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Using weak measurements to extract the Z_2 index of a topological insulator

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Recently, there has been an interest in applying the concept of weak values and weak measurements to condensed matter systems. Here a weak measurement protocol is proposed for obtaining the Z_2 index of a topological insulator. The setup consists of a topological insulator with a hole pierced by a time dependent Aharonov-Bohm flux. A certain weak value (A_{gs}) associated with the time-integrated magnetization in the hole has a universal response to a small ambient magnetic field (B), namely, $A_{gs}B = 2\hbar$. This result is unaffected by disorder, interactions, and to a large extent, the speed of the flux threading. It hinges mainly on preventing the flux from leaking outside the hole, as well as being able to detect magnetization at a resolution of a few spins. A similar result may be obtained using only charge measurements, in a setup consisting of a double quantum dot weakly coupled to an LC circuit. Here one obtains $\langle \phi \rangle_{\text{weak}} Q_0 = 2\hbar$, where $\langle \phi \rangle_{\text{weak}}$ is a weak value associated with the flux on the inductor and Q_0 is the average capacitor charging. The universality of these results suggests that they may be used as a test bed for weak values in condensed matter physics.

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Topological insulators (TIs) have attracted much attention in recent years due to their novel bulk and surface properties [1,2]. In their bulk, these materials are insulating and certain twists in the bulk's band structure are characterized by topological indices. This implies, via a bulk-edge correspondence, that the surfaces of these materials are metallic, and have a strong coupling between momentum and spin. Such profound spin dependent effects, that do not require external magnetic fields, are potentially useful in the field of spintronics [3–5].

The topological character of a material may correspond to a quantized bulk response function, e.g., the quantized Hall conductance in the integer quantum Hall effect (QHE). This allows for the direct detection of the topological index of the material. However, TIs, which in two dimensions (2D) can be pictured as two stacked QHE layers with opposite magnetic fields [1,2], are not known to bear any quantized bulk response function or observable. As a result, experimental identification of a TI material is a more subtle task that relies on indirect evidence; for example, analysis for surface angle-resolved photoemission spectroscopy spectrum [1,6] and measurement of edge conductance [7].

Recently, there has been an interest in applying a different measuring scheme, based on the idea of weak values [8], to condensed matter systems [9–12]. This measurement scheme allows one to measure off-diagonal matrix elements of operators directly, and hence extract more information than is available from a standard measurement [13]. Also, under certain circumstances, weak values can be used to amplify a weak signal [10,14]. A typical setup is a double quantum dot on which one applies various perturbations to induce either a Stueckelberg-Landau-Zener (Zener) transition [11] or Rabi oscillations [9] between two charge states of the device. The signal of a charge detector which is weakly coupled to the device, can then achieve values which exceed the classically allowed ones, provided that one postselects only the measurements in which an unlikely outcome occurred. From an entirely different direction, a certain weak value (called the "strange correlator") has been used to identify power law correlations in symmetry protected topological phases [15]. However, measuring this weak value is unfeasible, as it would require waiting for an extremely unlikely event in which a quantum fluctuation in the topological insulator makes it appear as the ground state of a trivial insulator. In contrast, below we propose a more physical weak measurement which can be used to identify a TI.

In a geometry with closed boundary conditions, a TI hosts a single unavoided Zener transition driven by Aharonov-Bohm (AB) fluxes [16,17]. This transition occurs between the ground state and a magnetic excitation which resides on the boundary. Considering, for instance, a Corbino-disk geometry, the threading of a single AB flux quantum (ϕ_0) through the hole in the disk results in a single level crossing, which occurs exactly when half the flux is threaded. The final state after the threading is orthogonal to the ground state and contains some magnetization [18]. A unique feature of this crossing point, is that only two levels which form a Kramers pair are involved [16]. This ensures its persistence even as time reversal symmetry (TRS) respecting disorder and interactions are introduced [19,20]. Moreover, the orthogonality between the initial and final states remains unaltered also far away from the adiabatic limit, up to flux threading rates of roughly 1 THz [19,21]. More generally, a defining property of a TI (trivial band insulator) is that it hosts (does not host) such a transition [16,17,22].

In this work, a measurement protocol of the Z_2 index of a topological insulator is proposed, which exploits the above Zener transition with its high degree of robustness. We consider a detector which is weakly coupled to the boundary magnetization and measure its signal during a threading of the AB flux. At the end of the threading, a regular (strong) magnetization measurement is performed to determine the final state. The weak detector signal is then conditionally averaged on having a nonmagnetized final state. The result (A_{gs}), also known as a weak value (WV) [8], shows a quantized response to a small ambient magnetic field (B_i) which corresponds to the Z_2 invariant directly [see Eq. (8)]. Similarly quantized results are obtained for a double quantum dot coupled to a quantum LC circuit, this time relating the weak value of the flux on the inductor $\langle \phi \rangle_{\text{weak}}$, with the charge on the capacitor Q_0.

Let us begin with some background on the aforementioned Zener transition. For many purposes, a TI can be thought of as a double layer system wherein one layer consists of only spin-up ($s = 1$) electrons and is in an integer quantum Hall effect with a Hall conductance (σ_{xy}) of e^2/h and the other layer consists of only spin-down ($s = -1$) electrons and is in an integer quantum Hall effect with an opposite Hall conductance [2]. We focus on the inner edge of one layer in a Corbino-disk geometry, initially with no interactions and no disorder, such that the edge conserves the momentum parallel to it ($k_\parallel$). One then finds a branch of chiral modes confined to the boundary ($E^{\uparrow}_{k_\parallel}$) [23]. The sign of the slope of these chiral modes (sgn[$\partial_{k_\parallel} E^{\uparrow}_{k_\parallel}$]), is determined by the sign of σ_{xy}, or equivalently in our setup, by s. For a finite boundary of length L, the allowed momenta along this chiral branch are quantized to $k_\parallel(n) = \frac{2\pi(n+\phi/\phi_0)}{L}$ [24] with n being an integer and ϕ is the AB flux through the Corbino disk [see Fig. 1(b), red branch, and imagine that the crossing there is unavoided]. In the many-body ground state, all states with momenta $k_\parallel(n)$ such that $E^{\uparrow}_{k_\parallel(n)} < \mu$, where μ is the chemical potential, are occupied (full red circles).

Using the adiabatic approximation one finds that threading a single flux quantum ($\phi \to \phi + \phi_0$) changes the many-body ground state to a state in which all $k_\parallel(n)$ such that $E^{\uparrow}_{k_\parallel(n-1)} < \mu$, are occupied. The latter is an excited state with one additional spin-up electron. Considering the other, spin-down layer, the opposite effect would occur and one spin-down electron would be depleted from the boundary. In total, the number of electrons has not changed but a spin-flip excitation was created. Notably the outer edge would exhibit a similar yet opposite effect. For a large system and at low energies compared to the bulk gap, the edges are effectively two decoupled systems [2]. Correspondingly one may ignore the outer edge in all of the following.

A threading of an AB flux is a periodic cycle in parameter space (up to an insignificant gauge transformation) and the fact that the system does not come back to its ground state after such a cycle necessitates a level crossing. This level crossing occurs at $\phi = \phi_0/2$, when an empty level of the spin-down branch, which is being pushed down in energy by the flux, becomes degenerate with a full spin-up level, being pushed up by the flux. Consequently the excitation, consisting of shifting the electron between these two levels, would cost zero energy. Provided TRS is conserved, no TRS respecting operator can open a gap at this crossing point. This includes operators related to quench disorder and electron-electron interactions. Furthermore, one may naturally extend the definition of TRS to a time-dependent Hamiltonian ($H[t]$) via [16,19] $H[t] = \Theta H[-t]\Theta^{-1}$, where $\Theta = Kis_y$, where K denotes complex conjugation and s_y is a y-Pauli matrix acting on the spin degrees of freedom. This slightly stronger requirement ensures that an orthogonal state is obtained independently of the flux threading rate [19–21].

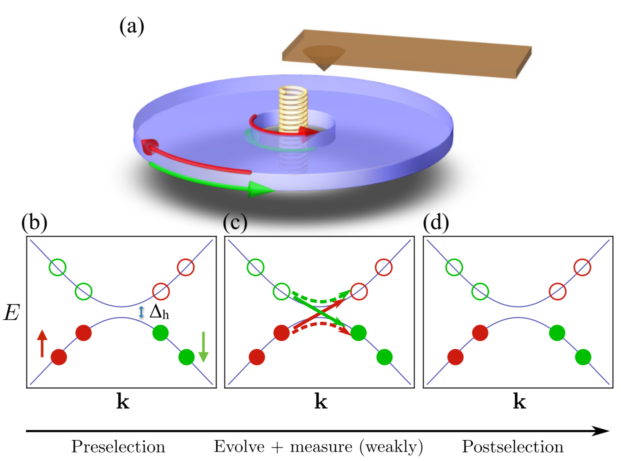

FIG. 1. (Color online) (a) An illustration of a 2D topological insulator in a Corbino-disk geometry. Each edge supports counterpropagating chiral modes of opposite spin (denoted by green and red counterpropagating arrows). Placing a solenoid within the disk allows for a time-dependent threading of an Aharonov-Bohm flux, and the buildup of magnetization at the edges. The magnetization at the inner edge should be measured using an accurate nanoscale scanning magnetometer [25–28], depicted by the tip (brown) above the sample. (b)–(d) Evolution of the system during the weak measurement protocol: The system is prepared in its ground state (b) and a very weak applied magnetic field induces a small gap, Δ_h. Once the flux is threaded, the levels start climbing/descending according to their spin (c). Next a detector which is weakly coupled to the magnetization on the edge is measured (A). For $\Delta_h \to 0$, the flux threading would typically induce a diabatic Zener transition [(c), solid lines] and occasionally no transition [(c), dashed lines]. If the former occurs, the measurement, A, is discarded, and if the latter occurs (d) it is registered. For a topological insulator the resulting conditional average diverges according to Eq. (8)

We begin by describing the measurement setup which consists of a TI and a detector coupled to each other in the presence of a small ambient magnetic field. The respective Hamiltonians are

$$H[t] = H_{\mathrm{TI}}[t] + H_{\mathrm{pert}} + H_{\mathrm{detector}}. \tag{1}$$

We have in mind a 2D TI in a Corbino-disk geometry [see Fig. 1(a)]. Its Hamiltonian ($H_{\mathrm{TI}}[t]$) depends on the AB flux, $\phi(t) = \phi_0 \frac{t}{t_f}$, which is generated by a solenoid situated within the hole [see Fig. 1(a)]. As can be seen, each threading of a ϕ_0 would induce the aforementioned magnetic excitations on both the inner and outer edges. Applying a small magnetic field B_i at the inner edge introduces the perturbation $H_{\mathrm{pert}} = B_i M_i$, where M_i is the total magnetization on the inner edge in the $\hat{i}$ direction. Generically, this would cause a small gap, Δ_h, between the counterpropagating spin edges [see Fig. 1(b)]. Below we will always consider the limit $\Delta^{-1} \ll t_f/\hbar \ll \Delta_h^{-1}$, where Δ is the bulk gap of the TI.

We propose to monitor the system evolution by coupling a weak detector to the magnetization at the inner edge. The detector is modeled as a harmonic oscillator with a low frequency, ω, whose momentum, P, is weakly coupled to the system, namely,

$$H_{\mathrm{detector}} = \frac{M\omega^2 X^2}{2} + \frac{P^2}{2M} + \lambda P M_i. \tag{2}$$

Notice, however, that in the setup suggested in Fig. 1(a) P is actually the vertical position operator of the cantilever.

Later we comment on how to choose λ and calibrate our measurement of X.

The measurement protocol begins with both system and detector in their respective ground states, i.e., the initial state is $|i\rangle = |i_S\rangle|i_D\rangle$. Pictorially, the system's ground state ($|i_S\rangle$) corresponds to Fig. 1(b). Next $\phi(t)$ is scanned, at a constant rate, from 0 to ϕ_0. Considering the limit of small λ, one can use first order time dependent perturbation and express the final state ($|f\rangle$) as

$$|f\rangle = \left(U_0^{t_f} + i\lambda\int_0^{t_f}\frac{dt}{\hbar}U_t^{t_f}M_z P U_0^t\right)|i\rangle,$$
$$U_{t_0}^{t_1} = \mathcal{T}e^{i\int_{t_0}^{t_1}\frac{dt}{\hbar}H_{\text{TI}}[t]+H_{\text{pert}}}, \tag{3}$$

where, for simplicity, we have assumed the time scales of the detector to be much longer than t_f, allowing us to ignore the free evolution of the detector. The final state can be readily evaluated on the product basis consisting of detector position basis $\{|x\rangle\}$ and many-body edge excitation spectrum $\{|m\rangle\}$,

$$\langle x|\langle m||f\rangle = \langle m|U_0^{t_f}|i_S\rangle\langle x|e^{\lambda\hbar^{-1}A_m P}|i_D\rangle + O(\lambda^2), \tag{4}$$

where

$$A_m = i\frac{\langle m|\int dt U_t^{t_f}M_i U_0^t|i_S\rangle}{\langle m|U_0^{t_f}|i_S\rangle} = \hbar\partial_{B_i}\log\left[\langle m|U_0^{t_f}|i_S\rangle\right]. \tag{5}$$

Notably in the last equality we exploited the fact that B_i couples to the same operator as P does. The A_m's are known as WVs, and for the reexponentiation ($e^{\lambda\hbar^{-1}A_m P} \approx 1 + \lambda\hbar^{-1}A_m P$) we have assumed that higher-order WVs are negligible [8,10,29]. Since the characteristic scale of P is $\sqrt{\hbar M\omega}$, the condition for a weak measurement is

$$\lambda A_m \ll \frac{1}{\sqrt{\hbar M\omega}}. \tag{6}$$

Following the weak measurement protocol [8] one now applies a second strong measurement that determines the final state of the system (postselection) in order to measure a specific WV. In our case, we consider a postselection on the ground state of the system at time t_f with $\phi = \phi_0$, i.e., $|m\rangle \equiv G_{\phi_0}|i_S\rangle$, where G_{ϕ_0} is a gauge transformation which inserts a flux quantum through the disk ($G_{\phi_0} = e^{i\theta}$, where θ is the angle along the disk). The weak measurement outcomes are collected conditional on the postselection outcome, i.e., if the system is not found to be in its ground state, the experiment outcome is ignored. Since both the initial and final states are ground states, we denote this weak value simply as A_{gs} from now on.

According to Eq. (4), the collapse of the system's state on $G_{\phi_0}|i_S\rangle$ leaves the detector in a pure state given by $e^{\lambda\hbar^{-1}A_{gs}P}|i_D\rangle + O(\lambda^2)$. Assuming for the moment that A_{gs} is purely imaginary, the ground state of the detector is simply shifted by λA_{gs} and consequently a standard measurement of X would give $\langle X\rangle_{\text{weak}} = \lambda A_{gs}$.

Let us turn to evaluate A_{gs}. Following Eq. (5), this amounts to evaluating $\langle i_S|G_{\phi_0}^\dagger U_0^{t_f}|i_S\rangle$. Taking the simplest level crossing model and assuming $\Delta_h t_f/\hbar \ll 1$ one may use the well known result [30,31]

$$\left|\langle i_S|G_{\phi_0}^\dagger U_0^{t_f}|i_S\rangle\right|^2 = c_0 B_i^2 + O\left(B_i^4\right), \tag{7}$$

where c_0 is some nonuniversal constant which depends, in particular, on the rate of flux threading, and the direction of the magnetic field ($\hat{i}$). As previously mentioned, taking a more realistic description of this transition, allowing, for example, several nearby energy levels, would not alter this result [19,20]. Comparatively, for a trivial band insulator, with no edge modes, the only energy scale is Δ with respect to which the experiment is adiabatic and consequently, the above probability changes to $1 - O(B_i^2)$. Plugging these expressions into Eq. (5), the nonuniversal contributions decouple and one obtains

$$A_{gs} = \frac{2\hbar\nu_2}{B_i} + O(1), \tag{8}$$

where $\nu_2 = 1(0)$ for a TI (band insulator). Roughly speaking, this follows from viewing the effective magnetic perturbation as $(B_i + \lambda P)$. Postselecting for an avoided Zener transition then means that as B_i decreases, a strong fluctuation in λP is required to assist an avoided transition.

Notably, however, A_{gs} turned out real. Consequently, one extra procedure is needed to witness its effect on the detector position. Reexpressing the detector state following the postselection as

$$\begin{aligned} e^{\lambda\hbar^{-1}A_{gs}P}|i_D\rangle &= \left(\frac{\hbar}{2\pi M\omega}\right)^{1/2}\int dk\exp\left[-\frac{\hbar k^2}{2M\omega} + \lambda A_{gs}k\right]|k\rangle \\ &= \left(\frac{\hbar}{2\pi M\omega}\right)^{1/2} \\ &\quad\times\int dk\exp\left[-\frac{\hbar[k-\lambda\hbar^{-1}A_{gs}M\omega]^2 + O(\lambda^2)}{2M\omega}\right]|k\rangle, \end{aligned} \tag{9}$$

one finds that up to corrections in λ^2, which we consistently neglect, the weak value simply shifts the momentum (P) by $\omega M\lambda\hbar^{-1}A_{gs}$. Waiting for the resulting coherent state of the detector to evolve for a quarter period ($\pi/2$) and then measuring X, yields

$$B_i\frac{\langle X\rangle_{\text{weak},\pi/2}}{\lambda} = 2\hbar\nu_2. \tag{10}$$

Equation (6) ensures that $\langle X\rangle_{\text{weak},\pi/2}$ is much smaller than the standard deviation of X. Notably all the quantities on the left-hand side are system-independent quantities associated with the detector and external perturbation, while the right-hand side is quantized in units of $\hbar$. This is the key result of this work.

A few comments are in order regarding the observability of the above result. First, obtaining a divergent weak value does not imply that the detector signal actually diverges, as that would mean that the measurement ceases to be weak [see Eq. (6)]. Instead, as A_{gs} diverges, λ must be reduced and the detector readout must be recalibrated using an independent classical source of magnetization. Alternatively stated, we treat $\frac{\langle X\rangle_{\text{weak},\pi/2}}{\lambda}$ as the calibrated detector readout and only in terms of this value would the signal appear divergent.

Second, the quantization depends on an accurate detection of the final state. Final state detection errors would cut off the divergent nature of a weak value [11]. Since we require single spin levels of detection, such errors are unavoidable with current technology although the field is progressing rapidly [27]. In this aspect, it would be beneficial to choose the axis along which the final state magnetization is measured parallel to the anticipated direction of magnetization.

Third, an error in the quantization of the pole's residue would be induced by tilting the direction of the perturbing field (B_i) with respect to that of the magnetization being measured (M_i). Less restrictively, all is required is that the TRS breaking perturbation couples to the same TRS breaking operator which is being weakly measured. This operator can be any TRS breaking operator and, in particular, may vary in space. To achieve such coupling, one may use an invasive magnetometer, such as a magnetic force microscope, to both generate the perturbation and measure it.

Fourth, since in practice one cannot take the limit of infinitely weak Δ_h (and therefore infinitely weak λ), almost unavoided Zener transitions may also contribute to the pole's residue in a nongeneric way. However, away from fine-tuned points and for small hole circumference, we do not expect such near degeneracies either for TIs or for trivial insulators.

Lastly, if $\langle X\rangle_{\text{weak}}$ is to contain any information about the final magnetization measurement, the operators M_i carried by $U_t^{t_f}$ up to time t_f should not commute those measuring the final state [9]. For a TI in which charge is the only conserved quantity, this is generically the case. However, given extra symmetries—for example, s_z conservation—the axis of the final magnetization measurement and M_i must not be both aligned along $\hat{z}$. These requirements were implicit in our treatment via the assumptions that the perturbation induces a finite Δ_h gap and that the final measurement distinguishes the ground state from the excited state.

The quantized residue obtained for the TI relied mainly on having an unavoided Zener transition and detector-perturbation alignment. Consequently, it may be observed also in different setups. For instance, one may consider a gate-voltage driven Zener transition between two charge states of a weakly coupled, spin polarized, double quantum dot [11], these two states being one electron on the left dot and no electrons on the right dot, and vice versa. Obtaining residue quantization in this setup requires a detector which couples to a tunneling operator (T) that transfers charge between the two dots. Formally this requires H_{detector} with M_i replaced by T. Furthermore, one should control the average value of the detector's momentum ($\langle P\rangle = \lambda^{-1}B_i$) as this effectively generates the analogous term to H_{pert}. Potentially, such a detector could be realized using an LC circuit whose charging (P in our notations) controls the opening of a quantum point contact between the two dots. Postselection must again be done for the unlikely outcome, being that the electron hopped between the dots.

Applying the analysis carried earlier in the above mesoscopic setup, the following dependence of the weak value of the flux on the inductor $\langle\phi\rangle_{\text{weak}}$ on the average charging bias of the capacitor (Q_0) is obtained:

$$\langle\phi\rangle_{\text{weak}} = \frac{2\hbar}{Q_0}, \tag{11}$$

where the weak measurement limit simply requires Q_0 to be larger than the zero-point quantum fluctuations of the charging of the capacitor. Of course being related to voltage driven charge transition, rather than an AB flux driven magnetic transition as before, this WV carries no topological meaning. It is also worth noting that a single transition of the latter type cannot be realized in a charge conserving system, although pairs of such transitions may occur [32]. This is due to the change time in reversal polarization as a function of the AB flux [16]. Considering achievable resonator frequencies [33,34] of $\omega_{LC} \approx 5$ Ghz, a quantum coherent weak measurement requires $t_f^{-1} \gg \omega_{LC} \approx 5\text{ Ghz} \gg k_b T/\hbar$ with t_f limited by the gap to higher energy levels which is controlled by the magnetic field. At one Tesla this implies $t_f^{-1} \ll 100$ Ghz. Initial and final state readouts can be achieved using single electron detectors such as an RF-SET [35], or one may weakly couple the dots to leads and deduce their charge from Coulomb blockade peaks in the conductance [36].

To conclude, a weak measurement was proposed which identifies Zener transitions through the residue associated with a divergent WV. Interestingly, this residue is quantized in units of $\hbar$. Since a defining property of a TI is the presence of a single flux-driven Zener transition between a Kramers pair [16], this residue yields the Z_2 topological index. A different Zener transition, driven by gate voltage, may be realized in a mesoscopic setup yielding similar results. This latter setup appears feasible already with current technology.

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