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Cyclotron-resonance-induced negative dc conductivity in a two-dimensional electron system on liquid helium

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We theoretically predict instability of a zero-dc-current state of the two-dimensional electron system formed on the surface of liquid helium induced by the cyclotron resonance (CR). This conclusion follows from the theoretical analysis of the dc magnetoconductivity which takes into account the contribution from radiation in an exact way. A many-electron model of the dynamic structure factor of the 2D Coulomb liquid is used to describe the influence of strong internal forces acting between electrons. For low electron densities and high amplitudes of the microwave field, the dc magnetoconductivity is shown to become negative in the vicinity of the CR which causes the instability. This effect is strongly suppressed by Coulomb forces in the region of high densities.

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Microwave-induced resistance oscillations and zero-resistance states (ZRS) observed in a two-dimensional (2D) electron gas subjected to a transverse magnetic field [1–3] represent a surprising discovery in condensed matter physics. A number of theoretical mechanisms have been proposed to explain these oscillations and ZRS [4–8]. Still, by now the origin of the phenomenon is controversial. A crucial result [9], independent of the details of the microscopic mechanism, is that the ZRS can be explained by an assumption that the longitudinal linear response conductivity σ_{xx} is negative in appropriate ranges of the magnetic field B. In this case, a zero-current state is unstable and the system spontaneously develops a nonvanishing local current density.

The above noted phenomena were observed in high-quality GaAs/AlGaAs heterostructures. There is another extremely clean 2D electron system formed on the surface of liquid helium which exhibits remarkable quantum magnetotransport phenomena [10] although it is a nondegenerate system of strongly interacting electrons. Since electron gas degeneracy is not a crucial point for some of the theoretical mechanisms explaining ZRS in semiconductor systems, these mechanisms potentially can be applied to surface electrons (SEs) on liquid helium as well. Complementary studies of these phenomena in the system of SEs on liquid helium could help with identification of the origin of ZRS observed in GaAs/AlGaAs heterostructures.

It should be noted that another kind of magnetoconductivity oscillations and ZRS (zero-σ_{xx} states to be exact) was already observed in the system of SEs on liquid helium [11,12] when the energy of excitation of the second surface subband $(\Delta_2 - \Delta_1)$ was tuned to the resonance with the microwave (MW) frequency. These phenomena were explained [13] by nonequilibrium population of the second surface subband which triggers quasielastic intersubband decay processes accompanied by electron scattering against or along the driving force, depending on the ratio $(\Delta_2 - \Delta_1)/\hbar\omega_c$ (here ω_c is the cyclotron frequency). An important evidence for identification of the mechanism of oscillations and ZRS observed for SEs on liquid helium was recently found in studies of the Coulombic effect on positions of conductivity extrema [14].

The most frequently discussed mechanism of negative conductivity effects called the displacement mechanism was proposed already in 1969 by Ryzhii [15]. In this model, an electron scattering by an impurity potential accompanied by absorption of a photon is the origin of the negative linear response conductivity. In recent developments of the model [16,17], the contribution from radiation is taken into account exactly which is important for high MW powers. Even though in semiconductor systems other mechanisms reportedly [18,19] could give a stronger effect, it is very attractive to study the displacement model for the 2D electron system on liquid helium.

In the displacement model, the strength of the effect depends on the product of two dimensionless parameters:

$$\lambda = \frac{eE_{\rm ac}^{(0)} l_B}{\hbar\omega}\,, \quad \varkappa = \frac{\omega_c^2}{|\omega^2 - \omega_c^2|}, \tag{1}$$

where $E_{\rm ac}^{(0)}$ is the amplitude of the MW field, and $l_B = \sqrt{\hbar c/eB}$ is the magnetic length. For a fixed ratio ω/ω_c, the value of $\varkappa$ is the same in both semiconductor and SE systems. The effective mass of SEs is very close to the free electron mass m_e, while the effective mass of semiconductor electrons is much smaller: $m_e^* \simeq 0.064 m_e$. Therefore, at fixed $E_{\rm ac}^{(0)}$ and ω, in experiments with SEs on liquid helium λ is smaller than it is for semiconductor electrons by the factor $\sqrt{m_e^*/m_e} \simeq 1/4$, and multiple magnetoconductivity oscillations with $\omega/\omega_c \gtrsim 2$ are expected to be very small. The reduction of λ can be well compensated by approaching the cyclotron resonance (CR) condition $\omega_c \to \omega$, which increases $\varkappa$. Thus, for SEs on liquid helium, negative dc conductivity (similar to that of the semiconductor model) could be expected in the vicinity of the CR. Since the average Coulomb interaction energy per SE U_C is usually much larger than the average kinetic energy, a many-electron treatment of the displacement model is required.

In this work, we report results of a theory of the displacement mechanism of negative dc conductivity applied to SEs on liquid helium interacting with capillary-wave quanta (ripplons). The ac electric field is taken into account in an exact way similarly to Refs. [16,17]. Scattering with ripplons is described using a perturbation theory. Strong Coulomb interaction is taken into account employing a model based on the dynamic structure factor (DSF) of a 2D electron liquid. We found that results of the many-electron treatment drastically

depend on SE density n_s, which allows us to predict the range of n_s where negative conductivity effects can be observed.

We will use the Landau gauge in which a momentum in the y direction (p_y) is a good quantum number. The dc and ac electric fields ($\mathbf{E}_{\rm dc}$ and $\mathbf{E}_{\rm ac}$) are taken to be parallel to the x axis. The ac field $E_{\rm ac}(t) = E_{\rm ac}^{(0)} \cos \omega t$. In the absence of scatterers, the exact solution of the single-electron Hamiltonian can be written as [16,20]

$$\psi_{n,X}(x,y,t) = e^{i\vartheta(x,y,t)} \exp\left\{ i \frac{X}{l_B} \beta \sin \omega t \right\} \times \exp\left(-i X y / l_B^2\right) \varphi_n(x - X - \xi(t), t), \quad (2)$$

where

$$X = -\frac{c p_y}{eB} - \frac{e E_{\rm dc}}{m_e \omega_c^2}, \quad \beta = \lambda \frac{\omega_c^2}{\left(\omega^2 - \omega_c^2\right)}, \quad (3)$$

$\varphi_n(x,t)$ is the well-known solution for the unforced quantum harmonic oscillator, n is the Landau level index, and $\xi(t)$ is the classical solution of the forced harmonic oscillator, $\xi = e E_{\rm ac}^{(0)} \cos \omega t / m(\omega^2 - \omega_c^2)$. The resonant denominator of β originates from $\dot{\xi}$. The exact expression for $\vartheta(x,y,t)$ is not important in the following treatment.

The interaction with ripplons causes electron scattering between different states given in Eq. (2). The dc conductivity σ_{xx} of SEs can be found from the equation for current density $j_x = -e n_s l_B^2 \sum_{\mathbf{q}} q_y \bar{w}_{\mathbf{q}}$, where $\bar{w}_{\mathbf{q}}$ is the average probability of electron scattering with the momentum exchange $\hbar\mathbf{q}$, and $l_B^2 q_y$ represents a change of the orbit center number X for such a process. The effective collision frequency $\nu_{\rm eff}$, entering the usual conductivity form, is found as

$$\nu_{\rm eff} = -\frac{1}{m_e V_H} \sum_{\mathbf{q}} \hbar q_y \bar{w}_{\mathbf{q}}(V_H), \quad (4)$$

where $V_H = c E_{\rm dc}/B$ is the absolute value of the Hall velocity. The $\bar{w}_{\mathbf{q}}$ depends on $E_{\rm dc}$ because, in addition to $\varepsilon_n = \hbar\omega_c (n + 1/2)$, we have a term $e E_{\rm dc} X$. For nondegenerate electrons, the probability $w_{\mathbf{q}}$ is independent of the quantum number X due to $e E_{\rm dc}(X' - X) = \hbar q_y V_H$. In this case, $w_{\mathbf{q}}$ can be averaged over Landau level numbers only, assuming an equilibrium distribution $Z_\parallel^{-1} \exp(-\varepsilon_n/T_e)$ (here $Z_\parallel$ is the partition function).

In a single-electron treatment, $\bar{w}_{\mathbf{q}}$ can be found in terms of the DSF of a nondegenerate 2D electron gas

$$S(q,\Omega) = \frac{2}{\pi \hbar Z_\parallel} \sum_{n,n'} I_{n,n'}^2(x_q) \times \int d\varepsilon e^{-\varepsilon/T_e} g_n(\varepsilon)\, g_{n'}(\varepsilon + \hbar\Omega), \quad (5)$$

where $g_n(\varepsilon) = -{\rm Im} G_n(\varepsilon)$ represents the Landau level density of states, $G_n(\varepsilon)$ is the single-electron Green's function, $x_q = q^2 l_B^2/2$,

$$I_{n,n'}(x) = \sqrt{\frac{\min(n,n')!}{\max(n,n')!}} x^{\frac{|n'-n|}{2}} e^{-\frac{x}{2}} L_{\min(n,n')}^{|n'-n|}(x), \quad (6)$$

and $L_n^m(x)$ are the associated Laguerre polynomials. This representation is similar to that of the theory of thermal neutron (or x-ray) scattering by solids, where the scattering cross section of a particle flux is expressed in terms of the DSF of the target.

Comparing with the case $E_{\rm ac}^{(0)} = 0$, matrix elements, describing electron scattering, contain the additional factor $\exp(-i q_y l_B \beta \sin \omega t)$. Using the expansion $e^{iz\sin\phi} = \sum_m J_m(z) e^{im\phi}$ [here $J_m(z)$ is the Bessel function], the procedure of finding scattering probabilities can be reduced to a quite usual treatment. Then, for ripplon creation (index $+$) and destruction (index $-$) processes, $\bar{w}_{\mathbf{q}}^{(\pm)}(V_H)$ can be found as

$$\bar{w}_{\mathbf{q}}^{(\pm)}(V_H) = \frac{|\bar{C}_{\mathbf{q}}^{\pm}|^2}{\hbar^2} \sum_{m=-\infty}^{\infty} J_m^2(\beta q_y l_B) \times S(q, -q_y V_H + m\omega \mp \omega_{r,q}), \quad (7)$$

where $\bar{C}_q^{\pm} = V_{r,q} Q_q [N_q^{(r)} + 1/2 \pm 1/2]^{1/2}$, $N_q^{(r)}$ is the ripplon distribution function, $V_{r,q}$ is the electron-ripplon coupling [10], $Q_q = \sqrt{\hbar q/2\rho\omega_{r,q}}$, $\omega_{r,q} \simeq \sqrt{\alpha/\rho} q^{3/2}$, and α and ρ are the surface tension and mass density of liquid helium, respectively.

Generally, the structure of Eq. (7) is similar to that found for other scattering mechanisms important for semiconductor electrons [16,17,21]. The main advantage of the form of Eq. (7) is that we can employ the properties of the equilibrium DSF and model the effect of Coulomb interaction using the DSF of strongly interacting SEs. Such a possibility appears under the condition $l_B \ll a$ (here a is a typical electron spacing) which allows us to consider a fluctuational electric field $\mathbf{E}_{\rm f}$, acting on a particular electron, as a quasiuniform field [22]. This reduces the many-electron problem to a single-electron dynamics. Even in this limit, the situation remains to be very complicated; still an accurate form of the DSF of the 2D Coulomb liquid in a magnetic field can be found [10]:

$$S(q,\Omega) = \frac{2\sqrt{\pi}}{Z_\parallel} \sum_{n,n'} \frac{I_{n,n'}^2}{\gamma_{n,n'}} \exp\left[-\frac{\varepsilon_n}{T_e} - P_{n,n'}(\Omega) \right], \quad (8)$$

where

$$P_{n,n'} = \frac{[\Omega - (n'-n)\omega_c - \phi_n]^2}{\gamma_{n,n'}^2}, \quad \phi_n = \frac{\Gamma_n^2 + x_q \Gamma_C^2}{4 T_e \hbar}, \quad (9)$$

$$\hbar\gamma_{n,n'} = \sqrt{\frac{\Gamma_n^2 + \Gamma_{n'}^2}{2} + x_q \Gamma_C^2}, \quad (10)$$

Γ_n is the collision broadening of Landau levels, $\Gamma_C = \sqrt{2} e E_{\rm f}^{(0)} l_B$, and $E_{\rm f}^{(0)} \simeq 3\sqrt{T_e} n_s^{3/4}$ is the typical fluctuational electric field [23] under the condition $U_C/T > 10$.

Remarkably, the DSF of the Coulomb liquid with strong interaction given in Eq. (8) is similar to the DSF of noninteracting electrons (the latter corresponds to the regime $\Gamma_C \ll \Gamma_n$). The proportionality factor $1/\gamma_{n,n'}$ reflects the singular nature of the magnetotransport in 2D systems. For $\Gamma_C = 0$, eventually, it leads to the enhancement factor $\hbar\omega_c/\Gamma_n$ of the SCBA theory, which describes the effect of multiple electron scattering. The fluctuational electric field drives an electron from a scatterer which reduces multiple scattering by increasing $\gamma_{n,n'}$ given in Eq. (10). As the function of frequency, the DSF has maxima near Landau excitation energies. The fluctuational field introduces an additional broadening of these maxima $\sqrt{x_q}\Gamma_C$ and the shift in their positions $\phi_C = x_q \Gamma_C^2/4 T_e \hbar$. This form of the DSF describes well the magnetotransport

properties of SEs [10] even those induced by the intersubband MW resonance [14].

In most cases, the ripplon energy can be disregarded in the frequency argument of the DSF which allows us to consider electron scattering as quasielastic (in this limit $\bar{C}_q^{\pm} \to \bar{C}_{\mathbf{q}} = V_{r,q} Q_q \sqrt{T/\hbar\omega_{r,q}}$). Then, using the property of the equilibrium DSF, $S(q,-\Omega) = \exp(-\hbar\Omega/T_e) S(q,\Omega)$, the effective collision frequency can be represented as the sum $\nu_{\rm eff} = \sum_{m=0}^{\infty} \nu_m$, where

$$\nu_0 = \frac{1}{m_e T_e} \sum_{\mathbf{q}} q_y^2 |\bar{C}_{\mathbf{q}}|^2 J_0^2(\beta q_y l_B) S(q,0), \tag{11}$$

and

$$\nu_m = \frac{2}{m_e \hbar} \sum_{\mathbf{q}} q_y^2 |\bar{C}_{\mathbf{q}}|^2 J_m^2(\beta q_y l_B) \times \left[(1 - e^{-\frac{m\hbar\omega}{T_e}}) S'(q, m\omega) + \frac{\hbar}{T_e} e^{-\frac{m\hbar\omega}{T_e}} S(q, m\omega) \right] \tag{12}$$

for $m > 0$. The derivative $S' \equiv \partial S/\partial\Omega$ appears because of the linear expansion of the function $\bar{w}_{\mathbf{q}}(V_H)$ in Eq. (4). Equation (11) follows from the relationship $S'(q,0) = (\hbar/2T_e) S(q,0)$. Usually, the second term in square brackets of Eq. (12) is very small. In the following, we shall consider the regime of sharp maxima of $S(q,\Omega)$, realized at $\gamma_{n,n'} \ll \omega_c$.

Since λ, entering the definition of $|\beta| = \lambda\varkappa$, is usually very small, we shall concentrate on effects induced by the CR, when ω_c is quite close to ω. Sometimes we shall use a damping form $\sqrt{(\omega^2 - \omega_c^2)^2 + 4\gamma_*^2\omega^2}$ instead of $|\omega^2 - \omega_c^2|$ in the denominator of $\varkappa$ given in Eq. (1). This denominator originates from the classical equation; therefore, it is reasonable to set the damping parameter γ_* to its classical value ν_{cl}.

The Eq. (11) indicates that ν_0, as the function of $\omega - \omega_c$, has a symmetrical minimum at $\omega_c = \omega$. To the contrary, the next term ν_1 and the following terms ν_m with $m > 1$ have an asymmetrical shape of a derivative of a maximum affected by the symmetrical factor $J_m^2(\lambda\varkappa q_y l_B)$. For noninteracting electrons, $S'(q,\omega)$ has a negative minimum at $\omega - \omega_c = \gamma_{0,1}/\sqrt{2}$. At this point, the whole sum $\nu_{\rm eff}$ becomes negative already at $\lambda = 1.4 \times 10^{-3}$ due to the proximity of the CR condition and the extreme sharpness of Landau levels. This estimate is very promising for experimental studies of negative conductivity effects in the system of SEs on liquid helium.

At the chosen value of $\omega - \omega_c$, the sum over m converges quite rapidly. Still, each next term in the sum of $\nu_{\rm eff}$ has its own minimum which is closer to the point $\omega_c = \omega$. This follows from Eqs. (8) and (9): for each energy exchange $\hbar\Omega = \hbar m\omega$ there is a term with $n' - n = m$ having a sharp maximum near $\omega_c = \omega$ (the sharpness of the maximum increases with m). The derivative of such a term contributes to ν_m of Eq. (12). Since the effect of $E_{\rm ac}$ eventually comes from $\dot{\xi}(t)$, an approach to the resonance condition is equivalent to an effective increase in $E_{\rm ac}$ which explains the importance of multiphoton terms. Thus, in the vicinity of the resonance, a substantial number of ν_m should be taken into account. This situation is illustrated in Fig. 1 where partial sums $\sum_{m=0}^{m_{\max}} \nu_m$ with different $m_{\max}$ are shown as functions of B. Here, the damping parameter $\gamma_* = \nu_{cl}$, and the many-electron DSF is taken into account for

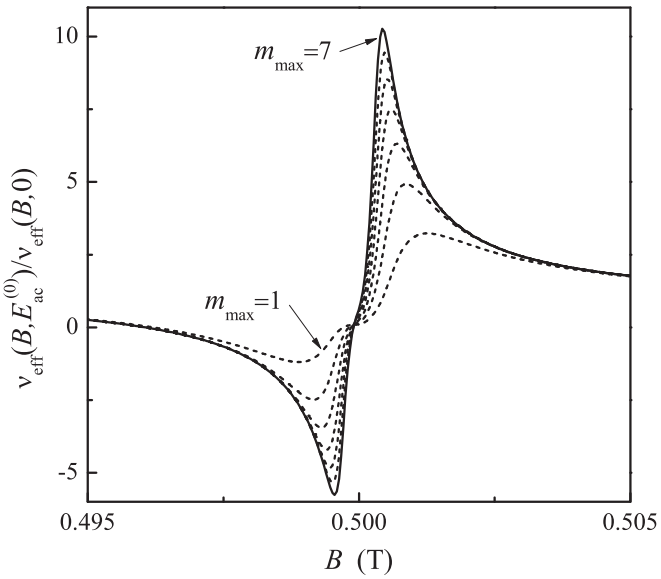

FIG. 1. Contributions from partial sums $\sum_{m=0}^{m_{\max}} \nu_m$ to $\nu_{\rm eff}$ normalized vs the magnetic field B for a sequence of $m_{\max}$: from $m_{\max} = 1$ to $m_{\max} = 7$ (solid). The conditions are the following: $T = 0.2$ K (liquid ^{4}He), $n_s = 10^6$ cm^{-2}, and $E_{\rm ac}^{(0)} = 0.05$ V/cm.

$n_s = 1 \times 10^6$ cm^{-2}. It is clear that an inclusion of higher terms only enhances the effect of negative conductivity making the minimum deeper and shifting its position closer to the point $\omega_c = \omega$.

Figure 2 illustrates how Coulomb forces affect $\sigma_{xx}(B)$. Here it was instructive to set $\gamma_* = 0$. One can see that an increase in n_s strongly suppresses the conductivity minimum and maximum near the CR without substantial changes in their positions and broadening. This is contrary to the Coulombic

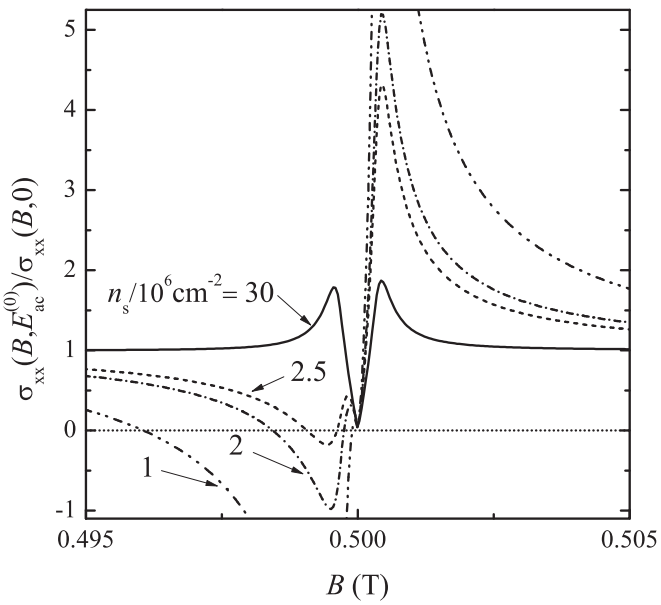

FIG. 2. The magnetoconductivity σ_{xx} normalized vs B for different electron densities: $n_s/10^6$ cm$^{-2} = 1$ (dash-dot-dotted), 2 (dash-dotted), 2.5 (dashed), and 30 (solid). Here $m_{\max} = 7$. Other conditions are the same as in Fig. 1.

effect reported previously for intersubband displacement mechanism [24] and for conductivity oscillations with $\omega/\omega_c \geqslant 2$ caused by one-photon assisted scattering [25].

For $n_s \gtrsim 2 \times 10^6$ cm^{-2}, an additional maximum is formed at $\omega - \omega_c > 0$. Both the minimum and the maximum of the region $\omega - \omega_c > 0$ are moving up when n_s increases. Eventually, the curve $\sigma_{xx}(B)$ obtains the shape with two maxima settled in the regions $\omega - \omega_c > 0$ and $\omega - \omega_c < 0$ (the latter one is higher) and with a strong minimum positioned between the maxima at $\omega - \omega_c$. It should be noted that a similar shape of the dc conductivity affected by the CR was experimentally observed for the vapor atom scattering regime [26]. An important conclusion which follows from Fig. 2 is that electron density should be rather small to obtain the negative conductivity regime giving rise to a state with $\sigma_{xx} = 0$ similar to ZRS in semiconductor systems.

Recent experiments [27] indicate an unusually large expansion of the electron system in a lateral direction, which cannot be understood in the framework of the generally accepted effective electron temperature approximation. Electron densities used in this experiment were rather high $n_s > 40 \times 10^6$ cm^{-2} which does not allow us to use directly an explanation based on the negative dc conductivity. Still, for electron temperatures T_e estimated there, the system enters the regime $U_c/T_e < 1$, where the fluctuational field model fails and the single-electron treatment could be a much better approximation. In this regime, the negative dc conductivity could appear even for a high n_s, because ν_0 decreases with T_e stronger than the sign-changing terms ν_m with $m \geqslant 1$. In a Corbino geometry, the formation of a steady ring current j, making $\sigma_{xx}(j) = 0$, should be accompanied by a lateral redistribution of SEs. Predictions on properties of the ring current are possible only in a nonlinear (in $E_{\rm dc}$) treatment which requires separate investigations. Nevertheless, the present theory allows us to formulate experimental conditions, where negative dc conductivity effects can be observed.

In summary, we have investigated theoretically the influence of cyclotron resonant excitation on the dc magnetoconductivity of the highly correlated 2D electron system formed on the surface of liquid helium. In the low electron density region ($n_s \lesssim 10^6$ cm^{-2}), where the Coulomb interaction is weak enough, the dc magnetoconductivity is shown to reach negative values for quite usual amplitudes of the MW field which causes instability of the zero-dc-current state. The Coulomb interaction, increasing with electron density, is shown to eliminate this effect, and at high densities (above 10^7 cm^{-2}) the theory presented here yields the dc conductivity behavior which is similar to that observed in experiments. We found also that in the high-density range, instability could be triggered by electron heating which restores the applicability of the single-electron theory.

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