Quantum corrections to the conductivity of itinerant antiferromagnets

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We present a systematic calculation of the effects of scattering of electrons off spin waves on electron transport properties in itinerant antiferromagnetic thin films in two and three dimensions. We study various regimes set by the parameters related to the spin-wave gap, exchange energy, as well as the exchange splitting, in addition to the scales set by temperature and disorder. We find an interaction-induced quantum correction to the conductivity linear in temperature, similar to that obtained recently for ferromagnetic systems within a certain regime of disorder, although the disorder dependence is different. In addition, we explore the phase relaxation rates and the associated weak-localization corrections for both small and large spin-wave gaps. We obtain a wide variety of temperature and disorder dependence for various parameter regimes. These results should provide an alternative way to study magnetic properties of thin antiferromagnetic films, for which neutron scattering measurements could be difficult, by direct transport measurements.

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I. INTRODUCTION

Quantum corrections to conductivity due to weak localization and Coulomb interaction effects [1,2] have now been observed in numerous disordered metallic systems [3,4]. These corrections have a logarithmic temperature (T) dependence in two dimensions (2D). In three dimesions (3D), the weak-localization corrections have a $T^{1/2}$ dependence while the Coulomb interaction corrections are $T^{3/2}$. In the context of spintronics, quantum corrections to transport of charge carriers in dilute magnetic semiconductor films have been studied extensively [5,6]. In contrast, for itinerant metallic magnetic systems, spin-wave mediated interaction corrections to the conductivity (labeled Altshuler-Aronov correction [2] or "AA" correction in the following) have only started to be explored. This work has been motivated by challenging experimental data on the conductivity of magnetically ordered films of typically a few nanometers thickness. In the case of ferromagnetic Fe films, it was found that 2D weak-localization corrections appear in both the longitudinal and the Hall conductivities [7]. The variation of the prefactor of the logarithm in temperature in the Hall conductivity has been successfully explained by the interplay of skew scattering and side-jump mechanisms producing an anomalous Hall effect [8]. By contrast, in ferromagnetic Gd films, a localizing contribution to the conductivity linear in temperature has been observed in addition to a logarithmically varying term [9,10]. While the latter is well accounted for by the usual weak-localization and AA contribution for 2D systems, the linear T term was shown to arise from scattering off ferromagnetic spin-wave excitations, as presented in the following. These quantum corrections are sizable, in the case of Gd films they are of order 10%. In particular, the linear temperature dependence was observed in thin 2D disordered ferromagnetic films of Gd where the spin-wave gap is negligible. On the other hand, since the spin-wave gap, the exchange interaction, as well as the exchange splitting introduce additional energy scales that compete with the usual scales set by temperature and disorder, quantum corrections to the conductivity from scattering of electrons off of spin waves can be expected to have a wide range of distinctive behavior in different regimes that are not available in nonmagnetic systems. It is observed that even for thin-film geometries, the system may effectively be in the three-dimensional regime, if the phase-breaking length is less than the film thickness, which may happen on account of the strong scattering off spin-wave excitations [11–13]. Indeed, recent studies of thin antiferromagnetic films of Mn, with a spin-wave gap of about 15–20 K, have revealed unusual temperature as well as disorder dependence [14] that shows the importance of exploring the different regimes in these systems. These distinct behaviors also provide an opportunity to study certain magnetic properties of thin films by direct transport measurements done in specific regimes, specially when neutron scattering measurements are difficult. For example in Ref. [14], it was possible to extract both the spin-wave gap and the exchange energy from the detailed study of the temperature and disorder dependence of the quantum correction to the conductivity. In addition, it turns out that antiferromagnetic thin Mn films can be made highly disordered without going through the metal-insulator transition. In this regime, a nonuniversal weak localization correction in 3D was found. The observed corrections to the conductivity are large, of order 50%. Thus, specifically for antiferromagnetic systems, one expects a wide variety of temperature as well as disorder dependence of the phase-breaking lengths at different regimes in the large available parameter space.

However, while phase-breaking lengths and corrections to conductivity for ferromagnets have been studied before [9,13,15], and some results for antiferromagnetic films have been reported while explaining experimental observations [14], a systematic study of the antiferromagnetic systems is still missing. In this work, we provide some details of results used in Ref. [14], as well as results in parameter regimes not yet reported elsewhere. In particular, we obtain a linear T dependence for the quantum correction to the conductivity for antiferromagnetic systems just like the corresponding ferromagnetic case obtained before. We take the opportunity to provide some details of the results obtained in Ref. [9] on the disorder dependence of conductivity for a 2D ferromagnetic film at large disorder, and then show how the disorder dependence changes for an antiferromagnetic system. We systematically study the phase-breaking time and associated weak-localization corrections where the inverse phase-breaking time can be larger or smaller compared to the spin-wave gap in 2D as well as 3D systems. We show how the dispersion relation for antiferromagnets allows for an interplay with disorder that is qualitatively different compared to a ferromagnet, giving rise to weak-localization effects. Instead of numerically evaluating the temperature or disorder dependence accurately, we consider various limiting cases where analytic expressions allow one to have a more detailed understanding of the system. For example, in a numerical plot, a crossover temperature dependence, where the temperature is of the order of the spin-wave gap, can be hard to distinguish from a fractional power law; we show how both can be achieved in different parameter regimes.

The paper is organized as follows. In Sec. II, we introduce the model Hamiltonian and obtain the formula for the "AA" quantum correction to conductivity due to scattering of electrons off of spin waves for magnetic systems with arbitrary spin-wave gap, exchange energy, and exchange splitting. The general formula obtained is then used in Sec. III to obtain the temperature and disorder dependence of the conductivity for both ferromagnetic and antiferromagnetic systems. In Sec. IV, we obtain the temperature and disorder dependence of the inelastic lifetime due to scattering off spin waves in an antiferromagnetic system in various parameter regimes. These results are then used to obtain the weak-localization corrections in Sec. V. Section VI contains a brief summary and discussions. The Appendix contains some details of the results used in Sec. II regarding calculations of the conductivity diagrams shown in Fig. 1.

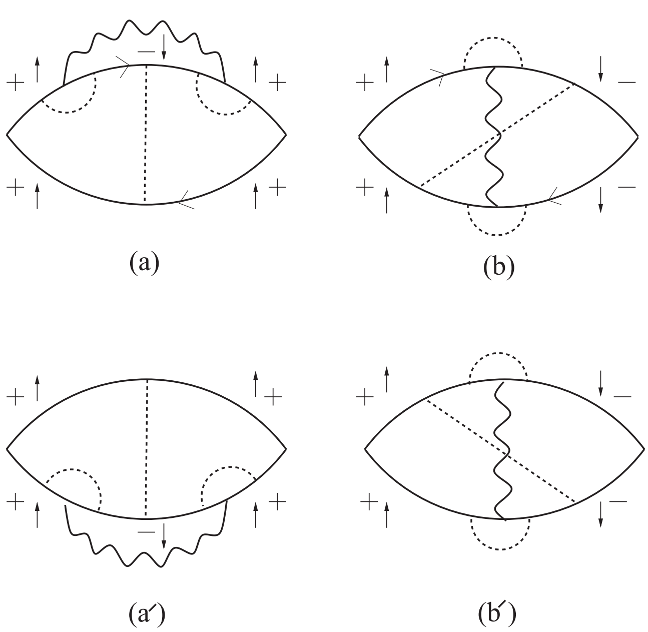

FIG. 1. Diagrams representing spin-wave contributions to the conductivity. Solid lines represent impurity averaged Green's functions G_σ, while the broken lines are the "diffusons" $\Gamma_{\sigma\sigma'}$. The wavy lines are the effective spin-wave-mediated interactions $v(q,\omega)$.

II. SPIN-WAVE-INDUCED AA CORRECTION TO THE CONDUCTIVITY

Our starting Hamiltonian for itinerant electrons scattering from impurities as well as spin waves is [13]

$$H = \sum_{\mathbf{k}\sigma}\left(\epsilon_{\mathbf{k}} - \frac{1}{2}\sigma B\right)c^+_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}/\sigma,\mathbf{j}} V(\mathbf{k}-\mathbf{k}')e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_j}c^+_{\mathbf{k}/\sigma}c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}}\omega_{\mathbf{q}}a^+_{\mathbf{q}}a_{\mathbf{q}} + J\sum_{\mathbf{q},\mathbf{k}}[a^+_{\mathbf{q}}c^+_{\mathbf{k}+\mathbf{q}\downarrow}c_{\mathbf{k}\uparrow} + \text{H.c.}], \quad (2.1)$$

where the annihilation and creation operators c_k, c^+_k refer to electrons and a_q, a^+_q refer to spin waves, J is the effective spin-exchange interaction, and B is the exchange splitting. The potential V models the impurity scattering. For the moment, we keep the derivation valid for both ferromagnets and antiferromagnets by choosing the spin wave to have a dispersion relation (at not too large wave vectors)

$$\omega_q = \Delta + A_\alpha q^\alpha, \quad (2.2)$$

where Δ is the spin-wave gap and A_α is the spin stiffness. Here, $\alpha = 2$ for ferromagnetic spin waves while $\alpha = 1$ for antiferromagnets. There are many different regimes, owing to the several characteristic energy scales. The energy scales entering the Hamiltonian are (1) Fermi energy E_F (typically several eV), (2) exchange coupling energy $\bar{J} = nJ$ (where n is the electron density; nJ is typically of order 0.1 eV), (3) exchange splitting B (0.001 to 0.1 eV), (4) spin-wave gap Δ (of order 1 meV). Then, we have two derived energy scales (5) the impurity scattering rate $1/\tau$ (in the range 0.05 to 0.5 E_F), and (6) the phase-breaking rate $1/\tau_\phi$ (of order $0.1/\tau$). These should be compared with the thermal energy T. We will be interested in the parameter regime where E_F is the largest energy scale. The exchange energy $\bar{J}$ is typically the next largest energy scale. The exchange splitting B and the spin-wave gap Δ may be larger or smaller, and both may be larger or smaller than the derived energy scales $1/\tau$ or $1/\tau_\phi$. The spin stiffness is approximately related to the exchange energy by $A_\alpha k_F^\alpha = \bar{J}$. The spin-wave propagator is

$$S_{\uparrow\downarrow}(q,\omega_n) = 1/[i\omega_n - \omega_q + i\gamma(q)] = [S_{\downarrow\uparrow}]^*, \quad (2.3)$$

where the star denotes complex conjugation, $\omega_n = 2\pi nT$ is the bosonic Matsubara frequency, and $\gamma(q)$ is a phenomenological damping parameter that depends on q as some power q^β with $\beta \geqslant 2$. The spin-wave-mediated effective interaction is given by

$$v(q,\omega_l) = nJ^2[S_{\uparrow\downarrow}(q,\omega_l) + S_{\downarrow\uparrow}(q,\omega_l)] = -\frac{2nJ^2\omega_q}{(|\omega_l| + \gamma)^2 + \omega_q^2}, \quad (2.4)$$

which is attractive. Here, n is the density of conduction electrons.

The conductivity diagrams are shown in Fig. 1. For finite Zeeman splitting B, the spin-dependent Green's functions G_σ for $\sigma = \uparrow,\downarrow$ have different energies, $G_\sigma^{-1}(k,\omega_n) = i\omega_n - \epsilon_{k\sigma} + \frac{i}{2\tau}\text{sign}(\omega_n)$. We choose $\epsilon_{k\uparrow} = \epsilon_{k\downarrow} - B$. The particle-hole (ph) diffuson propagator in the presence of a finite B is given by [13]

$$\Gamma^{+-}_{\uparrow\downarrow}(q,\omega_l,B) = \frac{1}{2\pi N_0\tau}\frac{1}{1-\frac{\tilde{\tau}^*}{\tau}+\frac{\tilde{\tau}^{*2}}{\tau}(|\omega_l|+\tilde{D}^*q^2)} = \frac{1}{2\pi N_0\tau}\frac{\tau}{\tilde{\tau}^{*2}}\frac{1}{|\omega_l|+\tilde{D}^*q^2+\tilde{\delta}^*}, \tag{2.5}$$

where we have defined

$$\tilde{D} \equiv \frac{1}{d}v_F^2\tilde{\tau};\quad \frac{1}{\tilde{\tau}} \equiv \frac{1}{\tau} - iB;\quad \tilde{\delta} \equiv \frac{\tau}{\tilde{\tau}^2} - \frac{1}{\tilde{\tau}} = -iB(1-iB\tau). \tag{2.6}$$

We only show the evaluation of Fig. 1(a) here in some detail and leave the rest for the Appendix. Using standard rules of diagrams [1,2] we have, for Fig. 1(a),

$$L^{1(a)} = -T\sum_{\epsilon_n}T\sum_{\omega_l}\sum_q C_1^2C_2^2 \times \Gamma^2_{\uparrow\downarrow}(q,\omega_l)\Gamma_{\uparrow\downarrow}(q,\omega_l-\Omega_m)v(q,\omega_l) \times \Theta[\epsilon_n(\epsilon_n-\Omega_m)]\Theta[-\epsilon_n(\epsilon_n-\omega_l)]. \tag{2.7}$$

Here,

$$\begin{aligned} C_1 &\equiv \sum_k (G^+_{k\uparrow})^2G^-_{k-q\downarrow}\frac{k_x}{m} \\ &\approx \sum_k (G^+_{k\uparrow})^2\left[G^-_{k\downarrow} + \left(i|\omega_l| - \frac{kq}{m}\right)(G^-_{k\downarrow})^2 + \cdots\right]\frac{k_x}{m} \\ &= \sum_k (G^+_{k\uparrow})^2(G^-_{k\downarrow})^2\left\langle\frac{-k_x^2}{m^2}\right\rangle_\theta q_x \\ &= -4\pi N_0\tilde{D}^*\tilde{\tau}^{*2}q_x, \end{aligned} \tag{2.8}$$

where the angular average is obtained from $\langle k_x^2\rangle_\theta = \frac{1}{d}k_F^2$, and

$$C_2 \equiv \sum_k G^+_{k\uparrow}G^-_{k-q\downarrow} \approx 2\pi N_0\tilde{\tau}^*[1-|\omega_l|\tilde{\tau}^* - \tilde{D}^*q^2\tilde{\tau}^*]. \tag{2.9}$$

In the above we used $\epsilon_n > 0$, so that $\epsilon_n - \Omega_m > 0$, $\epsilon_n - \omega_l < 0$, such that

$$T\sum_{\epsilon_n=\Omega_m}^{\omega_l} = (\omega_l - \Omega_m);\quad \omega_l > \Omega_m. \tag{2.10}$$

On the other hand, for $\epsilon_n < 0$, we have $\epsilon_n - \Omega_m < 0$, $\epsilon_n - \omega_l > 0$, such that

$$T\sum_{\epsilon_n=\omega_l}^{0} = |\omega_l|;\quad \omega_l < 0. \tag{2.11}$$

In this case, C_1 and C_2 are given by the above expressions with $G^\pm$ replaced by $G^\mp$, which essentially replaces C_1 and C_2 by their complex conjugates. We thus obtain, for Fig. 1(a),

$$L^{1(a)} = -T\sum_{\omega_l>\Omega_m}\omega^-_{lm}\sum_q q^2F(q,\omega_l)K(q,\omega^-_{lm}) - T\sum_{\omega_l>0}|\omega_l|\sum_q q^2F^*(q,-\omega_l)K^*(q,-\omega^+_{lm}), \tag{2.12}$$

where we have defined

$$\omega^\pm_{lm} \equiv \omega_l \pm \Omega_m, \tag{2.13}$$

$$F(q,\omega_l) = 4\pi N_0\tilde{D}^{*2}[1-|\omega_l|\tilde{\tau}^* - \tilde{D}^*q^2\tilde{\tau}^*]^2 \times \frac{v(q,\omega_l)}{(|\omega_l|+\tilde{D}^*q^2+\tilde{\delta}^*)^2} \tag{2.14}$$

and

$$K(q,\omega^-_{lm}) = \frac{1}{|\omega^-_{lm}|+\tilde{D}^*q^2+\tilde{\delta}^*}. \tag{2.15}$$

Figure 1(a′) has the interaction on the lower line. As shown in the Appendix, it gives

$$L^{1(a')} = (L^{1(a)})^*. \tag{2.16}$$

Figures 1(b) and 1(1b′) are also evaluated similarly in the Appendix. The results are

$$L^{1(b)} = T\sum_{\omega_l>\Omega_m}(\omega^-_{lm})\sum_q q^2F(q,\omega_l)K(q,\omega^-_{lm}) + T\sum_{\omega_l>\Omega_m}(\omega^-_{lm})\sum_q q^2F^*(q,-\omega_l)K^*(q,-\omega^+_{lm}),$$
$$L^{1(b')} = (L^{1(b)})^*. \tag{2.17}$$

Adding all contributions from the four diagrams we get

$$\begin{aligned} L^{(\text{sum})} &= T\sum_{\omega_l>0}\omega_l\sum_q q^2F^*(q,-\omega_l)K^*(q,-\omega^+_{lm}) \\ &\quad + T\sum_{\omega_l>\Omega_m}(\omega^-_{lm})\sum_q q^2F^*(q,-\omega_l)K^*(q,\omega^+_{lm}) + \text{c.c.} \\ &= -T\left[\sum_{\omega_l=0}^{\Omega_m}\omega_l + \sum_{\omega_l=\Omega_m}^{\infty}\Omega_m\right]C(\omega_l,\Omega_m) + \text{c.c.}, \end{aligned} \tag{2.18}$$

where we have defined

$$C(\omega_l,\Omega_m) = \widetilde{C}_0\sum_q[1-|\omega_l|\tilde{\tau} - \tilde{D}q^2\tilde{\tau}]^2 \times \frac{q^2v(q,\omega_l)}{(|\omega_l|+\tilde{D}q^2+\tilde{\delta})^2(|\omega^+_{lm}|+\tilde{D}q^2+\tilde{\delta})} \tag{2.19}$$

with

$$\widetilde{C}_0 = 4\pi N_0\tilde{D}^2. \tag{2.20}$$

We now define $\widetilde{C}(\omega_l,\Omega_m) = C(\omega_l,\Omega_m) + \text{c.c.}$, then we can follow Ref. [2] to analytically continue to the complex ω plane and obtain

$$L = -\frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega\, \omega \coth\frac{\omega}{2T} \times [\widetilde{C}[-i(\omega+\Omega),-i\Omega] - \widetilde{C}(-i\omega,-i\Omega)]. \quad (2.21)$$

The conductivity correction is the $\Omega \to 0$ limit after dividing by Ω, which gives

$$\delta\sigma = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega\, \widetilde{C}(-i\omega,0)\frac{\partial}{\partial\omega}\left[\omega \coth\frac{\omega}{2T}\right]. \quad (2.22)$$

This agrees with the Altshuler-Aronov correction (or the "AA correction") in Ref. [2] when $\widetilde{C}$ obtained for spin-wave-mediated interaction here is replaced by the corresponding function for Coulomb interactions.

III. TEMPERATURE AND DISORDER DEPENDENCE OF CONDUCTIVITY

We will use Eq. (2.22) to evaluate the leading temperature and disorder dependence of the conductivity. We confine our discussion to the low-temperature $T \ll B$ and strong disorder $B\tau \ll 1$ regimes.

A. Temperature dependence

As seen from expression (2.22), the leading temperature dependence is generated in the frequency range $0 < \omega < 2T$, where we may expand $\coth x \approx 1/x + x/3$ for $x \ll 1$ and write

$$\delta\sigma \approx \frac{1}{2\pi}\frac{1}{3T}\int_0^{2T} \omega\, d\omega \widetilde{C}(-i\omega,0). \quad (3.1)$$

For $B \gg T$ (which is the generic regime for low temperatures in the several Kelvin range or below), the diffusons are cut off by the finite B term and the only ω dependence to F and K comes from the spin-wave interaction $v(q,\omega)$. In this case, we write

$$\widetilde{C}(-i\omega) \approx \widetilde{C}_0 \int_0^{q_0} (dq)\frac{q^2 v(q,-i\omega)}{(\tilde{D}q^2+\tilde{\delta})^3} + \text{c.c.}, \quad (3.2)$$

where $(dq) = d^D q/(2\pi)^D$ for dimensions $D = 2,3$. Here, $q_0 \approx l^{-1}$ is a cutoff related to the inverse mean-free path $l^{-1} = 1/(v_F\tau)$. Since the diffusons do not contribute in a singular fashion, and $\omega_q = \Delta + A_\alpha q^\alpha$, the dominant contribution to the q integral comes from $q_0 > q > (T/A_\alpha)^{1/\alpha}$. In that case, the ω dependence of $v(q,\omega)$ can be neglected. This immediately leads to a linear T dependence:

$$\delta\sigma \approx \frac{2\widetilde{C}_0}{3\pi} T \int_0^{q_0} (dq)\frac{q^2 v(q,\omega=0)}{(\tilde{D}q^2+\tilde{\delta})^3}. \quad (3.3)$$

Note that the linear T dependence is independent of the dimensionality, and should be valid for both ferromagnets and antiferromagnets. It was experimentally observed in ferromagnetic thin Gd films [9]. [We take this opportunity to point out that there is a misprint in Ref. [9], where the inequality after Eq. (10) should be $B\tau \ll 1$ in order to obtain the given disorder dependence.]

We note here that in their theoretical evaluations, Ref. [15] did not obtain a linear T dependence in any regime of a ferromagnet. The difference seems to appear from the choice of the cutoff $q_c \equiv \sqrt{T/A_2}$, using $\alpha = 2$ for ferromagnets. While Ref. [15] assumes the upper cutoff for the q integral to be q_c, we take the upper limit to be given by $q_0 = 1/l$, with $q_c \ll 1/l$ such that the major contribution of the q integral comes from $q_0 > q > q_c$. In this case, $q_c^2 = T/A_2 < 1/l^2$ implies $T/\overline{J} < 1/(k_F l)^2$, where $\overline{J} = A_2 k_F^2$. Typically, $T/\overline{J} \sim 10^{-2}$, which implies that for a wide range of disorder the above inequality is well satisfied. As pointed out in Ref. [15], the large disorder limit considered here allows us to ignore additional nondiffuson diagrams considered there.

B. Disorder dependence

We will basically use Eq. (3.3) to evaluate the disorder dependence of the conductivity correction in various limiting cases in two and three dimensions. We will ignore the spin-wave damping, so that the spin-wave-mediated interaction is

$$v(q\omega) = -\frac{2nJ^2\omega_q}{\omega^2+\omega_q^2}. \quad (3.4)$$

We recall that the system parameters entering here are $\widetilde{C}_0 \approx 4\pi N_0\tilde{D}^2$, $\tilde{\delta} = -iB(1-iB\tau)$, $\tilde{D} = \frac{D}{1-iB\tau}$. Note that up to this point, the formulas are valid for both ferromagnets and antiferromagnets, differentiated only by the q dependence in the dispersion relation.

1. Ferromagnetic spin waves: 2D

Since Eq. (3.3) is valid for ferromagnets as well, we take this opportunity to provide some details for obtaining the results given in Ref. [9]. Consistent with Ref. [9], we will consider a 2D system, in the limit where the exchange splitting B is large compared to Δ but $B\tau \ll 1$ (note: there is a misprint in Ref. [9], which states the opposite limit $B\tau \gg 1$). Then,

$$\delta\sigma \approx \frac{2\widetilde{C}_0}{3\pi}T(-2nJ^2)\frac{1}{A_2\tilde{D}^3}I_1,$$
$$I_1 = \int_0^{q_0} dq\frac{q^3}{(b+q^2)^3(c_2+q^2)}, \quad (3.5)$$

where

$$b \equiv \frac{\tilde{\delta}}{\tilde{D}}, \quad c_\alpha \equiv \frac{\Delta}{A_\alpha}. \quad (3.6)$$

While the integral can be done exactly, we will be interested in the limit of strong disorder b, $c_2 < (q_0)^2$, when we may take $q_0 \to \infty$ and $D < A_2$, giving the leading term as

$$I_1 = \frac{1}{2b^2}, \quad q_0 \to \infty, \quad b \gg c_2. \quad (3.7)$$

In this limit, the conductivity becomes

$$\delta\sigma \approx -\frac{8}{3}N_0 nJ^2\frac{T}{A_2}\text{Re}\left(\frac{\tilde{D}}{\tilde{\delta}^2}\right), \quad (3.8)$$

where we have used the definitions of $\widetilde{C}_0$ and b. Considering furthermore the limit $B\tau \ll 1$, when

$$\text{Re}\left(\frac{\tilde{D}}{\tilde{\delta}^2}\right)\lim_{B\tau\ll 1} = -\frac{D}{B^2}, \quad (3.9)$$

we finally obtain

$$\delta\sigma \approx \frac{16}{3} N_0 J \frac{nJ}{B} \frac{\epsilon_F}{B} \frac{T}{A_2 k_F^2} (\epsilon_F \tau), \quad B\tau \ll 1 \tag{3.10}$$

where $D = \frac{1}{2} v_F^2 \tau$ and $k_F v_F = 2\epsilon_F$ ($\hbar = 1$) have been used. This agrees with [9] for negligible damping ($\gamma = 0$) (apart from the misprint about the $B\tau$ limit mentioned before), where it was shown to agree well with the experimental data.

2. Antiferromagnetic spin waves

The antiferromagnetic spin wave is characterized by $\omega_q = \Delta + A_1 q$. We will consider both the 3D and 2D cases, but only in the limits of large spin-wave gap Δ and large disorder $B\tau \ll 1$.

Case I: 3D, $B\tau \ll 1$: In this limit we start with

$$\delta\sigma \approx \frac{2\tilde{C}_0}{3\pi} T(-2nJ^2) \frac{1}{A_1 \tilde{D}^3} I_2,$$
$$I_2 = \int_0^{q_0} dq \frac{q^4}{(b+q^2)^3 (c_1+q)}, \tag{3.11}$$

where b and c_1 are defined in (3.6). Since $q_0 \gg (c_1, \sqrt{b})$ we may take $q_0 \to \infty$ and also $b \gg c_1^2$, when

$$I_2 \approx \frac{1}{4b}, \quad q_0 \to \infty, \quad b \gg c_1^2. \tag{3.12}$$

Then, the contribution to the conductivity is

$$\delta\sigma \approx -\frac{8}{3} N_0 n J^2 \frac{T}{A_1} \mathrm{Re} \frac{1}{\tilde{\delta}}. \tag{3.13}$$

Using the 3D values $N_0 = \frac{mk_F}{\pi^2}$, $n = \frac{k_F^3}{3\pi^2}$ this gives

$$\delta\sigma \approx -8 \frac{T\bar{J}}{v_F} \mathrm{Re} \frac{1}{\tilde{\delta}}, \quad \bar{J} = nJ \tag{3.14}$$

where we defined the spin-exchange energy scale $\bar{J} = nJ$ and used $A_1 \approx v_F$. In the limit $B\tau \ll 1$ using $\mathrm{Re}(1/\tilde{\delta}) \to -\tau$, we finally get

$$\delta\sigma \approx 4 \frac{\bar{J}}{\epsilon_F} \frac{T}{\epsilon_F} (\epsilon_F \tau) k_F. \tag{3.15}$$

Comparing this result in the case of Mn films with the contribution from the weak-localization mechanism considered in the next section, it is found to be smaller by two orders of magnitude and therefore has not been observed in the experiments reported in [14].

Case II: 2D, $B\tau \ll 1$. In 2D, we have

$$\delta\sigma \approx \frac{2\tilde{C}_0}{3\pi} T(-2nJ^2) \frac{1}{A_1 \tilde{D}^3} I_3,$$
$$I_3 = \int_0^{q_0} dq \frac{q^3}{(b+q^2)^3 (c_1+q)}. \tag{3.16}$$

In the regime under consideration, the integral is simply

$$I_3 \approx \frac{\pi}{16} \frac{1}{b^{3/2}}, \quad q_0 \to \infty, \quad b \gg c_1^2. \tag{3.17}$$

The corresponding conductivity is

$$\delta\sigma \approx -\frac{\pi}{3} N_0 n J^2 \frac{T}{A_1} \mathrm{Re} \left(\frac{\tilde{D}}{\tilde{\delta}^3} \right)^{1/2}. \tag{3.18}$$

In the limit $B\tau \ll 1$, we finally get

$$\delta\sigma \approx \frac{\pi}{3} \frac{\bar{J}}{\epsilon_F} \frac{\bar{J}}{B} \sqrt{\frac{\epsilon_F}{B}} \frac{T}{A_1 k_F} \sqrt{\epsilon_F \tau}. \tag{3.19}$$

Thus, the disorder dependence in 2D for antiferromagnets ($\sqrt{\epsilon_F \tau}$) is different from that for ferromagnets ($\epsilon_F \tau$).

IV. INELASTIC LIFETIME FOR ANTIFERROMAGNETIC SYSTEMS

In order to consider the conductivity corrections due to weak localization, we will need the inelastic lifetime caused by scattering off spin waves. Some ferromagnetic cases have been considered in Ref. [13]. Here, we consider the antiferromagnetic case. The inelastic lifetime due to scattering off spin waves is given by [13]

$$\frac{\hbar}{\tau_\phi} = \frac{4}{\pi\hbar} n J^2 \int_0^{q_0} \frac{q^{d-1} dq}{\sinh \beta\omega_q} \frac{Dq^2 + 1/\tau_\phi}{(Dq^2 + 1/\tau_\phi)^2 + \omega_q^2}, \tag{4.1}$$

where we substitute the dispersion law for antiferromagnetic spin waves $\hbar\omega_q = \Delta + A_1 q$. We will consider both the limits $\hbar/\tau_\phi \ll \Delta$ and $\hbar/\tau_\phi \gg \Delta$. Considering that the sinh function acts as an exponential cutoff of the q integral at $q \sim q_1$, where $\omega_{q_1} \sim 2T$, we distinguish in the following the two cases $q_1 < q_0$ and $q_1 > q_0$.

Note that for the ferromagnetic case, ω_q in the spin-wave interaction term has the same q dependence as Dq^2 in the diffusion term, while it has a lower power for the antiferromagnetic case. If dominant contributions come from large q, then the antiferromagnetic systems will be much more dominated by disorder, eventually leading to more interesting weak-localization effects.

A. Large spin-wave gap

We consider the case $\hbar/\tau_\phi \ll \Delta$ first. Depending on the temperature, this may have different possibilities, as shown in the following.

Case I: 3D, $\hbar/\tau_\phi \ll \Delta$, $T > \Delta$, $q_1 < q_0$. Consider 3D systems. In the limit $\hbar/\tau_\phi \ll \Delta$ we can neglect the $1/\tau_\phi$ terms inside the integral. Then, the inelastic lifetime is given by

$$\frac{\hbar}{\tau_\phi} = \frac{4}{\pi\hbar} n J^2 \int_0^{q_0} \frac{q^2 dq}{\sinh \beta\omega_q} \frac{Dq^2}{(Dq^2)^2 + \omega_q^2} = \frac{4\bar{J}}{\pi} K_1, \tag{4.2}$$

where we defined the dimensionless quantity K_1:

$$K_1 \equiv \frac{\bar{J}}{\hbar n} \int_0^{q_0} \frac{q^2 dq}{\sinh \frac{\Delta + A_1 q}{T}} \frac{Dq^2}{(Dq^2)^2 + (\Delta + A_1 q)^2}. \tag{4.3}$$

The sinh function will cut off the integral at $(\Delta + A_1 q)/T \leqslant 2$ or $q \leqslant (2T - \Delta)/A_1$, if $2T > \Delta$ and if $(2T - \Delta)/A_1 < q_0$. We will first consider this limit. (Note that in the opposite limit $2T \ll \Delta$, the T dependence is exponential.) We may then replace the upper limit of the integral as

$$K_1 \approx \frac{\bar{J}}{\hbar n D} \int_0^{\frac{2T-\Delta}{A}} \frac{q^4 dq}{\sinh\left[\frac{A_1}{T}\left(q + \frac{\Delta}{A_1}\right)\right]} \times \frac{1}{q^4 + \frac{A_1^2}{D^2}\left(q + \frac{\Delta}{A_1}\right)^2}. \tag{4.4}$$

In addition, if $\frac{\Delta}{A_1} < \frac{2T-\Delta}{A_1}$ or equivalently $\Delta < T$, then we split the integral into two pieces: $K_1 = K_{11} + K_{12}$ with

$$K_{11} \approx \frac{\bar{J}}{\hbar n D}\int_0^{\frac{\Delta}{A_1}} \frac{q^4 dq}{\sinh\frac{\Delta}{T}}\frac{1}{q^4+\frac{\Delta^2}{D^2}} = \frac{\bar{J}}{\hbar n D}\frac{1}{\sinh\frac{\Delta}{T}}K_{00}, \tag{4.5}$$

where

$$K_{00} \equiv \int_0^{\frac{\Delta}{A_1}} \frac{q^4 dq}{q^4+\frac{\Delta^2}{D^2}}. \tag{4.6}$$

(Note: so far we have used $\hbar = 1$.)

Putting $\hbar$ back and defining $\bar{\Delta} = \Delta/\hbar$, we change variables to $u = Dq^2/\bar{\Delta}$, giving

$$K_{00} = \frac{\hbar}{2}\frac{\Delta^2}{A_1^3}\frac{1}{x^{3/2}}I(x), \quad I(x) \equiv \int_0^x \frac{u^{3/2}du}{1+u^2}, \tag{4.7}$$

where

$$x \equiv \frac{\hbar D\Delta}{A_1^2}. \tag{4.8}$$

While the exact result for this integral is available, if lengthy, the limits can be obtained simply:

$$I(x) \sim \int_0^x u^{3/2}du = \frac{2}{5}x^{5/2}, \quad x \ll 1$$
$$I(x) \sim \int_0^x u^{-1/2}du = 2x^{1/2}, \quad x \gg 1. \tag{4.9}$$

The second part of the integral is typically small. We can estimate it as

$$K_{12} \approx \frac{\bar{J}}{\hbar n D}\int_{\frac{\Delta}{A}}^{\frac{2T-\Delta}{A}} \frac{dq}{\sinh\frac{A_1 q}{T}}\frac{q^2}{q^2+\frac{A_1^2}{D^2}} = \frac{\bar{J}}{\hbar n D}\frac{T}{A_1}\int_{\frac{\Delta}{T}}^{\frac{2T-\Delta}{T}} \frac{dy}{\sinh y}\frac{y^2}{y^2+\frac{A_1^2}{D^2T^2}}, \tag{4.10}$$

where we changed the integration variable to $y = qA_1/T$. For $x \gg 1$, we have $A_1^2/DT = \Delta/Tx \ll \Delta/T$ and in this limit we have

$$K_{12} = \frac{\bar{J}}{\hbar n D}\frac{T}{A_1}\int_{\frac{\Delta}{T}}^{\frac{2T-\Delta}{T}} \frac{dy}{\sinh y} = \frac{T}{DA_1}\ln\frac{\tanh[1-\Delta/(2T)]}{\tanh[\Delta/(2T)]}. \tag{4.11}$$

In the opposite limit $x \ll 1$, $A_1^2/DT = \Delta/Tx \gg \Delta/T$, the integrals may only be done numerically to get the leading T dependence.

Assuming the first part dominates, we can estimate the inelastic length L_ϕ as follows. Rewrite

$$x = \frac{\hbar D\Delta}{A_1^2} = \frac{\hbar D k_F^2\Delta}{\bar{J}^2}, \quad \bar{J} \equiv A_1 k_F$$
$$n = \frac{k_F^3}{3\pi^2}; \quad J = \frac{\bar{J}}{n}, \quad nJ^2 = \frac{3\pi^2 A_1^2}{k_F}. \tag{4.12}$$

Then,

$$\frac{\hbar}{\tau_\phi} \approx \frac{6\pi}{\sinh\frac{\Delta}{T}}\frac{\Delta^2}{\bar{J}}\frac{1}{x^{3/2}}I(x). \tag{4.13}$$

The inelastic length is given by

$$L_\phi = \sqrt{D\tau_\phi}. \tag{4.14}$$

Writing D in terms of x as $D = \frac{\bar{J}^2}{\hbar k_F^2\Delta}x$, we rewrite

$$k_F L_\phi = \left(\frac{\bar{J}}{\Delta}\right)^{3/2}\left(\frac{\sinh\frac{\Delta}{T}}{6\pi}\right)^{1/2}\left(\frac{x^{5/2}}{I(x)}\right)^{1/2}. \tag{4.15}$$

In the two limits x large and small we get

$$k_F L_\phi \approx \left(\frac{\bar{J}}{\Delta}\right)^{3/2}\left(\frac{\sinh\frac{\Delta}{T}}{12\pi}\right)^{1/2}x, \quad x \gg 1$$
$$\approx \left(\frac{\bar{J}}{\Delta}\right)^{3/2}\left(\frac{5\sinh\frac{\Delta}{T}}{12\pi}\right)^{1/2}, \quad x \ll 1. \tag{4.16}$$

Since x decreases with increasing disorder, L_ϕ decreases with increasing disorder and then saturates at a disorder independent value for $x \ll 1$ given by the ratios $\bar{J}/\Delta$ and Δ/T. This is the regime studied in Ref. [14].

Case II: 3D, $\hbar/\tau_\phi \ll \Delta$, $T > \Delta$, $q_1 > q_0$. In this case in 3D

$$\frac{\hbar}{\tau_\phi} \approx \frac{4}{\pi\hbar}nJ^2\frac{1}{\sinh\frac{\Delta}{T}}K_{01}, \tag{4.17}$$

where

$$K_{01} \equiv \int_0^{q_0=1/l} q^2 dq\frac{Dq^2}{(Dq^2)^2+\bar{\Delta}^2}, \quad \Delta/A_1 \ll q_0. \tag{4.18}$$

This leads to

$$K_{01} = \frac{27}{2\hbar^2}\frac{\Delta^2}{v_F^3}\bar{x}^{3/2}I(\bar{x}), \quad \bar{x} \equiv \frac{\hbar D q_0^2}{\Delta} = \frac{\hbar v_F^2}{9\Delta D} \tag{4.19}$$

giving

$$\frac{1}{\tau_\phi} = \frac{81\pi}{4\hbar\sinh\frac{\Delta}{T}}\frac{\Delta^2\bar{J}^2}{\epsilon_F^3}\bar{x}^{3/2}I(\bar{x}). \tag{4.20}$$

The corresponding inelastic length is given by

$$k_F L_\phi = \frac{4}{9}\left(\frac{\epsilon_F^5}{\Delta^3\bar{J}^2}\right)^{1/2}\left(\frac{\sinh\frac{\Delta}{T}}{\pi}\right)^{1/2}\left(\frac{1}{\bar{x}^{3/2}I(\bar{x})}\right)^{1/2}. \tag{4.21}$$

In the two limits, the disorder dependence is given by

$$k_F L_\phi \sim \frac{1}{\bar{x}} \sim D, \quad \bar{x} \gg 1$$
$$\sim \frac{1}{\bar{x}^2} \sim D^2, \quad \bar{x} \ll 1. \tag{4.22}$$

Case III: 2D, $\hbar/\tau_\phi \ll \Delta$, $T < \Delta$. As mentioned above, the temperature dependence is exponential in this limit. We will consider this limit in two dimensions, where the corresponding weak-localization effect can be important. The quantity K_1 in Eq. (4.3) is now approximated as

$$K_1 \approx \frac{\bar{J}}{\hbar n}\int_0^\infty dq\, e^{-\frac{\Delta}{T}-\frac{Aq}{T}}\frac{Dq^3}{(Dq^2)^2+(\Delta+A_1q)^2}$$
$$= \frac{\bar{J}}{\hbar n}e^{-\frac{\Delta}{T}}\int_0^\infty dy\frac{D\left(\frac{T}{A_1}\right)^3 y^3 e^{-y}}{D^2\left(\frac{T}{A_1}\right)^4 y^4+(\Delta+Ty)^2}. \quad (4.23)$$

For $\Delta > T$ we can ignore the term Ty compared to Δ in the denominator since the exponential term limits $y < 1$. Also, we will consider the regime where $D(T/A)^2y^2$ can be neglected compared to Δ, as is true for $T/\bar{J} \ll 1$. In this limit,

$$K_1 \approx e^{-\frac{\Delta}{T}}\left(\frac{T}{Ak_F}\right)^4\frac{\bar{J}Dk_F^2}{\Delta^2} \quad (4.24)$$

leading to

$$\frac{1}{\tau_\phi} \approx e^{-\frac{\Delta}{T}}\frac{T^4}{\bar{J}^2\Delta^2}(\epsilon_F\tau)\epsilon_F. \quad (4.25)$$

The corresponding result in 3D will have a different T-dependent prefactor to the exponential. Similarly, the ferromagnetic case in 2D will have a logarithmic T dependence as a prefactor to the exponential.

B. Small spin-wave gap

We now consider the case $\hbar/\tau_\phi \gg \Delta$. In this limit, we use $\hbar\omega_q \approx A_1q$ and $\sinh(\beta\omega_q) \sim A_1q/T$, and there will be self-consistent equations because the integrals will have a lower cutoff at $1/L_\phi$.

Case I: 3D, $\hbar/\tau_\phi \gg \Delta$. For 3D, we have

$$\frac{\hbar}{\tau_\phi} \approx \frac{4}{\pi\hbar}nJ^2\frac{T}{AD}K_0', \quad K_0' \equiv \int_{1/L_\phi}^{q_0}\frac{q\,dq}{q^2+\left(\frac{A}{\hbar D}\right)^2}. \quad (4.26)$$

The interesting limit is when $\hbar D/Al \gg 1$. In this case,

$$\frac{\hbar}{\tau_\phi} \approx \frac{4}{\pi\hbar}nJ^2\frac{T}{AD}\ln\left(\frac{L_\phi}{l}\right), \quad \frac{\hbar D}{Al} = \frac{2}{3}\frac{\epsilon_F}{\bar{J}} \gg 1. \quad (4.27)$$

To leading order, we replace $L_\phi/l = \tau_\phi/\tau$ in the argument of the logarithm by the prefactor, giving

$$\frac{\tau}{\tau_\phi} \approx 9\pi\frac{\bar{J}T}{\epsilon_F^2}\ln\left(\frac{\epsilon_F^2}{9\pi\bar{J}T}\right). \quad (4.28)$$

Case II: 2D, $\hbar/\tau_\phi \gg \Delta$. In contrast, in 2D,

$$\frac{\hbar}{\tau_\phi} \approx \frac{4}{\pi\hbar}nJ^2\frac{T}{A^2}\left[\arctan\frac{\hbar D}{Al} - \arctan\frac{\hbar D}{AL_\phi}\right]. \quad (4.29)$$

Again, the interesting limit is

$$\frac{\hbar}{\tau_\phi} \approx 3T\frac{\bar{J}}{\epsilon_F}\frac{L_\phi}{l}, \quad \frac{\hbar D}{Al} \gg 1. \quad (4.30)$$

Replacing $L_\phi = \sqrt{D\tau_\phi}$ and solving for τ_ϕ gives

$$\frac{\hbar}{\tau_\phi} \approx \left(\frac{\hbar}{2\tau}\right)^{1/3}\left(\frac{\bar{J}}{\epsilon_F}\right)^{2/3}T^{2/3}. \quad (4.31)$$

V. WEAK LOCALIZATION

The observation of weak-localization (WL) contributions to the conductivity of magnetically ordered films is somewhat surprising since one may expect a strong internal magnetic field cutting off the WL contribution. Nonetheless, WL effects have been observed in ferromagnetic nickel films [16], thin Fe films [7], and ferromagnetic GaMnAs nanostructures [17]. There are at least two reasons for that. For once, the inelastic scattering rate $1/\tau_\phi$ is unusually large on account of the strong scattering off spin waves (see above), such that in the temperature regime considered in these experiments the relevant cutoff is $1/\tau_\phi$ and not the magnetic induction. Second, the demagnetization field of the film geometry compensates the internal magnetic induction almost completely.

Given the inelastic lifetime, the weak-localization correction to the conductivity is found as

$$\delta\sigma \sim \sqrt{\frac{\tau}{\tau_\phi}}, \quad 3\text{D}$$
$$\sim \ln\frac{\tau}{\tau_\phi}, \quad 2\text{D}. \quad (5.1)$$

We will consider two limiting cases.

A. Large spin-wave gap

When the spin-wave gap is large compared to the inverse inelastic lifetime, i.e., $\frac{\hbar}{\tau_\phi} \ll \Delta$, there are three interesting possibilities.

Case I: 3D, $T > \Delta$, $q_1 < q_0$, strong disorder. Using the corresponding results for the inelastic lifetimes,

$$\frac{\tau}{\tau_\phi} \approx \frac{\tau}{\hbar}\frac{6\pi}{\sinh\frac{\Delta}{T}}\frac{\Delta^2}{\bar{J}}\frac{1}{x^{3/2}}I(x). \quad (5.2)$$

Inserting the limiting behavior of $I(x)$ for small and large x, we get

$$\frac{\tau}{\tau_\phi} \approx \frac{2}{15}\left(\frac{\Delta}{\bar{J}}\right)^3(k_Fl)^2\frac{6\pi}{\sinh\frac{\Delta}{T}}, \quad x \ll 1$$
$$\approx \frac{3}{4}\frac{\Delta\bar{J}}{\epsilon_F^2}\frac{6\pi}{\sinh\frac{\Delta}{T}}, \quad x \gg 1. \quad (5.3)$$

Then, the weak-localization correction is

$$\delta\sigma \approx \sqrt{\frac{4}{5}}\left(\frac{\Delta}{\bar{J}}\right)^{3/2}(k_Fl)\sqrt{\frac{\pi}{\sinh\frac{\Delta}{T}}}, \quad x \ll 1$$
$$\approx \sqrt{\frac{9}{2}}\left(\frac{\Delta\bar{J}}{\epsilon_F^2}\right)^{1/2}\sqrt{\frac{\pi}{\sinh\frac{\Delta}{T}}}, \quad x \gg 1. \quad (5.4)$$

Note that

$$x = \frac{\hbar Dk_F^2\Delta}{\bar{J}^2} = \frac{2}{3}\frac{\Delta\epsilon_F}{\bar{J}^2}(k_Fl). \quad (5.5)$$

The T dependence in the crossover regime $T \sim \Delta$ might look like a fractional power law. This was observed recently experimentally in Ref. [14].

Case II: 3D, $T > \Delta$, $q_1 > q_0$, moderate disorder. In this limit,

$$\frac{\tau}{\tau_\phi} = \frac{81\pi\tau}{4\hbar \sinh\frac{\Delta}{T}} \frac{\Delta^2 \bar{J}^2}{\epsilon_F^3} \bar{x}^{3/2} I(\bar{x}),$$
$$\bar{x} \equiv \frac{\hbar D}{\Delta l^2} = \frac{1}{3}\frac{\hbar}{\Delta\tau}. \tag{5.6}$$

In the two limits of large and small $\bar{x}$, the localization corrections are given by

$$\delta\sigma \approx \sqrt{\frac{4}{5}}\left(\frac{\bar{J}}{\Delta}\right)\frac{1}{(k_F l)^{3/2}}\sqrt{\frac{\pi}{\sinh\frac{\Delta}{T}}}, \quad \bar{x} \ll 1$$
$$\approx 3\sqrt{2}\left(\frac{\bar{J}}{\epsilon_F}\right)\frac{1}{k_F l}\sqrt{\frac{\pi}{\sinh\frac{\Delta}{T}}}, \quad \bar{x} \gg 1. \tag{5.7}$$

Case III: 2D, $T < \Delta$. Here, we have the thermally activated behavior of $\frac{1}{\tau_\phi}$, which when substituted into the logarithm leads to

$$\delta\sigma \sim \ln e^{-\Delta/T} \sim -\frac{\Delta}{T} \tag{5.8}$$

a distinctive behavior, which should be observable in experiment.

B. Small spin-wave gap

In the regime where the spin-wave gap is smaller than the inverse inelastic lifetime $\frac{\hbar}{\tau_\phi} \gg \Delta$, one arrives at a self-consistent equation and we will only consider the limit $\frac{\hbar D}{\Delta l} \gg 1$ in 3D. Then, from Eq. (4.28) the conductivity correction is

$$\delta\sigma \approx \sqrt{9\pi\frac{\bar{J}T}{\epsilon_F^2}\ln\left(\frac{\epsilon_F^2}{9\pi\bar{J}T}\right)}. \tag{5.9}$$

This is another unusual T dependence in 3D that should be observable.

C. Very strong disorder

Finally, we like to mention a renormalization of the prefactor of the standard weak-localization expression in the case of a metallic 3D system close to the metal-insulator transition, when the conductivity σ already strongly deviates from the Drude result σ_0. As discussed in Ref. [14], the WL correction to the conductivity gets renormalized as

$$\frac{\delta\sigma_\square}{L_{00}} = \frac{\sigma}{\sigma_0}\frac{t}{L_\phi}. \tag{5.10}$$

Here, $\delta\sigma_\square = t\delta\sigma$, with t denoting the film thickness, and $L_{00} = e^2/\pi h$ is the conductance quantum. The ratio σ/σ_0 is accessible through the sheet resistance R_0 at $T = 0$,

$$\frac{\sigma}{\sigma_0} \approx \frac{2}{1+\sqrt{1+(R_0/R_c)^2}} \tag{5.11}$$

with R_c a characteristic sheet resistance of the order of e^2/h. The above is based on a self-consistent theory of Anderson localization, which is known to give a good description of the 3D conductivity near the Anderson transition, but still outside of the critical regime [18].

VI. SUMMARY

In this work, we have systematically evaluated the temperature and disorder dependence of the quantum corrections to the conductivity due to scattering of electrons off of spin waves in itinerant antiferromagnetic systems. First, we considered the interaction-induced AA correction. We showed how a linear T correction arises naturally in some specific parameter range in both ferromagnetic and antiferromagnetic systems. However, the disorder dependence is different in antiferromagnetic systems compared to ferromagnets. We provide details of the derivation of the crossover temperature behavior reported in Ref. [14], which allowed the extraction of the spin-wave gap and the exchange-energy parameters from the temperature and disorder dependence of the conductivity of thin Mn films. Second, we reconsider the inelastic scattering, or phase-breaking rate, induced by scattering of electrons off spin waves. This quantity determines the weak-localization correction to the conductivity. We show how the temperature and disorder dependence in the various parameter regimes can be very different, and provides an opportunity to study magnetic properties of thin films in 2D or 3D by direct transport measurements.

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APPENDIX: EVALUATION OF THE CONDUCTIVITY DIAGRAMS

Here, we show the details of evaluating Figs. 1(a′), 1(b), and 1(b′).

Diagram 1(a′) has the interaction on the lower line. It gives

$$L^{1(a')} = -T\sum_{\epsilon_n} T\sum_{\omega_l}\sum_q C_1^2 C_2^2 \Gamma_{\uparrow\downarrow}^2(q,\omega_l)\Gamma_{\uparrow\downarrow}(q,\omega_{lm}^+)$$
$$\times v(q,\omega_l)\Theta[\epsilon_n(\epsilon_n - \Omega_m)]$$
$$\times \Theta[-\epsilon_n(\epsilon_n - \omega_{lm}^+)]. \tag{A1}$$

For $\epsilon_n > 0$, C_1 and C_2 are given by Eqs. (2.8) and (2.9), but for $\epsilon_n - \Omega_m > 0$ and $\epsilon_n - \Omega_m - \omega_l < 0$, the sum over ϵ_n becomes

$$T\sum_{\epsilon_n=\Omega_m}^{\omega_l+\Omega_m} = \omega_l, \quad \omega_l > 0. \tag{A2}$$

For $\epsilon_n < 0$, we have $\epsilon_n - \Omega_m < 0$, $\epsilon_n - \Omega_m - \omega_l > 0$, such that

$$T\sum_{\epsilon_n=\omega_l+\Omega_m}^{0} = -(\omega_l + \Omega_m), \quad \omega_l < 0 \tag{A3}$$

and C_1 and C_2 replaced by complex conjugates as before. Thus, we have for diagram 1(a′)

$$L^{1(a')} = -T \sum_{\omega_l>0} \omega_l \sum_q q^2 F(q,\omega_l) K(q,\omega_{lm}^+) - T \sum_{\omega_l>\Omega_m} \omega_{lm}^- \sum_q q^2 F^*(q,-\omega_l) K^*(q,-\omega_{lm}^-). \quad \text{(A4)}$$

Since F and K depend on the frequency only as the absolute value, i.e., $F(q,\omega_l) = F(q,-\omega_l)$ and $K(q,\omega_{lm}^+) = K(q,-\omega_{lm}^+)$, we see that

$$L^{1(a')} = (L^{1(a)})^*. \quad \text{(A5)}$$

We now consider Fig. 1(b). It is given by

$$L^{1(b)} = -T \sum_{\epsilon_n} T \sum_{\omega_l} \sum_q C_1 C_3 C_2^2 \Gamma_{\uparrow\downarrow}^2(q,\omega_l) \Gamma_{\uparrow\downarrow}(q,\omega_{lm}^-) \times v(q,\omega_l) \Theta[\epsilon_n(\epsilon_n - \Omega_m)] \Theta[-\epsilon_n(\epsilon_n - \omega_l)] \times \Theta[(\epsilon_n - \omega_l)(\epsilon_n - \omega_{lm}^+)], \quad \text{(A6)}$$

where for $\epsilon_n > 0$,

$$C_3 \equiv \sum_k (G_{k\uparrow}^+)(G_{k-q\downarrow}^-)^2 \frac{k_x}{m} \approx \sum_k (G_{k\uparrow}^+)\left[G_{k\downarrow}^- + \left(i|\omega_l| - \frac{kq}{m}\right)(G_{k\downarrow}^-)^2 + \cdots\right]^2 \frac{k_x}{m} = \sum_k (G_{k\uparrow}^+)(G_{k\downarrow}^-)^3 \left\langle \frac{-2k_x^2}{m^2} \right\rangle_\theta q_x = +4\pi N_0 \tilde{D}^* \tilde{\tau}^{*2} q_x \quad \text{(A7)}$$

such that $C_3 = -C_1$. In the above we used $\epsilon_n > 0$, so that $\epsilon_n - \Omega_m > 0$, $\epsilon_n - \omega_l < 0$, and $\epsilon_n - \Omega_m - \omega_l < 0$ such that

$$T \sum_{\epsilon_n=\Omega_m}^{\omega_l} = (\omega_l - \Omega_m), \quad \omega_l > \Omega_m. \quad \text{(A8)}$$

On the other hand, for $\epsilon_n < 0$, we have $\epsilon_n - \Omega_m < 0$, $\epsilon_n - \omega_l > 0$, and $\epsilon_n - \Omega_m - \omega_l > 0$ such that

$$T \sum_{\epsilon_n=(\omega_l+\Omega_m)}^{0} = -(\omega_l + \Omega_m), \quad \omega_l < -\Omega_m. \quad \text{(A9)}$$

We thus obtain, for Fig. 1(b),

$$L^{1(b)} = T \sum_{\omega_l>\Omega_m} \omega_{lm}^- \sum_q q^2 F(q,\omega_l) K(q,\omega_{lm}^-) + T \sum_{\omega_l>\Omega_m} \omega_{lm}^- \sum_q q^2 F^*(q,-\omega_l) K^*(q,-\omega_{lm}^+). \quad \text{(A10)}$$

Figure 1(b′) can be evaluated just as Fig. 1(a′'), and the result is that

$$L^{1(b')} = (L^{1(b)})^*. \quad \text{(A11)}$$

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