Spin effects induced by thermal perturbation in a normal metal/magnetic insulator system

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Using one of the methods of quantum nonequilibrium statistical physics, we have investigated the spin transport transverse to the normal metal/ferromagnetic insulator interface in hybrid nanostructures. An approximation of the effective parameters, when each of the interacting subsystems (electron spin, magnon, and phonon) is characterized by its own effective temperature, has been considered. The generalized Bloch equations which describe the spin-wave current propagation in the dielectric have been derived. Finally, two sides of the spin transport "coin" have been revealed: the diffusive nature of the magnon motion and magnon relaxation processes, responsible for the spin pumping, and the spin-torque effect.

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I. INTRODUCTION

One of the central issues of spintronics is the generation and control of spin currents in solids. There are different methods to implement the spin current: optical, magnetic, and using an electric current. The latter is especially important for use in devices [1] when spin-polarized charge carriers are injected from a ferromagnetic material to a nonmagnetic material. At that, in the nonmagnetic material at the spin diffusion length, spin accumulation arises. With an external perturbation usually affecting the kinetic degrees of freedom, the interaction between translational (kinetic) and spin degrees of freedom plays the main role in the formation of the spin response to the external perturbation. Combined electric dipole resonance can serve as an example of such an effect. In this case, the interaction of conduction electrons and an alternating electric field results in resonance at the Zeeman frequency [2]. Another example of such a response is the spin Hall effect (SHE) [3,4] that is exhibited as a spin current perpendicular to both the normal current and the spin accumulation. There also exist mechanisms of interaction with external fields whose energy is simultaneously transferred to both electronic subsystems (kinetic and spin) [5]. It has been found that thermal perturbations may also cause the spin effects to occur. The first effect that has opened a new direction in spintronics, the influence of thermal perturbations on spin effects, is the spin Seebeck effect (SSE) [6,7]. It has turned out to be inherent in conducting crystals of $Ni_{81}Fe_{19}$. Afterwards, the SSE could be observed in various materials such as (Ga,Mn)As [7] semiconductors and metallic Co_2MnSi ferromagnets [8]. Besides, later the spin Nernst effect (or the thermal spin Hall effect), the spin Peltier effect, and others have been discovered [9–14]. As noted in the work [15], there is much in common between the spin effects implemented in an electric or an inhomogeneous temperature field. So, in spintronics, studying the interaction between charge and spin currents, the new direction appears—spin caloritronics [16,17]. As far back as in the late 20th century, a few theoretical aspects of these interactions were discussed [18].

Studying of the SSE in a nonconducting magnet in the system of a nonmagnetic conductor/magnetic insulator (N/F) $LaY_2Fe_5O_{12}$ [19] has shown that this effect cannot be described by standard approaches with regard to a description of thermoelectric effects [18]. As distinct from conducting crystals where the transfer of the spin angular momentum is due to band charge carriers, the spin Seebeck effect can be realized in nonconducting magnetic materials through the excitation of a localized spin system. In this case, the excitations (magnons) underlying the spin-wave current cause the transfer of the angular momentum. Thus, conducting crystals and a nonconductive magnet differ from each other in the type of spin current, namely, a spin-wave current. It is a new type of spin current. The system of localized spin can be deviated from its equilibrium state in various ways. In the experiment [19], this has been achieved by passing an electric current through the nonmagnetic conductor Pt, which results in the spin accumulation and the spin current in it. The interaction of the spin-polarized electrons with the localized spins at the interface (N/F) is accompanied by the creation (or annihilation) of magnons, which in turn gives rise to a perturbation of the magnetic subsystem. Due to small spin-wave damping, the spin-wave current propagates at much greater distances than the electron spin current. This fact makes the effect possible to apply in practice [20,21].

An important role in the study of thermal perturbations is played by a lattice (phonons). Indeed, an inhomogeneous temperature field may cause a deviation of both the localized spin subsystem (m) and the phonon subsystem (p) from their equilibrium state. If the nonequilibrium state of each subsystem is characterized by an effective temperature T_i, $i = m, p$, the nonequilibrium state of the system as a whole can be clearly described by a set of parameters T_i. The difference in the effective temperatures of the subsystems can lead to the realization of the effect—the transfer of the angular momentum from the magnetic subsystem to the lattice or to the electron spin subsystem [22,23] and vice versa. Thermal perturbations are also responsible for the drag effects, whose role is essential in a range of not-very-high temperatures [24,25]. As the authors [26] believe, it is the thermal perturbations which produce the observable anomaly in the temperature dependence of the SSE.

The spin effects in nonconductive magnetic materials under thermal perturbations were studied in several papers [22,27–29]. In this case, the theoretical description largely boiled down to the consideration of the evolution of the localized moment subsystem. Given thermal fluctuations

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derived from the fluctuation-dissipative theorem, the authors of the works mentioned above modeled the localized moments' dynamics by the phenomenological Landau-Lifshitz-Gilbert equation. As to the spin-density dynamics in a nonmagnetic material, it is described by the Bloch equation with phenomenological spin relaxation frequencies. A spin current that was injected into the nonmagnetic material (N) was calculated as the average value of the rate of change of the spin density in a nonmagnetic material. Spin-charge kinetics in the systems (F/N), ($F/N/F$) under a temperature gradient was thoroughly covered in Ref. [30] in terms of a simple phenomenological model and the simplest approximations.

The study of the spin-thermal effects requires a correct description of the thermal perturbation. There are several possible scenarios accounting for such perturbations. For example, the reaction of weakly nonequilibrium systems on a disturbance of the thermal type can be represented as a Fourier transform of time correlation functions of appropriate flux operators over the statistical equilibrium state of the system [31]. The structure of admittances of such a type is analogous to expressions for the transport coefficients arising in the theory of equilibrium systems as the response to a mechanical perturbation. These expressions can be represented as an additional summand in the system's Hamiltonian. That is why the response to a thermal perturbation is often found using indirect methods. Such cases require introducing fictitious external forces identical to thermal perturbations exerted on the system [32]. Response to the thermal perturbations can also be calculated based on the Onsager hypothesis [33] about of the nature of fluctuation damping, or by using the local equilibrium distribution as an initial condition for the statistical operator of the system to be found [34].

The response of weakly nonequilibrium systems to the thermal perturbations can be universally constructed through the method of the nonequilibrium statistical operator (NSO) and its modifications [35]. The latter represent the NSO as a functional of the local equilibrium distribution. The method allows calculating the response of not only equilibrium but also strongly nonequilibrium systems described in terms of roughened macroscopic variables. Within this method, the kinetic coefficients are expressed via the Fourier transform of time correlation functions over the statistical distribution which describes the unperturbed nonequilibrium process. Naturally, in this case, electron scattering needs to be taken into account.

In the given paper, we consider the spin-thermal effects in a normal conductor/magnetic insulator, using the NSO method to describe thermal perturbations. The article is organized as follows. The first part formulates the model at hand, represents a Hamiltonian of the system, and introduces basic operators and their microscopic equations of motion. The second part of the work covers constructing both the nonequilibrium entropy operator, including the thermal perturbations of the system, and the NSO. The third section of the article focuses on an analysis of macroscopic equations describing the spin-thermal effects.

II. THE HAMILTONIAN

Our model consists of a normal conductor (N) and a ferromagnetic insulator (F). We hold that, in the first conductor, the spin accumulation takes place. It can be obtained and implemented in various ways, for example, by using the spin Hall effect. The conduction electrons of the normal metal are coupled with the localized spin subsystem of the ferromagnetic insulator through exchange interaction. The inelastic scattering of the electrons by the localized spins, accompanied by magnon emission or absorption, alters the electron spin orientation and unbalances the localized spin system. Consider the magnon scattering mechanism by the example of the magnon-phonon interaction. We assume the ferromagnetic insulator to be in a nonuniform temperature field. Let the system of conduction electrons in the normal conductor consist of two subsystems: the kinetic and spin. In this case, the first is characterized by an equilibrium temperature T and the second (the spin subsystem) by a temperature T_s. The symbol T_m designates a temperature of the localized spin subsystem, and T_p a temperature of the lattice (phonons).

The Hamiltonian of the system (N/F) can be represented as $H = H_N + H_F + H_L$. Here

$$H_N = \int d\mathbf{x}\,[\,H_k(\mathbf{x}) + H_s(\mathbf{x})\,],$$
$$H_k(\mathbf{x}) = \sum_j \left\{ \frac{p_j^2}{2m}, \delta(\mathbf{x}-\mathbf{x}_j) \right\}, \tag{1}$$
$$H_s(\mathbf{x}) = -\hbar\omega_s \sum_j s_j^z\, \delta(\mathbf{x}-\mathbf{x}_j).$$

The integration is over the volume occupied by (NM). s_j^z and $p_j^\gamma = \hbar k_j^\gamma$ are components of the spin and momentum of the jth electron operators, respectively. $\omega_s = g_s\mu_0 H/\hbar$ is the Zeeman precession frequency of free electrons in an external magnetic field directed along the axis z. g_s, μ_0 are the effective spectroscopic splitting factor for electrons and the Bohr magneton, respectively, $\{A,B\} = (AB + BA)/2$, and

$$H_F = \int d\mathbf{x}\,[H_m(\mathbf{x}) + H_{ms}(\mathbf{x})] \tag{2}$$

is the Hamiltonian of the localized spins subsystem. $H_m(\mathbf{x})$ is the energy density operator of the magnetic subsystem. It involves a sum of the exchange (over the nearest neighbors) H_{SS} and Zeeman energy H_S:

$$H_{SS} = -J\sum_{j\delta} \mathbf{S}_j\mathbf{S}_{j+\delta}, \quad H_S = -\hbar\omega_m \sum_j S_j^z, \tag{3}$$

where J is the exchange integral, and $\omega_m = g_m\mu_0 H/\hbar$. $H_{ms}(\mathbf{x})$ is the energy density operator of interaction between the s and m subsystems at the interface. Then

$$H_{ms} = -J_0 \sum_j \int d\mathbf{x}\,\mathbf{s}(\mathbf{x})\,\mathbf{S}(\mathbf{R}_j)\,\delta(\mathbf{x}-\mathbf{R}_j), \tag{4}$$

where J_0 is the exchange integral, $\mathbf{S}(\mathbf{R}_j)$ being the operator of the localized spin with the coordinate $\mathbf{R}_j$ at the interface. The integration in (2) is over the volume occupied (F).

$H_L = H_p + H_{pm}$ is the lattice Hamiltonian,

$$H_p = \sum_{\mathbf{q},\lambda} \hbar\Omega_{\mathbf{q},\lambda}\, c^+_{\mathbf{q},\lambda}\, c_{\mathbf{q},\lambda},$$

where $c^{+}_{\mathbf{q},\lambda}\,(c_{\mathbf{q},\lambda})$ is the phonon creation (annihilation) Bose operator with wave vector $\mathbf{q}$, the polarization vector λ, and the phonon frequency $\Omega_{\mathbf{q},\lambda}$. H_{pm} is the operator of interaction between the localized spins and phonons. The explicit form of interaction H_{pm}, we present below:

$$H_L = \int d\mathbf{x}\,[H_p(\mathbf{x}) + H_{pm}(\mathbf{x})\,].$$

In the future, we are interested in the evolution of the magnetic subsystem, so the scattering of conduction electrons by phonons for simplicity are omitted from consideration.

III. THE ENTROPY OPERATOR

To analyze the kinetics of the spin-thermal effects, we employ a scheme developed in the NSO method applied to the case of a small deviation of the system from the equilibrium Gibbs distribution $\rho_0 = \exp\{-S_0\}$. The entropy S_0 of the equilibrium system with the Hamiltonian H can be written as

$$S_0 = \Phi_0 + \beta(H_k + H_s - \mu\, n) + \beta(H_m + H_p + H_{ms} + H_{mp}),$$

where $\beta^{-1} = T$ is the equilibrium temperature of the system.

In terms of average densities, to the nonequilibrium state of the system there corresponds the entropy operator

$$\begin{aligned} S(t) = \Phi(t) + \int d\mathbf{x} \Bigg\{ & \beta \left[H_k(\mathbf{x}) - \sum_{\alpha} \mu^{\alpha}(\mathbf{x},t) n^{\alpha}(\mathbf{x},t) \right] \\ & + \beta_s(\mathbf{x},t) H_s(\mathbf{x},t) + \beta\, H_p(\mathbf{x},t) \\ & + \beta_m(\mathbf{x},t)[H_m(\mathbf{x},t) + H_{ms}(\mathbf{x},t) + H_{pm}(\mathbf{x},t)] \Bigg\} \\ \simeq S_0 + \delta S(t), \end{aligned} \tag{5}$$

where $\Phi(t)$ is the Massieu-Planck functional; $\beta_s(\mathbf{x},t)$, $\beta_m(\mathbf{x},t)$ are the local equilibrium values of the inverse temperature of the (s), (m) subsystems, respectively. $\mu^{\alpha}(\mathbf{x},t)$ is the local equilibrium value of the chemical potential of electrons with spin $\alpha = \uparrow,\downarrow$, and

$$n^{\alpha}(\mathbf{x}) = \sum_j \delta\left(\mathbf{x} - \mathbf{x}^{\alpha}_j\right)$$

is the operator of the particle number density with the spin α. $n(\mathbf{x}) = n^{\uparrow}(\mathbf{x}) + n^{\downarrow}(\mathbf{x})$.

The operator $\delta S(t) = \int d\mathbf{x}\delta S(\mathbf{x},t)$ describes the system deviation from its equilibrium state:

$$\begin{aligned} \delta S(t) = \Delta \int d\mathbf{x} \Bigg\{ & \delta\beta_s(\mathbf{x},t) H_s(\mathbf{x}) - \beta \sum_{\alpha} \delta\mu^{\alpha}(\mathbf{x},t)\, n^{\alpha}(\mathbf{x}) \\ & + \delta\beta_m(\mathbf{x},t)[H_m(\mathbf{x}) + H_{ms}(\mathbf{x}) + H_{pm}(\mathbf{x})] \Bigg\}, \\ \Delta A = A - \langle A\rangle_0, \quad & \langle \ldots\rangle_0 = Sp\,(\ldots \rho_0), \\ \langle \ldots\rangle^t = Sp\,(\ldots \rho(t)), \quad & \rho_q(t) = \exp\{-S(t)\}, \\ \delta\mu^{\alpha}(\mathbf{x},t) = \mu^{\alpha}(\mathbf{x},t) - \mu, \quad & \delta\beta_i(\mathbf{x},t) = \beta_i(\mathbf{x},t) - \beta, \end{aligned} \tag{6}$$

where $(i = s, m)$. The quantities $\delta\mu^{\alpha}(\mathbf{x},t)$,$\delta\beta_m(\mathbf{x},t)$ have the meaning of nonequilibrium additions to the chemical potential μ and deviations of the magnetic subsystem temperature from the equilibrium temperature $\beta^{-1} = T$.

In the linear approximation the deviation from the equilibrium, the NSO (or the density matrix) $\rho(t)$ can be written as follows [35]:

$$\rho(t) = \rho_q(t) + \int_{-\infty}^{0} dt_1\, e^{\epsilon t_1} \int_0^1 d\tau\, \rho_0^{\tau} \dot{S}(t + t_1, t_1) \rho_0^{-\tau}\, \rho_0. \tag{7}$$

Here $\rho_q(t)$ is the quasiequilibrium statistical operator, $\dot{S}(t)$ being the entropy production operator:

$$\dot{S}(t) = \delta\dot{S}(t) = \frac{\partial S(t)}{\partial t} + \frac{1}{i\hbar}[S(t), H].$$

A further algorithm for constructing the operator $\rho(t)$ reduces to finding the entropy production operator. Commuting the operators $n^{\alpha}(\mathbf{x})$, $H_s(\mathbf{x})$ with the Hamiltonian H, we find the operator equations of motion,

$$\dot{n}^{\alpha}(\mathbf{x}) = -\nabla\, I_{n^{\alpha}}(\mathbf{x}) + \dot{n}^{\alpha}_{(ms)}(\mathbf{x}), \tag{8}$$

$$\dot{H}_s(\mathbf{x}) = -\nabla\, I_{H_s}(\mathbf{x}) + \dot{H}_{s(ms)}(\mathbf{x}). \tag{9}$$

Here $I_{n^{\alpha}}(\mathbf{x}) = \frac{1}{m}\sum_j \{p_j, \delta(\mathbf{x} - \mathbf{x}^{\alpha}_j)\}$ is the flux density of particles with spin α. $\dot{n}^{\alpha}_{(ms)}(\mathbf{x})$ determines the rate of change of the number of electrons with spin α due to the spin-flip electron scattering by the (m) subsystem.

Similarly, $I_{H_s}(\mathbf{x}) = -\hbar\omega_s \frac{1}{m}\sum_{j,\alpha} s^z_j \{p_j, \delta(\mathbf{x} - \mathbf{x}^{\alpha}_j)\}$ determines the flux density of the Zeeman energy. $\dot{H}_{s(sm)}(\mathbf{x})$ is the rate of change of the electron (s)-subsystem local energy due to their interaction with magnons (m):

$$\dot{A}_{\lambda(\gamma)}(\mathbf{x}) = (i\hbar)^{-1}[A_{\lambda}(\mathbf{x}), H_{\gamma}]. \tag{10}$$

Revealing the spin index explicitly, we write the expression for the spin current density $J_s(\mathbf{x})$:

$$\begin{aligned} J_s(\mathbf{x}) &= \frac{\{\dot{n}^{\uparrow}(\mathbf{x}) - \dot{n}^{\downarrow}(\mathbf{x})\}}{2} = -\nabla\, I_{s^z}(\mathbf{x}) + \dot{s}^z_{(ms)}(\mathbf{x}), \\ I_{s^z}(\mathbf{x}) &= [I^{\uparrow}_s(\mathbf{x}) - I^{\downarrow}_s(\mathbf{x})]/2, \\ \dot{s}^z_{(ms)}(\mathbf{x}) &= [\dot{n}^{\uparrow}_{(ms)}(\mathbf{x}) - \dot{n}^{\downarrow}_{(ms)}(\mathbf{x})]/2. \end{aligned} \tag{11}$$

From the expression (11) it follows that $J_s(\mathbf{x})$ contains two parts: collisionless $I_{s^z}(\mathbf{x})$ and collisional $\dot{s}^z_{(ms)}(\mathbf{x})$. The former is due to the flux of particles with different spin orientations; the latter is determined by the spin-flip scattering.

Let us turn to the consideration of the magnetic subsystem. Using the method of Holstein-Primakov [36], the Hamiltonian of localized spins subsystem can be rewritten with spin-wave (magnon) variables (using the operators creation $b^{+}_{\mathbf{k}}$ and annihilation $b_{\mathbf{k}}$). Expressed in terms of the magnon operators, the Zeeman and exchange interactions are of the form

$$H_S = -\hbar\omega_d N S + \hbar\omega_d \sum_{\mathbf{k}} b^{+}_{\mathbf{k}}\, b_{\mathbf{k}}, \tag{12}$$

$$\begin{aligned} H_{SS} &\simeq -JNzS^2 + \sum_{\mathbf{k}} 2JzS(1 - \gamma_{\mathbf{k}}) b^{+}_{\mathbf{k}}\, b_{\mathbf{k}}, \\ \gamma_{\mathbf{k}} &= (1/z)\sum_{\delta} \exp\{i\mathbf{k}\,\boldsymbol{\delta}\}. \end{aligned} \tag{13}$$

Here N is the number of localized moments, with the spin S,z the number of the nearest neighbors. $b^{+}_{\mathbf{k}}\, b_{\mathbf{k}}$ are the

creation and annihilation operators for magnons with the wave vector $\mathbf{k}$. For $|\mathbf{k}\,\boldsymbol{\delta}| \ll 1$, we have $z(1-\gamma_{\mathbf{k}}) \approx (1/2)\sum_{\boldsymbol{\delta}}(\mathbf{k}\,\boldsymbol{\delta})^2$, i.e., $\omega_{\mathbf{k}} = JS\sum_{\boldsymbol{\delta}}(\mathbf{k}\,\boldsymbol{\delta})^2$. For body-centered and face-centered cubic lattices with a lattice constant equal to a, we have $\omega_{\mathbf{k}} = 2JS(ka)^2$. In other words, the contribution of the exchange interaction for the magnon frequency has the same form as the de Broglie dispersion law for free particles of mass m^*:

$$\omega_k = \frac{\hbar}{2m^*}\,k^2, \quad m^* = \frac{1}{2JSa^2}.$$

Thus, regarding the magnon gas as free, we arrive at

$$H_m = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} b^+_{\mathbf{k}}\, b_{\mathbf{k}}.$$

This expression can be interpreted as the sum of the energies of the quasiparticles-ferromagnons having the quasimomentum $\hbar\mathbf{k}$ with their own effective mass m^* and the magnetic moment [37].

The Hamiltonian of the magnon-phonon interaction can be found by considering the possibility of displacement of the magnetic ions from their equilibrium positions. In the linear order in the displacement the Hamiltonian of a magnon-phonon interaction can be represented as [28,38]

$$H_{mp} = \sum_{\mathbf{q},\mathbf{p}} C_{\mathbf{p},\mathbf{q}}\,(c^+_{-\mathbf{q}} + c_{\mathbf{q}})\, b^+_{\mathbf{p}+\mathbf{q}}\, b_{\mathbf{p}}. \tag{14}$$

Here

$$C_{\mathbf{p},\mathbf{q}} \sim \frac{C(\hbar\omega_{\mathbf{p}})}{v_p}\sqrt{\frac{\hbar v_{\mathbf{q}}}{2N_F\, M_i}}, \tag{15}$$

where C is the dimensionless magnon-phonon coupling constant, N_F is the number of lattice sites in the domain, M_i is the ion mass, and $v_{\mathbf{q}} = v_p q$ is the phonon energy for the phonon velocity v_p.

The exchange interaction (4) includes both the elastic electron scattering by the localized spins and the inelastic scattering process, with the former preserving the spin orientation and the latter happening with the electron spin-flip and the creation or annihilation for magnons. Using the creation and annihilation operators for electrons and magnons, we rewrite the inelastic part of the exchange interaction in the form

$$H_{ms} = -J^* \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \{b^+_{\mathbf{q}}\, a^+_{\mathbf{k}\uparrow}\, a_{\mathbf{k}'\downarrow} + b_{\mathbf{q}}\, a^+_{\mathbf{k}'\downarrow}\, a_{\mathbf{k}\uparrow}\}\,\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}, \tag{16}$$

where $a^+_{\mathbf{k}\alpha}$ ($a_{\mathbf{k}\alpha}$) are the creation (annihilation) operators for electrons with a certain spin value $\alpha = \uparrow, \downarrow$. The inelastic part of the exchange interaction (H_{ms}) leads to the angular momentum transfer between electrons of the normal metal and magnons of the ferromagnetic insulator. Obviously, nonequilibrium of one of the subsystems (electronic or magnon) makes the magnon/spin current via the interface. At the same time, the total angular momentum conservation law provides the boundary condition for the current at the metal-insulator interface: the continuity of the spin current.

Thus, the equation of motion for the magnetic subsystem can be written as

$$\dot{H}_m(\mathbf{x}) = -\nabla\, I_{H_m}(\mathbf{x}) + \dot{H}_{m(ms)}(\mathbf{x}) + \dot{H}_{m(mp)}(\mathbf{x}). \tag{17}$$

Here $I_{H_m}(\mathbf{x}) = -\hbar\omega_m\, I_{S^z}(\mathbf{x})$ is the flux density of the magnon energy in the (m) subsystem. The remaining summands on the right-hand side of the equation are responsible for the magnon-phonon scattering processes. Finally, the equation of motion for the lattice subsystem has the form

$$\dot{H}_p(\mathbf{x}) = -\nabla\, I_{H_p}(\mathbf{x}) + \dot{H}_{p(mp)}(\mathbf{x}). \tag{18}$$

Using the operator equations of motion, we can construct an expression for the entropy production operator $\dot{S}(t)$ in a linear approximation over gradients of spatial inhomogeneities of temperature and chemical potential [39], integrating by parts the summands containing the divergence of fluxes. Having discarded the surface integrals, we write the entropy production operator in the form

$$\begin{aligned}\dot{S}(t) = \Delta\int d\mathbf{x}\Bigg\{&-\beta\sum_{\alpha}\left[I_{n^\alpha}(\mathbf{x})\nabla\mu^\alpha(\mathbf{x},t) + \delta\mu^\alpha(\mathbf{x},t)\dot{n}^\alpha_{(ms)}(\mathbf{x})\right]\\ &+ I_{H_m}(\mathbf{x})\nabla\beta_m(\mathbf{x},t) + [\beta_s(\mathbf{x},t) - \beta_m(\mathbf{x},t)]\dot{H}_{s(ms)}(\mathbf{x})\\ &+ [\beta_m(\mathbf{x},t) - \beta]\dot{H}_{m(pm)}(\mathbf{x})\Bigg\}.\end{aligned} \tag{19}$$

By revealing the spin index in the first two summands on the right-hand side of the formula (19), we get

$$\sum_{\alpha} I_{n^\alpha}(\mathbf{x})\nabla\mu^\alpha(\mathbf{x},t) = I_{s^z}(\mathbf{x})\,\nabla\,\mu_s(\mathbf{x},t), \tag{20}$$

$$\begin{aligned}\sum_{\alpha}\delta\mu^\alpha(\mathbf{x},t)\dot{n}^\alpha_{(ms)}(\mathbf{x}) &= \mu_s(\mathbf{x},t)\dot{s}^z_{(ms)}(\mathbf{x})\\ &= -\mu_s(\mathbf{x},t)\dot{S}^z_{(ms)}(\mathbf{x}).\end{aligned} \tag{21}$$

Here $\mu_s = \mu^\uparrow - \mu^\downarrow$. It is obvious that the expression (21) provides the boundary conditions for the collisional part of the spin current at the interface. The expressions (20), (11) imply that the heterogeneity in the spin accumulation distribution governs the collisionless part of the spin current.

Before proceeding to constructing the equations for the averages, we show that the entropy production operator $\dot{S}(t)$ can be represented both through the spin accumulation $\mu_s(\mathbf{x},t)$ and through the spin temperature $\beta_s(\mathbf{x},t)$. For this purpose, we find a linear relationship between the deviations of thermodynamic coordinates and thermodynamic forces from their equilibrium values. In this approximation, it is sufficient to put $\rho(t) \sim \rho_q(t)$ and expand the quasiequilibrium operator $\rho_q(t) = \exp\{-S(t)\}$ in powers of $\delta S(t)$. Then we come up with the result

$$\begin{aligned}\delta\langle H_s(\mathbf{x})\rangle^t &= -\int d\mathbf{x}'\Bigg\{\delta\beta_s(\mathbf{x}',t)(H_s(\mathbf{x}),\, H_s(\mathbf{x}'))_0\\ &\qquad - \beta\sum_{\alpha}\delta\mu^\alpha(\mathbf{x}',t)(H_s(\mathbf{x}),n^\alpha(\mathbf{x}'))_0\Bigg\},\\ \delta\langle n(\mathbf{x})\rangle^t &= -\int d\mathbf{x}'\Big\{\delta\beta_s(\mathbf{x}',t)(n(\mathbf{x}),H_s(\mathbf{x}'))_0\\ &\qquad - \beta\sum_{\alpha}\delta\mu^\alpha(\mathbf{x}',t)(n(\mathbf{x}),n^\alpha(\mathbf{x}'))_0\Bigg\},\end{aligned} \tag{22}$$

where

$$\delta\langle A\rangle^t = \langle A\rangle^t - \langle A\rangle_0,$$
$$(A,B)_0 = \int_0^1 d\lambda Sp\{A\rho_0^\lambda \Delta B\, \rho_0^{1-\lambda}\}.$$

Going over to the Fourier components of the spatial coordinates,

$$A(\mathbf{x}) = \int \frac{d\mathbf{q}}{(2\pi)^3} A(\mathbf{q})\, e^{i\mathbf{q}\mathbf{x}}, \quad A(\mathbf{q}) = \int d\mathbf{x} A(\mathbf{x})\, e^{-i\mathbf{q}\mathbf{x}},$$

and taking into account that $\langle n\rangle^t = \langle n\rangle_0$, we obtain

$$\beta\mu_s(\mathbf{q},t) = \delta\beta_s(\mathbf{q},t)\,\frac{(n(\mathbf{q}),H_s(-\mathbf{q}))_0}{(n(\mathbf{q}),n(-\mathbf{q}))_0}. \tag{23}$$

Equation (23) defines the relationship between the spin temperature T_s and the spin accumulation μ_s. In the long-wavelength limit $\mathbf{q} = 0$ and the steady state, we obtain

$$\mu_s = -(\hbar\omega_s/2)\,(1 - T/T_S). \tag{24}$$

Substituting the expression (23) into the equation for $\delta\langle H_s(\mathbf{x})\rangle^t$, we get

$$\begin{aligned}\delta\langle H_s(\mathbf{q})\rangle^t &= -\delta\beta_s(\mathbf{q},t)\,(\hbar\omega_s)^2\, C_{zz}(\mathbf{q}),\\ C_{zz}(\mathbf{q}) &= (s^z(\mathbf{q}),s^z(-\mathbf{q}))_0 - \frac{(n(\mathbf{q}),s^z(-\mathbf{q}))_0^2}{(n(\mathbf{q}),n(-\mathbf{q}))_0}.\end{aligned} \tag{25}$$

The desired relationship between the correlation function C_{zz} and the differential paramagnetic susceptibility χ_s [40] of the electron gas is given by

$$\chi_s = \frac{\partial}{\partial H}\, g_s\mu_b\langle s^z\rangle^t = \beta(g_s\mu_b)^2\, C_{zz}. \tag{26}$$

IV. MACROSCOPIC EQUATIONS

Let us construct the equations for the averages, which have the meaning of local conservation laws of the average energy density for the (s) and (m) subsystems and of the particle number density. Inserting the entropy production operator into the expression for the NSO (7), we average the operator equation (11) for the spin current $\langle J_s(\mathbf{x})\rangle^t = (d/dt)\langle s^z(\mathbf{x})\rangle^t$. Then we arrive at

$$\begin{aligned}\langle\overline{J_s(\mathbf{x})}\rangle^t = \int d\mathbf{x}'\big\{ &D^\gamma_{s^z s^z}(\mathbf{x},\mathbf{x}')\beta\,\nabla^\gamma\mu_s(\mathbf{x}') + [\beta_m(\mathbf{x}') - \beta_s(\mathbf{x}')]\\ &\times \hbar\omega_s\, L^z_{z,(ms)}(\mathbf{x},\mathbf{x}') - \beta\,\mu_s(\mathbf{x}')\, L^z_{z,(ms)}(\mathbf{x},\mathbf{x}')\big\},\end{aligned} \tag{27}$$

where

$$\begin{aligned}L^z_{z,(ms)}(\mathbf{x},\mathbf{x}') &= \int_{-\infty}^0 dt'\, e^{\epsilon t'}\left(\dot s^z_{(ms)}(\mathbf{x})\,,\,\dot s^z_{(ms)}(\mathbf{x}',t')\right)_0,\\ D^\gamma_{s^z s^z}(\mathbf{x},\mathbf{x}') &= \int_{-\infty}^0 dt'\, e^{\epsilon t'}\sum_\lambda \nabla^\lambda\left(I^\lambda_{s^z}(\mathbf{x}),I^\gamma_{s^z}(\mathbf{x}',t')\right)_0.\end{aligned} \tag{28}$$

Here $\overline{A}$ denotes time averaging, $\overline{\beta_i(\mathbf{x},t+t')} = \beta_i(\mathbf{x}),(i = s,m)$; $\overline{\mu(\mathbf{x},t+t')} = \mu(\mathbf{x})$.

We have derived the generalized Bloch equation that describes the motion of the spin magnetization density—the spin diffusion and relaxation processes. The first summand on the right-hand side of the equation reflects the former process; the second summand is responsible for the latter caused by the electron-magnetic impurity interaction at the interface. In doing so, we have made an allowance for the effects of temporal and spatial dispersion of the spin diffusion tensor and spin relaxation frequency. It should be pointed out that the resulting expressions for the transport coefficients $L^z_{z,(ms)}(\mathbf{x},\mathbf{x}'), D^\gamma_{s^z s^z}(\mathbf{x},\mathbf{x}')$ are suitable for both classical and quantizing magnetic fields and free from assumptions about the nature of the spectrum, the the kind of statistics, etc. Similar expressions for the spin relaxation frequency are also inherent in the theory describing the spin-lattice relaxation of a nonequilibrium spin system with magnetic resonance saturation [40].

Averaging the operator equation (17), we have

$$\langle\dot H_m(\mathbf{x})\rangle^t = -\nabla\,\langle I_{H_m}(\mathbf{x})\rangle^t + \langle\dot H_{m(ms)}(\mathbf{x})\rangle^t + \langle\dot H_{m(mp)}(\mathbf{x})\rangle^t. \tag{29}$$

Or averaging time

$$\begin{aligned}\langle\overline{\dot H_m(\mathbf{x})}\rangle^t = \int d\mathbf{x}'\Big\{&\sum_\gamma D^\gamma_{H_m H_m}(\mathbf{x},\mathbf{x}')\nabla^\gamma\beta_m(\mathbf{x}')\\ &- \beta\,\mu_s(\mathbf{x}')/(\hbar\omega_m)\, L^m_{m,(ms)}(\mathbf{x},\mathbf{x}')\\ &+ [\beta_m(\mathbf{x}') - \beta_s(\mathbf{x}')]\,(\omega_s/\omega_m)\, L^m_{m,(ms)}(\mathbf{x},\mathbf{x}')\\ &+ [\beta_m(\mathbf{x}') - \beta]\, L^m_{m(mp)}(\mathbf{x},\mathbf{x}')\Big\},\end{aligned} \tag{30}$$

where

$$L^m_{m(ml)}(\mathbf{x},\mathbf{x}') = \int_{-\infty}^0 dt'\, e^{\epsilon t'}\,(\dot H_{m(ml)}(\mathbf{x})\,,\,\dot H_{m(ml)}(\mathbf{x}',t'))_0, \tag{31}$$

$$D^\gamma_{H_m H_m}(\mathbf{x},\mathbf{x}') = \int_{-\infty}^0 dt'\, e^{\epsilon t'}\sum_\lambda \nabla^\lambda\left(I^\lambda_{H_m}(\mathbf{x}),I^\gamma_{H_m}(\mathbf{x}',t')\right)_0. \tag{32}$$

The first term on the right-hand side of Eq. (29) is the energy density change in the magnon subsystem due to the temperature gradient, which in turn leads to the magnon flux. As in the case of the spin electron subsystem, this term describes the magnon diffusion. The second and third terms of the expression (29) are responsible for the impact of the electronic (spin) and lattice (phonon) subsystems on the magnon energy change through their interaction. The role of the spin electron subsystem reduces to the generation and annihilation of the magnons by the inelastic spin-flip electron scattering at the normal metal/ferrodielectric interface. According to (24), this contribution is proportional to the spin accumulation μ_s. The phonon subsystem proves to affect the magnon energy change in a twofold manner. On the one hand, the magnon-phonon scattering processes make themselves felt in the energy relaxation behavior of the magnon subsystem; on the other hand, the phonon subsystem often acts as a "heating" of the heat/charge transfer processes by means of drag effects [24]. It should be emphasized that the booster role can also be studied by the NSO method. The energy transfer efficiency between the phonon and magnon subsystems depends on temperatures of the appropriate subsystems and the degree of their mutual

interaction. As can easily be seen from (30), a nonzero contribution to the spin-wave current is the consequence of the difference in the temperatures of the above subsystems. Equation (30) implies that, depending upon its direction, the magnon flux produced by a temperature gradient can give rise to the angular momentum transfer from the magnon system to the electron system. Besides, the heat fluxes contribute to the emergence both of the thermally induced spin-torque effect [41–43] and of the spin-torque effect generated by magnons [44,45].

V. CONCLUSIONS

Using one of the methods of quantum nonequilibrium statistical physics (NSO), we investigated the spin transport in hybrid nanostructures: normal metal/ferromagnetic insulator. An approximation of the effective parameters, when each of the interacting subsystems (electron spins, magnons, phonons) is characterized by its effective temperatures was considered. We constructed macroscopic equations for the spin current caused by both the unbalanced spin subsystem and an inhomogeneous temperature field in the ferromagnetic insulator. We derived the generalized Bloch equations which describe the spin and spin-wave current propagation in the system. At that, these allow for both the diffusive nature of the magnon motion and the magnon relaxation processes responsible for the spin pumping and the spin-torque effect.

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APPENDIX A

Let us give the results coming from the expressions for the spin diffusion tensor (28), including the temporal and spatial dispersion, and find their relationship with the conductivity tensor components. The correlation functions of the fluxes in formula (28) involve the Fourier components of the isothermal Green's functions:

$$G_{AB}(t) = \theta(-t)\, e^{\epsilon t}\, (A,\, B(t))_0 = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{i\omega t}\, G_{BA}(\omega). \tag{A1}$$

Differentiating (A1) over t, we obtain the chain of equations

$$\begin{gathered}\left(\frac{\partial}{\partial t} - \epsilon - i\omega_0\right) G = -\delta(t)(B,A)_0 + G_1,\\ \left(\frac{\partial}{\partial t} - \epsilon - i\omega_0\right) G_1 = -\delta(t)(B,\dot{A})_0 - G_2,\\ \cdots\cdots\cdots\cdots,\\ G(t) = G_{AB}(t), \quad G_1(t) = \theta(-t)\, e^{\epsilon t}\, (A,\, \dot{B}(t))_0.\\ G_2(t) = \theta(-t)\, e^{\epsilon t}\, (\dot{A},\, \dot{B}(t))_0.\end{gathered} \tag{A2}$$

Here $\dot{A} = (i\hbar)^{-1}[A, H_V]$, where H_V is the scatterer Hamiltonian. The formal solution of the chain is

$$G_{AB}(\omega) = [M_{AB}(\omega) + \epsilon - i(\omega - \omega_0)]^{-1}(A,B)_0, \tag{A3}$$

where $M(\omega) = G_1\, G^{-1}$ is the mass operator for the Green's function.

Let us consider the frequency dispersion by the example of the longitudinal spin diffusion coefficient. Restricting ourselves to the Born approximation of the interaction with the lattice in the expression for the mass operator, we have

$$D_{zz}^{zz} = \frac{1}{C_{zz}} \frac{\left(I_{s^z}^z, I_{s^z}^z\right)_0}{\nu_{zz}^{zz}(\omega) - i\omega}. \tag{A4}$$

Calculating the correlation functions $(A,B)_0$, we come up with

$$\left(I_{s^z}^z, I_{s^z}^z\right)_0 = n/2m, \quad C_{zz} = \frac{n}{8} \frac{F_{-1/2}(\mu/T)}{F_{1/2}(\mu/T)},$$

where $F_m(x)$ are the Fermi integrals. The expression ν_{zz}^{zz} coincides exactly with the formula for the relaxation frequency ν_p of the electron momentum [46]. Thus, the longitudinal spin diffusion coefficient is given by

$$D_{zz}^{zz} = D_0 \frac{\nu_p}{\nu_p - i\omega}, \quad D_0 = \frac{2T}{\nu_p} \frac{F_{-1/2}(\mu/T)}{F_{1/2}(\mu/T)}. \tag{A5}$$

The components of the spin diffusion tensor can be expressed in terms of the components of the conductivity tensor $\sigma_{ik}(\mathbf{q},\omega)$, which in our notation is

$$\sigma_{\gamma k}(\mathbf{q}) = \frac{e^2}{T} \int_{-\infty}^{0} dt e^{(\epsilon - i\omega)t} \left(I_N^{\gamma}(\mathbf{q}), I_N^k(-\mathbf{q},t)\right). \tag{A6}$$

Simple calculations show that

$$D_{ik}^{zz}(\mathbf{q},\omega) = \frac{T}{4e^2\, C_{zz}(q)}\, \sigma_{ik}(\mathbf{q},\omega).$$

APPENDIX B

The calculation of the correlation function is as follows:

$$\begin{aligned} R &= \int_{-\infty}^{0} e^{\epsilon t_1} dt_1\, (\dot{H}_{m(mp)}, \dot{H}_{m(mp)})\\ &= \int_{-\infty}^{0} e^{\epsilon t_1} dt_1 \int_0^1 d\lambda\, Sp(\dot{H}_{m(mp)} \dot{H}_{m(mp)}(T)\rho_0)\\ T &= i\hbar\beta\lambda + t_1. \end{aligned} \tag{B1}$$

The commutator $\dot{H}_{m(mp)}$ is

$$\dot{H}_{m(mp)} = \frac{1}{i\hbar} \sum_{\mathbf{pq}} C_{\mathbf{p},\mathbf{q}}\, (\omega_{\mathbf{p+q}} - \omega_{\mathbf{p}})\, (c_{-\mathbf{q}}^{+} + c_{\mathbf{q}})\, b_{\mathbf{p+q}}^{+} b_{\mathbf{p}}. \tag{B2}$$

By substituting (B2) in the correlation function and averaging operators, we obtain

$$\begin{aligned} R = \frac{2\pi}{\hbar} \sum_{\mathbf{pq}} & C_{\mathbf{p},\mathbf{q}}\, C_{\mathbf{p+q},\mathbf{q}}\, (\omega_{\mathbf{p+q}} - \omega_{\mathbf{p}})^2\\ & \times f_{\mathbf{p+q}}\, (1 - f_{\mathbf{p}}) \{ n_{\mathbf{q}}\, \delta(\varepsilon_{\mathbf{p+q}} - \varepsilon_{\mathbf{p}} + E_{-\mathbf{q}})\\ & + (1 + n_{\mathbf{q}})\, \delta(\varepsilon_{\mathbf{p+q}} - \varepsilon_{\mathbf{p}} - E_{\mathbf{q}}) \}, \end{aligned} \tag{B3}$$

where $f_{\mathbf{p}}, n_{\mathbf{q}}$ are the distribution functions of phonons and magnons:

$$f_{\mathbf{p}} = \frac{1}{e^{\beta\varepsilon_{\mathbf{p}}} - 1}, \quad n_{\mathbf{q}} = \frac{1}{e^{\beta E_{\mathbf{q}}} - 1}.$$

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