Theory of magnetothermoelectric phenomena in high-mobility two-dimensional electron systems under microwave irradiation

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The response of two-dimensional electron gas to a temperature gradient in perpendicular magnetic field under steady-state microwave irradiation is studied theoretically. The electric currents induced by the temperature gradient and the thermopower coefficients are calculated taking into account both diffusive and phonon-drag mechanisms. The modification of thermopower by microwaves takes place because of Landau quantization of the electron energy spectrum and is governed by the microscopic mechanisms which are similar to those responsible for microwave-induced oscillations of electrical resistivity. The magnetic-field dependence of microwave-induced corrections to phonon-drag thermopower is determined by mixing of phonon resonance frequencies with radiation frequency, which leads to interference oscillations. The transverse thermopower is modified by microwave irradiation much stronger than the longitudinal one. Apart from showing prominent microwave-induced oscillations as a function of magnetic field, the transverse thermopower appears to be highly sensitive to the direction of linear polarization of microwave radiation.

I. INTRODUCTION

Electron transport in two-dimensional (2D) electron systems placed in a perpendicular magnetic field remains one of the most important subjects in condensed matter physics. Recently, it was established that a variety of interesting transport phenomena takes place [1] in the region of moderately strong magnetic field, where the Shubnikov–de Haas oscillations of the electrical resistivity are suppressed because of thermal smearing of the Fermi level. In particular, there are several kinds of magnetoresistance oscillations [1] observed in high-mobility 2D systems such as GaAs quantum wells, strained Ge quantum wells, and electrons on liquid helium surface. Among these phenomena, the microwave-induced resistance oscillations (MIRO) appearing under microwave (MW) irradiation of 2D electron gas [2–5] are studied most extensively. Their origin is briefly described as follows. In the presence of the MW excitation, when absorption and emission of radiation quanta by the electron system take place, both the distribution function and scattering probabilities of electrons are modified. These modifications correlate with the oscillating density of electron states owing to Landau quantization in the magnetic field B, thus leading to corresponding oscillating contributions to resistivity determined by the ratio of the radiation frequency ω to the cyclotron frequency $\omega_c = |e|B/mc$. The period and phase of MIRO, as well as the temperature and power dependence of their amplitudes, are in agreement with this physical picture supported by a detailed consideration of microscopic mechanisms of MIRO in the past years [6–11]. According to both experiment and theory, the MW irradiation strongly affects the longitudinal (dissipative) resistivity and has a much weaker effect on the transverse (Hall) resistivity. More recent experiments uncover the existence of small corrections, sensitive to the direction of the electric field of microwaves (MW polarization), to both longitudinal and Hall resistivities [12,13], also in general agreement with theory.

Apart from its influence on electrical resistivity, the MW excitation is expected to modify other transport coefficients

of 2D electrons, for the same reasons as explained above. The thermoelectric coefficients are of special interest in this connection. The study of thermoelectric phenomena in magnetic fields has a long history, and the fundamentals of this topic, with applications to bulk conductors, are reviewed in Ref. [14]. The electrical response to temperature gradient ∇T is described by the longitudinal (Seebeck) and transverse (Nernst-Ettingshausen) components of thermoelectric power (briefly, thermopower). These coefficients are determined by two mechanisms: the diffusive one, when electrons are directly driven by the diffusion force due to temperature gradient in electron gas, and the phonon-drag one, when electrons are driven by a frictional force between them and phonons propagating along the temperature gradient. The contribution of both these mechanisms in magnetothermopower of 2D electron systems has been studied in a number of theoretical and experimental works [15–21] (see also review paper Ref. [22] and references therein). The quantum effects are commonly observed in strong magnetic fields, where Shubnikov–de Haas oscillations of thermopower coefficients take place [22]. In high-mobility GaAs quantum wells, the phonon-drag thermopower shows another kind of quantum oscillations, related to resonant phonon-assisted scattering of electrons between Landau levels [20]. This occurs in the region of moderately strong magnetic fields, below the onset of the Shubnikov–de Haas oscillations, which is favorable for observation of MW-induced quantum effects.

There are two main ways in which the MW irradiation can influence the thermopower. First, this irradiation leads to nonequilibrium electron distribution that has nontrivial dependence not only on electron energy but also on the temperature of electron gas. Both the diffusive and phonon-drag contributions to thermoelectric coefficients should be sensitive to such changes. Next, the MW irradiation in the presence of magnetic field considerably influences electron-phonon scattering. This causes an effect on electrical resistivity [23] under conditions when the probability of electron-phonon scattering is comparable to that of elastic scattering by impurities. At temperatures below 4.2 K these conditions

are realized only in very pure 2D systems. In contrast, the effect of microwaves on electron-phonon scattering is always important for thermoelectric properties, since the phonon-drag mechanism determined by this scattering gives a very significant [15], if not a major, contribution to thermopower of 2D electrons.

The above consideration also suggests that in spite of the same microscopic mechanisms involved in both cases, the effect of microwaves on magnetothermoelectric coefficients of 2D electrons should be different from their effect on magnetoresistance. The classical Mott relation between the diffusive current responses to temperature gradient and to electric field is not expected to be valid under MW excitation, even for the moderately strong magnetic fields. Moreover, one may presume that both longitudinal and transverse components of thermopower oscillate with magnetic field in a way different from MIRO, and their dependence on MW polarization is also different. Therefore, there is enough motivation for a theoretical study of the influence of MW irradiation on thermopower of 2D electron systems in perpendicular magnetic field. The present paper is devoted to this previously unexplored problem.

In the linear response regime considered in the following, the current density **j** is given by the general expression

$$\mathbf{j} = \hat{\sigma}\mathbf{E} - \hat{\beta}\nabla T, \tag{1}$$

where **E** is the electric field in the plane (x,y) of the 2D electron system. It is assumed that the 2D system is macroscopically homogeneous so that the chemical potential μ does not depend on 2D coordinate **r**. Under conditions when no conduction currents flow in the system, one gets $\mathbf{E} = \hat{\alpha}\nabla T$. The thermopower tensor $\hat{\alpha}$ describes the voltage drop as a result of temperature gradient. It is given by $\hat{\alpha} = \hat{\rho}\hat{\beta}$, where the resistivity tensor $\hat{\rho}$ is the matrix inverse of the conductivity tensor $\hat{\sigma}$. The theoretical approach presented below is based on calculation of thermoelectric tensor $\hat{\beta}$ in the presence of the ac field of microwaves by using the method of the quantum Boltzmann equation [1,8,10,23] established in the previous calculations of the conductivity tensor $\hat{\sigma}$. As both $\hat{\beta}$ and $\hat{\sigma}$ are known, the thermopower is found straightforwardly. The results are presented for the case of moderately strong magnetic field, when the Shubnikov–de Haas oscillations are still suppressed, but quantum oscillations due to Landau quantization exist in high-mobility 2D systems. Such oscillations are caused by inelastic scattering of electrons between Landau levels as a result of electron interaction with acoustic phonons of a resonant frequency ω_{ph} (magnetophonon effect [20,24–29]) and with microwaves of frequency ω. These two kinds of inelastic processes actually interfere, leading to combined frequencies $\omega_{ph} \pm \omega$ whose ratio to ω_c determines the periodicity of the quantum magneto-oscillations [23]. As shown below, such oscillations exist in both longitudinal and transverse thermopower caused by the phonon drag, while the diffusion part of the thermopower follows the MIRO periodicity determined by the single frequency ω. The phonon-drag part of the MW-induced contribution to transverse thermopower is found to be comparable with that of longitudinal thermopower. Since the transverse thermopower is much smaller than the longitudinal one in classically strong magnetic fields, it is dramatically affected by MW irradiation, demonstrating giant

microwave-induced oscillations and a high sensitivity to the direction of MW polarization.

The paper is organized as follows. Section II describes the main formalism including description of ac electric field generated by incident electromagnetic radiation, electric current in the presence of temperature gradient, and kinetic equation for 2D electrons with collision integrals for electron-impurity and electron-phonon interactions. In Sec. III the kinetic equation is solved and the tensor $\hat{\beta}$ is presented and discussed both for the equilibrium case and under MW irradiation. Section IV contains expressions for longitudinal and transverse components of thermopower tensor $\hat{\alpha}$, their discussion, and presentation of the results of numerical calculations of these components as functions of magnetic field and polarization angle. More discussion and concluding remarks are given in the last section.

II. GENERAL FORMALISM

Throughout the paper, one uses the system of units where Planck's constant $\hbar$ and Boltzmann constant k_B are both set to unity. The electron spectrum is assumed to be isotropic and parabolic, with effective mass m. The Zeeman splitting of electron states is neglected.

Consider a monochromatic electromagnetic wave normally incident on the surface containing a 2D layer (the direction of incidence coincides with the direction of the magnetic field, along the z axis). The electric field of this wave near the layer is written, in the general form, as

$$\begin{aligned}\mathbf{E}_t^{(i)} &= E_\omega^{(i)}\mathrm{Re}[\mathbf{e}e^{-i\omega t}]\\ &= \frac{E_\omega^{(i)}}{\sqrt{2}}\mathrm{Re}\left\{\left[\mathbf{e}_-\begin{pmatrix}1\\i\end{pmatrix} + \mathbf{e}_+\begin{pmatrix}1\\-i\end{pmatrix}\right]e^{-i\omega t}\right\},\end{aligned} \tag{2}$$

where **e** is the polarization vector. The second part of this equation represents the wave as a sum of two circularly polarized waves, $\mathbf{e}_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2} = \kappa_\pm e^{\pm i\chi}$; χ is the angle between the main axis of polarization of $\mathbf{E}_t^{(i)}$ and the x axis, and $\kappa_\pm$ are real numbers characterizing ellipticity of the incident wave (they are normalized according to $\kappa_+^2 + \kappa_-^2 = 1$). A circular polarization means that either κ_+ or κ_- is equal to zero. In the case of linear polarization, $\kappa_+ = \kappa_- = 1/\sqrt{2}$ so that $\mathbf{e}_\pm = e^{\pm i\chi}/\sqrt{2}$. The screening of electromagnetic waves due to the presence of free carriers in the 2D layer changes the polarization angle and ellipticity [30,31], so the electric field in the layer, $\mathbf{E}_t$, differs from $\mathbf{E}_t^{(i)}$ and has the following form:

$$\mathbf{E}_t = \frac{E_\omega}{\sqrt{2}}\mathrm{Re}\left\{\left[(\omega-\omega_c)s_-\begin{pmatrix}1\\i\end{pmatrix} + (\omega+\omega_c)s_+\begin{pmatrix}1\\-i\end{pmatrix}\right]e^{-i\omega t}\right\}, \tag{3}$$

where

$$s_\pm = \frac{\mathbf{e}_\pm}{\omega \pm \omega_c + i\omega_p}. \tag{4}$$

Here ω_p is the radiative decay rate that determines the cyclotron line broadening because of the electrodynamic screening effect. It is given by $\omega_p = 2\pi e^2 n_s/mc\sqrt{\epsilon^*}$, where n_s is the electron density, $\sqrt{\epsilon^*} = (1 + \sqrt{\epsilon})/2$, and ϵ is the dielectric permittivity of the sample material. Next,

$E_\omega = E_\omega^{(i)}/\sqrt{\epsilon^*}$. In Eqs. (3) and (4), it is assumed that transport relaxation rate, ν_{tr}, which determines electron mobility, is much smaller than either $|\omega \pm \omega_c|$ or ω_p. The relation $\nu_{tr} \ll \omega_p$ is a very good approximation for high-mobility samples with typical electron density $n_s > 10^{11}$ cm^{-2}.

Apart from the ac field $\mathbf{E}_t$, the electron system is driven by a weak static (dc) field $\mathbf{E}$. To take into account the influence of both these fields on 2D electrons, it is very convenient to use a transition to the moving coordinate frame (see Ref. [10] and references therein). Then, the quantum kinetic equation for electrons derived by using Keldysh formalism for nonequilibrium electron systems (see details in Refs. [10,23]) contains the effect of external fields only in the collision integral. The radiation power is assumed to be weak enough to neglect the influence of microwaves on the energy spectrum of electrons: the spectrum remains isotropic and the density of states is not affected by the radiation. Further, the magnetic field is assumed to be weak enough so there is a large number of Landau levels under the Fermi energy. The kinetic equation written for the electron distribution function $f_{\varepsilon\varphi}$ averaged over the period $2\pi/\omega$ takes the form

$$\frac{\mathbf{p}_{\varepsilon\varphi}}{m} \cdot \nabla f_{\varepsilon\varphi} + \omega_c \frac{\partial f_{\varepsilon\varphi}}{\partial \varphi} = J_{\varepsilon\varphi}, \;\; J_{\varepsilon\varphi} = J_{\varepsilon\varphi}^{im} + J_{\varepsilon\varphi}^{ph}, \quad (5)$$

where ε is the electron energy, $\mathbf{p}_{\varepsilon\varphi} = p_\varepsilon(\cos\varphi, \sin\varphi)$ with $p_\varepsilon = \sqrt{2m\varepsilon}$ is the electron momentum in the 2D layer plane, and φ is the angle of this momentum. Since the dependence of all quantities on the spatial coordinate $\mathbf{r}$ is considered as a parametric one, the coordinate index at the distribution function and collision integrals is omitted. The density of electric current is given by the expression

$$\mathbf{j} = \frac{e}{\pi} \int d\varepsilon D_\varepsilon \int_0^{2\pi} \frac{d\varphi}{2\pi} \mathbf{p}_{\varepsilon\varphi} f_{\varepsilon\varphi} - \sigma_\perp \hat{\epsilon} \mathbf{E} - c\hat{\epsilon} \nabla M_z, \quad (6)$$

where $\sigma_\perp = e^2 n_s / m\omega_c = |e|n_s c/B$ is the classical Hall conductivity and D_ε is the density of electron states expressed in the units m/π. Next, $\hat{\epsilon} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is the antisymmetric unit matrix in the space of 2D Cartesian indices. The last term in the expression (6) is given by the spatial gradient of magnetic moment $\mathbf{M}$ of electrons per unit square. This moment arises because of diamagnetic currents circulating in the electron system. Actually, the last term in Eq. (6) does not contribute to the *total* current across any finite sample. However, the necessity of taking into account this term (its bulk analog is $-c[\nabla \times \mathbf{M}]$) in the expression for the *local* current density was emphasized in studies of magnetothermoelectric phenomena a long time ago [14,32]. Being expressed through the distribution function, the magnetic moment comprises two terms:

$$M_z = -\frac{m}{\pi B} \int d\varepsilon [D_\varepsilon \varepsilon - \Pi_\varepsilon] f_\varepsilon, \quad (7)$$

where f_ε is the isotropic (averaged over φ) part of electron distribution function, and $\Pi_\varepsilon = \int_{-\infty}^{\varepsilon} d\varepsilon' D_{\varepsilon'}$ is the antiderivative of D_ε. In the ideal 2D electron system, the first and the second terms in M_z correspond to magnetization due to bulk and edge currents, respectively [33]. In the case of the equilibrium Fermi distribution function $f_\varepsilon = \{\exp[(\varepsilon - \mu)/T] + 1\}^{-1}$, it is easy to transform Eq. (7) to

a well-known thermodynamic expression $M_z = -\partial\Omega/\partial B$, where $\Omega = -(Tm/\pi) \int d\varepsilon D_\varepsilon \ln\{1 + \exp[(\mu - \varepsilon)/T]\}$ is the thermodynamic potential per unit area.

In the absence of any collisions, $J_{\varepsilon\varphi} = 0$, the local current is nondissipative, $\mathbf{j} = \mathbf{j}^{(n)}$, where

$$\mathbf{j}^{(n)} = -\frac{cm}{\pi B} \int d\varepsilon \Pi_\varepsilon \hat{\epsilon} \nabla f_\varepsilon - \sigma_\perp \hat{\epsilon} \mathbf{E}. \quad (8)$$

If coordinate dependence of f_ε exists solely due to temperature gradient, one has $\nabla f_\varepsilon = (\partial f_\varepsilon/\partial T)\nabla T$. The integral term in Eq. (8) is reduced to nondissipative thermoelectric current $-\hat{\beta}\nabla T$ flowing perpendicular to ∇T, with $\hat{\beta} = (cm/\pi B) \int d\varepsilon \Pi_\varepsilon(\partial f_\varepsilon/\partial T)\hat{\epsilon}$. If chemical potential μ entering f_ε also depends on coordinate, the integral in Eq. (8) produces an additional term proportional to $\nabla\mu$. This term, with the aid of the identity $\partial f_\varepsilon/\partial\mu = -\partial f_\varepsilon/\partial\varepsilon$, can be combined with the last term of Eq. (8), leading to the form $\sigma_\perp \hat{\epsilon} \nabla\zeta$, where $\zeta = \Phi + \mu/e$ is the electrochemical potential and Φ is the electrostatic potential determining the electric field $\mathbf{E} = -\nabla\Phi$. The electric field or, in general, the gradient of the electrochemical potential induced as a result of a temperature gradient is derived from the expression $\mathbf{j}^{(n)} = 0$. This leads to diagonal thermopower tensor $\hat{\alpha} = \hat{1}\alpha$, where $\hat{1}$ is the unit 2×2 matrix and

$$\alpha = \frac{m}{\pi e n_s} \int d\varepsilon \Pi_\varepsilon \frac{\partial f_\varepsilon}{\partial T}. \quad (9)$$

Substituting the equilibrium distribution function into Eq. (9), one gets the well-known result

$$\alpha = -\frac{S}{|e|n_s}, \; S = -\frac{\partial\Omega}{\partial T}, \quad (10)$$

where S is the entropy of 2D electron gas per unit area. For strongly degenerate electron gas, $\mu = \varepsilon_F$, one has $S = (\pi^2/3)n_s T/\varepsilon_F$.

The collision integrals $J_{\varepsilon\varphi}^{im}$ and $J_{\varepsilon\varphi}^{ph}$ standing in Eq. (5) describe, respectively, electron-impurity and electron-phonon scattering [23]:

$$J_{\varepsilon\varphi}^{im} = \int_0^{2\pi} \frac{d\varphi'}{2\pi} \sum_{n=-\infty}^{\infty} \nu(|\mathbf{q}_{\varepsilon n}|)[J_n(|\mathbf{R}_\omega \cdot \mathbf{q}_{\varepsilon n}|)]^2$$
$$\times D_{\varepsilon + n\omega + \gamma_n}[f_{\varepsilon + n\omega + \gamma_n \varphi'} - f_{\varepsilon\varphi}], \quad (11)$$

$$J_{\varepsilon\varphi}^{ph} = \int_0^{2\pi} \frac{d\varphi'}{2\pi} \sum_\lambda \int_{-\infty}^{\infty} \frac{dq_z}{2\pi} m$$
$$\times \sum_{n=-\infty}^{\infty} \left\{ M_{\lambda\mathbf{Q}^-} [J_n(|\mathbf{R}_\omega \cdot \mathbf{q}_{\varepsilon n}^-|)]^2 [(N_{\lambda\mathbf{Q}^-} + f_{\varepsilon\varphi}) \right.$$
$$\times f_{\varepsilon - \omega_{\lambda\mathbf{Q}^-} + n\omega + \gamma_n^- \varphi'} - (N_{\lambda\mathbf{Q}^-} + 1)f_{\varepsilon\varphi}]$$
$$\times D_{\varepsilon - \omega_{\lambda\mathbf{Q}^-} + n\omega + \gamma_n^-} + M_{\lambda\mathbf{Q}^+}[J_n(|\mathbf{R}_\omega \cdot \mathbf{q}_{\varepsilon n}^+|)]^2$$
$$\times [(N_{\lambda - \mathbf{Q}^+} + 1 - f_{\varepsilon\varphi})f_{\varepsilon + \omega_{\lambda\mathbf{Q}^+} + n\omega + \gamma_n^+ \varphi'}$$
$$\left. - N_{\lambda - \mathbf{Q}^+} f_{\varepsilon\varphi}]D_{\varepsilon + \omega_{\lambda\mathbf{Q}^+} + n\omega + \gamma_n^+} \right\}, \quad (12)$$

where J_n is the Bessel function, $\nu(q) = mw(q)$ is the isotropic elastic scattering rate expressed through the Fourier transform $w(q)$ of the correlation function of random potential of impurities, $\mathbf{q}_{\varepsilon n} = \mathbf{p}_{\varepsilon\varphi} - \mathbf{p}_{\varepsilon+n\omega\varphi'}$ is the momentum transferred in scattering in the presence of ac field, and $\mathbf{R}_\omega$ is a complex vector describing the coupling of the electron system to this field:

$$\mathbf{R}_\omega = \frac{eE_\omega}{\sqrt{2}m\omega}(s_+ + s_-,(s_+ - s_-)/i). \qquad (13)$$

The interaction with phonons is considered under approximation of bulk phonon modes. The phonons are characterized by the mode index λ and three-dimensional phonon momentum $\mathbf{Q}$ with out-of-plane component q_z. The squared matrix element of the electron-phonon interaction potential is represented as $M_{\lambda\mathbf{Q}} = C_{\lambda\mathbf{Q}}I_{q_z}$. The squared overlap integral $I_{q_z} = |\langle 0|e^{iq_z z}|0\rangle|^2$ is determined by the confinement potential which defines the ground state of 2D electrons, $|0\rangle$. The function $C_{\lambda\mathbf{Q}}$ characterizes electron-phonon scattering in the bulk. The in-plane momenta transferred in electron-phonon collisions, $\mathbf{q}_{\varepsilon n}^\pm$, are found from the equation $\mathbf{q}_{\varepsilon n}^\pm = \mathbf{p}_{\varepsilon\varphi} - \mathbf{p}_{\varepsilon\pm\omega_{\lambda\mathbf{Q}^\pm}+n\omega\varphi'}$, where $\mathbf{Q}^\pm = (\mathbf{q}_{\varepsilon n}^\pm, q_z)$ and $\omega_{\lambda\mathbf{Q}}$ is the phonon frequency. The effect of the static electric field on the collision integrals is given by the energies $\gamma_n = \mathbf{V}_D \cdot \mathbf{q}_{\varepsilon n}$ and $\gamma_n^\pm = \mathbf{V}_D \cdot \mathbf{q}_{\varepsilon n}^\pm$, where $\mathbf{V}_D = c[\mathbf{E}\times\mathbf{B}]/B^2 = (c/B)\hat{e}\mathbf{E}$ is the drift velocity in the crossed electric and magnetic fields.

In the case of electrons interacting with long-wavelength acoustic phonons in cubic lattice, the expressions for $C_{\lambda\mathbf{Q}}$ and dynamical equations needed for determination of $\omega_{\lambda\mathbf{Q}}$ are the following:

$$C_{\lambda\mathbf{Q}} = \frac{1}{2\rho_M\omega_{\lambda\mathbf{Q}}}\left[\mathcal{D}^2\sum_{ij}\mathrm{e}_{\lambda\mathbf{Q}i}\mathrm{e}_{\lambda\mathbf{Q}j}q_iq_j\right.$$
$$\left.+\frac{(eh_{14})^2}{Q^4}\sum_{ijk,i'j'k'}\kappa_{ijk}\kappa_{i'j'k'}\mathrm{e}_{\lambda\mathbf{Q}k}\mathrm{e}_{\lambda\mathbf{Q}k'}q_iq_jq_{i'}q_{j'}\right], \quad (14)$$

$$\sum_j\left[K_{ij}(\mathbf{Q}) - \delta_{ij}\rho_M\omega_{\lambda\mathbf{Q}}^2\right]\mathrm{e}_{\lambda\mathbf{Q}j} = 0, \qquad (15)$$

$$K_{ij}(\mathbf{Q}) = [(c_{11} - c_{44})q_i^2 + c_{44}Q^2]\delta_{ij} + (c_{12} + c_{44})$$
$$\times q_iq_j(1 - \delta_{ij}). \qquad (16)$$

Here $\mathcal{D}$ is the deformation potential constant, h_{14} is the piezoelectric coupling constant, and ρ_M is the material density. The sums are taken over Cartesian coordinate indices. The coefficient κ_{ijk} is equal to unity if all the indices i, j, k are different and equal to zero otherwise. Next, $\mathrm{e}_{\lambda\mathbf{Q}i}$ are the components of the unit vector of the mode polarization, and $K_{ij}(\mathbf{Q})$ is the dynamical matrix expressed through the elastic constants c_{11}, c_{12}, and c_{44}.

Finally, $N_{\lambda\mathbf{Q}}$ in Eq. (12) is the distribution function of phonons. In the presence of thermal gradients this function depends not only on the frequency $\omega_{\lambda\mathbf{Q}}$ but also on the direction of $\mathbf{Q}$. In the general case, $N_{\lambda\mathbf{Q}}$ can be represented as a sum of symmetric (s) and antisymmetric (a) parts satisfying the relations $N_{\lambda-\mathbf{Q}}^s = N_{\lambda\mathbf{Q}}^s$ and $N_{\lambda-\mathbf{Q}}^a = -N_{\lambda\mathbf{Q}}^a$, respectively. The drag of electrons by phonons is caused by the antisymmetric

part. In the linear regime, N^a is proportional to ∇T while N^s is reduced to the equilibrium distribution function. In particular, one often uses the following form [34]:

$$N_{\lambda\mathbf{Q}} = N_{\omega_{\lambda\mathbf{Q}}} + \frac{\partial N_{\omega_{\lambda\mathbf{Q}}}}{\partial\omega_{\lambda\mathbf{Q}}}\frac{\omega_{\lambda\mathbf{Q}}}{T}\tau_\lambda\mathbf{u}_{\lambda\mathbf{Q}}\cdot\nabla T, \qquad (17)$$

obtained from a linearized kinetic equation for phonons in the relaxation time approximation. Here $N_{\omega_{\lambda\mathbf{Q}}} = [\exp(\omega_{\lambda\mathbf{Q}}/T) - 1]^{-1}$ is the equilibrium (Planck) distribution function, τ_λ is the relaxation time of phonons, and $\mathbf{u}_{\lambda\mathbf{Q}} = \partial\omega_{\lambda\mathbf{Q}}/\partial\mathbf{Q}$ is the phonon group velocity. Notice that a simple expression $\mathbf{u}_{\lambda\mathbf{Q}} = s_\lambda\mathbf{Q}/Q$ relating the group velocity to the sound velocity s_λ is valid only in the isotropic approximation. For elastic waves in real cubic crystals the direction of $\mathbf{u}_{\lambda\mathbf{Q}}$ does not generally coincide with the direction of $\mathbf{Q}$, though the symmetry relation $\mathbf{u}_{\lambda-\mathbf{Q}} = -\mathbf{u}_{\lambda\mathbf{Q}}$ is always valid. Substituting Eq. (17) into the expression for the collision integral $J_{\varepsilon\varphi}^{ph}$, it is convenient to write the latter as a sum of two parts,

$$J_{\varepsilon\varphi}^{ph} = J_{\varepsilon\varphi}^{ph(0)} + J_{\varepsilon\varphi}^{ph(1)}, \qquad (18)$$

where $J_{\varepsilon\varphi}^{ph(0)}$ contains the equilibrium phonon distribution $N_{\omega_{\lambda\mathbf{Q}}}$ only, while $J_{\varepsilon\varphi}^{ph(1)}$ is determined by the anisotropic nonequilibrium correction to phonon distribution [second term in Eq. (17)] and is proportional to ∇T. The second term in Eq. (18) is responsible for the phonon-drag contribution to electric current.

By using Eqs. (11) and (12), one can directly check the identity $\int d\varepsilon D_\varepsilon \int d\varphi J_{\varepsilon\varphi} = 0$ expressing the electron conservation requirement. It is worth emphasizing that the collision integrals Eqs. (11) and (12) are written in the general form valid for an arbitrary relation between radiation frequency ω, phonon frequency $\omega_{\lambda\mathbf{Q}}$, and electron energy ε. In Ref. [23] the collision integrals are written in a simpler form valid under the assumptions $\omega \ll \varepsilon$ and $\omega_{\lambda\mathbf{Q}} \ll \varepsilon$. For degenerate electron gas, the electrons contributing to electric current have energies ε close to the Fermi energy, and these assumptions usually work very well for microwave frequencies and acoustic phonon scattering. However, in the problem of diffusive thermocurrent an extra accuracy is required, so the terms of the first order in ω/ε are to be retained at least in the isotropic part of the electron-impurity collision integral [see Eq. (25) below].

III. SOLUTION OF KINETIC EQUATION

When searching for the response to temperature gradients only, the effect of the dc field in the collision integrals is omitted, $\gamma_n = \gamma_n^\pm = 0$. It is also assumed that the main cause of momentum relaxation of electrons comes from electron-impurity scattering rather than from electron-phonon scattering. In GaAs quantum wells with electron mobility of about 10^6 cm²/V s this approximation holds at low temperatures $T < 10$ K (for GaAs quantum well of typical width 20 nm the phonon-limited mobility is estimated as 1.3×10^7 cm²/V s at $T = 4.2$ K and 3.8×10^6 cm²/V s at $T = 10$ K). Thus, one may neglect the contribution $J_{\varepsilon\varphi}^{ph(0)}$ in comparison to $J_{\varepsilon\varphi}^{im}$, but the contribution $J_{\varepsilon\varphi}^{ph(1)}$ leading to phonon drag must be retained. It is convenient to expand the distribution

function in the angular harmonics, $f_{\varepsilon\varphi} = \sum_k f_{\varepsilon k} e^{ik\varphi}$. The electric current density given by Eq. (6) is determined by the components with $k = \pm 1$. Only the effects linear in MW power are considered below. The distribution function is represented as a sum of two terms, $f_{\varepsilon k}^{(0)} + f_{\varepsilon k}^{(MW)}$, where $f_{\varepsilon k}^{(MW)}$ is proportional to MW power. For $k \neq 0$ the first term is given by the expression comprising the diffusive and phonon-drag parts:

$$f_{\varepsilon k}^{(0)} = -\frac{1}{ik\omega_c + \nu^{(k)} D_\varepsilon} \left\{ \frac{p_\varepsilon}{2m} \left(\frac{\partial f_{\varepsilon k+1}}{\partial T} \nabla_+ T + \frac{\partial f_{\varepsilon k-1}}{\partial T} \nabla_- T \right) \right.$$
$$+ \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{2\pi} \frac{d\varphi'}{2\pi} \sum_{k'} e^{i(k'-k)\varphi}$$
$$\left. \times \hat{\mathcal{M}} \left\{ \sum_{l=\pm 1} l D_{\varepsilon - l\omega_{\lambda \mathbf{Q}}} (f_{\varepsilon - l\omega_{\lambda \mathbf{Q}} k'} e^{-ik'\theta} - f_{\varepsilon k'}) \right\} \right\}, \quad (19)$$

where $\nabla_\pm = \nabla_x \pm i\nabla_y$, $\nu^{(k)} = \overline{\nu_\theta[1 - \cos(k\theta)]}$ (the line over the expression denotes angular averaging), and $\nu_\theta = \nu[2p_\varepsilon \sin(\theta/2)]$. The integral operator $\hat{\mathcal{M}}$ is proportional to

temperature gradient and defined as

$$\hat{\mathcal{M}}\{A\} = \sum_\lambda \int_{-\infty}^\infty \frac{dq_z}{2\pi} m M_{\lambda \mathbf{Q}} \tau_\lambda F\left(\frac{\omega_{\lambda \mathbf{Q}}}{2T}\right)$$
$$\times \left[\frac{1}{Q^2} \mathbf{q} \cdot \nabla T + \frac{1}{q^2 \omega_{\lambda \mathbf{Q}}} \frac{\partial \omega_{\lambda \mathbf{Q}}}{\partial \varphi_q} \mathbf{q} \cdot \hat{\epsilon} \nabla T \right] A, \quad (20)$$

with $F(x) = [x/\sinh(x)]^2$. It is taken into account that $\omega_{\lambda \mathbf{Q}} \ll \varepsilon$, which allows one to use the quasielastic approximation, when the transferred 2D momentum $\mathbf{q}_{\varepsilon n}^\pm$ is replaced by $\mathbf{q}$, with absolute value $q = 2p_\varepsilon \sin(\theta/2)$ depending on electron energy and scattering angle $\theta = \varphi - \varphi'$. The angle of the vector $\mathbf{q}$ is $\varphi_q = \pi/2 + \phi$, where $\phi = (\varphi + \varphi')/2$. The phonon frequency can be written as $\omega_{\lambda \mathbf{Q}} = s_{\lambda \mathbf{Q}} Q$, where $Q = \sqrt{q^2 + q_z^2}$ and $s_{\lambda \mathbf{Q}}$ is the sound velocity that depends on the mode index and direction of vector $\mathbf{Q}$. If the quantum well is grown in the [001] crystallographic direction, as assumed in the following, both $\omega_{\lambda \mathbf{Q}}$ and $M_{\lambda \mathbf{Q}}$ are periodic in φ_q with the period $\pi/2$.

To find $f_{\varepsilon k}^{(MW)}$ with the accuracy up to the linear terms in MW power, only the contributions with low-order, $|n| \leqslant 1$, Bessel functions J_n are to be taken in Eqs. (11) and (12). Physically, this corresponds to a neglect of multiphoton absorption processes. If $k \neq 0$, then

$$f_{\varepsilon k}^{(MW)} = \frac{P_\omega(\varepsilon)/4}{ik\omega_c + \nu^{(k)} D_\varepsilon} \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{2\pi} \frac{d\varphi'}{2\pi} (1 - \cos\theta)[1 - be^{2i\phi} - b^* e^{-2i\phi}] \sum_{k'} e^{i(k'-k)\varphi}$$

$$\times \sum_{n=\pm 1} \left\{ \nu_\theta [D_{\varepsilon + n\omega}(f_{\varepsilon + n\omega k'} e^{-ik'\theta} - f_{\varepsilon k'}) - D_\varepsilon f_{\varepsilon k'}(e^{-ik'\theta} - 1)] \right.$$

$$\left. - \hat{\mathcal{M}} \left\{ \sum_{l=\pm 1} l [D_{\varepsilon - l\omega_{\lambda \mathbf{Q}} + n\omega}(f_{\varepsilon - l\omega_{\lambda \mathbf{Q}} + n\omega k'} e^{-ik'\theta} - f_{\varepsilon k'}) - D_{\varepsilon - l\omega_{\lambda \mathbf{Q}}}(f_{\varepsilon - l\omega_{\lambda \mathbf{Q}} k'} e^{-ik'\theta} - f_{\varepsilon k'})] \right\} \right\}, \quad (21)$$

where

$$P_\omega(\varepsilon) = \frac{2e^2 E_\omega^2 \varepsilon}{m\omega^2}(|s_+|^2 + |s_-|^2) \quad (22)$$

is the dimensionless function proportional to MW power [see Eq. (4) for definition of $s_\pm$] and

$$b = \frac{s_- s_+^*}{(|s_+|^2 + |s_-|^2)} \quad (23)$$

is a complex dimensionless coefficient which depends on the direction of MW polarization and determines the sensitivity of transport properties of electrons to this direction. The neglect of multiphoton processes implies $P_\omega(\varepsilon) \ll 1$. The expression (21) comprises both electron-impurity and electron-phonon parts, though only the electron-phonon part is essential below.

To find the isotropic ($k = 0$) part of the distribution function, it is necessary to include electron-electron scattering into consideration. Though the corresponding collision integral J_ε^{ee} is not written in Eq. (5) explicitly, it can be found in Refs. [9,23]. The kinetic equation is written as

$$J_\varepsilon^{im} + J_\varepsilon^{ph(0)} + J_\varepsilon^{ee} = 0, \quad (24)$$

where only the isotropic part of the distribution function is retained under the collision integrals. It is essential that J_ε^{ee} is not affected by MW irradiation, while J_ε^{im} is nonzero only in the presence of MW irradiation. The distribution f_ε is represented as a sum of smooth part $f_\varepsilon^{(0)}$ and rapidly oscillating part $f_\varepsilon^{(MW)}$. The smooth part is controlled by electron-electron scattering and, therefore, can be approximated by a heated Fermi distribution with effective electron temperature T_e; the latter is to be found from the energy balance equation $\int d\varepsilon D_\varepsilon \varepsilon [J_\varepsilon^{im} + J_\varepsilon^{ph(0)}] = 0$. For the oscillating part, one gets the following expression:

$$f_\varepsilon^{(MW)} = \frac{P_\omega(\varepsilon)}{4} \tau_{in} \nu_{tr} \sum_{n=\pm 1} \left(1 + \frac{n\omega}{2\varepsilon}(1 - \mathcal{Z}_{tr}) \right)$$
$$\times \delta D_{\varepsilon + n\omega}(f_{\varepsilon + n\omega} - f_\varepsilon), \quad (25)$$

where $\delta D_\varepsilon = D_\varepsilon - 1$, $\nu_{tr} = \tau_{tr}^{-1} = \nu^{(\pm 1)}$ is the transport relaxation rate, and

$$\mathcal{Z}_{tr} = \frac{\partial \ln \tau_{tr}}{\partial \ln \varepsilon} \quad (26)$$

is the logarithmic derivative of the transport time over energy. The inelastic scattering time τ_{in} entering Eq. (25) describes relaxation of the isotropic oscillating part of electron

distribution [9]. This relaxation is caused mostly by electron-electron scattering and scales with temperature as $\tau_{\text{in}} \propto T_e^{-2}$.

The electric current is given by Eq. (6), where the distribution function, found from Eqs. (19), (21), and (25) with the accuracy up to the terms linear in both ∇T and P_ω, is substituted. The thermoelectric tensor $\hat{\beta}$ determining the thermocurrent is represented below as a sum of four parts:

$$\hat{\beta} = \hat{\beta}_d^{(0)} + \hat{\beta}_p^{(0)} + \hat{\beta}_d^{(\text{MW})} + \hat{\beta}_p^{(\text{MW})}, \tag{27}$$

where diffusive (d) and phonon drag (p) parts are written separately. Two first terms correspond to dark thermocurrent, in the absence of MW irradiation, while the next two terms are MW-induced corrections. While $\hat{\beta}_d^{(0)}$ and $\hat{\beta}_p^{(0)}$ are determined only by $f_{\varepsilon k}^{(0)}$ from Eq. (19), the MW-induced parts are found in a more elaborate way, by combining together the results given by Eqs. (19), (21), and (25), as described in Sec. III B.

A. Dark thermocurrent

In the absence of MWs, the linear response to temperature gradient is found from Eq. (19) for $k = \pm 1$ with isotropic ($k' = 0$) distribution functions substituted in the right-hand side. The thermoelectric coefficients are given by the following expressions:

$$\hat{\beta}_d^{(0)} = \frac{|e|}{\pi} \int d\varepsilon \, \frac{\partial f_\varepsilon^{(0)}}{\partial T_e} \frac{\omega_c \Pi_\varepsilon \hat{\epsilon} - \nu_{\text{tr}} \varepsilon D_\varepsilon^2 \hat{1}}{\omega_c^2 + \nu_{\text{tr}}^2 D_\varepsilon^2} \tag{28}$$

and

$$\hat{\beta}_p^{(0)} = \frac{|e|}{2\pi} \int d\varepsilon \, \frac{\omega_c \varepsilon D_\varepsilon \hat{\epsilon} - \nu_{\text{tr}} \varepsilon D_\varepsilon^2 \hat{1}}{\omega_c^2 + \nu_{\text{tr}}^2 D_\varepsilon^2}$$
$$\times \hat{\mathcal{P}}_1 \left\{ \sum_{l=\pm 1} l D_{\varepsilon - l\omega_{\lambda \mathbf{Q}}} \left(f_{\varepsilon - l\omega_{\lambda \mathbf{Q}}}^{(0)} - f_\varepsilon^{(0)} \right) \omega_{\lambda \mathbf{Q}}^{-1} \right\}, \tag{29}$$

where $\hat{\mathcal{P}}_n$ is the integral operator defined as

$$\hat{\mathcal{P}}_n\{A\} = \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\varphi_q}{2\pi} \sum_\lambda \int_0^\infty \frac{dq_z}{\pi}$$
$$\times (1 - \cos\theta)^n m^2 M_{\lambda \mathbf{Q}} \tau_\lambda F\left(\frac{\omega_{\lambda \mathbf{Q}}}{2T} \right) \frac{2\omega_{\lambda \mathbf{Q}}}{Q^2} A. \tag{30}$$

The matrices given by Eqs. (28) and (29) contain diagonal symmetric ($\propto \hat{1}$) and nondiagonal antisymmetric ($\propto \hat{\epsilon}$) parts, so their symmetry is the same as the symmetry of the electrical conductivity.

The expressions (28) and (29) describe the thermoelectric tensor in a wide region of temperatures and magnetic fields. Quantum oscillations of $\hat{\beta}_d^{(0)}$ and $\hat{\beta}_p^{(0)}$ occur because of the oscillating dependence of the density of states, D_ε. In the following, the approximation of overlapping Landau levels is used: $D_\varepsilon = 1 - 2d\cos(2\pi\varepsilon/\omega_c)$, where $d = \exp(-\pi/|\omega_c|\tau)$ is the Dingle factor ($d \ll 1$) and τ is the quantum lifetime of electrons, given at low temperatures by $\tau = 1/\overline{\nu_\theta}$. Apart from the condition $d \ll 1$, the validity of the expression for D_ε implies $\varepsilon\tau \gg 1$. Under the same requirements, $\Pi_\varepsilon = \varepsilon - (\omega_c/\pi)d\sin(2\pi\varepsilon/\omega_c)$. The integrals over energy in Eqs. (28) and (29) are calculated below under the assumption of strongly degenerate electron gas, and the quantum effects up to the second order in the Dingle factors are retained. To take into

account energy dependence of the Dingle factor due to energy dependence of τ, the logarithmic derivative $\mathcal{Z} = \partial \ln \tau / \partial \ln \varepsilon$ is introduced. The diffusive part is given by the following expression:

$$\hat{\beta}_d^{(0)} = \frac{\pi |e| T_e}{3(\omega_c^2 + \nu_{\text{tr}}^2)} \left\{ \omega_c \left[1 + \frac{2\mathcal{Z}_{\text{tr}} \nu_{\text{tr}}^2}{\omega_c^2 + \nu_{\text{tr}}^2} + 6d \cos \frac{2\pi\varepsilon_F}{\omega_c} \frac{\mathcal{B}}{X} \right] \hat{\epsilon} \right.$$
$$- \nu_{\text{tr}} \left[1 - \mathcal{Z}_{\text{tr}} \frac{\omega_c^2 - \nu_{\text{tr}}^2}{\omega_c^2 + \nu_{\text{tr}}^2} - 12d \frac{\varepsilon_F}{\pi T_e} \mathcal{B} \sin \frac{2\pi\varepsilon_F}{\omega_c} \right.$$
$$\left. \left. + 2d^2 \left(1 - \mathcal{Z}_{\text{tr}} + \frac{2\pi\mathcal{Z}}{|\omega_c|\tau} \right) \right] \hat{1} \right\} \tag{31}$$

with $X = 2\pi^2 T_e/\omega_c$ and $\mathcal{B} = \partial(X/\sinh X)/\partial X = (1 - X \coth X)/\sinh X$. All energy-dependent quantities, namely $\nu_{\text{tr}}, \tau, \mathcal{Z}_{\text{tr}},$ and $\mathcal{Z}$, in Eq. (31) are taken at $\varepsilon = \varepsilon_F$. The classical terms and the quantum term proportional to d in the diagonal part of $\hat{\beta}_d^{(0)}$ have been reported previously [22].

Calculating the phonon-drag part from Eq. (29) under the same approximations, one gets the result

$$\hat{\beta}_p^{(0)} = \frac{|e| n_s}{m(\omega_c^2 + \nu_{\text{tr}}^2)} \left\{ \omega_c [\Gamma_1 + 2d^2 \Gamma_{c1}] \hat{\epsilon} \right.$$
$$- \nu_{\text{tr}} [\Gamma_1(1 + 2d^2) + 4d^2 \Gamma_{c1}] \hat{1}$$
$$\left. - 4d\Gamma_{s1} \frac{X}{\sinh X} \cos \frac{2\pi\varepsilon_F}{\omega_c} [\omega_c \hat{\epsilon} - (3/2)\nu_{\text{tr}} \hat{1}] \right\}. \tag{32}$$

Similarly to Eq. (31), this expression contains both classical terms and quantum terms proportional to d and d^2. The dimensionless functions Γ_i used here and below are defined as

$$\begin{pmatrix} \Gamma_n \\ \Gamma_{cn} \\ \Gamma_{sn} \end{pmatrix} = \hat{\mathcal{P}}_n^F \left\{ \begin{array}{c} 1 \\ \cos \frac{2\pi\omega_{\lambda\mathbf{Q}}}{\omega_c} \\ \frac{\omega_c}{2\pi\omega_{\lambda\mathbf{Q}}} \sin \frac{2\pi\omega_{\lambda\mathbf{Q}}}{\omega_c} \end{array} \right\}, \tag{33}$$

where $\hat{\mathcal{P}}_n^F$ denotes $\hat{\mathcal{P}}_n$ at $\varepsilon = \varepsilon_F$. The function Γ_1 determines the *classical* contribution [34] to phonon-drag thermoelectric response and does not depend on the magnetic field. This contribution has been considered previously in the isotropic approximation for the phonon spectrum, when there are one longitudinal phonon branch with velocity s_l and two transverse branches with velocity s_t. For high temperatures, when T exceeds both $s_\lambda p_F$ and $\pi s_\lambda/a$ (p_F is the Fermi momentum and a is the quantum well width), Γ_1 is temperature-independent. At low temperatures, $T \ll s_\lambda p_F$, the Bloch-Gruneisen transport regime is realized, when electron scattering by phonons occurs at small angles, $\theta \ll 1$. In this regime [35], Γ_1 scales with temperature as T^2 (or as T^4 if only the deformation-potential mechanism of electron-phonon interaction is present). The functions Γ_{cn} and Γ_{sn} standing in the quantum contributions depend on the magnetic field and can be analytically calculated only in certain limits (see Appendix).

The terms $\propto d$ in Eqs. (31) and (32) describe the Shubnikov–de Haas oscillations of the thermocurrent. The forthcoming consideration, however, is focused at the case of $|\omega_c| \ll 2\pi^2 T_e$, which means that $X/\sinh X$ is exponentially small so the Shubnikov–de Haas oscillations are suppressed and the quantum corrections are given by the terms $\propto d^2$ only. Using $n_s = m\varepsilon_F/\pi$, one may check that

the tensor (31) under these conditions satisfies the Mott relation $\hat{\beta}_d^{(0)} = -(\pi^2 T_e/3|e|)(\partial \hat{\sigma}/\partial \varepsilon_F)$, where $\hat{\sigma} = \hat{1}\sigma_d - \hat{\epsilon}\sigma_\perp$ is the conductivity tensor whose components are $\sigma_d = e^2 n_s \nu_{\text{tr}}(1 + 2d^2)/[m(\omega_c^2 + \nu_{\text{tr}}^2)]$ and $\sigma_\perp = e^2 n_s \omega_c/[m(\omega_c^2 + \nu_{\text{tr}}^2)]$. The quantum corrections both in these expressions and in Eq. (31) are essential only in the classically strong magnetic fields, so the terms $\propto (\nu_{\text{tr}}/\omega_c)d^2$ are neglected in comparison to the terms $\propto d^2$.

The diffusive thermoelectric coefficients do not oscillate before the onset of Shubnikov–de Haas oscillations. In contrast, quantum magneto-oscillations of phonon-drag thermoelectric coefficients persist under the assumed condition $|\omega_c| \ll 2\pi^2 T_e$, because of the presence of Γ_{c1} in $\hat{\beta}_p^{(0)}$. Indeed, the oscillating nature of the function $\cos(2\pi\omega_{\lambda\mathbf{Q}}/\omega_c)$ is not completely washed out after the integration under $\hat{\mathcal{P}}$. The major contribution to such integrals comes from the region of variables around $q_z = 0$ and $\theta = \pi$, which physically corresponds to backscattering of electrons as a result of emission or absorption of phonons moving in the quantum well plane; the wave number of these phonons is close to $2p_F$. Thus, there exist resonant phonon frequencies, roughly estimated as $2p_F s_\lambda$, which lead to magneto-oscillations of phonon-drag thermopower observed [20] in high-mobility samples. With decreasing temperature, the oscillations are exponentially suppressed in the Bloch-Gruneisen regime (see Appendix). The same kinds of oscillations are observed in electrical resistivity; they are known as acoustic magnetophonon oscillations or phonon-induced resistance oscillations [24–29].

B. Microwave-induced thermocurrent

The distribution functions $f_{\varepsilon 1}$ and $f_{\varepsilon -1}$ determining the electric current under MW irradiation are to be found up to the terms linear in $P_\omega(\varepsilon)$. There are two types of such MW-induced contributions. The direct ones are obtained in two ways: (i) by calculating $f_{\varepsilon \pm 1}^{(\text{MW})}$ from Eq. (21), where the isotropic distribution function $f_\varepsilon^{(0)}$ is retained under the integral (only the phonon part is essential), and (ii) by calculating $f_{\varepsilon \pm 1}^{(0)}$ from Eq. (19), where the isotropic MW-induced distribution function $f_\varepsilon^{(\text{MW})}$ is placed in the right-hand side. The indirect contributions assume calculation of $f_{\varepsilon \pm 1}^{(0)}$ and $f_{\varepsilon \pm 1}^{(\text{MW})}$ by substituting anisotropic parts of $f_{\varepsilon k}^{(\text{MW})}$ and $f_{\varepsilon k}^{(0)}$, respectively, in the right-hand sides of Eq. (19) and Eq. (21). A similar technique has been used for calculation of the MW-induced conductivity. Following the notations of Ref. [10], one may denote the direct contributions (i) and (ii) as the "displacement" and "inelastic" ones, respectively, and the indirect contributions as the "quadrupole" ones. Strictly speaking, there exists one more indirect contribution called the "photovoltaic" one [10], which is determined by the MW-generated time-dependent part of the distribution function and cannot be obtained from the kinetic equation Eq. (5) because the latter is written for time-independent $f_{\varepsilon\varphi}$. The indirect contributions to $\hat{\beta}$ begin with the terms of the order ν_{tr}/ω_c compared to direct contributions. Since all the MW-induced contributions are of quantum nature and important only in the region of classically strong magnetic field, $\omega_c \gg \nu_{\text{tr}}$, the indirect contributions are less significant than the direct ones and can be safely neglected in the thermopower coefficients

presented in the next section. Therefore, the attention below is focused at the direct contributions only.

The current is calculated in the regime when electron gas is degenerate. Within the required accuracy, the solution of Eq. (25) is given by the following expression:

$$f_\varepsilon^{(\text{MW})} = \frac{d}{2}P_\omega(\varepsilon)\tau_{\text{in}}\nu_{\text{tr}}\left\{\sin\frac{2\pi\varepsilon}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}\right.$$
$$\times\left(f_{\varepsilon+\omega}^{(0)} - f_{\varepsilon-\omega}^{(0)}\right) - \cos\frac{2\pi\varepsilon}{\omega_c}\cos\frac{2\pi\omega}{\omega_c}$$
$$\times\left[f_{\varepsilon+\omega}^{(0)} + f_{\varepsilon-\omega}^{(0)} - 2f_\varepsilon^{(0)}\right.$$
$$\left.\left.+\frac{\omega}{2\varepsilon}\left(1 - \mathcal{Z}_{\text{tr}} + \frac{2\pi\mathcal{Z}}{|\omega_c|\tau}\right)\left(f_{\varepsilon+\omega}^{(0)} - f_{\varepsilon-\omega}^{(0)}\right)\right]\right\}.$$
(34)

The first term of this expression gives the main contribution sufficient for calculation of the MW-induced resistance [9]. The second term represents a correction of the order ω/ε, which is necessary for calculation of the diffusive thermopower. The term proportional to the factor $\sin(2\pi\omega/\omega_c)$ in Eq. (34) also enters Eqs. (35), (37), (43), (45), and (48) below, where the contribution of inelastic mechanism is present. This factor reflects the property [9] that the strongest modification of the electron distribution function under MW irradiation in the presence of weak Landau quantization occurs when $\omega/\omega_c = n \pm 1/4$ (n is an integer). The correction proportional to the factor $\cos(2\pi\omega/\omega_c)$ appears because the resonance absorption of MW radiation at $\omega/\omega_c = n$ also has an effect on the distribution function.

The consideration below assumes the approximation $|\omega_c| \ll 2\pi^2 T_e$, when Shubnikov–de Haas oscillations are thermally averaged out. This dramatically simplifies calculation of the integrals over energy because one can average the products of rapidly oscillating functions such as D_ε and $f_\varepsilon^{(\text{MW})}$ over the period ω_c before integration over the energy. After substituting Eq. (34) into the first part of the right-hand side of Eq. (19) and calculating the current according to Eq. (6) [one may equally use Eq. (28) with $f_\varepsilon^{(0)}$ replaced by $f_\varepsilon^{(\text{MW})}$], the diffusive part of $\hat{\beta}$ takes the form

$$\hat{\beta}_d^{(\text{MW})} = \frac{|e|d^2\tau_{\text{in}}\nu_{\text{tr}}\omega^2 P_\omega \mathcal{T}_{\text{in}}}{\pi T_e(\omega_c^2 + \nu_{\text{tr}}^2)}$$
$$\times\left[\frac{\omega_c^2}{2\pi\omega}\sin\frac{2\pi\omega}{\omega_c}\hat{\epsilon} - \nu_{\text{tr}}(1 - \mathcal{Z}_{\text{tr}})\cos\frac{2\pi\omega}{\omega_c}\hat{1}\right],$$
(35)

where all energy-dependent quantities are taken at $\varepsilon = \varepsilon_F$, in particular, $P_\omega \equiv P_\omega(\varepsilon_F)$. The main contribution to the derivative over temperature in Eq. (19) comes from temperature dependence of the inelastic relaxation time, expressed through the logarithmic derivative

$$\mathcal{T}_{\text{in}} = \frac{\partial \ln \tau_{\text{in}}}{\partial \ln T_e} \simeq -2.$$
(36)

The factor $\sin(2\pi\omega/\omega_c)$ typical for MW-induced conductivity [9] does not appear in the diagonal part of thermoelectric tensor Eq. (35), because of different dependence of the

diffusive thermoelectric current on electron energy distribution as compared to the drift current. The first term of $f_\varepsilon^{(\text{MW})}$ is averaged out in the diagonal components of $\hat{\beta}_d^{(\text{MW})}$, while the second term of $f_\varepsilon^{(\text{MW})}$, proportional to $\cos(2\pi\omega/\omega_c)$, survives this averaging.

For the phonon-drag part of $\hat{\beta}$ the result is the following:

$$
\begin{aligned}
\hat{\beta}_p^{(\text{MW})} = \frac{2|e|n_s d^2 P_\omega}{m(\omega_c^2 + \nu_{\text{tr}}^2)} &\left\{ \nu_{\text{tr}}\tau_{\text{in}}\Gamma_{s1}\frac{2\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c} \right. \\
&\times[-\omega_c\hat{\epsilon} + (3/2)\nu_{\text{tr}}\hat{1}] \\
&+ \left(\Gamma_{c2}\sin^2\frac{\pi\omega}{\omega_c} + \Gamma_{s2}\frac{\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}\right) \\
&\times[\omega_c(-\hat{\epsilon} + \hat{g}_0) + 2\nu_{\text{tr}}(\hat{1} + \hat{h}_0)] \\
&+ \left(\tilde{\Gamma}_{c2}\sin^2\frac{\pi\omega}{\omega_c} + \tilde{\Gamma}_{s2}\frac{\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}\right) \\
&\left. \times [\omega_c\hat{g}_1 + 2\nu_{\text{tr}}\hat{h}_1] \right\},
\end{aligned}
\tag{37}
$$

where

$$
\begin{aligned}
\hat{g}_0 &= b'\hat{\sigma}_x + b''\hat{\sigma}_z, \quad \hat{g}_1 = b'\hat{\sigma}_x - b''\hat{\sigma}_z, \\
\hat{h}_0 &= b'\hat{\sigma}_z - b''\hat{\sigma}_x, \quad \hat{h}_1 = b'\hat{\sigma}_z + b''\hat{\sigma}_x;
\end{aligned}
\tag{38}
$$

$\hat{\sigma}_z = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ and $\hat{\sigma}_x = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$ are the Pauli matrices, while b' and b'' denote real and imaginary parts of b [see Eq. (23)], respectively. The quantities $\tilde{\Gamma}_i$ differ from Γ_i by placing the factor

$$
\cos(4\varphi_q) - \sin(4\varphi_q)\left(1 + \frac{q_z^2}{4p_\varepsilon^2\sin^2\theta/2}\right)\frac{1}{\omega_{\lambda\mathbf{Q}}}\frac{\partial\omega_{\lambda\mathbf{Q}}}{\partial\varphi_q}
\tag{39}
$$

under the integral operator $\hat{\mathcal{P}}$ in Eq. (33). In the general case, both terms in Eq. (39) are essential for calculation of $\tilde{\Gamma}_i$. In the isotropic approximation for phonon spectrum the second term in Eq. (39) vanishes, but $\tilde{\Gamma}_i$ is still nonzero, because the piezoelectric-potential part of $M_{\lambda\mathbf{Q}}$ remains angular-dependent. If the anisotropy of the phonon spectrum is weak, the second term in Eq. (39) can be neglected in the calculation of the piezoelectric-potential contribution.

The expression (37) includes contributions from both inelastic (first term) and displacement (second and third terms) mechanisms. The inelastic-mechanism contribution can be obtained from Eq. (29) after replacing $f_\varepsilon^{(0)}$ with $f_\varepsilon^{(\text{MW})}$. The displacement-mechanism contribution has a form similar to that of the MW-induced contribution to conductivity [8], as it contains the factors $\sin(2\pi\omega/\omega_c)$ and $\sin^2(\pi\omega/\omega_c)$. The first of these factors has extrema at $\omega/\omega_c = n \pm 1/4$, corresponding to the conditions of maximal displacement of electrons along the effective drag force or against this force under photon-assisted scattering, similar to the case of a response to dc field [7]. The second factor describes the enhancement of photon-assisted scattering probabilities in the resonance, $\omega/\omega_c = n$, and their suppression in the antiresonance, $\omega/\omega_c = n + 1/2$. The displacement-mechanism contribution depends on MW polarization direction through the terms with the matrices of Eq. (38).

The fundamental difference between $\hat{\beta}_p^{(\text{MW})}$ and the MW-induced contribution to conductivity is given by the factors Γ_i and $\tilde{\Gamma}_i$, which are not merely constants but functions of the magnetic field describing the magnetophonon oscillations. The products of these magnetophonon oscillating factors by the MW-induced oscillating factors $\sin(2\pi\omega/\omega_c)$ and $\sin^2(\pi\omega/\omega_c)$ physically correspond to the interference of these two kinds of oscillations and can be viewed as a result of photon and phonon frequency mixing in the scattering probabilities.

The phonon-drag part of $\hat{\beta}$ depends on electron temperature T_e through the inelastic scattering time τ_{in}. The quantities Γ_i and $\tilde{\Gamma}_i$ are determined by the lattice temperature T.

There is an important question of whether the thermoelectric tensor $\hat{\beta}$ satisfies the symmetry with respect to time inversion (Onsager symmetry). In the absence of microwaves, this symmetry, of course, is satisfied. Under MW irradiation, when electrons are out of equilibrium, the Onsager symmetry can be broken [10]. In application to the problem of electrons in the presence of electromagnetic waves, the time inversion implies, apart from the magnetic field reversal $\omega_c \to -\omega_c$, the transformations $\mathbf{e} \to \mathbf{e}^*$ and $\mathbf{k} \to -\mathbf{k}$, where $\mathbf{k}$ is the wave vector of the electromagnetic wave. $\mathbf{e} \to \mathbf{e}^*$ means that $e_\pm \to e_\mp^*$, which is equivalent to $\kappa_\pm \to \kappa_\mp$ (see the beginning of Sec. II), while $\mathbf{k} \to -\mathbf{k}$ means that the sign at ω_p in Eq. (4) for $s_\pm$ is inverted, as follows from reversibility of the wave transmission problem [30] employed for derivation of Eq. (3). Therefore, the denominator in Eq. (4) transforms as $\omega \pm \omega_c + i\omega_p \to \omega \mp \omega_c - i\omega_p$, which results in $s_\pm \to s_\mp^*$ under the time inversion. The Onsager symmetry relation takes the form

$$
\beta_{ij}(\omega_c, s_-, s_+) = \beta_{ji}(-\omega_c, s_+^*, s_-^*),
\tag{40}
$$

and similar relations can be written for the other transport coefficients including the conductivity. From $s_\pm \to s_\mp^*$ one can see that both $|s_+|^2 + |s_-|^2$ and $s_-s_+^*$ are invariants with respect to time inversion; thus the function b defined by Eq. (23) is also an invariant. The MW-induced part of $\hat{\beta}$ given by Eq. (37) does contain the terms violating the Onsager symmetry Eq. (40); these are the terms at the matrices $\hat{g}_0$ and $\hat{g}_1$. These terms are invariant under permutation of Cartesian indices.

IV. THERMOPOWER COEFFICIENTS

Having found $\hat{\beta}$, one may calculate the thermopower tensor $\hat{\alpha}$, which is presented below as a sum of dark and MW-induced parts, $\hat{\alpha} = \hat{\alpha}^{(0)} + \hat{\alpha}^{(\text{MW})}$. Because of the presence of terms which depend on MW polarization, this tensor is a general matrix.

In the absence of MW irradiation, $\hat{\alpha}$ has the same symmetries as the resistivity tensor, $\alpha_{xx}^{(0)} = \alpha_{yy}^{(0)}$ and $\alpha_{xy}^{(0)} = -\alpha_{yx}^{(0)}$. By using the expressions $\rho_{xy}^{(0)} = m\omega_c/e^2 n_s$ and $\rho_{xx}^{(0)} = m\nu_{\text{tr}}(1 + 2d^2)/e^2 n_s$ together with Eqs. (31) and (32), where the Shubnikov–de Haas terms are neglected, one obtains, within the accuracy up to d^2, the following results:

$$
\alpha_{xx}^{(0)} = -\frac{\pi^2}{3|e|}\frac{T_e}{\varepsilon_F}\left(1 + \frac{\mathcal{Z}_{\text{tr}}\nu_{\text{tr}}^2}{\omega_c^2 + \nu_{\text{tr}}^2}\right) - \frac{1}{|e|}(\Gamma_1 + 2d^2\Gamma_{c1}), \tag{41}
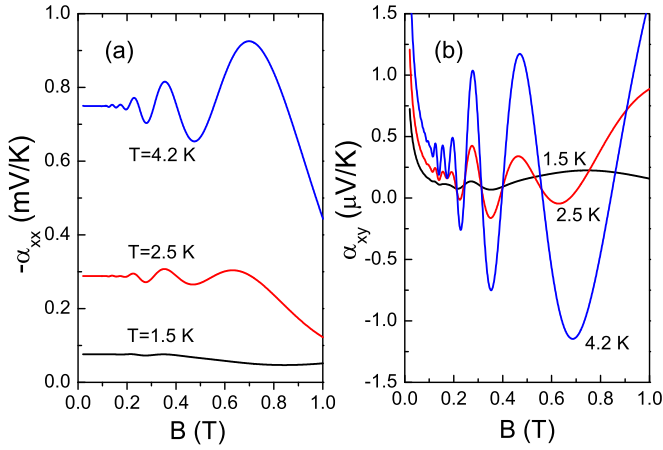$$

FIG. 1. (Color online) Longitudinal (a) and transverse (b) thermopower in the absence of microwave excitation plotted for three temperatures. The calculations are done for a GaAs quantum well of width 14 nm with electron density $n_s = 5 \times 10^{11}$ cm^{-2} and mobility 2×10^6 cm^2/V s. The quantum lifetime of electrons is $\tau = 7$ ps.

$$\alpha_{xy}^{(0)} = \frac{\omega_c \nu_{\mathrm{tr}}}{\omega_c^2 + \nu_{\mathrm{tr}}^2} \left\{ \frac{\pi^2}{3|e|} \frac{T_e}{\varepsilon_F} \left[\mathcal{Z}_{\mathrm{tr}} + 2d^2 \left(\mathcal{Z}_{\mathrm{tr}} - \frac{2\pi \mathcal{Z}}{|\omega_c|\tau} \right) \right] \right.$$
$$\left. - \frac{1}{|e|} 2d^2 \Gamma_{c1} \right\}. \tag{42}$$

In the classical case, the thermopower coefficients have the usual forms found in literature [22]. The Landau quantization leads to additional terms proportional to d^2. In the phonon-drag part of thermopower, these terms are determined by the function Γ_{c1} oscillating with the magnetic field. Because of these quantum corrections, the transverse phonon-drag thermopower is nonzero. In Fig. 1 the longitudinal and transverse thermopower are plotted as functions of magnetic field for a rectangular GaAs quantum well of width 14 nm, with electron density $n_s = 5 \times 10^{11}$ cm^{-2} and mobility 2×10^6 cm^2/V s. The quantum lifetime $\tau = 7$ ps is assumed, which corresponds to the ratio $\tau_{\mathrm{tr}}/\tau \simeq 11$. The phonon scattering time τ_λ is chosen as $0.2\,\mu$s for each mode, which approximately corresponds to a 1 mm mean-free path for phonons [36]. The elastic coefficients for GaAs in units 10^{11} dyn/cm^2 are $c_{11} = 12.17$, $c_{12} = 5.46$, and $c_{44} = 6.16$. The deformation potential, piezoelectric coefficient, and density are $\mathcal{D} = 7.17$ eV, $h_{14} = 1.2$ V/nm, and $\rho = 5.317$ g/cm^3, respectively. The energy dependence of the transport time and quantum lifetime is assumed to be $\propto \varepsilon^{3/2}$ and $\propto \varepsilon^{1/2}$, respectively, which corresponds to $\nu(q) \propto \exp(-l_c q)$ under the condition of small-angle scattering, when $l_c p_F \gg 1$. The oscillations of the thermopower coefficients are caused by magnetophonon resonances. At low temperature, the oscillations are barely visible because the system falls into the Bloch-Gruneisen regime, but they are essential at higher temperatures. The last peak of $\alpha_{xx}^{(0)}$ is due to the scattering of electrons by high-energy (longitudinal) phonons; this peak disappears first with lowering temperature. The nonoscillating, proportional to $1/B$, part of $\alpha_{xy}^{(0)}$ is determined by the diffusive contribution.

Let us consider now the thermopower coefficients in the presence of MW excitation. While Eqs. (41) and (42) are valid for both classically strong and classically weak magnetic

fields, the MW-induced contributions are important only in the limit of classically strong magnetic fields. For this reason, only a part of the terms presented in Eqs. (35) and (37) are essential for calculation of thermopower in this limit. In particular, the longitudinal thermopower in classically strong magnetic fields is written simply as $\alpha_{xx} = \rho_{xy}\beta_{yx}$. The influence of microwaves on Hall resistivity ρ_{xy} is weak [12], so α_{xx} is directly determined by β_{yx}. Neglecting the contributions of higher order in $\nu_{\mathrm{tr}}/\omega_c$ in Eqs. (35) and (37), one obtains

$$\alpha_{xx}^{(\mathrm{MW})} = \frac{2d^2 P_\omega}{|e|} \left\{ \nu_{\mathrm{tr}}\tau_{\mathrm{in}} \frac{2\pi\omega}{\omega_c} \sin\frac{2\pi\omega}{\omega_c} \left(\Gamma_{s1} - \frac{\mathcal{T}_{\mathrm{in}}\omega_c^2}{8\pi^2 \varepsilon_F T_e} \right) \right.$$
$$+ [(1 + b')\Gamma_{c2} + b'\tilde{\Gamma}_{c2}] \sin^2\frac{\pi\omega}{\omega_c}$$
$$\left. + [(1 + b')\Gamma_{s2} + b'\tilde{\Gamma}_{s2}] \frac{\pi\omega}{\omega_c} \sin\frac{2\pi\omega}{\omega_c} \right\}, \tag{43}$$

while $\alpha_{yy}^{(\mathrm{MW})}$ differs from this expression by changing the sign at b'. The first term in Eq. (43) is caused by modification of the isotropic distribution function of electrons by microwaves (inelastic mechanism) and includes both the phonon-drag and the diffusive contributions. Since the diffusive term increases with decreasing temperature, it may become comparable to the phonon-drag one. However, inevitable heating of electron gas by microwaves tends to hinder the contribution of the diffusive term. The remaining terms in Eq. (43) describe the phonon-drag thermopower caused by the displacement mechanism. They contain contributions proportional to b', which change the symmetry of the thermopower coefficients. The dependence of these contributions on the polarization angle χ can be illustrated for the case of linear polarization of the incident wave, when b is represented in the form

$$b = \frac{1}{2} e^{-2i\chi} \frac{\omega^2 - \omega_c^2 + \omega_p^2 - 2i\omega_c\omega_p}{\omega^2 + \omega_c^2 + \omega_p^2}. \tag{44}$$

Since $b' \equiv \mathrm{Re}(b)$ contains the terms both even and odd in magnetic field, α_{xx}, in general, is not symmetric in B (the reversal of magnetic field means alteration of the sign of ω_c in all equations). The "inelastic" contribution in Eq. (43) should dominate at low enough temperatures, when $\nu_{\mathrm{tr}}\tau_{\mathrm{in}} > 1$. The "displacement" terms become more important with increasing temperature. It is worth emphasizing that the oscillations in these terms due to the factor $\sin^2(\pi\omega/\omega_c)$ are comparable by amplitude with the oscillations due to the factor $\sin(2\pi\omega/\omega_c)$. This behavior is in contrast with that for MW-induced resistance. In the resistance, the contribution at $\sin(2\pi\omega/\omega_c)$ dominates because it overcomes the oscillating part of $\sin^2(\pi\omega/\omega_c)$ by the factor $2\pi\omega/\omega_c$ which is numerically large in the region $\omega > \omega_c$ where MIRO are observed. As a consequence, the MW-induced resistance magneto-oscillations due to the displacement mechanism are very similar to the magneto-oscillations due to the inelastic mechanism [9], so these two mechanisms are difficult to separate experimentally. In the phonon-drag thermopower, the contributions at $\sin(2\pi\omega/\omega_c)$ and $\sin^2(\pi\omega/\omega_c)$ are proportional to the functions Γ_{s2} and Γ_{c2}, respectively, and Γ_{c2} is larger than Γ_{s2}. Moreover, $\Gamma_{c2} \gg \Gamma_{s2}$ in the region of low magnetic fields, $|\omega_c| \ll 4\pi p_F s_\lambda$; see the Appendix. The ratio of the amplitudes of $\sin(2\pi\omega/\omega_c)$ and $\sin^2(\pi\omega/\omega_c)$ oscillations in the "displacement" part of the

thermopower is estimated as $\omega/2p_F s_\lambda$, which is of the order of unity for typical electron densities and MW frequencies. The same is true for the "displacement" contribution to transverse thermopower described below by Eq. (47).

A more careful analysis is required for evaluation of the transverse (Nernst-Ettingshausen) thermopower, because the latter is determined by both diagonal and nondiagonal parts of $\hat{\beta}$ and is sensitive to MW-induced modifications of the longitudinal resistivity. Indeed, $\alpha_{xy} = \rho_{xy}\beta_{yy} + \rho_{xx}\beta_{xy}$. The influence of microwaves on ρ_{xy} is weak and not essential for determination of α_{xy}, while their influence on ρ_{xx} is strong. Under the assumed condition that the electron-impurity scattering is more important than electron-phonon scattering, the longitudinal resistivity correction due to MW irradiation is written as [9]

$$\rho_{xx}^{(\mathrm{MW})} = -\frac{2d^2 m \nu_{\mathrm{tr}}^2 \tau_{\mathrm{in}}}{e^2 n_s} P_\omega \frac{2\pi\omega}{\omega_c} \sin\frac{2\pi\omega}{\omega_c}, \qquad (45)$$

and $\rho_{yy}^{(\mathrm{MW})} = \rho_{xx}^{(\mathrm{MW})}$. Equation (45) implies that $\rho_{xx}^{(\mathrm{MW})}$ is governed by the inelastic mechanism. The displacement mechanism for electron-impurity scattering is less important at low temperatures, especially in the case of small-angle scattering processes relevant for high-mobility 2D systems [9]. In contrast, for electron-phonon scattering determining phonon-drag thermopower, the displacement mechanism is significant under the condition $\omega_{\lambda\mathbf{Q}} \leqslant 2T$ when the main contribution to oscillating functions Γ_{c2} and Γ_{s2} comes from large-angle scattering processes (backscattering). Among the "displacement" terms contributing into the transverse thermopower $\alpha_{xy}^{(\mathrm{MW})}$ there is a strong polarization-dependent term coming from the diagonal part of the matrices $\hat{g}_0$ and $\hat{g}_1$ in Eq. (37). The other contributions to $\alpha_{xy}^{(\mathrm{MW})}$ contain a small factor $\nu_{\mathrm{tr}}/\omega_c$. Out of them, only the "inelastic" ones can compete with the mentioned polarization-dependent contribution. Therefore, with the assumed accuracy up to d^2, the result is written as a sum of two terms:

$$\alpha_{xy}^{(\mathrm{MW})} \simeq \Delta\alpha_{xy}\sin(2\chi + \eta_{\mathbf{B}}) + \alpha_{xy}^{\mathrm{in}}, \qquad (46)$$

where

$$\Delta\alpha_{xy} = \frac{2d^2 P_\omega}{|e|}|b|\left[(\Gamma_{c2} - \tilde{\Gamma}_{c2})\sin^2\frac{\pi\omega}{\omega_c}\right.$$
$$\left. + (\Gamma_{s2} - \tilde{\Gamma}_{s2})\frac{\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}\right], \qquad (47)$$

and

$$\alpha_{xy}^{\mathrm{in}} = -\frac{2d^2 P_\omega \nu_{\mathrm{tr}}^2 \tau_{\mathrm{in}}}{|e|\omega_c}\left[\frac{2\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}\left(\Gamma_1 + \frac{\pi^2 T_e}{3\varepsilon_F} - \frac{\Gamma_{s1}}{2}\right)\right.$$
$$\left. - \frac{\omega^2 \mathcal{T}_{\mathrm{in}}}{2T_e\varepsilon_F}\left(\frac{\omega_c}{2\pi\omega}\sin\frac{2\pi\omega}{\omega_c} - (1 - \mathcal{Z}_{\mathrm{tr}})\cos\frac{2\pi\omega}{\omega_c}\right)\right]. \qquad (48)$$

To obtain $\alpha_{yx}^{(\mathrm{MW})}$, one should change the sign at the second term in Eq. (46). Since the effects under consideration are linear in MW intensity, the polarization-dependent term is a harmonic function of the doubled polarization angle; a similar angular dependence is expected for electrical resistivity [37]. This term is characterized by the amplitude $\Delta\alpha_{xy}$ and the phase angle $\eta_{\mathbf{B}}$ which are, respectively, a symmetric and an antisymmetric

function of the magnetic field. For linear polarization, when Eq. (44) is valid, the phase angle is defined as $\tan\eta_{\mathbf{B}} = 2\omega_c\omega_p/(\omega^2 - \omega_c^2 + \omega_p^2)$. One may introduce the effective polarization angle $\chi_{\mathbf{B}} = \chi + \eta_{\mathbf{B}}/2$ describing the direction of the ac electric field in the 2D plane, which is different from the polarization of the incident wave. The polarization-dependent term, in general, is not antisymmetric under reversal of $\mathbf{B}$, though for special orientation of the incident ac field along x or y axes the symmetry property $\alpha_{xy}^{(\mathrm{MW})}(\mathbf{B}) = -\alpha_{xy}^{(\mathrm{MW})}(-\mathbf{B})$ is preserved. If the angle $\chi_{\mathbf{B}}$ is equal to $\pi/2$ or 0, which means that the electric field in the 2D plane is polarized along y or x axes (i.e., along or perpendicular to the temperature gradient), the polarization-dependent term is equal to zero. The contribution of this term can be experimentally distinguished from the other contributions by its dependence on the polarization.

The polarization-independent term given by Eq. (48) contains several contributions of different origin, though all of them are caused by the inelastic mechanism. The first part [the first line of Eq. (48)] comprises three different contributions. The first one, at Γ_1, comes from the MW-induced correction to resistance if the thermoelectric current is due to the phonon-drag mechanism. The second contribution comes from the MW-induced correction to resistance if the thermoelectric current is due to the diffusive mechanism. These two contributions can be distinguished from each other by their temperature dependence. At low temperatures (roughly estimated as $T_e < 0.5$ K), the second contribution can exceed the first one, as it decreases with T_e slower [see Eq. (A11) for low-temperature behavior of Γ_1]. However, the MW heating of electron gas renders this regime practically unrealizable. The third contribution, at Γ_{s1}, is caused by the MW-induced correction to the phonon-drag part of thermoelectric tensor. In contrast to the first and second contributions, this one contains magnetophonon oscillations. However, in the region of fields where these oscillations exist, $|\omega_c| < 2p_F s_\lambda$, the term $\Gamma_{s1}/2$ is much smaller than Γ_1. The second part [the last line of Eq. (48)] contains the contributions due to MW-induced correction to diffusive part of thermoelectric tensor. This part does not exceed the contribution proportional to $\pi^2 T_e/3\varepsilon_F$ in the second line of Eq. (48) under the assumed condition $|\omega_c| \ll 2\pi^2 T_e$. Therefore, the contribution proportional to Γ_1 dominates over the others in Eq. (48) in the relevant region of parameters. This means that magneto-oscillations of $\alpha_{xy}^{\mathrm{in}}$ are determined only by the ratio ω/ω_c and are similar to MIRO. The magneto-oscillations of the polarization-dependent term are more complicated, because they also have the magnetophonon constituent due to the factors $\Gamma_{c2} - \tilde{\Gamma}_{c2}$ and $\Gamma_{s2} - \tilde{\Gamma}_{s2}$ [see Eq. (47), Fig. 6, and its discussion below]. Therefore, the two terms in Eq. (46) can be distinguished from each other not only by polarization dependence and B-inversion symmetry but also by the behavior of magneto-oscillations.

It is important to emphasize that the components of the thermopower tensor given by Eqs. (43) and (46) do not violate the Onsager symmetry. This fact requires an explanation in view of the observation (see the end of Sec. III) that some terms in $\hat{\beta}$ violate this symmetry. Indeed, $\hat{\alpha}$ is formed as a result of matrix multiplication of $\hat{\rho}$ and $\hat{\beta}$ and its full form does contain terms violating the Onsager symmetry. However, such terms are small in comparison to the terms included in Eqs. (43) and (46), so they are neglected.
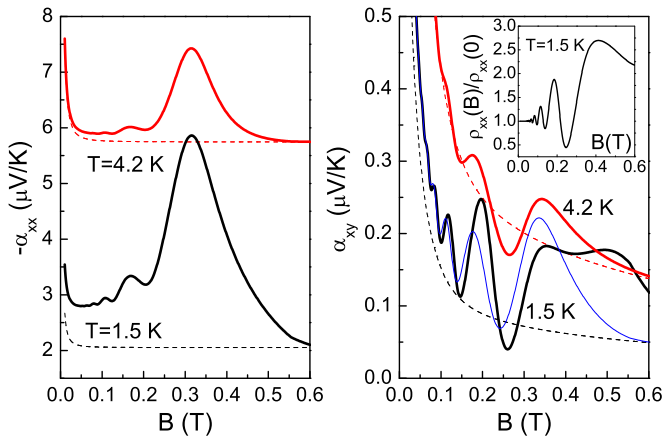
FIG. 2. (Color online) Longitudinal (left) and transverse (right) *diffusive* thermopower at $T = 1.5$ K and $T = 4.2$ K under the linearly polarized MW excitation of frequency 130 GHz and electric field $E_\omega = 2$ V/cm. The parameters of the system are the same as in Fig. 1. The dashed lines show the dark thermopower (no MW excitation). The narrow solid line in the right-hand part shows the result of approximation $\alpha_{xy}^{(MW)} \simeq \rho_{xx}^{(MW)} \beta_{xy}^{(0)}$ for $T = 1.5$ K. The inset presents the calculated behavior of the longitudinal resistance.
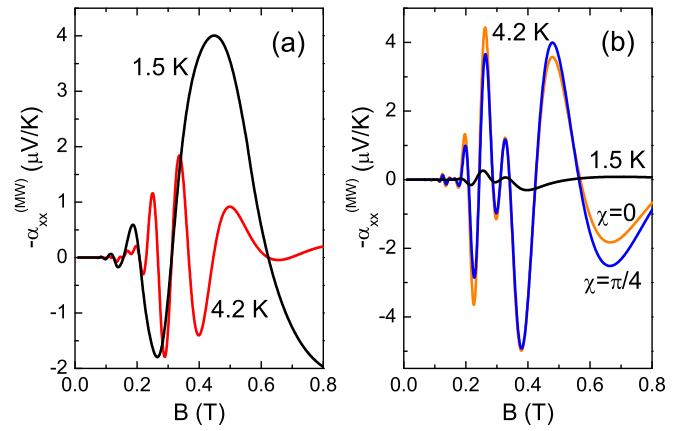

FIG. 3. (Color online) Microwave-induced corrections to longitudinal thermopower at $T = 1.5$ K and $T = 4.2$ K due to inelastic (a) and displacement (b) mechanisms, for linearly polarized MW excitation of frequency 130 GHz and electric field $E_\omega = 2$ V/cm. The parameters of the system are the same as in Fig. 1. Two plots for $T = 4.2$ K in (b) correspond to two angles of MW polarization.

Coming to presentation of numerical results, let us consider first the diffusive contribution to thermopower coefficients. This contribution is given by Eqs. (41), (42), (43), and (46), where all Γ_i and $\tilde{\Gamma}_i$ are set to zero. The inelastic scattering time here and below is estimated according to [9] $\tau_{in} = \varepsilon_F/T^2$. The diffusive thermopower is not sensitive to MW polarization. The longitudinal diffusive thermopower α_{xx} is modified by the microwaves in two ways: through the heating of 2D electrons and through the quantum correction in Eq. (43). The calculations (see Fig. 2) demonstrate that the heating mechanism is more essential. In particular, it leads to a peak at cyclotron absorption frequency and to oscillations at small B caused by the oscillations of absorbed MW power due to Landau quantization. The transverse diffusive thermopower α_{xy}, in contrast, is considerably affected by the MW-induced quantum corrections from Eq. (48). Among these corrections there is a term $\rho_{xx}^{(MW)} \beta_{xy}^{(0)}$, whose oscillations directly reproduce the MIRO pattern shown in the inset of Fig. 2. The calculations demonstrate that the other terms, those in the last line of Eq. (48), are equally important, although their contribution becomes weaker with increasing temperature.

Consider now the influence of microwaves on the thermopower coefficients in the presence of both diffusive and phonon drag mechanisms. Theoretical and experimental studies of GaAs quantum wells show that for temperatures above 0.5 K the phonon-drag contribution dominates over the diffusive one. Consequently, the behavior of thermopower is governed mostly by the influence of MW excitation on the phonon-drag contribution. For the typical parameters of MW excitation, the oscillating quantum corrections given by Eq. (43) are of the order of several μV/K. The partial contributions due to inelastic mechanism [the first term in Eq. (43)] and displacement mechanism (the remaining terms) are shown in Fig. 3. The role of the displacement mechanism increases with increasing temperature. At low temperatures

(Bloch-Gruneisen regime), the period of the oscillations is determined by the ratio ω/ω_c. With increasing temperature, the magnetophonon resonances become important and the picture of oscillations becomes more rich. The sensitivity of the displacement mechanism to MW polarization is illustrated by plotting its contribution for two angles of electric field of the incident wave, $\chi = 0$ and $\chi = \pi/4$.

However, the relative change of the longitudinal component α_{xx} under MW irradiation is not strong. The terms due to phonon drag in Eq. (43) are proportional to the functions Γ_{s1}, Γ_{c2}, and Γ_{s2}, which are small in comparison to Γ_1 in the important region of parameters $|\omega_c| < 2p_F s_{\lambda \mathbf{Q}}$ and $|\omega_c| \ll 2\pi^2 T_e$, where magnetophonon oscillations take place but Shubnikov–de Haas oscillations are suppressed (see a more detailed comparison in the Appendix). The ratio of the relative change of α_{xx} due to MW irradiation to the relative change of the resistivity ρ_{xx} is estimated by a small factor Γ_{s1}/Γ_1. This means that even in the case when MW-induced resistance oscillations are strong, the MW-induced oscillations of the longitudinal thermopower still may be weak. The magnetic-field dependence of α_{xx} at low temperature is presented in Fig. 4(a). For $T = 1.5$ K one can see changes in the oscillation picture, in particular, inversion of the minimum around 0.18 T and a considerable enhancement of the last peak. The vertical shift of α_{xx} as a whole with respect to $\alpha_{xx}^{(0)}$ is caused mostly by the diffusive mechanism contribution, due to heating of electrons by microwaves; see Fig. 2. With increasing temperature, the relative effect of microwaves on α_{xx} becomes weaker because $\alpha_{xx}^{(0)}$ increases faster than $\alpha_{xx}^{(MW)}$.

The transverse thermopower α_{xy}, in contrast, is strongly changed by microwaves, because the dark thermopower $\alpha_{xy}^{(0)}$ is small itself. At low temperature [see Fig. 4(b)] the modification is almost entirely governed by the oscillations of resistivity, which means that the approximation $\alpha_{xy}^{(MW)} \simeq \rho_{xx}^{(MW)} \beta_{xy}^{(0)}$ works well. This approximation is no longer valid when temperature increases and the polarization-dependent contribution, the first term in the expression Eq. (46), becomes significant. This is
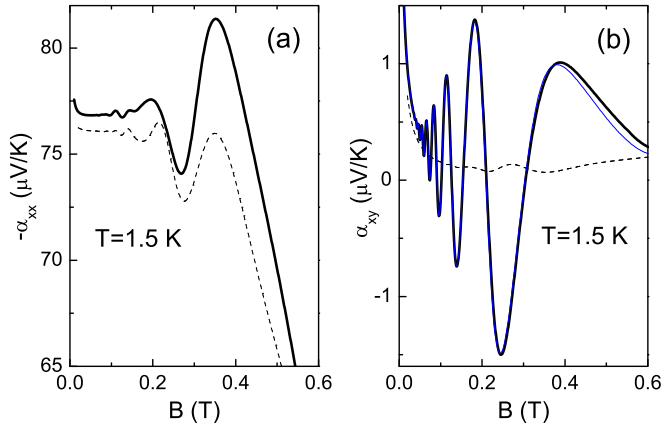
FIG. 4. (Color online) Longitudinal (a) and transverse (b) thermopower at $T = 1.5$ K under the linearly polarized MW excitation of frequency 130 GHz and electric field $E_\omega = 2$ V/cm. The parameters of the system are the same as in Fig. 1. The dashed lines show the dark thermopower. The narrow solid line shows the result of approximation $\alpha_{xy}^{(MW)} \simeq \rho_{xx}^{(MW)} \beta_{xy}^{(0)}$ for transverse thermopower.

FIG. 6. (Color online) Magnetic-field dependence of polarization-sensitive part of transverse thermopower at different temperatures, for the MW excitation of frequency 130 GHz and electric field $E_\omega = 2$ V/cm. The parameters of the system are the same as in Fig. 1. The inset shows dependence of thermopower on the polarization angle at $B = 0.5$ T.

demonstrated in Fig. 5, where α_{xy} is plotted for two directions of ac electric field: along the x axis ($\chi = 0$) and at the angle of $\pi/4$ to this axis. With increasing B, when the ratio ν_{tr}/ω_c becomes smaller, α_{xy} deviates from the simple dependence $\propto \rho_{xx}^{(MW)}$ and becomes strongly sensitive to polarization.

The polarization dependence of α_{xy} for different magnetic fields is characterized by the amplitude $\Delta \alpha_{xy}$ given by Eq. (47). This function is plotted in Fig. 6 for different temperatures. The complicated oscillating behavior of $\Delta \alpha_{xy}$ is caused by the interference of magnetophonon oscillations with microwave-induced oscillations. At small T, when the system is in the Bloch-Gruneisen regime, $\Delta \alpha_{xy}$ is small. With increasing T, $\Delta \alpha_{xy}$ increases and saturates around 10–15 K. The inset in Fig. 6 shows how the rotation of the MW polarization angle changes the total transverse thermopower.
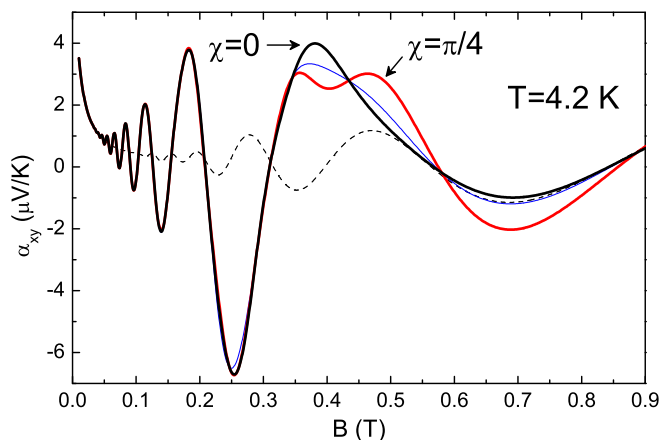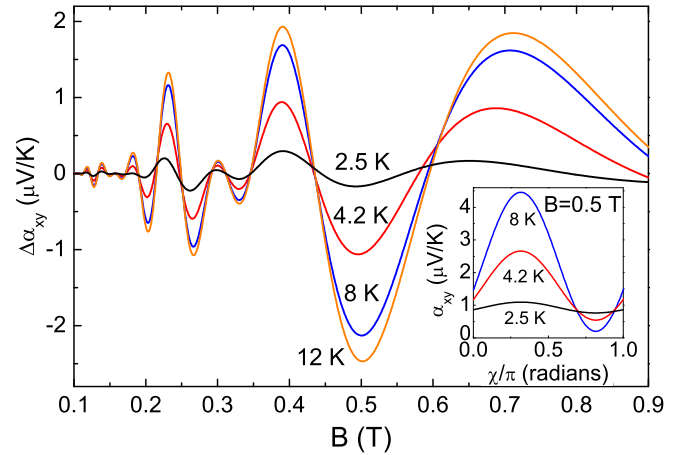

FIG. 5. (Color online) Transverse thermopower at $T = 4.2$ K under the MW excitation of frequency 130 GHz and electric field $E_\omega = 2$ V/cm, for two different directions of linear polarization of incident wave. The parameters of the system are the same as in Fig. 1. The dashed line shows the dark thermopower. The narrow solid line shows the result of approximation $\alpha_{xy}^{(MW)} \simeq \rho_{xx}^{(MW)} \beta_{xy}^{(0)}$.

The relative contribution of the polarization-dependent part can be further enhanced at higher MW intensity and at higher mobility, because the second term in Eq. (46) is proportional to the factor $\nu_{tr}^2 \tau_{in}$ which goes down when inelastic scattering time $\tau_{in} \propto T_e^{-2}$ decreases because of microwave heating of electron gas and when the transport scattering rate ν_{tr} (inversely proportional to the mobility) decreases.

In the case of circular polarization or nonpolarized radiation (chaotic polarization) the polarization-dependent term vanishes and $\alpha_{xy}^{(MW)}$ is determined by the second term in Eq. (46). Since the most important part of this term is given by $\rho_{xx}^{(MW)} \beta_{xy}^{(0)}$, the oscillations of transverse thermopower under these conditions follow the MW-induced resistance oscillations.

The longitudinal and transverse thermopower components α_{xx} and α_{xy} are directly measured in the Hall bars. The longitudinal thermopower can also be measured in the Corbino disk geometry [38]. In this case, polarization-dependent terms do not appear and the voltage between inner and outer contacts is determined by the thermopower $\alpha_d = \beta_d/\sigma_d$, where β_d and σ_d are the diagonal parts of the tensors $\hat{\beta}$ and $\hat{\sigma}$ in the absence of MW polarization. Since σ_d is modified by microwaves stronger than β_d, the behavior of thermopower in MW-irradiated Corbino disks is determined mostly by MW-induced oscillations of σ_d.

The theory developed in this paper does not take into account temperature dependence of the density of states. Such a dependence appears mostly due to contribution of electron-electron scattering into the inverse quantum lifetime $1/\tau$ (see Ref. [11] and references therein). This effect leads to an exponential suppression of all quantum contributions in the transport coefficients, including those considered above, with increasing T_e. Formally, this occurs because the Dingle factor d acquires a multiplier $\exp(-\pi/\tau_{ee}(T_e)|\omega_c|)$, where $1/\tau_{ee}(T_e) \sim T_e^2/\varepsilon_F$. This effect tends to decrease the quantum part of dark thermopower and MW-induced corrections to thermopower with increasing temperature. Since the main (phonon-drag)

contribution to thermopower, in contrast, increases with increasing temperature at $T < p_F s_\lambda$, it is important to investigate possible competition of these opposite trends in the quantum (proportional to d^2) terms in thermopower. Assuming that $T_e \simeq T$, the exponential dependence of these terms on temperature in the Bloch-Gruneisen regime ($T \ll p_F s_\lambda$) is written as $e^{-\Phi_T}$, where $\Phi_T \simeq 2\pi T^2/\varepsilon_F |\omega_c| + 2p_F s_\lambda/T$ is a nonmonotonic function of temperature. This function decreases at $T < T_0$ and increases at $T > T_0$, where $T_0 \simeq p_F s_\lambda (|\omega_c|/4\pi m s_\lambda^2)^{1/3}$. Since the estimate for GaAs gives $T_0 > p_F s_\lambda$ even for magnetic fields as small as 0.05 T, one may conclude that the temperature dependence of the density of states does not alter the thermal increase of the quantum contributions to thermopower at $T < p_F s_\lambda$. However, at $T > p_F s_\lambda$ all these contributions, both in the dark thermopower and MW-induced corrections, decrease with temperature instead of going to saturation.

V. DISCUSSION AND CONCLUSIONS

The influence of MW irradiation on the energy distribution of electrons and on electron scattering by phonons and impurities has a profound effect on transport properties of 2D electron systems in perpendicular magnetic field. While the effect of microwaves on the electrical resistance is widely studied, the related behavior of the other kinetic coefficients has not received proper attention. This paper reports a theoretical study of possible MW-induced quantum effects in thermopower. Such effects can exist in the samples with high electron mobility in the moderately strong magnetic fields, that is, under the same conditions when the MW-induced quantum oscillations of the electrical resistance are observed.

In contrast to electrical resistance, which at low temperatures is determined by electron-impurity scattering, the thermopower is determined mostly by electron-phonon scattering, through the phonon drag mechanism. The theory of phonon-drag thermoelectric response in quantizing magnetic fields remains an issue of interest even under quasiequilibrium conditions, in the absence of MW irradiation. A further development of such theory is presented in this paper. In particular, an anisotropy of the acoustic phonon spectrum has been taken into account and analytical expressions valid in the regime of overlapping Landau levels with the accuracy up to the square of the Dingle factor have been derived; see Eqs. (32), (33), (41), and (42). The theory gives a clear picture of the origin of magnetophonon oscillations observed [20] in the longitudinal thermopower of high-mobility GaAs quantum wells and predicts similar oscillations in the transverse thermopower (Fig. 1). For typical parameters of GaAs wells, the oscillations are clearly visible for temperatures above 2 K, while at lower temperatures they become exponentially suppressed because the Bloch-Gruneisen regime is reached. In the experiment [20], however, the oscillations were resolved between 0.5 K and 1 K. This discrepancy can be explained by taking into account that the phonon distribution function in the experiments on thermopower is not reduced to the form of Eq. (17) commonly applied by theorists. Even at low temperatures of the sample, there can exist high-energy phonons able to cause backscattering of electrons. Indeed, since the phonon mean-free path at low temperatures is very large (of 1 mm scale), it is quite possible that such

high-energy phonons may arrive at the 2D system directly from the heater, via ballistic propagation. Another possible reason, which is especially relevant at low temperatures, is that the modification of phonon distribution function is strong and cannot be represented in the form of a small correction linear in temperature gradient. In any case, a quantitative agreement with experiment can be reached only if the phonon distribution is known. The theory presented in this paper can be generalized to the case of arbitrary phonon distribution by substituting the antisymmetric part of actual phonon distribution function instead of the second term in Eq. (17).

The influence of MW irradiation on the longitudinal α_{xx} and transverse α_{xy} components of the thermopower has been studied above by using the approved methods applied earlier to calculation of the resistivity. It is found that the MW irradiation has a considerable effect on both these components. In contrast, for electrical resistance the microwaves strongly modify only the longitudinal component ρ_{xx}. Both the diffusive and phonon-drag contributions to thermopower are shown to be affected by MW irradiation. The MW-induced quantum corrections to diffusive thermopower increase with decreasing electron temperature, in contrast to classical diffusive thermopower, which is proportional to this temperature. However, since the phonon-drag contribution dominates, the MW-induced quantum corrections to phonon-drag thermopower appear to be more important. These effects are of the order of several μV/K for typical parameters of the 2D system and MW excitation, and can be detected experimentally. The oscillating behavior of MW-induced corrections as functions of the magnetic field reflects the properties of electron scattering by phonons under conditions when the electron distribution function acquires an MW-induced oscillating component (inelastic mechanism) and when MW-assisted scattering takes place (displacement mechanism). Both these mechanisms are important, and both provide a mixing of resonant phonon frequencies with MW frequency ω, thereby leading to interference oscillations of the thermopower.

In terms of relative values, the MW-induced changes in the longitudinal thermopower are much smaller than the corresponding effect in the resistivity. In contrast, the relative MW-induced changes in the transverse thermopower are large, because in the classically strong magnetic fields the transverse thermopower itself is much smaller than the longitudinal one. At lower temperatures and weaker magnetic fields, the oscillations of transverse thermopower α_{xy} follow the picture of MW-induced resistance oscillations (MIRO) [Fig. 4(b)]. As the temperature and magnetic field increase, the oscillations of α_{xy} no longer follow the MIRO picture and become strongly sensitive to polarization of the incident wave. The polarization dependence of α_{xy} is much stronger than the corresponding dependence of the electrical resistivity under MW irradiation. These findings may stimulate experimental studies of the transverse thermopower of MW-irradiated 2D electron gas.

The appearance of a large polarization-dependent term in the MW-induced transverse thermopower is one of the main results of the present study. The nature of this effect can be easily understood by considering the collisionless approximation (no electron-impurity scattering, $\nu_{tr} = 0$), when

the transverse thermopower does not appear without MW irradiation. The drag of electrons by the phonons drifting along the temperature gradient ∇T can be described [22] in terms of a dragging force due to effective electric field $\mathbf{E}_{ph} \propto \nabla T$. The electrons in the magnetic field are drifting perpendicular to $\mathbf{E}_{ph}$. To compensate this drift, a real electric field $\mathbf{E} = -\mathbf{E}_{ph}$ develops. Thus, the longitudinal thermopower is equal to $|\mathbf{E}|/|\nabla T|$ while the transverse thermopower is zero. When a polarized ac field is applied to the system, the effective electric field $\mathbf{E}_{ph}$, in general, is not directed along ∇T and becomes sensitive to polarization. This occurs because $\mathbf{E}_{ph}$ is formed as a result of electron-phonon interaction assisted by emission and absorption of radiation quanta, and this interaction is stronger when the in-plane components of phonon momenta are parallel to the polarization-dependent vector $\mathbf{R}_\omega$; see Eqs. (12) and (13). Consequently, the real electric field $\mathbf{E} = -\mathbf{E}_{ph}$ is not parallel to ∇T, which means that there exists a transverse component of thermopower. This component is given by the first term in Eq. (46). Beyond the collisionless approximation, the other, polarization-independent terms in α_{xy} are also important. A larger relative contribution of the polarization-dependent term is expected in 2D electron systems with higher mobility (smaller ν_{tr}).

An important issue left beyond the above consideration is the behavior of thermopower at zero longitudinal resistance. In high-mobility 2D systems, intensive MW irradiation leads to a remarkable phenomenon of zero-resistance states [3–5], which means that the longitudinal resistance vanishes in certain intervals of magnetic fields corresponding to MIRO minima at lower MW intensity. This effect is often explained (see Ref. [1] and references therein) as a result of the instability of homogeneous current flow under condition of negative local resistance, which leads to spontaneous formation of domains with different directions of the currents and Hall fields. Since the longitudinal resistivity formally enters the expression for thermopower and, as shown above, considerably affects the transverse thermopower in the presence of MW irradiation, the magnetic-field dependence should demonstrate the regions of nearly constant α_{xy} in the intervals of $\rho_{xx} = 0$, while α_{xx} is not expected to be sensitive to zero-resistance states. Of course, this conclusion looks somewhat naive, because the presence of domains may affect the behavior of measured thermopower. It is not clear, however, which kind of domain picture is realized under zero-resistance state conditions in thermoelectric experiments, when there is no electric currents through the contacts. Future studies should shed light on this particularly interesting problem.

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APPENDIX: ASYMPTOTIC BEHAVIOR OF THE FUNCTIONS $\mathbf{\Gamma}_i$ AND $\tilde{\mathbf{\Gamma}}_i$

In the approximation of the isotropic phonon spectrum, the integral over the polar angle φ_q in the operator $\hat{\mathcal{P}}_n$ can be

carried out analytically, and Eq. (33) is reduced to the form

$$\begin{pmatrix} \Gamma_n \\ \Gamma_{cn} \\ \Gamma_{sn} \end{pmatrix} = \frac{m^2}{\rho_M} \int_0^\pi \frac{d\theta}{\pi} (1-\cos\theta)^n \int_0^\infty \frac{dq_z}{\pi} I_{q_z}$$

$$\times \sum_{\lambda=l,t} \tau_\lambda G_\lambda F\left(\frac{s_\lambda Q}{2T}\right) \begin{pmatrix} 1 \\ \cos\frac{2\pi s_\lambda Q}{\omega_c} \\ \frac{\omega_c}{2\pi s_\lambda Q}\sin\frac{2\pi s_\lambda Q}{\omega_c} \end{pmatrix},$$

(A1)

where $Q = \sqrt{q^2 + q_z^2}$, $q = 2p_F \sin(\theta/2)$, $G_l = \mathcal{D}^2 + (eh_{14})^2 9q^4 q_z^2/2Q^8$, and $G_t = (eh_{14})^2(8q^2 q_z^4 + q^6)/2Q^8$. For $\tilde{\Gamma}_i$ one should replace G_l and G_t by $\tilde{G}_l = -(eh_{14})^2 9q^4 q_z^2/4Q^8$ and $\tilde{G}_t = (eh_{14})^2(8q^4 q_z^2 - q^6)/4Q^8$, respectively. The functions G_t and $\tilde{G}_t$ describe interaction of electrons with transverse phonon modes due to piezoelectric-potential mechanism, while G_l and $\tilde{G}_l$ describe interaction with longitudinal phonon modes due to both deformation potential and piezoelectric-potential mechanisms. Analytical expressions for the functions Γ_{cn}, Γ_{sn}, $\tilde{\Gamma}_{cn}$, and $\tilde{\Gamma}_{sn}$ calculated from Eq. (A1) are given below in some limiting cases.

In the limit $\omega_c \ll 4\pi s_\lambda p_F$, when $\cos(2\pi s_\lambda Q/\omega_c)$ and $\sin(2\pi s_\lambda Q/\omega_c)$ are rapidly oscillating functions of θ and q_z/p_F, the main contribution to the integrals in Eq. (A1) comes from the region of small q_z, when $I_{q_z} \simeq 1$, and from two regions of θ around $\theta = 0$ (corresponding to forward scattering of electrons) and $\theta = \pi$ (backscattering), because these are the regions of slowest variation of Q as a function of θ and q_z. Under the requirement $|\omega_c| \ll 2\pi^2 T$, which is already stated as the condition when the Shubnikov–de Haas oscillations are suppressed, one obtains

$$\Gamma_{c1} = \frac{\gamma_t}{2\epsilon_t} F\left(\frac{s_t p_F}{T}\right)\cos\epsilon_t - \frac{59\gamma_t}{2^8\epsilon_t^2} - \frac{45\gamma_l}{2^8\epsilon_l^2}$$
$$+ 4\frac{\gamma_l}{\epsilon_l}\left[F\left(\frac{s_l p_F}{T}\right)\cos\epsilon_l + \frac{3}{\epsilon_l^3}\right]\left(\frac{\mathcal{D}p_F}{eh_{14}}\right)^2, \quad (A2)$$

$$\Gamma_{c2} = \frac{\gamma_t}{\epsilon_t} F\left(\frac{s_t p_F}{T}\right)\cos\epsilon_t + \frac{261\gamma_t}{2^7\epsilon_t^4} + \frac{189\gamma_l}{2^7\epsilon_l^4}$$
$$+ 8\frac{\gamma_l}{\epsilon_l}\left[F\left(\frac{s_l p_F}{T}\right)\cos\epsilon_l - \frac{45}{\epsilon_l^5}\right]\left(\frac{\mathcal{D}p_F}{eh_{14}}\right)^2, \quad (A3)$$

$$\Gamma_{s1} = \frac{\gamma_t}{2\epsilon_t^2} F\left(\frac{s_t p_F}{T}\right)\sin\epsilon_t + \frac{59\gamma_t}{2^8\epsilon_t^2} + \frac{45\gamma_l}{2^8\epsilon_l^2}$$
$$+ 4\frac{\gamma_l}{\epsilon_l^2}\left[F\left(\frac{s_l p_F}{T}\right)\sin\epsilon_l - \frac{1}{\epsilon_l^2}\right]\left(\frac{\mathcal{D}p_F}{eh_{14}}\right)^2, \quad (A4)$$

$$\Gamma_{s2} = \frac{\gamma_t}{\epsilon_t^2} F\left(\frac{s_t p_F}{T}\right)\sin\epsilon_t - \frac{87\gamma_t}{2^7\epsilon_t^4} - \frac{63\gamma_l}{2^7\epsilon_t^4}$$
$$+ 8\frac{\gamma_l}{\epsilon_l^2}\left[F\left(\frac{s_l p_F}{T}\right)\sin\epsilon_l + \frac{9}{\epsilon_l^4}\right]\left(\frac{\mathcal{D}p_F}{eh_{14}}\right)^2, \quad (A5)$$

$$\tilde{\Gamma}_{c1} = -\frac{\gamma_t}{4\epsilon_t} F\left(\frac{s_t p_F}{T}\right)\cos\epsilon_t - \frac{5}{2^9}\left(\frac{\gamma_t}{\epsilon_t^2} - \frac{9\gamma_l}{\epsilon_l^2}\right), \quad (A6)$$

$$\tilde{\Gamma}_{c2} = -\frac{\gamma_t}{2\epsilon_t} F\left(\frac{s_t p_F}{T}\right)\cos\epsilon_t - \frac{21}{2^8}\left(\frac{\gamma_t}{\epsilon_t^4} + \frac{9\gamma_l}{\epsilon_l^4}\right), \quad (A7)$$

$$\tilde{\Gamma}_{s1} = -\frac{\gamma_t}{4\epsilon_t^2} F\left(\frac{s_t p_F}{T}\right) \sin\epsilon_t + \frac{5}{2^9}\left(\frac{\gamma_t}{\epsilon_t^2} - \frac{9\gamma_l}{\epsilon_l^2}\right), \quad \text{(A8)}$$

$$\tilde{\Gamma}_{s2} = -\frac{\gamma_t}{2\epsilon_t^2} F\left(\frac{s_t p_F}{T}\right) \sin\epsilon_t + \frac{7}{2^8}\left(\frac{\gamma_t}{\epsilon_t^4} + \frac{9\gamma_l}{\epsilon_l^4}\right), \quad \text{(A9)}$$

where

$$\gamma_\lambda = \frac{\tau_\lambda m^2 (eh_{14})^2}{\pi \rho_M p_F}, \quad \epsilon_\lambda = \frac{4\pi s_\lambda p_F}{|\omega_c|}. \quad \text{(A10)}$$

For comparison, it is useful to present also the expression for Γ_1:

$$\Gamma_1 = \frac{177\zeta(3)}{2^9}\gamma_t\left(\frac{T}{s_t p_F}\right)^2 + \frac{135\zeta(3)}{2^9}\gamma_l\left(\frac{T}{s_l p_F}\right)^2$$
$$+ \left(\frac{\mathcal{D}p_F}{eh_{14}}\right)^2 15\zeta(5)\gamma_l\left(\frac{T}{s_l p_F}\right)^4, \quad \text{(A11)}$$

where $\zeta(k)$ is the Riemann zeta function. This expression is valid in the limit of $T \ll s_\lambda p_F$ and can be used for order-of-value estimates at $T \simeq s_\lambda p_F$.

From the definition (A10), the applicability region for Eqs. (A2)–(A9) can be written as $\epsilon_\lambda \gg 1$. The magneto-oscillations of the functions described by Eqs. (A2)–(A9) occur because of the terms with $\cos\epsilon_\lambda$ and $\sin\epsilon_\lambda$. The amplitudes of these oscillating terms are always much smaller than Γ_1 of Eq. (A11) in the case $\epsilon_\lambda \gg 1$. If $T \simeq s_\lambda p_F$, this smallness is given by the factors ϵ_λ^{-1} for Γ_{c1}, Γ_{c2}, $\tilde{\Gamma}_{c1}$, and $\tilde{\Gamma}_{c2}$ and ϵ_λ^{-2} for Γ_{s1}, Γ_{s2}, $\tilde{\Gamma}_{s1}$, and $\tilde{\Gamma}_{s2}$. With lowering T, the oscillations are exponentially suppressed because of $F(s_\lambda p_F/T) \simeq (2s_\lambda p_F/T)^2 \exp(-2s_\lambda p_F/T)$ at $T \ll s_\lambda p_F$. In the case of strong exponential suppression, the absolute values of the functions given by Eqs. (A2)–(A9) are determined by their nonoscillating parts which are proportional to powers of ω_c. The nonoscillating parts of $n = 1$ functions (Γ_{c1}, Γ_{s1}, $\tilde{\Gamma}_{c1}$, and $\tilde{\Gamma}_{s1}$) are much smaller than Γ_1 due to parameters $(\omega_c/2\pi^2 T)^2$ for the piezoelectric-potential contribution and $(\omega_c/2\pi^2 T)^4$ for the deformation-potential contribution. The nonoscillating parts of $n = 2$ functions (Γ_{c2}, Γ_{s2}, $\tilde{\Gamma}_{c2}$, and $\tilde{\Gamma}_{s2}$) contain extra small factors ϵ_λ^{-2}, because these functions are much smaller than $n = 1$ functions at small-angle scattering, $\theta \ll 1$.

In stronger magnetic fields, when ω_c is comparable to $4\pi s_\lambda p_F$, analytical expressions can be obtained at $T > s_\lambda p_F$ and under a wide-well approximation, the latter means that the quantum well width a is much larger than π/p_F so that the convergence of the integral over q_z takes place at $q_z \ll p_F$ and is governed by the function I_{q_z}. Introducing $q_0 = \pi^{-1}\int_0^\infty dq_z I_{q_z}$ (for infinitely deep rectangular well

$q_0 = 3/2a$), one obtains

$$\Gamma_{cn} = \frac{2^n m^2 q_0}{\rho_M}\left[(-1)^n \tau_l \mathcal{D}^2 \mathcal{I}_{2n}(\epsilon_l)\right.$$
$$\left. + (-1)^{n-1}\tau_t \frac{(eh_{14})^2}{8p_F^2}\mathcal{I}_{2n-2}(\epsilon_t)\right], \quad \text{(A12)}$$

$$\Gamma_{sn} = \frac{2^n m^2 q_0}{\rho_M}\left[(-1)^n \tau_l \frac{\mathcal{D}^2}{\epsilon_l}\mathcal{I}_{2n-1}(\epsilon_l)\right.$$
$$\left. + (-1)^{n-1}\tau_t \frac{(eh_{14})^2}{8p_F^2 \epsilon_t}\mathcal{I}_{2n-3}(\epsilon_t)\right], \quad \text{(A13)}$$

$$\tilde{\Gamma}_{cn} = -\frac{2^n m^2 q_0 (eh_{14})^2 \tau_t}{16 p_F^2 \rho_M}(-1)^{n-1}\mathcal{I}_{2n-2}(\epsilon_t), \quad \text{(A14)}$$

$$\tilde{\Gamma}_{sn} = -\frac{2^n m^2 q_0 (eh_{14})^2 \tau_t}{16 p_F^2 \rho_M \epsilon_t}(-1)^{n-1}\mathcal{I}_{2n-3}(\epsilon_t), \quad \text{(A15)}$$

where

$$\mathcal{I}_k(x) = \frac{d^k J_0(x)}{dx^k} \quad \text{(A16)}$$

is the kth-order derivative of the Bessel function $J_0(x)$. Such derivatives can be expressed through the other Bessel functions $J_i(x)$. In the special case of Γ_{s1}, there is a term with the function $\mathcal{I}_{-1}(\epsilon_t)$, which should be treated as the antiderivative of $J_0(\epsilon_t)$. This term is expressed through the Bessel functions and Struve functions $\mathbf{H}_i$:

$$\mathcal{I}_{-1}(x) \equiv \int_0^x dx' J_0(x') = x J_0(x)$$
$$+ \frac{\pi}{2}x[J_1(x)\mathbf{H}_0(x) - J_0(x)\mathbf{H}_1(x)]. \quad \text{(A17)}$$

In the regime of validity of Eqs. (A12)–(A15) the function Γ_1 is given by

$$\Gamma_1 = \frac{m^2 q_0}{\rho_M}\left[\tau_l \mathcal{D}^2 + \tau_t \frac{(eh_{14})^2}{4p_F^2}\right]. \quad \text{(A18)}$$

For large arguments ϵ_λ, the functions (A12)–(A15) are reduced to combinations of oscillating factors $\sin\epsilon_\lambda$ and $\cos\epsilon_\lambda$, similarly to the case described by Eqs. (A2)–(A9), and are small in comparison to Γ_1. If $\epsilon_\lambda \simeq 1$, these functions become comparable to Γ_1. Actually, the wide-well limit $a \gg \pi/p_F$ is hardly attainable for single-subband occupation in the quantum well. The expressions (A12)–(A15) are nevertheless useful for estimates of the maximal possible values of the quantities Γ_i and $\tilde{\Gamma}_i$.

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