applied parallel to the junction plane. The TLSs located in the interlayer of a junction are shown. (b) The typical dependence of the Josephson critical current on the magnetic field for a small Josephson junction [Eq. (4)].

where the parameter $\gamma(\Phi)$ determines magnetic field induced suppression of the Josephson critical current as

$$I_c(\Phi) = I_{c0}\gamma(\Phi) = I_{c0}\left|\frac{\phi_0}{\pi\Phi}\sin\left(\frac{\pi\Phi}{\phi_0}\right)\right|. \tag{4}$$

The typical dependence of the Josephson critical current on the magnetic field, $I_c(\Phi)$, is shown in Fig. 1(b). Here, E_{J0} and ω_{p0} are the Josephson coupling energy and the Josephson plasma frequency in the absence of magnetic field, accordingly. The normalized dc bias $j = I/I_{c0}$ allows one to effectively tune (decrease) the potential relief [see Eq. (3)] for the Josephson phase.

The Lagrangian of two-level defects distributed in an insulator layer of the Josephson junction reads as

$$L_{TLS} = \sum_i \frac{m}{2}[\dot{\Psi}_i]^2 - U(\Psi_i), \tag{5}$$

where Ψ_i and $U(\Psi_i)$ are correspondingly the degree of freedom (the relative angle between the dipole momentum and the z axis) and a double-well potential characterizing a single TLS; m is the effective mass of the TLSs.

A most important interaction of TLSs with the Josephson phase is the dipole-electric field interaction [15] which reads as $U_{int} = -\vec{d}\vec{E}$, where $\vec{d}$ is the dipole moment of TLS and $\vec{E}$ is the intrinsic electric field of a Josephson junction. Such an interaction leads to the additional term L_{int} in the total

Lagrangian

$$L_{\rm int} = \frac{E_{J0}\eta}{\omega_{p0}} \sum_i \dot{\Psi}_i(t)\chi(t), \tag{6}$$

where the dimensionless parameter $\eta \simeq \frac{d\omega_{p0}}{DI_{c0}}$ (D is the interlayer thickness of a Josephson junction) determines the interaction strength of the Josephson phase with a single TLS. The total Lagrangian of the Josephson junction containing the TLSs is written as

$$L = L_J + L_{TLS} + L_{\rm int}. \tag{7}$$

The dynamics of the Josephson phase $\chi(t)$ interacting with the TLSs is described by the nonlinear differential equation

$$-\frac{1}{\omega_{p0}^2}\ddot{\chi}(t) + V'(\chi) = \frac{\eta}{\omega_{p0}} \sum_i \dot{\Psi}_i(t), \tag{8}$$

where the potential $V(\chi) = \gamma(\Phi)[1 - \cos(\chi)] - j\chi$.

To solve this equation we represent the Josephson phase $\chi(t)$ as a sum of low- and high-frequency terms, i.e., $\chi(t) = \frac{\pi}{2} + \tilde{\chi}(t) + \xi(t)$. The high-frequency term $\xi(t)$ displays the classical behavior, and it is found as

$$\xi(t) = \frac{\eta}{\omega_{p0}} \sum_i \int_0^\infty d\tau_1 G(t - t_1)\dot{\Psi}_i(t_1), \tag{9}$$

where $G(t)$ is the Green's function of the linearized equation (8) that reads a:

$$-\frac{1}{\omega_{p0}^2}\ddot{Y}(t) + \gamma(\Phi)\tilde{\chi}Y(t) = \delta(t). \tag{10}$$

Explicitly the Green's function $G(t)$ is written as

$$G(t) = \int \frac{d\omega}{2\pi} G(\omega)e^{i\omega t} = \int \frac{d\omega}{2\pi} e^{i\omega t} \frac{\omega_{p0}^2}{\omega^2 - \omega_{p0}^2\gamma(\Phi)\tilde{\chi} + i\alpha\omega}. \tag{11}$$

Here, we introduce the parameter α in order to describe the dissipative effects in the Josephson phase dynamics.

Notice here, that in Eqs. (9) and (11) we neglect a small suppression of the function $G(t)$ that is due to the back influence of the Josephson phase dynamics on the dynamics of TLSs. This is valid as the energy difference between two low-lying states is much smaller than other characteristic energies. Indeed, the back influence effect induces the switching between the two states of TLS. The matrix element of this process is small as $\Delta/(\hbar\tilde{\omega}) \ll 1$, where $\tilde{\omega}$ is the frequency of small oscillations in equilibrium positions of the TLS. In this case, the dynamics of quantum degrees of freedom Ψ_i weakly depends on $\xi(t)$, and TLSs just show the quantum Rabi oscillations with the frequencies Δ_i.

Due to a nonlinearity of the potential $V(\chi)$ the high-frequency term $\xi(t)$ leads to the effective resonant reduction of the potential barrier as

$$V_{eff} = V(\tilde{\chi}) - \frac{\xi^2(\tilde{\chi})}{2}\tilde{\chi}. \tag{12}$$

The equilibrium value of $\tilde{\chi}$ is determined by an external bias current as

$$\tilde{\chi}_0 = \sqrt{2\delta} = \sqrt{\frac{2[j - \gamma(\Phi)]}{\gamma(\Phi)}}. \tag{13}$$

III. MQT AND CROSSOVER TEMPERATURE T_{cr}: INFLUENCE OF QUANTUM (RABI) OSCILLATIONS OF TLSs

The Josephson phase escape in the MQT regime is determined by the quantum-mechanical tunneling of the Josephson phase $\tilde{\chi}$ through the effective stationary (independent on time) barrier, $V_{\rm eff}$. By making use of the quasiclassical analysis of the MQT [3,18] we obtain the escape rate of the Josephson phase in the MQT regime as

$$\Gamma(\delta) \simeq \exp\left\{-\frac{36E_{J0}\gamma^{1/2}(\Phi)}{5\hbar\omega_{p0}}\{2[\delta - \xi^2(\tilde{\chi}_0)/2)]\}^{5/4}\right\}. \tag{14}$$

Equation (14) indicates that the enhancement of Γ is determined by the value of ξ^2.

Taking into account that the typical values of $\delta = \delta_0$ allowing non-negligible escape of the Josephson phase $\tilde{\chi}$ are determined by the condition $\Gamma(\delta) \simeq 1$, we obtain the transcendent equation for δ_0,

$$\delta_0 - \xi^2(\delta_0)/2 \simeq \left(\frac{\hbar\omega_{p0}}{E_{J0}}\right)^{4/5} \gamma^{-2/5}(\Phi), \tag{15}$$

and the crossover temperature T_{cr} is written as

$$T_{cr} = \frac{\hbar\omega_{p0}\gamma^{1/2}(\Phi)}{2\pi k_B}(2\delta_0)^{1/4}. \tag{16}$$

Since in the absence of TLSs the parameter $\xi = 0$ we arrive at Eq. (1) for the magnetic field dependence of T_{cr}. It reads explicitly as

$$T_{cr} = \frac{2^{1/4}\hbar\omega_{p0}}{2\pi k_B}\left(\frac{\hbar\omega_{p0}}{E_{J0}}\right)^{1/5}\gamma^{2/5}(\Phi). \tag{17}$$

Such a smooth dependence is shown in Fig. 2 by a blue (black) line. The deviation from Eq. (17) is also small in the regime as the Josephson phase weakly interacts with a set of linear oscillators because in this case the function $G(t)$ does not display the resonant behavior.

The situation changes drastically as we turn to the interaction of the Josephson phase with the coherent TLSs. At low temperatures the TLSs display coherent quantum Rabi oscillations, and quantitatively the coherent dynamics of TLSs is described by the Bloch equations [19]. In this case we obtain

$$\xi^2 = \frac{\eta^2}{\omega_{p0}^2} \sum_i \int dt_1 \int dt_2 \dot{G}(t - t_1)\dot{G}(t - t_2)K_i(t_1 - t_2), \tag{18}$$

where the time-dependent correlation function $K_i(t_1 - t_2) = \langle\Psi_i(t_1)\Psi_i(t_2)\rangle$ of coherent TLSs has been obtained in Refs. [19–21] as

$$K_i^0(t) = \Psi_0^2 e^{-i\Delta t/\hbar - \tilde{\Gamma}|t|}, \tag{19}$$

where $\pm\Psi_0$ are the values of Ψ in two quantum states, the Δ_i is the energy splitting of the ith TLS, and the parameter $\tilde{\Gamma}$ describes the decay rate of quantum Rabi oscillations.

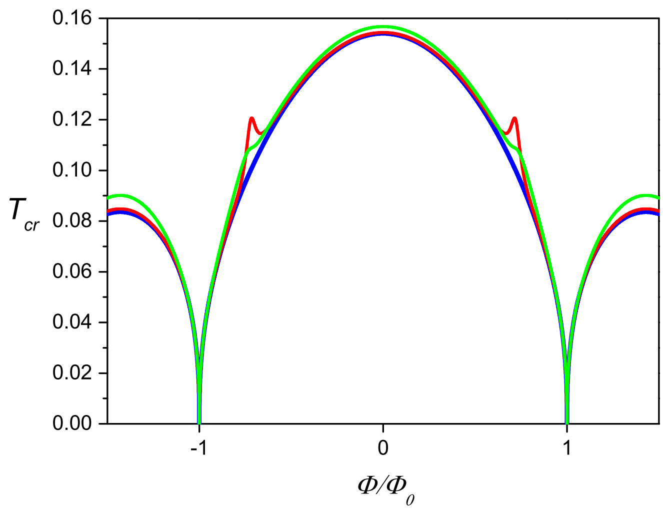


FIG. 2. (Color online) The typical dependencies of the crossover temperature T_{cr} on the magnetic field: the interaction with TLSs is absent [the blue (black) line]; a weak interaction with the TLSs having the same splitting energies Δ [the red (gray) line]; a weak interaction with the TLSs having randomly distributed splitting energies in the range between $\Delta_{\max}$ and $\Delta_{\min}$ [the green (light-gray) line]. The parameters $\Delta/(2\pi k_B) = \Delta_{\max}/(2\pi k_B) = 0.1$ K and $\Delta_{\min}/(2\pi k_B) = 0.02$ K have been used. The quality factor of the resonance $\Delta/\tilde{\alpha}$ is equal to 20. The coupling between a single TLS and the Josephson phase, $\eta\Psi_0 = 7\times 10^{-4}$, and a number of coherent TLSs $N = 100$ have been chosen.

Equation (19) is valid as $k_B T < \Delta$ and two-level systems display the weakly decaying coherent Rabi oscillations. In the opposite case, $k_B T > \Delta$, random switchings occur between two stable states, and the contribution of TLSs in the MQT dynamics becomes negligible. Substituting (19) in (18) we find that the parameter ξ has a resonant form and it is written as

$$\xi^2 = \pi\eta^2\Psi_0^2\omega_{p0}^2\frac{\alpha+\tilde{\Gamma}}{\alpha} \times \sum_i\{[\Delta_i - \hbar\omega_{p0}\gamma^{1/2}(\Phi)(2\delta_0)^{1/4}]^2 + \tilde{\alpha}^2\}^{-1}. \quad (20)$$

Here, the parameter $\tilde{\alpha} = \sqrt{2(\alpha^2+\tilde{\Gamma}^2)}$ determines the strength of the resonance, and it depends on all dissipative effects in both the TLSs and Josephson phase dynamics.

Next, analysis depends on the distribution of tunneling splitting energies Δ_i of TLSs in Josephson junctions. First, for simplicity we consider a case where all TLSs have the same splitting energies Δ. In this case all TLSs give the same contribution to ξ^2, and substituting (20) in (15) and (16) we obtain the narrow peak in the dependence of $T_{cr}(\Phi)$. In this case the width of the peak is determined by a small ratio $\tilde{\alpha}/\Delta$. The typical dependency of T_{cr} on the magnetic field is shown in Fig. 2 by a red (gray) line.

In a more realistic case as tunneling splittings Δ_i are randomly distributed in a wide range, i.e., $\Delta_{\min} < \Delta < \Delta_{\max}$, the enhancement of MQT occurs also in a wide range of magnetic fields Φ. The width of the peak is determined by "inhomogeneous broadening," and it can be much larger than in the case of a narrow distribution of tunneling splitting energies. The typical dependence of T_{cr} on the magnetic field for this case is shown in Fig. 2 by a green (light-gray) line.

IV. CONCLUSIONS

We theoretically studied the MQT phenomenon in the Josephson junction containing quantum TLSs and in the presence of an externally applied magnetic field. At high temperatures the thermal fluctuations induce the Josephson phase escape, and below the crossover temperature T_{cr} the Josephson phase escape is determined by the MQT. We focused on the dependence of T_{cr} on an externally applied magnetic field characterized by magnetic flux Φ. In the absence of TLSs the crossover temperature T_{cr} displays a smooth decrease with Φ in the range $-\phi_0 < \Phi < \phi_0$ [see Eqs. (1) and (17); the typical dependence is shown in Fig. 2 by a blue (black) line].

In the presence of even a weak interaction of the Josephson phase with the TLSs the resonant enhancement of the MQT was obtained. First of all, such an enhancement of the MQT leads to a substantial increase of the Josephson phase escape rate $\Gamma(\delta)$ [see Eq. (14)]. Another manifestation of this phenomenon is a narrow peak obtained in the dependence of $T_{cr}(\Phi)$ [see Eqs. (16), (15), and (20)].

The obtained resonant enhancement of the MQT is explained by a resonant suppression of the potential barrier for the Josephson phase escape which, in turn, is due to the presence of the coherent quantum (Rabi) oscillations in the equilibrium state of coherent quantum TLSs. Such an effect resembles a resonant suppression of the potential barrier in the Josephson junctions subject to an externally applied microwave radiation [13,14]. This enhancement is especially strong if the TLSs have the same splitting energies Δ. The typical dependence of $T_{cr}(\Phi)$ showing a narrow peak is shown in Fig. 2 by a red (gray) line. In the opposite case as the tunneling splittings display a wide distribution, we find that a narrow peak transforms in a "bump" in the dependence of $T_{cr}(\Phi)$ [see Fig. 2, green (light-gray) line], and an enhancement of the MQT occurs in a wide range of magnetic fields. A crucial condition allowing one to observe such a resonant enhancement of the MQT is the presence in Josephson junctions of a large amount of coherent TLSs, i.e., TLSs with the tunneling splitting $\Delta \geqslant k_B T$, and $\Delta \gg \Delta_0$, where Δ_0 is the energy difference between the minimums of a double-well potential characterizing the TLSs. The quality factors of both the Josephson junction and the TLSs have to be large.

The observation of these remarkable effects, namely, the resonant enhancement of the MQT and the crossover temperature T_{cr} can result in a strong impact on the properties of Josephson junctions at low temperatures, and it will provide an opportunity to study diverse coherent quantum phenomena in an ensemble of TLSs intrinsically present in Josephson junctions.

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