Depinning transition of bubble phases in a high Landau level

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(Received 28 December 2014; revised manuscript received 10 February 2015; published 2 March 2015)

In the higher Landau levels ($N > 0$) a reentrant integer quantum Hall effect (RIQHE) state, which resides at fractional filling factors but exhibits integer Hall plateaus, has been previously observed and studied extensively. The nonlinear dynamics of the RIQHE were measured by microwave resonance, with the results consistent with an electronic bubble phase pinned by impurities. We have carried out depinning experiments on the $N = 2$ bubble phases by using Corbino geometry, where depinning threshold values have been systematically measured as a function of magnetic fields and temperatures. Domain sizes and pinning potential of the bubble phases have been estimated from the nonlinear transport data.

DOI: 10.1103/PhysRevB.91.115301 PACS number(s): 73.43.−f, 73.20.Qt

I. INTRODUCTION

In condensed matter systems nonlinear dynamics are commonly associated with a class of translational-symmetry-broken phases, often termed electronic crystals. Archetypical examples include Wigner crystals (WCs), the striped phase in high-T_c superconductors, the charge order in organic materials, and charged colloidal crystals. Among these, nonlinear transport of the charge density wave (CDW) has been studied extensively [1,2], where the CDW is pinned by impurities in a small-bias regime. Its dynamics can be revealed by dramatic transport behaviors, such as threshold electric field (E_T) and conductance steps resulting from the depinning transition in an increasing bias. In the quantum Hall system which is realized in a two-dimensional electron gas (2DEG) under an intensive magnetic field, the WC, striped, and bubble phases have been observed [3–13]. In order to understand the nonlinear dynamics of these phases, it is of great interest to directly measure their critical transport parameters, such as thresholds for the depinning. However, such information is often obscured by off-diagonal components of the conductance tensor under magnetic fields.

In the $N = 1$ Landau level (LL), the even-denominator $\nu = 5/2$ [$\nu = n/(eB/h)$, where n is the electron density] fractional quantum Hall effect (FQHE) is generally believed to host non-Abelian quasiparticles that are potentially relevant to topological quantum computation [14,15]. In order to systematically study the correlation physics in the $N = 1$ LL, recent work has also been focused on the reentrant integer quantum Hall effect (RIQHE) around $\nu = 5/2$ [16–21]. In the $N = 2$ LL, the RIQHE between $\nu = 4$ and $\nu = 5$ integer quantum Hall effects (IQHE) has been observed in the square samples, with filling factors generally believed to be ∼4.25 and ∼4.75 [17,22,23]. For those two states, microwave experiments have confirmed the bubble phases by resonance in the real part of the diagonal conductivity [24]. Nonlinear transport measurement in these states also provides support for the insulating bubble phases [25,26], which are analogous to the WCs [27–29] but with multiple electrons per site [12]. In the WCs or bubble phases, the conductivity increases above E_T, consistent with a depinning transition. However, the values of E_T in WCs reported by different groups spanned orders of magnitude [30]. Although such a discrepancy is not fully understood, the mixture of longitudinal resistance and Hall resistance is thought to significantly influence the outcome. As shown here, such ambiguity could be completely avoided by using a Corbino sample [31].

In this article we report on the distinctive sliding transition of the bubble phases in the $N = 2$ LL of a very high-mobility 2DEG system, by measuring the E_T in Corbino samples. We observe remarkably strong pinning of the bubble phase which may suggest extremely small domain sizes of electron bubbles. The E_T as a function of temperatures and magnetic fields is determined. Based on the phase diagram of E_T, we discuss the form factor of bubbles and the critical phenomenon around E_T. We also demonstrate that nonlinear transport can be a useful tool to distinguish RIQHE and IQHE states.

II. SAMPLES AND EXPERIMENT

The measurements were made on two GaAs/AlGaAs heterostructures. The densities of Sample A and Sample B are 4.2×10^{11} and 2.8×10^{11} cm^{-2}, and their motilities are 2.1×10^7 and 2.8×10^7 cm V^{-1}s^{-1}, respectively. Sample A (B) has an inner diameter of 1.8 mm (1.4 mm) and an outer diameter of 2.0 mm (1.6 mm). Figure 1(a) shows the longitudinal resistance and the Hall resistance in a square sample (from the same wafer of Sample A), which shows corresponding integer Hall plateaus, confirming the RIQHE. The measurements were carried out in a dilution fridge with a base temperature of 6 mK (measured at the mixing chamber, T_{MC}) and a base electron temperature of about 25 mK. The differential conductivity was measured using a standard lock-in technique at 23 Hz, as shown in Figs. 1(b) and 1(c). The maximum ac voltage drop across Corbino samples was

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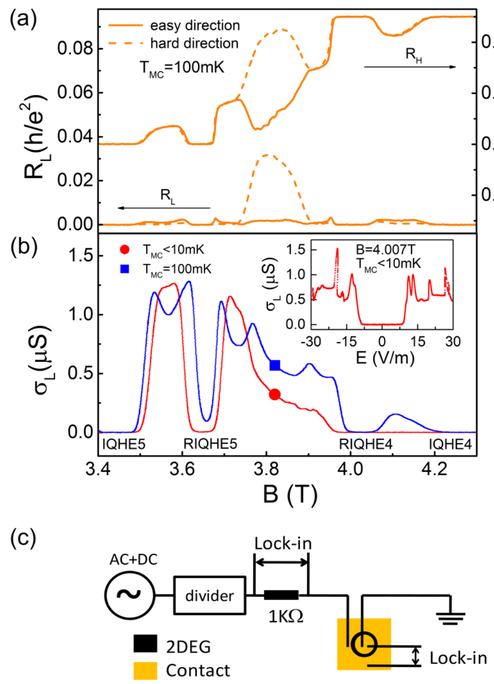

FIG. 1. (Color online) (a) Longitudinal and Hall resistance of a square sample made from the same wafer as Corbino Sample A. The signature of the RIQHE is clearly identified. (b) Longitudinal conductivity of Corbino Sample A at two temperatures, normalized by the sample geometry. Inset: An example of sliding for the RIQHE. (c) Differential conductivity measurement setup for Corbino samples. The electric field across the sample is provided by the dc voltage drop between inner and outer contacts divided by the 2DEG ring width.

50 μV. Samples were illuminated by a red light-emitting diode at 4.5 K and 15 μA for 1 h before measurements were taken.

III. CONDUCTANCE THRESHOLD

In Fig. 1(a), a square sample clearly shows the anisotropic transport at $\nu = 9/2$ [22,23] and the features of reentrant states at ∼4.25 and ∼4.75 filling factors. In a square or Hall bar geometry, the RIQHE is similar to the IQHE with a vanishing longitudinal resistance and, more interestingly, with an integer quantization of Hall resistance reentrant to the neighboring quantum Hall plateau. Being pinned by disorder, electrons in the partially filled LLs do not contribute to the RIQHE conductivities [16,17,20–23,32–35]. The resistance measurements in Fig. 1(a) confirm the previous observations of the RIQHE.

In the Corbino geometry, both RIQHE and IQHE exhibit zero conductivity, with edge states being shunted by electrodes. In Fig. 1(b), Corbino Sample A exhibits the insulating behavior for both IQHE and RIQHE at T_{MC} of 100 mK, and the RIQHE = 4 insulating state is separated from its neighboring IQHE = 4 insulating state by a conducting region in the magnetic fields. The RIQHE can also be identified from the comparison with the square sample made from the same wafer. The transport in the RIQHE with Corbino samples is free of Hall field contributions so that the electric field across the samples can be precisely controlled. In the RIQHE, when electric fields are applied to the Corbino samples by a dc bias,

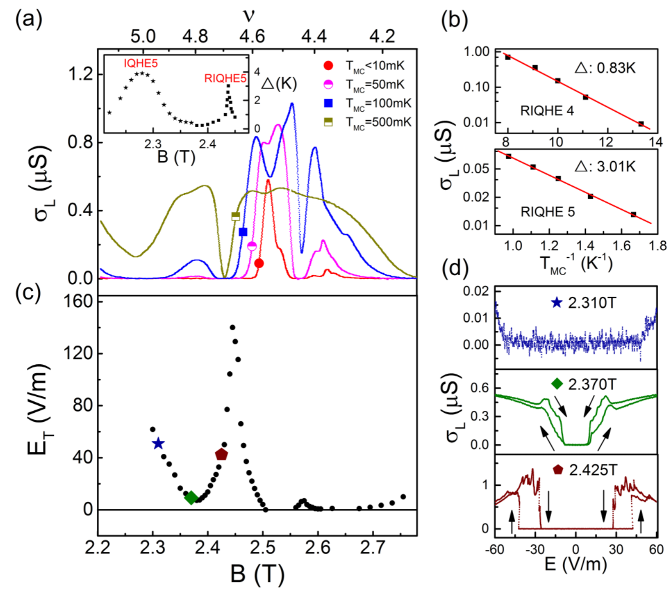

FIG. 2. (Color online) (a) Differential conductivity of Corbino Sample B as a function of the magnetic field at different temperatures. Inset: The energy gap Δ as a function of the magnetic field, showing a transition from IQHE to RIQHE. Below 2.375 T (symbol star), the energy gap is fitted by the equation $\sigma_L \propto \exp(-\Delta/2k_BT)$. Above 2.375 T (symbol square), the energy gap is fitted by the equation $\sigma_L \propto \exp(-\Delta/k_BT)$. (b) Examples of the energy gap of reentrant states, fitted by the equation $\sigma_L \propto \exp(-\Delta/k_BT)$. (c) Threshold electric field E_T as a function of the magnetic field at the lowest temperature. (d) From top to bottom: Examples of the IQHE breakdown, crossover behavior from IQHE = 5 to RIQHE = 5, and RIQHE depinning, respectively. The examples at three filling factors are labeled differently in panel (c).

the depinning transition from the insulating state appears at a certain threshold. The inset of Fig. 1(b) shows an example of nonlinear behaviors in the RIQHE. The rapid disappearance of the insulating state, the irregular differential conductivity above E_T, and the hysteresis are evidence of the depinning of the bubble phases. In this work, E_T is defined as the critical value for such a transition, experimentally determined in sweeping the sample dc voltage bias upward. Similar sliding behaviors in the RIQHE have been observed in a wide range of temperatures and in certain magnetic field regimes, for both samples. These observations conform to the characteristics of a large class of CDWs [1,2,4–6,8–13,36–39] in solids, where metastability and hysteresis are commonly observed at low temperatures [1,2].

IV. DOMAIN SIZE OF THE BUBBLE PHASES

The conductivity of Corbino Sample B at different temperatures is shown in Fig. 2(a). The characteristic of depinning is observed in this sample again. The lowest temperatures E_T in Sample A are 9.03 V/m (center of RIQHE = 4) and 5.94 V/m (center of RIQHE = 5). The lowest temperatures E_T in Sample B are 2.57 V/m (center of RIQHE = 4) and 140.0 V/m (center of RIQHE = 5). Bubbles form domains that slide under the electric field and domain size can be estimated from E_T [30]. The lattice constant of the bubble crystal is approximately $3.3R_c$, where cyclotron radius $R_c = \sqrt{2N+1} \cdot l$, with l being the magnetic length and N being the LL index [5]. Given the sample densities and the reentrant states we studied, the relevant lattice constant is around 100 nm. Based on Eq. (5.1) in Ref. [30], the ratios of domain size over lattice constant are ~13 (Sample A, RIQHE = 4), ~16 (Sample A, RIQHE = 5), ~20 (Sample B, RIQHE = 4), and ~3 (Sample B, RIQHE = 5) at the respective filling factors and at the lowest temperature, assuming one electron per bubble. If there are M electrons in a bubble, the domain size is enlarged by a factor of $\sqrt{M}$ [30,40]. A shrinking domain size leads to the disappearance of long-range order and implies competitions with noncrystalline structure, such as smectic and nematic states [9,13].

V. FORM FACTORS

The large difference in E_T between Samples A and B suggests that corresponding bubbles may have different form factors (the number of electrons per bubble), depending on the sample details. In Sample A, the filling factors and energy gaps for RIQHE = 4 and RIQHE = 5 are 4.25 and 4.70 and 1.56 K and 1.24 K, respectively. In Sample B, the filling factors and energy gaps for RIQHE = 4 and RIQHE = 5 are 4.41 and 4.67 and 0.83 K and 3.01 K, respectively [Fig. 2(b)]. Here the precise filling factors of the RIQHE are determined by the position of the minimum conductivity at elevated temperatures, when the bubble phases start to melt. Theoretical calculations suggest that the form factor depends on the partial filling factor [37,38,41]. The one-, two-, and three-electron bubble phases and the stripe phase are predicted in the $N = 2$ LL by Hartree-Fock theory but three-electron bubbles are not expected by the density matrix renormalization group calculations [41]. A filling factor of 4.41, which is closer to 9/2, for the RIQHE implies bubbles of three electrons, different from the two-electron bubbles commonly observed at ~4.25.

VI. PINNING POTENTIAL

Nonlinear responses of the longitudinal conductivity to electric fields are systematically measured in Sample B at different temperatures and magnetic fields. The phase diagram of the RIQHE, which is mapped according to E_T, clearly shows that RIQHE = 5 and IQHE = 5 are well separated [Fig. 3(a)] at high temperatures. Interestingly, the most stable filling factor in terms of the melting temperature is not the same for the highest E_T [Fig. 3(b)]. Therefore, the possibility of the thermal heating breakdown process for the nonlinear behavior of the RIQHE can be excluded. We can make a simple assumption that the domain pinning potential is comparable to the thermal activated energy plus the external electric energy, which is the charge under the electric field over the domain size. As a result, $M \times \mathrm{e} \times d \times E_T \sim -k_B T$, where M is the number of electrons per site and d is the characteristic domain size. The observed E_T at the center of the reentrant state shows a nearly negative linear dependence on the temperature in this work, indicating a domain size on the order of 0.1 to 1 μm at different reentrant states, the same order of magnitude as the domain sizes directly calculated from E_T at base temperature.

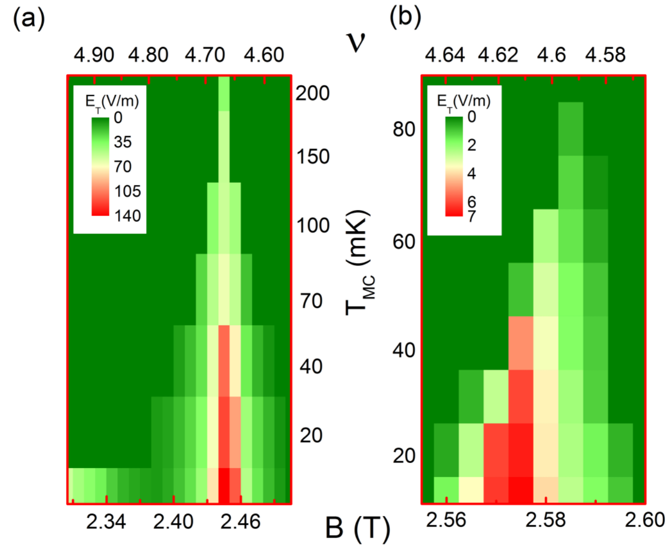

FIG. 3. (Color online) (a) *B-T* phase diagram of RIQHE = 5 for Sample B. Different colors represent different values of threshold electric fields. (b) *B-T* phase diagram of RIQHE = 4 for Sample B. Different colors represent different values of threshold electric fields.

The conductivity measurement also allows for the study of critical phenomenon around E_T [42]. As the electric field E is lowered towards E_T, the movement of the domain freezes and its velocity will behave as a power law exponent of the reduced field, where the reduced field is $(E - E_T)/E_T$. Mean field theory predicts the exponent of 3/2 [42]. The fitted exponents are 1.85 ± 0.02 (Sample A, RIQHE = 4), 1.96 ± 0.07 (Sample A, RIQHE = 5), 1.60 ± 0.02 (Sample B, RIQHE = 4), and 3.3 ± 0.6 (Sample B, RIQHE = 5) at the center of the reentrant state and at the lowest temperature, implying that the details of the interaction pertaining to this phase transition may play an important role.

VII. ONSET TEMPERATURE

A recent experimental study [17] points out that the onset temperatures of the RIQHE in the $N = 1$ and $N = 2$ LLs are inconsistent with the calculated cohesive energy [38,40] and suggests that the number of electrons per bubble in the $N = 2$ LL is likely different from what was previously predicted. In Ref. [17], the ratio of the reduced onset temperature between RIQHE = 4 and RIQHE = 5 is slightly larger than 1, which is similar to our results for Sample A. The reduced onset temperature is defined as $k_B T_c / E_c$, where T_c is the onset temperature determined by a sharp peak in longitudinal resistance [16] and E_c is the Coulomb energy. In Sample B, the reduced onset temperature is 11.2×10^{-4} for RIQHE = 4 and 57.4×10^{-4} for RIQHE = 5, where we use the disappearance of zero conductivity to determine the onset temperature. The reduced onset temperature provides a contrast for energy scales of different reentrant states. Judging from the different melting temperatures of the RIQHE between Sample A and Sample B, it is likely that the number of electrons per bubble depends on factors beyond the interaction between electrons and the magnetic field. Another recent experimental work on the $N = 1$ LL also suggests that bubble phases are more complicated than currently understood [21].

VIII. MERGING OF IQHE AND RIQHE

Figure 2(c) shows Sample B's E_T as a function of the magnetic field at the lowest temperature, where the RIQHE = 5 insulating state fully develops and merges with IQHE = 5. Similar behaviors are also observed near RIQHE = 4 in the other wafer, in both the Corbino and the square samples. At a higher temperature, the RIQHE and the IQHE are both zero-conductivity states, and they are separated by a conductivity peak, indicating an insulator-metal-insulator transition as expected. Given the different contact sizes and different methods used to fabricate the contacts, we can rule out the scenario that the merging results from a lack of electrochemical potential mixing between 2DEG and contacts. Mixed insulating phases between the bubble phase and the WCs have been proposed by both microwave and transport measurements [43,44]. Inhomogeneity of the 2DEG and the separation between IQHE regions and patches of electronic solids may contribute to the phenomenon we observed. Similar mixing of two different phases has been suggested near the filling factor $1/3$ state [45].

Before closing, we would like to point out that the RIQHE and the IQHE show different nonlinear transport characteristics. At the lowest temperature, RIQHE = 5 and IQHE = 5 states are indistinguishable in traditional transport. Note that the breakdown effect will also destroy the IQHE's insulating state [46]. Figure 2(d) shows, respectively, examples of nonlinear transport in the IQHE, transition from the IQHE to the RIQHE, and the RIQHE. Although the relevant E_T are similar, there are significant differences between breakdown and depinning. The conductivity in breakdown changes slowly and smoothly above E_T. The conductivity is zero from $\nu = 4.6$ to $\nu = 5.0$, and the nature of insulating can result from either the IQHE or the RIQHE, indicating a transition from a single-particle insulating state to a many-body insulating state with the same Chern number. Nonlinear transport presents evidence for both the insulating IQHE and the insulating RIQHE, and the transition between them.

IX. CONCLUSION

In summary, we have reported low-temperature transport experiments of the insulating states between $\nu = 4$ and $\nu = 5$ using Corbino samples, where the intrinsic conductivity threshold is determined. The results are remarkably consistent with a field-driven depinning transition of the bubble phases. The sliding of the reentrant states is studied at various temperatures and magnetic fields, which leads to a phase diagram of the RIQHE in the $N = 2$ LL. We note that the same experiments can be readily performed in the $N = 1$ LL, where the physics of the $5/2$ FQHE and nearby RIQHE states should be further revealed by nonlinear transport characteristics.

ACKNOWLEDGMENTS

We are grateful to Xincheng Xie, Fa Wang, Chi Zhang, and Junren Shi for their helpful comments. The work at PKU was funded by the NBRPC (Grant No. 2012CB921301) and the NSFC (Grants No. 11274020 and No. 11322435). The work at Rice was funded by the DOE (Grant No. DE-FG02-06ER46274). The work at Princeton was partially funded by the Gordon and Betty Moore Foundation as well as the National Science Foundation MRSEC Program through the Princeton Center for Complex Materials (Grant No. DMR-0819860).

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