# Aharonov-Bohm effect for excitons in a semiconductor quantum ring dressed by circularly polarized light

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We show theoretically that the strong coupling of circularly polarized photons to an exciton in ringlike semiconductor nanostructures results in physical nonequivalence of clockwise and counterclockwise exciton rotations in the ring. As a consequence, the stationary energy splitting of exciton states corresponding to these mutually opposite rotations appears. This excitonic Aharonov-Bohm effect depends on the intensity and frequency of the circularly polarized field and can be detected in state-of-the-art optical experiments.

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## I. INTRODUCTION

Progress in semiconductor nanotechnologies has led to developments in the fabrication of various mesoscopic objects, including quantum rings. The fundamental physical interest attracted by these systems arises from a wide variety of purely quantum-mechanical effects which can be observed in ringlike nanostructures. One of them is the Aharonov-Bohm (AB) effect arisen from the direct influence of the vector potential on the phase of the electron wave function [1,2]. In ringlike nanostructures pierced by a magnetic flux, this effect results in the energy splitting of electron states corresponding to mutually opposite directions of electronic rotation in the ring [3]. As a consequence, magnetic-flux-dependent oscillations of the conductance of the ring appear [4–9]. Since the AB effect takes place for both a single electron and many-particle quantum states [10], it can be observed for elementary excitations in semiconductor nanostructures as well. The simplest of them is an exciton which is a bound quantum state of a negative charged electron in the conduction band and a positive charged hole in the valence band. Manifestations of various excitonic effects in semiconductor ringlike structures, including the AB effect induced by a magnetic field, have attracted great attention of both theorists [11–22] and experimentalists [23–27].

Fundamentally, the AB effect is caused by the broken time-reversal symmetry in an electron system subjected to a magnetic flux. Namely, the flux breaks the equivalence of clockwise and counterclockwise electron rotation inside a ringlike structure, which results in the flux-controlled interference of the electron waves corresponding to these rotations. The similar broken equivalence of electron motion for mutually opposite directions caused by a magnetic field can take place in various nanostructures, including quantum wells [28], carbon nanotubes [29], and hybrid semiconductor/ferromagnet nanostructures [30]. However, the time-reversal symmetry can be broken not only by a magnetic flux but also by a circularly polarized electromagnetic field. Indeed, the field breaks the symmetry since the time reversal turns clockwise polarized photons into counterclockwise polarized ones and vice versa. polarized photons results in the magnetic-flux-like splitting of electron energy levels corresponding to mutually opposite electronic rotation in the ring [31] and oscillations of the ring conductance as a function of the intensity and frequency of the irradiation [32]. This phenomenon can be described in terms of a stationary artificial U(1) gauge field generated by the strong coupling between an electron and circularly polarized photons [32]. Therefore, various stationary phenomena similar to the AB effect can take place in ringlike electronic systems interacting with a circularly polarized electromagnetic field. As a consequence, unexplored quantum optical phenomena in semiconductor nanostructures appears. Although a theory of these AB-like phenomena in quantum rings has been elaborated for a single electron [31,32], the optically-induced AB effect for excitons still awaits detailed analysis. This paper is aimed to fill this gap in the theory, which lies at the border between quantum optics and physics of semiconductor nanostructures.

In quantum rings, the strong electron coupling to circularly

The paper is organized as follows. In Sec. II, the excitonphoton Hamiltonian is analyzed and solutions of the excitonphoton Schrödinger problem are found. In Sec. III, the energy spectrum of the dressed excitons is discussed and experimental sets to detect the effect are proposed. In Sec. IV, the conclusion is presented.

#### **II. MODEL**

An electron-hole pair in a one-dimensional quantum ring (see Fig. 1) can be described by the Hamiltonian

$$\hat{\mathcal{H}}_0 = -\frac{\hbar^2}{2m_h R^2} \frac{\partial^2}{\partial \varphi_h^2} - \frac{\hbar^2}{2m_e R^2} \frac{\partial^2}{\partial \varphi_e^2} + V(\varphi_e - \varphi_h), \quad (1)$$

where *R* is the radius of the ring,  $m_{e,h}$  is the effective mass of an electron (hole) in the ring,  $V(\varphi_e - \varphi_h)$  is the potential energy of hole-electron interaction, and  $\varphi_{e,h}$  are the azimuthal angles of the electron (hole) in the ring. Introducing the variables,

$$\varphi = rac{m_e \varphi_e + m_h \varphi_h}{m_e + m_h}, \quad \theta = \varphi_e - \varphi_h,$$



Quantum ring

FIG. 1. (Color online) Sketch of an exciton-field system in a quantum ring under consideration. The exciton coupling to the circularly polarized electromagnetic field results in physical nonequivalence of exciton states corresponding to clockwise and counterclockwise rotations of the exciton as a whole along the ring (shown by the arrows). These exciton states are described by mutually opposite angular momenta m and -m along the ring axis.

the Hamiltonian (1) can be rewritten as

$$\hat{\mathcal{H}}_0 = -\frac{\hbar^2}{2MR^2} \frac{\partial^2}{\partial\varphi^2} - \frac{\hbar^2}{2\mu R^2} \frac{\partial^2}{\partial\theta^2} + V(\theta), \qquad (2)$$

where  $M = m_e + m_h$  is the exciton mass, and  $\mu = m_e m_h / M$  is the reduced exciton mass. The eigenfunctions of the stationary Schrödinger equation with the Hamiltonian (2) have the form

$$\psi_{nm}(\varphi,\theta) = \chi_n(\theta) \frac{e^{im\varphi}}{\sqrt{2\pi}},\tag{3}$$

where the function  $\chi(\theta)$  meets the Schrödinger equation

$$-\frac{\hbar^2}{2\mu R^2}\frac{\partial^2 \chi_n(\theta)}{\partial \theta^2} + V(\theta)\chi_n(\theta) = \varepsilon_n \chi_n(\theta), \qquad (4)$$

 $m = 0, \pm 1, \pm 2, \dots$  is the exciton angular momentum along the ring axis,  $n = 0, 1, 2, \dots$  is the principal quantum number of the exciton, and  $\varepsilon_n$  is the exciton binding energy. Correspondingly, the full energy of the exciton reads as

$$\varepsilon_{n,m} = \varepsilon_n + \frac{\hbar^2 m^2}{2MR^2},\tag{5}$$

where the second term is the kinetic energy of rotational motion of the exciton in the ring.

Let the ring be subjected to a circularly polarized electromagnetic wave with the frequency  $\omega$ , which propagates along the ring axis (see Fig. 1). Then the full Hamiltonian of the exciton-photon system, including both the field energy  $\hbar \omega \hat{a}^{\dagger} \hat{a}$ and the exciton Hamiltonian  $\hat{\mathcal{H}}_0$  is

$$\hat{\mathcal{H}} = \hbar \omega \hat{a}^{\dagger} \hat{a} + \hat{\mathcal{H}}_0 + \hat{U}, \qquad (6)$$

where  $\hat{a}$  and  $\hat{a}^{\dagger}$  are the operators of photon annihilation and creation, respectively, written in the Schrödinger representation (the representation of occupation numbers), and  $\hat{U}$  is the operator of exciton-photon interaction. Generalizing the operator of electron-photon interaction in a quantum ring [31] for the considered case of electron-hole pair, we can write this

operator as

$$\hat{U} = \frac{iq_e R}{2} \sqrt{\frac{\hbar\omega}{\epsilon_0 V_0}} [(e^{-i\varphi_e} - e^{-i\varphi_h})\hat{a}^{\dagger} + (e^{i\varphi_h} - e^{i\varphi_e})\hat{a}], \quad (7)$$

where  $q_e$  is the electron charge,  $V_0$  is the quantization volume, and  $\epsilon_0$  is the vacuum permittivity. To describe the excitonphoton system, let us use the notation  $|n,m,N\rangle$  which indicates that the electromagnetic field is in a quantum state with the photon occupation number N = 1,2,3,..., and the exciton is in a quantum state with the wave function (3). The electronphoton states  $|n,m,N\rangle$  are true eigenstates of the Hamiltonian

$$\hat{\mathcal{H}}_{R}^{(0)} = \hbar \omega \hat{a}^{\dagger} \hat{a} + \hat{\mathcal{H}}_{0},$$

which describes the noninteracting exciton-photon system. Correspondingly, their energy spectrum is

$$\varepsilon_{n,m,N}^{(0)} = N\hbar\omega + \varepsilon_{n,m}.$$

In order to find the energy spectrum of the full electron-photon Hamiltonian (6), let us use the conventional perturbation theory, considering the term (7) as a perturbation with the matrix elements  $\langle n',m',N'|\hat{U}|n,m,N\rangle$ . Taking into account in Eq. (7) that  $\varphi_e = \varphi + m_h \theta/M$  and  $\varphi_h = \varphi - m_e \theta/M$ , these matrix elements read as

$$\langle n',m',N'|U|n,m,N\rangle$$
  
=  $eR\sqrt{\frac{\hbar\omega}{\epsilon_0 V_0}}[I_{n'n}\sqrt{N+1}\delta_{m,m'+1}\delta_{N,N'-1}$   
 $-I_{n'n}^*\sqrt{N}\delta_{m,m'-1}\delta_{N,N'+1}],$  (8)

where

$$I_{n'n} = \int_{-\pi}^{\pi} \chi_{n'}^*(\theta) \chi_n(\theta) e^{-i(m_h - m_e)\theta/2M} \sin(\theta/2) d\theta.$$

Performing trivial calculations within the second order of the perturbation theory, we can derive eigenenergies of the exciton-photon Hamiltonian (6),

$$\varepsilon_{n,m,N} = \varepsilon_{n,m,N}^{(0)} + \sum_{n'} \left[ \frac{|\langle n', m+1, N-1 | \hat{U} | n, m, N \rangle|^2}{\varepsilon_{n,m,N}^{(0)} - \varepsilon_{n',m+1,N-1}^{(0)}} + \frac{|\langle n', m-1, N+1 | \hat{U} | n, m, N \rangle|^2}{\varepsilon_{n,m,N}^{(0)} - \varepsilon_{n',m-1,N+1}^{(0)}} \right].$$
(9)

Since Eq. (9) is derived within the second order of the perturbation theory, it describes the problem correctly if the energy differences in denominators of all terms lie far from zero. In what follows, we have to keep in mind that all parameters of the problem must lie far from these resonant points.

The energy spectrum of exciton-photon system (9) can be written formally as  $\varepsilon_{n,m,N} = N\hbar\omega + \tilde{\varepsilon}_{n,m,N}$ , where the first term is the field energy. Following the conventional terminology of quantum optics [33,34], the second term  $\tilde{\varepsilon}_{n,m,N}$ is the energy spectrum of the exciton dressed by the circularly polarized field (dressing field). Restricting our analysis by the most interesting case of classically strong dressing field  $(N \gg 1)$ , we arrive from Eq. (9) to the sought energy spectrum of dressed exciton,

$$\widetilde{\varepsilon}_{n,m} = \sum_{n'} \left[ \frac{(q_e E_0 R)^2 |I_{nn'}|^2}{\varepsilon_{n,m} - \varepsilon_{n',m+1} + \hbar\omega} + \frac{(q_e E_0 R)^2 |I_{nn'}|^2}{\varepsilon_{n,m} - \varepsilon_{n',m-1} - \hbar\omega} \right],\tag{10}$$

where  $E_0 = \sqrt{N\hbar\omega/\epsilon_0 V_0}$  is the classical amplitude of electric field of the electromagnetic wave. It is apparent that dressed exciton states with mutually opposite angular momenta, m and -m, have different energies (10). Physically, this should be treated as a field-induced nonequivalence of clockwise and counterclockwise exciton rotations in the ring. As a consequence, the excitonic Aharonov-Bohm effect induced by the circularly polarized field appears. In order to simplify the calculation of the field-induced splitting,  $\Delta \tilde{\varepsilon}_{n,m} = \tilde{\varepsilon}_{n,m} - \tilde{\varepsilon}_{n,m}$  $\tilde{\varepsilon}_{n,-m}$ , we will restrict our consideration to the case of the ground exciton state with n = 0. Let us assume that the characteristic binding energy of exciton,  $q_e^2/4\pi\epsilon_0 R^2$ , is much more than both the characteristic energy of rotational exciton motion,  $\hbar^2 |m|/2MR^2$ , and the photon energy  $\hbar\omega$ . Then we can neglect the field-induced mixing of exciton states with  $n' \neq 0$  in Eq. (10). As a result, we arrive from Eq. (10) to the field-induced splitting of exciton states with mutually opposite angular momenta,

$$\Delta \widetilde{\varepsilon}_{0,m} = \left| \int_{-\pi}^{\pi} |\chi_0(\theta)|^2 \sin\left(\frac{m_h - m_e}{2M}\theta\right) \sin\left(\frac{\theta}{2}\right) d\theta \right|^2 \\ \times \left[ \frac{2\hbar\omega(q_e E_0 R)^2}{\varepsilon_R^2 (1 - 2m)^2 - (\hbar\omega)^2} - \frac{2\hbar\omega(q_e E_0 R)^2}{\varepsilon_R^2 (1 + 2m)^2 - (\hbar\omega)^2} \right], \tag{11}$$

where  $\varepsilon_R = \hbar^2/2MR^2$  is the characteristic energy of exciton rotation. In order to calculate the integral in Eq. (11), we have to solve the Schrödinger equation (4) and find the wave function  $\chi_0(\theta)$ . Approximating the electron-hole interaction potential  $V(\theta)$  in Eq. (4) by the delta-function [12] and assuming the characteristic exciton size,  $a = \hbar/\sqrt{8\mu\varepsilon_0}$ , to be much less than the ring length  $2\pi R$ , we can write the splitting (11) in the final form

$$\Delta \tilde{\varepsilon}_{0,m} = \frac{\hbar \omega}{2} \left( \frac{m_h - m_e}{M} \right)^2 (eE_0 a)^2 \left[ \frac{1}{\varepsilon_R^2 (1 - 2m)^2 - (\hbar \omega)^2} - \frac{1}{\varepsilon_R^2 (1 + 2m)^2 - (\hbar \omega)^2} \right].$$
 (12)

It should be stressed that the simplest delta-function model [12] leads to reasonable results. This follows formally from the fact that the final expression (12) does not depend on model parameters: It depends only on the exciton binding energy  $\varepsilon_0$  which should be treated as a phenomenological parameter. We checked that numerical calculation using the Coulomb potential gives very similar results to those obtained analytically for the case of the delta potential if the binding energy  $\varepsilon_0$  is kept the same.

Let us estimate the main limitation of the model onedimensional Hamiltonian (1) which neglects the exciton motion in the radial direction. It can be important since the radial motion weakens the AB effect in wide rings [21]. Let a ring with the radius *R* have the width  $\Delta R$ . It follows from the numerical calculations that amplitudes of the AB oscillations for the case of  $R/\Delta R > 5$  and for the case of ideal one-dimensional ring ( $\Delta R \rightarrow 0$ ) are almost identical [21]. Therefore, the one-dimensional Hamiltonian (1) correctly describes the solved AB problem for typical semiconductor rings with radius *R* in the tens of nanometers and width  $\Delta R$ in the nanometer range.

## **III. RESULTS AND DISCUSSION**

The field-induced splitting (11) and (12) vanishes if the electron mass is equal to the hole mass,  $m_e = m_h$ . Physically, this can be explained in terms of an artificial U(1) gauge field produced by the coupling of a charged particle to circularly polarized photons [32]. Since the artificial field [32] depends on a particle mass, it interacts differently with an electron and a hole in the case of  $m_e \neq m_h$ . As a consequence, the splitting (11) and (12) is nonzero in the case of  $m_e \neq m_h$ , though an exciton is electrically neutral as a whole. In the case of  $m_e = m_h$ , the artificial gauge field interacts equally with both electron and hole. However, signs of the interaction are different for the electron and the hole since electrical charges of electron and hole are opposite. Therefore, the interaction of the artificial gauge field with an exciton is zero in the case of  $m_e = m_h$ .

The splitting (12) for exciton states with the angular momenta m = 1 and m = -1 in a GaAs quantum ring is presented graphically in Figs. 2 and 3 for various intensities of the dressing field,  $I_0 = \epsilon_0 E_0^2 c$ . The used effective masses of the electron and holes in GaAs,  $m_e/m_0 = 0.063$  and  $m_h/m_0 =$ 0.51, are taken from Ref. [35], where  $m_0$  is the mass of the electron in a vacuum. In Fig. 2, the splitting  $\Delta \varepsilon = \tilde{\varepsilon}_{0,1} - \tilde{\varepsilon}_{0,-1}$ is plotted as a function of the exciton binding energy  $\varepsilon_0$  which depends on the confinement potential of a quantum ring [13]. It is apparent that the splitting decreases with increasing of the binding energy. Physically, this is a consequence of decreasing the exciton size *a*. Indeed, an exciton with a very small size



FIG. 2. (Color online) The energy splitting of the exciton states with angular momenta m = 1 and m = -1 in a GaAs ring with the radius R = 9.6 nm as a function of the exciton binding energy  $\varepsilon_0$  for a circularly polarized dressing field with the frequency  $\omega = 1050$  GHz and different intensities  $I_0$ .



FIG. 3. (Color online) The energy splitting of exciton states with angular momenta m = 1 and m = -1 in a GaAs ring with the radius R = 9.6 nm as a function of the field intensity  $I_0$  and the field frequency  $\omega$  for different binding energies of the exciton: (a)  $\varepsilon_0 = 2 \text{ meV}$ ; (b)  $\varepsilon_0 = 4 \text{ meV}$ ; (c)  $\varepsilon_0 = 6 \text{ meV}$ ; (d)  $\varepsilon_0 = 8 \text{ meV}$ . The physically relevant areas of the field parameters, which correspond to applicability of the basic expressions derived within the perturbation theory, lie below the dashed lines.

looks like an electrically neutral particle from the viewpoint of the dressing electromagnetic field. As a consequence, the splitting (12) is small for small excitons.

It follows from Figs. 2 and 3 that the typical splitting is of  $\mu$ eV scale for stationary irradiation intensities of tens W/cm<sup>2</sup>. This splitting is comparable to the Lamb shift in atoms and can be detected experimentally by optical methods. It order to increase the splitting, the irradiation intensity  $I_0$ should also be increased. However, the increasing of stationary irradiation can fluidize a semiconductor ring. To avoid the fluidizing, it is reasonable to use narrow pulses of a strong dressing field which splits exciton states and narrow pulses of a weak probing field which detects the splitting. This wellknown pump-and-probe methodology has been elaborated long ago and is commonly used to observe quantum optics effects—particularly modifications of the energy spectrum of dressed electrons arisen from the optical Stark effect—in semiconductor structures (see, e.g., Refs. [36–38]). Within this approach, giant dressing fields (up to GW/cm<sup>2</sup>) can be applied to semiconductor structures. As a consequence, the splitting (12) can be of meV scale in state-of-the-art optical experiments.

It should be stressed that the discussed effect is qualitatively different from those arisen from absorption of circularly polarized light in quantum rings (see, e.g., Refs. [39-41]). Namely, the absorption of photons with nonzero angular momentum by electrons leads to the transfer of angular momentum from light to electrons in a ring. Correspondingly, photoinduced currents in the ring appear [39–41]. Since this effect is caused by light absorption, it can be described within the classical electrodynamics of ring-shaped conductors. In contrast, we consider the Aharonov-Bohm effect induced by light in the regime of electromagnetic dressing, when absorption of real photons is absent. To be more specific, the discussed AB effect arises from light-induced changing phase of electron wave function, which results in the appearance of the artificial gauge field [32] and shifts exciton energy levels in the ring. Evidently, this purely quantum phenomenon cannot be described within classical physics.

#### **IV. CONCLUSION**

Summarizing the aforesaid, we predict a quantumoptical phenomenon in semiconductor ringlike nanostructures. Namely, a high-frequency circularly polarized electromagnetic wave splits the energy levels of excitons in a semiconductor quantum ring. This effect should be treated as an opticallyinduced Aharonov-Bohm effect for excitons and can be observed in quantum rings using modern experimental technics. It should be noted that, besides semiconductor quantum rings, perspective objects for observing the discussed effect are ringlike semiconductor structures such as carbon nanotubes.

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