

Transport properties of a two-dimensional electron gas dressed by lightS. Morina,^{1,2} O. V. Kibis,^{3,1,*} A. A. Pervishko,¹ and I. A. Shelykh^{1,2,4}¹*Division of Physics and Applied Physics, Nanyang Technological University 637371, Singapore*²*Science Institute, University of Iceland, Dunhagi 3, IS-107, Reykjavik, Iceland*³*Department of Applied and Theoretical Physics, Novosibirsk State Technical University, Karl Marx Avenue 20, Novosibirsk 630073, Russia*⁴*ITMO University, St. Petersburg 197101, Russia*

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We show theoretically that the strong interaction of a two-dimensional electron gas (2DEG) with a dressing electromagnetic field drastically changes its transport properties. Particularly, the dressing field leads to a giant increase of conductivity (which can reach thousands of percents), resulting in nontrivial oscillating dependence of conductivity on the field intensity, and suppressing the weak localization of 2DEG. As a consequence, the developed theory opens an unexplored way to control transport properties of 2DEG by a strong high-frequency electromagnetic field. From an experimental viewpoint, this theory is applicable directly to quantum wells exposed to a laser-generated electromagnetic wave.

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I. INTRODUCTION

Transport properties of a two-dimensional electron gas (2DEG) in nanostructures exposed to a high-frequency electromagnetic field have been studied in the deep past and taken a deserved place in textbooks (see, e.g., Refs. [1–3]). However, the most attention in previous studies on the subject was paid to the regime of weak light-matter interaction. Following the conventional terminology of quantum optics, in this regime an electron energy spectrum is assumed to be unperturbed by photons. Correspondingly, the weak electron-photon interaction results only in electron transitions between unperturbed electron states, which are accompanied by absorption and emission of photons. As a consequence, the regime of weak electron-photon interaction in solids leads to photovoltaic effects, high-frequency conductivity, and other well-known electronic transport phenomena which are accompanied by absorption of field energy by conduction electrons. However, the interaction between electrons and a strong electromagnetic field (the regime of strong light-matter interaction) cannot be described as a weak perturbation. In this case, the system “electron + electromagnetic field” should be considered as a whole. Such a bound electron-photon system, which was called “electron dressed by photons” (dressed electron), became a commonly used model in modern physics [4,5]. For a long time the main objects for studying the physical properties of dressed electrons were atoms and molecules. The field-induced modification of the energy spectrum and wave functions of dressed electrons—the so-called dynamic Stark effect—was discovered in atoms many years ago [6] and has been studied in detail in various atomic and molecular systems. These studies of strong electron-photon processes formed such an exciting field of modern physics as quantum optics [4,5]. In nanostructures the research activity in the area of quantum optics was focused on exciton-polaritonic effects in microcavities with quantum wells [7–13] and quantum dots [14–16], physical properties of dressed electrons in graphene [17–19] and quantum wires [20], a variety of

technological applications [21], including novel types of the lasers [22,23], optical switches and logic gates [24,25], all-optical integrated circuits [26], and others. As to transport properties of dressed 2DEG, they are still waiting for detailed research. The present article is aimed to partially fill this interdisciplinary gap which takes place at the border between physics of nanostructures and quantum optics.

II. MODEL

For definiteness we will restrict our consideration to the case of 2DEG with a parabolic electron energy spectrum

$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m}, \quad (1)$$

where \mathbf{k} is the electron wave vector, and m is the electron effective mass. Let the 2DEG be subjected to a plane monochromatic electromagnetic wave propagating perpendicularly to the 2DEG plane (see Fig. 1). In what follows, we will assume that the wave frequency ω meets two conditions. First, the wave frequency is far from resonant electron frequencies corresponding to interband electron transitions and, therefore, the interband absorption of the wave by the 2DEG is absent. Second, the wave frequency is high enough in order to satisfy the inequality

$$\omega\tau_0 \gg 1, \quad (2)$$

where τ_0 is the electron relaxation time in an unirradiated 2DEG. It is well known that the intraband (collisional) absorption of wave energy by conduction electrons is negligibly small under condition (2) (see, e.g., Refs. [27–29]). Thus, the considered electromagnetic wave can be treated as a purely dressing (nonabsorbable) field. It follows from the basic principles of quantum optics that the strong coupling of electrons to such a dressing field leads to the renormalization of all physical quantities describing the electrons [4,5]. Particularly, it is well known that a high-frequency electromagnetic field can strongly affect the scattering of conduction electrons and change electronic transport characteristics [30,31]. Recently this approach was extended for the case of a two-dimensional electron gas subjected to a purely dressing field which cannot

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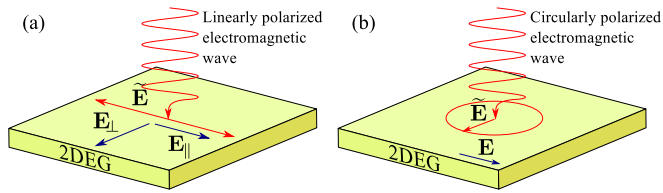


FIG. 1. (Color online) Sketch of the system under consideration: The 2DEG dressed by (a) a linearly polarized high-frequency field $\tilde{\mathbf{E}}$ and (b) a circularly polarized one in the presence of a stationary in-plane electric field $\mathbf{E} = (E_{\parallel}, E_{\perp})$.

be absorbed and emitted by conduction electrons [29]. Namely, the scattering probability of a dressed electron between electron states with wave vectors \mathbf{k} and \mathbf{k}' per unit time has the form [29]

$$w_{\mathbf{k}'\mathbf{k}} = \frac{2\pi}{\hbar} J_0^2(f_{\mathbf{k}'\mathbf{k}}) |U_{\mathbf{k}'\mathbf{k}}|^2 \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}), \quad (3)$$

where $J_0(z)$ is the zeroth order Bessel function of the first kind, and $U_{\mathbf{k}'\mathbf{k}}$ is the matrix element of the scattering potential $U(\mathbf{r})$, which arises from a macroscopically large number of scatterers in the conductor. In the case of a linearly polarized dressing field [29], the argument of the Bessel function is given by

$$f_{\mathbf{k}'\mathbf{k}} = \frac{e\mathbf{E}_{\omega}(\mathbf{k} - \mathbf{k}')}{m\omega^2}, \quad (4)$$

where E_{ω} is the amplitude of the electric field of the wave, $\tilde{\mathbf{E}}(t) = \mathbf{E}_{\omega} \sin \omega t$. In the case of a circularly polarized dressing field (see the Appendix), this argument is

$$f_{\mathbf{k}'\mathbf{k}} = \frac{2E_{\omega}ek}{m\omega^2} \sin\left(\frac{\theta_{\mathbf{k}'\mathbf{k}}}{2}\right), \quad (5)$$

where $\theta_{\mathbf{k}'\mathbf{k}} = \widehat{(\mathbf{k}', \mathbf{k})}$ is the angle between electron wave vectors \mathbf{k} and \mathbf{k}' . The formal difference between the scattering probability of a dressed electron (3) and the conventional expression for the scattering probability of a bare electron [32] consists of the Bessel-function factor $J_0^2(f_{\mathbf{k}'\mathbf{k}})$, where $f_{\mathbf{k}'\mathbf{k}}$ depends on the dressing field amplitude E_{ω} and the dressing field frequency ω [see Eqs. (4) and (5)]. Just this factor results in nontrivial dependence of electronic transport properties on the dressing field. In what follows, we will focus our attention on the conductivity and the weak localization of dressed 2DEG.

III. CONDUCTIVITY OF DRESSED 2DEG

Assuming the temperature to be zero, let us apply a stationary (dc) electric field \mathbf{E} to the 2DEG. It follows from the conventional Boltzmann equation for conduction electrons (see, e.g., Ref. [33]) that electric current density \mathbf{J} is given by the well-known expression

$$\mathbf{J} = \frac{e^2}{2\pi^2} \int_{\mathbf{k}} [\mathbf{E} \cdot \mathbf{v}(\mathbf{k})] \tau(\mathbf{k}) \mathbf{v}(\mathbf{k}) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) d^2\mathbf{k}, \quad (6)$$

where $v(\mathbf{k}) = (1/\hbar)\nabla_{\mathbf{k}}\varepsilon_{\mathbf{k}}$ is the electron velocity, $\varepsilon_F = \hbar^2\mathbf{k}_F^2/2m$ is the electron Fermi energy, and $\tau(\mathbf{k})$ is the relaxation time. In the most general case of anisotropic electron

scattering, this relaxation time is given by the equation [34]

$$\frac{1}{\tau(\mathbf{k})} = \sum_{\mathbf{k}'} \left[1 - \frac{\tau(\mathbf{k}')\mathbf{E} \cdot \mathbf{v}(\mathbf{k}')}{\tau(\mathbf{k})\mathbf{E} \cdot \mathbf{v}(\mathbf{k})} \right] w_{\mathbf{k}'\mathbf{k}}. \quad (7)$$

Since absorption of a high-frequency field satisfying inequality (2) is negligibly small, the field does not change the equilibrium distribution function of electrons [29]. Therefore, the field influences on the stationary (dc) transport of dressed electrons only through the renormalization of the scattering probability $w_{\mathbf{k}'\mathbf{k}}$ given by Eq. (3). It should be stressed that condition (2) is crucial in order to consider a 2DEG subjected to the field as an equilibrium system. Otherwise, the photon-assisted scattering of electrons is accompanied by absorption of field energy and leads to heating 2DEG. In this case, there are no stationary transport characteristics of the 2DEG and the problem should be reformulated in terms of the nonequilibrium electron transport (see, e.g., Refs. 30,31).

Substituting the scattering probability of dressed electron (3) into Eq. (7), we can obtain from Eqs. (6) and (7) the conductivity of dressed 2DEG, $\sigma_{ij} = J_i/E_j$.

To simplify calculations, let us consider the electron scattering within the s -wave approximation [32], where the matrix elements $U_{\mathbf{k}'\mathbf{k}}$ do not depend on the angle $\theta_{\mathbf{k}'\mathbf{k}} = \widehat{(\mathbf{k}', \mathbf{k})}$. Substituting Eq. (5) into Eqs. (3), (6), and (7), we arrive at the isotropic conductivity of 2DEG dressed by circularly polarized field σ_c , which is given by the expression

$$\frac{\sigma_c}{\sigma_0} = 2\pi \left[\int_0^{2\pi} [1 - \cos\theta] J_0^2\left(\frac{2eE_{\omega}k_F}{m\omega^2} \sin\frac{\theta}{2}\right) d\theta \right]^{-1}, \quad (8)$$

where σ_0 is the conductivity of an unirradiated 2DEG. In the case of a linearly polarized dressing field, the conductivity tensor σ_{ij} has a simple diagonal form $\sigma_{ii} = (\sigma_{\parallel}, \sigma_{\perp})$ in the basis of two axes which are parallel (\parallel) and perpendicular (\perp) to the wave field $\tilde{\mathbf{E}}$, respectively. Substituting Eq. (4) into Eqs. (3), (6), and (7), we arrive at

$$\frac{\sigma_{\parallel}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_{\parallel}(\theta)}{\tau_0} \cos^2\theta d\theta \quad (9)$$

and

$$\frac{\sigma_{\perp}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_{\perp}(\theta)}{\tau_0} \sin^2\theta d\theta, \quad (10)$$

where $\tau_{\parallel}(\theta)$ and $\tau_{\perp}(\theta)$ are the relaxation time (7) at the Fermi energy for the dc electric field $\mathbf{E} = (E_{\parallel}, 0)$ and $\mathbf{E} = (0, E_{\perp})$, respectively [see Fig. 1(a)], and $\theta = \widehat{(\mathbf{k}, \mathbf{E})}$.

The conductivities of irradiated 2DEG [Eqs. (8)–(10)], which can be easily calculated numerically, are plotted in Fig. 2. It is seen that the irradiation of 2DEG by a dressing light results in a giant increase of conductivity, which can reach thousands of percents. Physically this increase follows from the fact that the dressing field significantly decreases the scattering probability of dressed 2DEG (3). From the mathematical viewpoint, this is a consequence of the rapid decrease of the Bessel function in Eq. (3) versus the dressing field amplitude E_{ω} . As to the oscillations of the plotted conductivity, this is a formal consequence of the oscillating behavior of the Bessel function in the probability (3). In order to give a qualitative physical explanation of this behavior of conductivity, we have to stress that the Born scattering

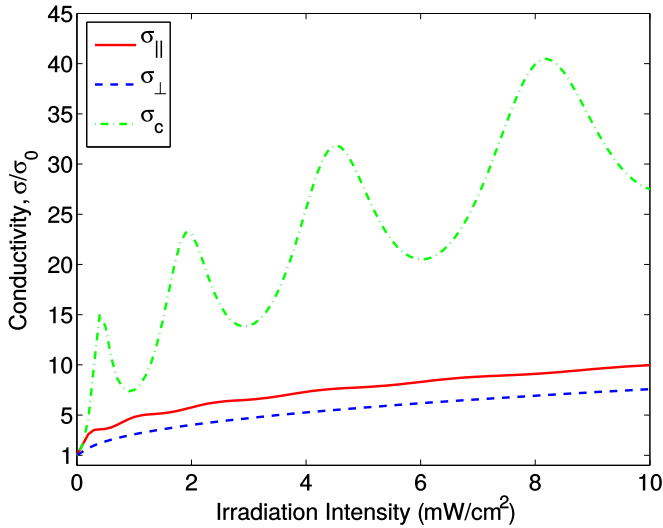


FIG. 2. (Color online) The conductivity of 2DEG in a GaAs quantum well irradiated by a dressing electromagnetic field with the frequency $\omega = 10^{11}$ rad/s at the temperature $T = 0$ for $\varepsilon_F = 10$ meV.

probability (3) is described by the overlap of wave functions of an incident electron with the wave vector \mathbf{k} and a scattered electron with the wave vector \mathbf{k}' in the area of scattering potential U . In turn, this overlap depends on the difference of electron phases in the high-frequency field, which are presented by the terms with $\sin \omega t$ and $\cos \omega t$ in Eq. (A2) (see also Eq. (6) in Ref. [29]). Since the terms depend on the field amplitude E_ω , the stationary overlap of the wave functions—and, correspondingly, the scattering probability (3)—can vanish for certain field amplitudes which correspond to the zeros of the Bessel function. As a consequence, this leads to both the increasing of conductivity and the oscillations of the conductivity, which are seen in Fig. 2.

The difference between the conductivities $\sigma_{||}$ and σ_{\perp} arises from the scattering anisotropy of a 2DEG dressed by a linearly polarized field, which follows directly from Eqs. (3) and (4). It should be stressed that the arguments of the Bessel function in the scattering probability (3) for the cases of linearly polarized field (4) and circularly polarized field (5) strongly differ from each other. Therefore, the integration over \mathbf{k} and \mathbf{k}' and in the scattering probability (3) substituted into the kinetic Boltzmann equation leads to strongly different plots in Fig. 2 for linear polarization and circular polarization. Particularly, the amplitude of oscillations of conductivity in the case of linear polarization is nonzero but very small as compared to the case of circular polarization. Therefore, the most appropriate experimental set for observing the oscillations should be based on using a circularly polarized light.

IV. WEAK LOCALIZATION OF DRESSED 2DEG

Multiple scattering of electrons in a conductor leads to the self-interference of electron waves propagating along a closed trajectory in mutually opposite—clockwise and counterclockwise—directions. As a result of the interference, the well-known weak localization (WL) of conduction electrons appears [35–37]. To find the WL correction to the

conductivity of dressed 2DEG, $\Delta\sigma$, one needs to consider all possible closed electron trajectories in the 2DEG irradiated by a dressing electromagnetic field. Going this way, the sought correction can be calculated using the conventional expression [35]

$$\frac{\Delta\sigma}{\sigma_0} = -\frac{\hbar}{m} \int_{\tau_F}^{\tau_\varphi} C(t) dt, \quad (11)$$

where $C(t)$ is the probability of finding an electron in the initial point of its trajectory at time t averaged over the 2DEG plane, τ_F is the mean free time of conduction electron at the Fermi energy, which is given by the expression

$$\frac{1}{\tau_F} = \sum_{\mathbf{k}_F} w_{\mathbf{k}_F \mathbf{k}'_F}, \quad (12)$$

and τ_φ is the effective breaking time of the electron phase. Generally the electron phase can be broken by both inelastic scattering and a magnetic field which destroy WL [38–43]. The breaking time of the electron phase caused by a magnetic field B is $\tau_{\varphi B} = \hbar/4eDB$, where D is the diffusion constant [43]. Correspondingly, the effective breaking time of the electron phase τ_φ can be written as $1/\tau_\varphi = 1/\tau_{\varphi\varepsilon} + 1/\tau_{\varphi B}$, where $\tau_{\varphi\varepsilon}$ is the characteristic breaking time arisen from inelastic processes. As to the mean free time of conduction electron τ_F , it can be found by substituting the scattering probability of dressed electron (3) into Eq. (12). It has been mentioned above that this scattering probability is anisotropic for 2DEG dressed by a linearly polarized field. Therefore, the time (12) depends on the initial electron wave vector \mathbf{k}_F . To describe this anisotropy, it is suitable to introduce the two mean free times $\tau_{F||}$ and $\tau_{F\perp}$ for the cases of $\mathbf{k}_F \parallel \tilde{\mathbf{E}}$ and $\mathbf{k}_F \perp \tilde{\mathbf{E}}$, respectively. Substituting these two times into Eq. (11), we arrive at the two WL corrections, $\Delta\sigma_{||}$ and $\Delta\sigma_{\perp}$. These two corrections describe the conductivity of dressed 2DEG for the cases of $\mathbf{E} = (E_{||}, 0)$ and $\mathbf{E} = (0, E_{\perp})$, respectively [see Fig. 1(a)]. Certainly, in the case of a circularly polarized dressing field, the anisotropy vanishes and the corresponding WL correction $\Delta\sigma_c$ does not depend on direction in the 2DEG plane.

Within the conventional theory of WL, the probability $C(t)$ in Eq. (11) can be described by the Feynman path integral [36,44]

$$C(t) = \int \mathcal{D}x \mathcal{D}y \exp \left[-\int_{-t/2}^{t/2} \frac{\dot{x}^2}{4D_x} + \frac{\dot{y}^2}{4D_y} - \frac{i}{\hbar} e \mathbf{r} [\tilde{\mathbf{E}}(t') - \tilde{\mathbf{E}}(-t')] dt' \right], \quad (13)$$

where the integration should be performed over closed electron trajectories per unit plane of 2DEG, and $\mathbf{r} = (x, y)$ is the electron radius vector in the 2DEG plane. In the integrand of the path integral (13), the first two terms in the exponent describe the kinetic energy of the diffusion propagation of a dressed electron along the path, where $D_{x,y}$ are the diffusion constants along the x, y axes. Generally these constants can be written as $D_x = v_F^2 \tau_{F||}/2$ and $D_y = v_F^2 \tau_{F\perp}/2$ (see, e.g., Ref. [43]), where v_F is the Fermi velocity, and the x axis is assumed to be directed along the electric field $\tilde{\mathbf{E}}$ of a linearly polarized dressing field. In the case of a circularly polarized dressing field, we have $\tau_{F||} = \tau_{F\perp}$ and, therefore, the x, y axes

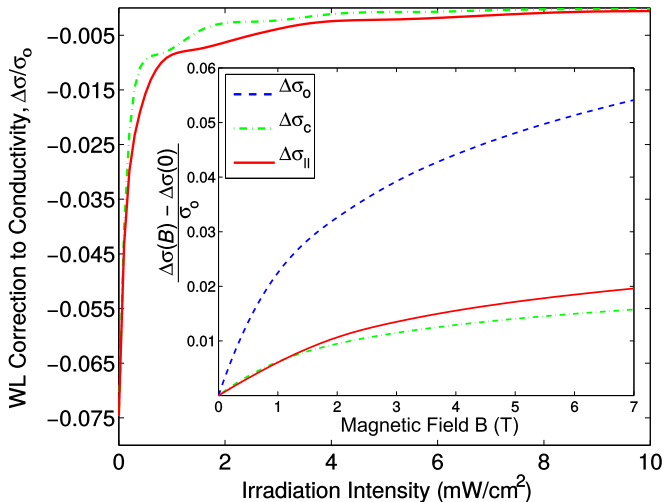


FIG. 3. (Color online) The dependence of weak localization corrections to conductivity of 2DEG on the intensity of irradiation by a dressing field. Calculations are performed for 2DEG in a GaAs quantum well at the temperature $T = 0$ for $\varepsilon_F = 10$ meV and $\tau_{\varphi\sigma} = 80$ ps. The inset demonstrates the dependence of the corrections on a magnetic field directed perpendicularly to the 2DEG in the presence of the irradiation with the intensity $I = 3$ mW/cm².

can be chosen arbitrarily. The last term in the exponent takes into account the potential energy of an electron in the dressing field and describes the self-interference of the dressed electron between the time-reversed trajectories. Performing the path integration in Eq. (13) within the conventional procedure [45] and substituting the obtained probability (13) into Eq. (11), we arrive at the WL corrections to the conductivity of dressed 2DEG, which are presented in Fig. 3.

It is seen in Fig. 3 that the irradiation of 2DEG by a dressing field leads to decreasing WL corrections to the conductivity. Physically this is a consequence of suppressed scattering in a dressed 2DEG, which has been mentioned above. As a result of the suppression, the probability of electron movement along a closed trajectory in the dressed 2DEG decreases and, correspondingly, WL corrections to the conductivity of dressed 2DEG decrease as well. It should be stressed that this effect differs significantly from the known suppression of WL due to intraband absorption of irradiation by conduction electrons [46–48]. Indeed, under condition (2) there is no absorption of a high-frequency field by conduction electrons. Therefore, the destruction of WL, which arises from the breaking of the electron phase in inelastic electron-photon processes [46–48], is negligibly small in the 2DEG under consideration. As a consequence of decreasing WL effects, the influence of a magnetic field on WL corrections to the conductivity for a dressed 2DEG is less than one for a bare 2DEG (see the inset in Fig. 3). It should be noted that the WL correction to the conductivity of 2DEG has a logarithmic dependence on a magnetic field [35]. Although the irradiation on the 2DEG reduces WL corrections, the behavior of conductivity of dressed 2DEG in a magnetic field obeys the same law. As to small oscillations visible in Fig. 3, they have the same nature as oscillations of conductivity pictured in Fig. 2. Anisotropy of weak localization in the case of a linearly

polarized field is within 1% and cannot be easily detected experimentally. Therefore, we have $\Delta\sigma_{\parallel} \approx \Delta\sigma_{\perp}$ for the plots in Fig. 3.

V. CONCLUSION

We demonstrated theoretically that the strong coupling of 2DEG to a dressing electromagnetic field results in a set of such unexpected transport phenomena as a giant increase of conductivity, oscillating behavior of conductivity versus intensity of dressing field, and suppression of weak localization effects. These phenomena open a way to control the transport properties of 2DEG by a strong high-frequency electromagnetic field and, therefore, form a basis for optoelectronic nanodevices.

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APPENDIX: SCATTERING OF 2DEG DRESSED BY A CIRCULARLY POLARIZED ELECTROMAGNETIC FIELD

For definiteness we will restrict our consideration to the case of a two-dimensional electron system subjected to a circularly polarized electromagnetic wave propagating perpendicularly to the system. Let the system lie in the plane (x, y) at $z = 0$ and the wave propagate along the z axis. Then the electron properties of the system in the absence of scatterers are described by the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{\mathcal{H}}_0 \psi, \quad (\text{A1})$$

with the Hamiltonian $\hat{\mathcal{H}}_0 = [\hat{\mathbf{p}} - e\mathbf{A}(t)]^2/2m$, where $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y)$ is the operator of electron momentum, m is the effective electron mass, e is the electron charge,

$$\mathbf{A}(t) = \mathbf{e}_x(E_\omega/\omega) \sin \omega t + \mathbf{e}_y(E_\omega/\omega) \cos \omega t$$

is the vector potential of the wave, E_ω is the electric field amplitude of the wave, ω is the frequency of the wave, and $\mathbf{e}_{x,y}$ are the unit vectors along the x, y axis. The Schrödinger equation (A1) can be solved accurately and the exact wave function of the electron is

$$\psi_{\mathbf{k}}(\mathbf{r}, t) = \exp \left[-i \left(\frac{\varepsilon_k t}{\hbar} + \frac{E_\omega^2 e^2 t}{2m\omega^2 \hbar} - \frac{E_\omega e k_y}{m\omega^2} \sin \omega t + \frac{E_\omega e k_x}{m\omega^2} \cos \omega t \right) \right] \varphi_{\mathbf{k}}(\mathbf{r}), \quad (\text{A2})$$

where $\varphi_{\mathbf{k}}(\mathbf{r}) = V^{-1/2} \exp(i\mathbf{k}\mathbf{r})$ is the plane electron wave, $\mathbf{k} = (k_x, k_y)$ is the electron wave vector, $\mathbf{r} = (x, y)$ is the electron radius vector, V is the normalization volume, and $\varepsilon_k = \hbar^2 k^2/2m$ is the energy spectrum of a free electron. Evidently the wave function (A2) can be easily verified by direct substitution into the Schrödinger equation (A1). Introducing the polar system $\{k, \varphi\}$, we can write the electron

wave vectors as $k_x = k \cos \varphi$ and $k_y = k \sin \varphi$. Then the wave function (A2) takes the simplest form

$$\psi_{\mathbf{k}}(\mathbf{r}, t) = \exp \left[-i \left(\frac{\varepsilon_k t}{\hbar} + \frac{E_\omega^2 e^2 t}{2m\omega^2 \hbar} + \frac{E_\omega e k}{m\omega^2} \cos(\omega t + \varphi) \right) \right] \varphi_{\mathbf{k}}(\mathbf{r}). \quad (\text{A3})$$

Let an electron move in a total scattering potential $U(\mathbf{r})$ in the presence of the same field. Then the wave function of the electron $\Psi(\mathbf{r}, t)$ satisfies the Schrödinger equation

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = [\hat{\mathcal{H}}_0 + U(\mathbf{r})] \Psi(\mathbf{r}, t). \quad (\text{A4})$$

Assuming the scattering potential energy $U(\mathbf{r})$ to be a small perturbation, we can apply the conventional perturbation theory to describe the electron scattering. Since the functions (A3) with different wave vectors \mathbf{k} form the complete function system for any time t , we can seek solutions of the Schrödinger equation (A4) as an expansion

$$\Psi(\mathbf{r}, t) = \sum_{\mathbf{k}'} a_{\mathbf{k}'}(t) \psi_{\mathbf{k}'}(\mathbf{r}, t). \quad (\text{A5})$$

Let an electron be in the state (A3) with the wave vector \mathbf{k} at the time $t = 0$. Correspondingly, $a_{\mathbf{k}'}(0) = \delta_{\mathbf{k}', \mathbf{k}}$, where $\delta_{\mathbf{k}', \mathbf{k}}$ is the Kronecker symbol. In what follows, we will assume that the wave frequency ω is large enough to satisfy the inequality

$$\omega \tau_0 \gg 1, \quad (\text{A6})$$

where τ_0 is the characteristic relaxation time of the conduction electron in the absence of the wave. Under the condition (A6), we can neglect the absorption (emission) of field energy by a scattered electron (see, e.g., Refs. [27,28]). Within this approximation, the scattering potential $U(\mathbf{r})$ mixes only electron states \mathbf{k} and \mathbf{k}' with the same energy $\varepsilon_k = \varepsilon_{k'}$ [29]. Therefore, it is enough to take into account only terms with $k' = k$ in the expansion (A5). Substituting the expansion (A5) into the Schrödinger equation (A4) and restricting the accuracy by the first order of perturbation theory, we arrive at the expression

$$a_{\mathbf{k}'}(t) = -i \frac{U_{\mathbf{k}\mathbf{k}'}}{\hbar} \int_0^t e^{i(\varepsilon_{k'} - \varepsilon_k)t'/\hbar} e^{i f_{\mathbf{k}\mathbf{k}'} \sin[\omega t' + (\varphi + \varphi')/2]} dt', \quad (\text{A7})$$

where

$$U_{\mathbf{k}\mathbf{k}'} = \langle \varphi_{\mathbf{k}'}(\mathbf{r}) | U(\mathbf{r}) | \varphi_{\mathbf{k}}(\mathbf{r}) \rangle \quad (\text{A8})$$

is the matrix element of the scattering potential,

$$f_{\mathbf{k}\mathbf{k}'} = \frac{2E_\omega e k}{m\omega^2} \sin\left(\frac{\theta}{2}\right), \quad (\text{A9})$$

and $\theta = \varphi - \varphi' = \widehat{(\mathbf{k}, \mathbf{k}'})$ is the scattering angle. Let us apply the Jacobi-Anger expansion,

$$e^{iz \sin \gamma} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\gamma},$$

in order to rewrite Eq. (A7) as

$$|a_{\mathbf{k}'}(t)|^2 = \frac{|U_{\mathbf{k}\mathbf{k}'}|^2}{\hbar^2} \left| \sum_{n=-\infty}^{\infty} J_n(f_{\mathbf{k}\mathbf{k}'}) e^{i(\varepsilon_{k'} - \varepsilon_k + n\hbar\omega)t/2\hbar} \times e^{in(\varphi + \varphi')/2} \int_{-t/2}^{t/2} e^{i(\varepsilon_{k'} - \varepsilon_k + n\hbar\omega)t'/\hbar} dt' \right|^2, \quad (\text{A10})$$

where $J_n(z)$ is the n th order Bessel function of the first kind. Since the integrals in Eq. (A10) for long time t turn into the δ function

$$\delta(\varepsilon) = \frac{1}{2\pi\hbar} \lim_{t \rightarrow \infty} \int_{-t/2}^{t/2} e^{i\varepsilon t'/\hbar} dt',$$

the expression (A10) takes the form

$$|a_{\mathbf{k}'}(t)|^2 = 4\pi^2 |U_{\mathbf{k}\mathbf{k}'}|^2 \sum_{n=-\infty}^{\infty} J_n^2(f_{\mathbf{k}\mathbf{k}'}) \delta^2(\varepsilon_{k'} - \varepsilon_k + n\hbar\omega). \quad (\text{A11})$$

To transform the square δ functions in Eq. (A11), we can apply the conventional procedure,

$$\delta^2(\varepsilon) = \delta(\varepsilon)\delta(0) = \frac{\delta(\varepsilon)}{2\pi\hbar} \lim_{t \rightarrow \infty} \int_{-t/2}^{t/2} e^{i0 \times t'/\hbar} dt' = \frac{\delta(\varepsilon)t}{2\pi\hbar}.$$

Keeping in mind that $\varepsilon_k = \varepsilon_{k'}$, the probability of the electron scattering between the states (A2) with the wave vectors \mathbf{k} and \mathbf{k}' per unit time, $w_{\mathbf{k}\mathbf{k}'} = d|a_{\mathbf{k}'}(t)|^2/dt$, is given by

$$w_{\mathbf{k}\mathbf{k}'} = \frac{2\pi}{\hbar} J_0^2(f_{\mathbf{k}\mathbf{k}'}) |U_{\mathbf{k}\mathbf{k}'}|^2 \delta(\varepsilon_{k'} - \varepsilon_k). \quad (\text{A12})$$

Since Eq. (A9) depends only on the electron energy $\varepsilon_k = \hbar^2 k^2/2m$ and the scattering angle θ , the probability (A12) describes the isotropic scattering which can be tuned by the wave amplitude E_ω and the wave frequency ω .

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