

Observation of incompressibility at $\nu = 4/11$ and $\nu = 5/13$

N. Samkharadze,¹ I. Arnold,¹ L. N. Pfeiffer,² K. W. West,² and G. A. Csáthy^{1,3}

¹*Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47907, USA*

²*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

³*Birck Nanotechnology Center, Purdue University, West Lafayette, Indiana 47907, USA*

(Received 27 December 2014; published 23 February 2015)

The region of filling factors $1/3 < \nu < 2/5$ is predicted to support new types of fractional quantum Hall states with topological order different from that of the Laughlin-Jain or the Moore-Read states. Incompressibility is a necessary condition for the formation of such novel topological states. We find that at 6.9 mK incompressibility develops only at $\nu = 4/11$ and $5/13$, while the states at $\nu = 6/17$ and $3/8$ remain compressible. Our observations at $\nu = 4/11$ and $5/13$ are first steps towards understanding emergent topological order in these fractional quantum Hall states.

DOI: [10.1103/PhysRevB.91.081109](https://doi.org/10.1103/PhysRevB.91.081109)

PACS number(s): 73.63.Hs, 73.43.Qt

The study of electronic systems with topological properties is one of the most active areas of research in contemporary condensed matter physics. This field of study was opened up by the discovery of the integer quantum Hall states (IQHSs) developing in the two-dimensional electron gas (2DEG) subjected to a perpendicular magnetic field (B) [1]. Topological phases distinct from the IQHSs also exist at zero magnetic field in topological insulators and topological superconductors [2]. We witnessed a rapid development of the theory of topological order evident in efforts to classify topological phases, to identify topological invariants, as well as to extend the theory beyond the known topological phases [3].

Strong interactions in many-body systems are recognized as being instrumental in the generation of emergent topological order. A textbook example of such emergent order is that of the conventional fractional quantum Hall states (FQHSs) described by Laughlin's wave function and Jain's theory of composite fermions (CFs) [4–6]. However, it is well known that electron-electron interactions also lead to FQHSs with order fundamentally different from that of the conventional Laughlin-Jain states. One example of such an exotic FQHS is the even-denominator FQHS at $\nu = 5/2$, which is thought to be described by a wave function of the Moore-Read type [7–10].

The observation of a depression, i.e., a local minimum, in the magnetoresistance R_{xx} at the filling factor $\nu = 4/11$ [11] gave a new impetus to the quest for an enlarged family of topological ground states. As noted in Ref. [11], the FQHS at $\nu = 4/11$ can be understood as a fractional quantum Hall effect of composite fermions. Early theoretical work suggested that, owing to the residual interactions between the CFs, at $\nu = 4/11$ the ground state is an unusual FQHS [12]. In contrast, subsequent work found that this state may be either a conventional FQHS [13–16] or even an electron crystal [17–20]. Work by Wójs, Yi, Quinn, and collaborators reinforced a novel FQHS at $\nu = 4/11$, termed the WYQ state [21–25]. Most recently the idea of novel topological order at $\nu = 4/11$ of the WYQ type received strong support from CF diagonalization over an extended Hilbert space [26] and which included a more accurate model of the interaction of CFs than the earlier used one [21,22]. Even though a trial wave function for the WYQ state remains yet to be constructed, calculations reveal that the WYQ state is topologically distinct

[21,22,26] from either the Laughlin-Jain [5,6] or the Moore-Read state [9]. This difference is quantified by a different shift, a topological invariant of the Haldane sphere [27,28].

Incompressibility is a necessary condition for establishing topological order. While the theoretical support favoring a FQHS with a novel topological order at $\nu = 4/11$ is compelling [21,22,26], an unambiguous experimental confirmation of incompressibility of the ground states at this filling factor is still missing. Incompressibility is signaled by the opening of an energy gap Δ in the density of states which in transport measurements manifests itself in an activated magnetoresistance of the form $R_{xx} \propto \exp(-\Delta/2k_B T)$. Here T is the temperature and k_B Boltzmann's constant. Testing for an activated behavior is of importance since a depression in R_{xx} may not always develop into an activated R_{xx} ; hence such a depression does not guarantee the development of a FQHS. Examples of filling factors at which a fractional quantum Hall ground state does not develop in spite of the presence of a depression in R_{xx} are $\nu = 1/7$ [29,30] and $2/11$ [30].

In this Rapid Communication we present a study of correlated states in the $1/3 < \nu < 2/5$ range at ultralow temperatures, a factor of 5 below that of previous studies at these filling factors [11]. We report an opening of an energy gap at $\nu = 4/11$ as seen in the observation of an activated magnetoresistance. Furthermore, the temperature dependence of magnetotransport at $\nu = 5/13$ is also consistent with the development of an incipient gap. Our results establish a fractional quantum Hall ground state at these two filling factors and open up the possibility for existence of novel topological order of the WYQ type in this region. However, despite considerable progress at $\nu = 4/11$ and $5/13$, the transport data in our sample at other filling factors of interest, such as $\nu = 3/8$ and $6/17$, do not exhibit signatures of an energy gap. The nature of the ground state at these two filling factors remains uncertain.

The sample used in this study is a symmetrically doped 56 nm wide GaAs/AlGaAs quantum well with electron density $8.3 \times 10^{10} \text{ cm}^{-2}$ and mobility $12 \times 10^6 \text{ cm}^2/\text{V s}$. When cooling the sample, it is of the essence to ensure that the electronic temperature follows that of the phonons. We achieved this in our experiment by the use of a He-3 immersion cell [31,32]. Electrons are thermalized by being passed through sintered silver heat exchangers directly attached onto the sample and

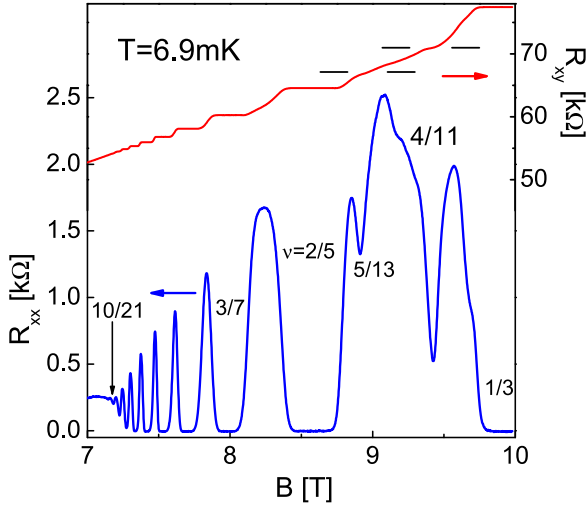


FIG. 1. (Color online) Longitudinal (R_{xx}) and Hall (R_{xy}) resistance in the filling factor range $1/3 < \nu < 1/2$ at 6.9 mK. Horizontal lines indicate the expected plateau-like behavior of the Hall resistance at $\nu = 4/11$ and $\nu = 5/13$.

which are immersed into the He-3 bath. The temperature of the He-3 bath is monitored via a quartz tuning fork thermometer, which provides magnetic-field-independent readings to the lowest temperatures [32].

Figure 1 shows the longitudinal magnetoresistance R_{xx} and the Hall resistance R_{xy} in the region of interest $1/3 < \nu < 1/2$ at 6.9 mK. A long series of conventional FQHSs [6] are observed at $\nu = n/(2n + 1)$ with n being an integer up to 10, demonstrating a sample of high quality. Between the two strong conventional FQHSs at $\nu = 1/3$ and $2/5$ we notice the presence of several features. For example, at $\nu = 4/11$ and $5/13$ we observe deep minima in the magnetoresistance and strong plateau-like inflections in the Hall resistance at $11h/4e^2$ and $13h/5e^2$, respectively. These features indicate the possibility of FQHSs at these filling factors.

A better appreciation of the features of R_{xx} can be gained from Fig. 2 by examining the region of interest $1/3 < \nu < 2/5$ on a magnified horizontal scale. At the relatively high temperature of 72 mK, there are four discernible depressions in the magnetoresistance curve at $\nu = 5/13, 3/8, 4/11$, and $6/17$ filling factors. These depressions are quite similar to those seen in Ref. [11] at 35 mK. However, as the temperature is decreased not all of these depressions evolve similarly. At $\nu = 4/11, 5/13$, and $3/8$ the depressions persist as T is lowered. In fact, the depressions at $\nu = 4/11$ and $5/13$ clearly get more pronounced at 6.9 mK. In contrast, the feature at $\nu = 6/17$ develops quite differently from those at the other three filling factors. As T is lowered, this depression significantly weakens at 25 mK and it has virtually disappeared at 15 mK. At the lowest achieved temperature of 6.9 mK, the R_{xx} shows only a weak inflection near $\nu = 6/17$, which is not separated from the $\nu = 1/3$ plateau with a local maximum. Similarly, R_{xy} at 6.9 mK does not show a quantization to $17h/(6e^2)$, but it is very close to $3h/e^2$ instead. This suggests that the system at the electronic filling factor $\nu = 6/17$ tends to localize in the limit of zero temperature, joining the $\nu = 1/3$ plateau. Thus we conclude that, in contrast to an earlier report [11], the ground

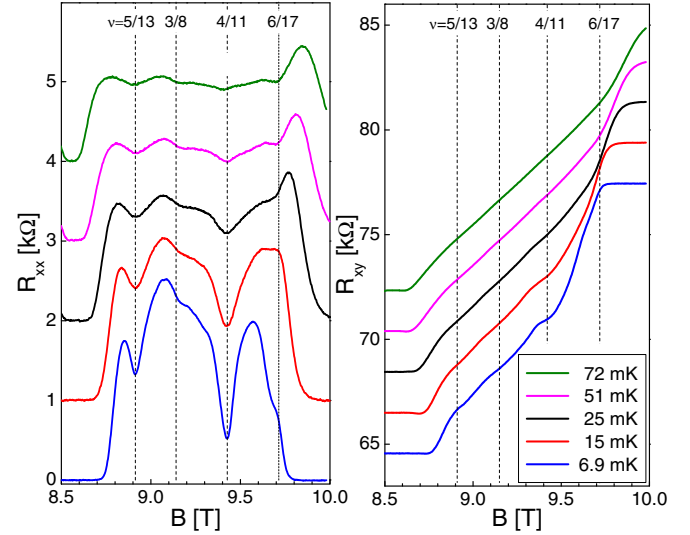


FIG. 2. (Color online) Waterfall plot of R_{xx} and R_{xy} curves at several different temperatures in the $1/3 < \nu < 2/5$ range. The dashed lines mark $\nu = 5/13, 3/8, 4/11$, and $6/17$ filling factors. Scales refer to the lowest-temperature data sets. For clarity, R_{xx} traces are shifted vertically by 1 k Ω , while R_{xy} traces are shifted by 2 k Ω each relative to the trace of lower temperature.

state at $\nu = 6/17$ in our sample is not a FQHS. However, such a ground state may develop in samples of lower disorder.

Further insight on the ground states at $\nu = 4/11, 5/13$, and $3/8$ emerges from Fig. 3, where R_{xx} is plotted against T . Since significant differences between R_{xx} at these three filling factors develop only below $T \approx 30$ mK, Fig. 3 demonstrates that accessing these low temperatures is a critical capability in the study of the $1/3 < \nu < 2/5$ region. We notice that as the temperature is lowered from 100 mK, R_{xx} at $\nu = 4/11$ first increases, then it drops significantly. These data are consistent with a vanishing R_{xx} in the $T = 0$ limit. Similarly, at the lowest temperatures, R_{xx} at $\nu = 5/13$ is also decreasing, albeit this decrease is not as pronounced as that at $\nu = 4/11$. The observed behavior at $\nu = 4/11$ and $5/13$ indicates incompressibility and, therefore, it establishes the ground state at these two filling factors is a genuine FQHS.

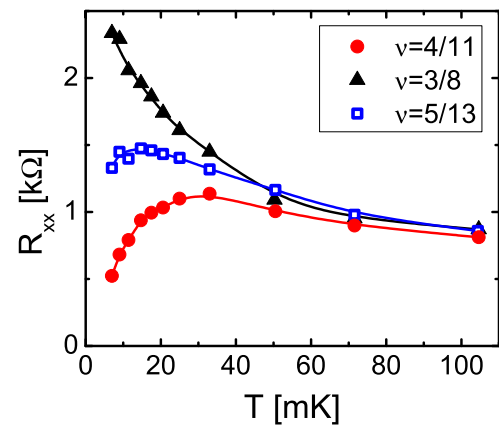


FIG. 3. (Color online) Dependence of magnetoresistance on temperature at the filling factors $\nu = 4/11, 3/8$, and $5/13$. Lines are guides to the eye.

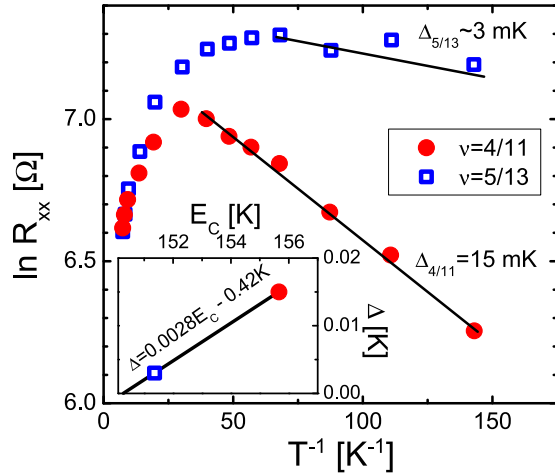


FIG. 4. (Color online) Arrhenius plot for R_{xx} at $\nu = 4/11$ and $\nu = 5/13$ filling factors. Straight lines are used as guides for the eye representing the activated regions. The inset shows the scaling of the experimentally extracted gaps Δ at $\nu = 4/11$ and $5/13$ states with the Coulomb energy E_C .

An examination of the temperature dependence of the magnetoresistance at $\nu = 4/11$ below $T = 30$ mK reveals that it is activated. Indeed, as shown in the Arrhenius plot Fig. 4, at the lowest temperatures we find a linear dependence of $\ln R_{xx}$ on $1/T$. The slope of this linear portion of the data yields an energy gap of $\Delta_{4/11} = 15$ mK. A similar analysis of the incipient incompressibility at $\nu = 5/13$ results in an upper bound of the energy gap $\Delta_{5/13} \simeq 3$ mK.

The FQHS at $\nu = 4/11$ is quite feeble as seen from its small energy gap $\Delta_{4/11} = 15$ mK. This value comes short by more than one order of magnitude when compared to the expected value $0.002E_C \approx 0.3$ K from recent numerical work [26]. Here $E_C = e^2/4\pi\epsilon l_B$ is the Coulomb energy, $l_B = \sqrt{\hbar/eB}$ is the magnetic length, and ϵ is the dielectric constant in GaAs. However, in Ref. [26] disorder effects are not included. In order to estimate the disorder-free, or intrinsic gap $\delta_{4/11}$ of the $\nu = 4/11$ FQHS, we employ a method of scaling of the gaps of a pair of particle-hole conjugate FQHSs with Coulomb energy [33]. This method has been previously used for gap estimations for the $\nu = 5/2$ FQHS, e.g., Refs. [33,34]. According to this method, the measured energy gaps scale with the Coulomb energy according to $\Delta^{\text{meas}} = \delta^{\text{int}} E_C - \Gamma$, where the term Γ is commonly referred to as the disorder broadening parameter. We assume that the $\nu = 4/11$ and $\nu = 5/13$ FQHSs are particle-hole conjugated, an assumption which may not hold for a dissimilar spin polarization. In the inset of Fig. 4 we plot the experimental gaps of $\nu = 4/11$ and $\nu = 5/13$ FQHSs against Coulomb energy. Fitting these two points, we extract the intrinsic gap $\delta^{4/11} \approx 0.0028$, which is of the same order of magnitude as 0.002, the value found in a recent numerical simulation [26]. One must note, however, that the numerical results do not include finite-thickness effects and are not extrapolated to the thermodynamic limit [26]. Within the assumptions used, the similarity of the intrinsic gap extracted from our data and that from numerics [26] raises the possibility of novel topological order of the WYQ type in the FQHS at $\nu = 4/11$ and at $5/13$.

So far we have established incompressibility, i.e., the formation of a gap, at $\nu = 4/11$ and the agreement of the measured and the calculated intrinsic gaps. These are necessary but not sufficient conditions for establishing the formation of the WYQ state at this filling factor. Another necessary condition for the formation of the WYQ state is a full spin polarization of CFs at this filling factor. Numerics cannot yet accurately predict in the thermodynamic limit the spin polarization field for realistic sample parameters [13–16,21,26,35]. Experimentally the tilted field technique may reveal the spin state of the FQHS at $\nu = 4/11$ [11]. However, because of the weakness of this state, convincing evidence of a spin transition remains an extremely challenging task.

In contrast to the temperature dependence of R_{xx} at $\nu = 4/11$ and $5/13$, that at $\nu = 3/8$ is very different. While the depression in R_{xx} is present at $\nu = 3/8$ to the lowest temperatures, its magnitude does not seem to develop significantly as the temperature is dropped. In addition, as seen in Fig. 3, R_{xx} at $\nu = 3/8$ continues to increase monotonically with a decreasing T . Furthermore, R_{xy} does not show signs of a developing plateau. The behavior of the magnetoresistance is thus inconsistent with an activated behavior and incompressibility despite the existence of a depression in R_{xx} at this filling factor. In the absence of incompressibility, the ground state at $\nu = 3/8$ and, as discussed earlier, at $6/17$ are not ascertained to be a FQHS.

A FQHS at $\nu = 2 + 3/8$ has been observed in several samples [31,32,36–40] and it is believed that this state, similarly to the $\nu = 5/2$ FQHS, is a paired CF state [41]. However, the formation of a FQHS at $\nu = 2 + 3/8$ in the second Landau level does not guarantee the formation of a FQHS at $\nu = 3/8$ in the lowest Landau level [19,20,42,43]. It is interesting to compare other two filling factors to the half-filled Landau levels in the lowest and second Landau level. At $\nu = 2 + 1/2 = 5/2$ a FQHS develops [7], whereas at $\nu = 1/2$ there is a compressible Fermi sea of CFs [44,45]. For these pairs of filling factors, the difference in the interactions between the CFs in different Landau levels clearly leads to very different ground states. It is thus not surprising that the presence of a FQHS at $\nu = 2 + 3/8$ may be accompanied by a compressible state at $\nu = 3/8$. Generally speaking, the conditions of the formation of paired CF states are currently not understood. Even though we do not observe incompressibility at $\nu = 3/8$, the possibility of opening of a gap at this filling factor in either higher-quality samples or at lower temperatures cannot be ruled out at this stage.

In conclusion, we performed ultralow-temperature measurements of the $1/3 < \nu < 2/5$ filling factor region, where some of fractional quantum Hall states are expected to have novel topological order. We presented compelling evidence that the ground states at $\nu = 4/11$ and $5/13$ are incompressible. Our observations ensure genuine fractional quantum Hall ground states and pave the way towards establishing exotic topological order at these two filling factors. In contrast, the ground state in our sample at two other filling factors in this region, namely at $\nu = 3/8$ and $6/17$, is compressible to the lowest temperatures accessed. The development of fractional quantum Hall states at the latter two filling factors remains an open question.

Note added. In the recent work by Pan *et al.* [46] incompressibility at $\nu = 4/11$ has been reported with an energy gap of 7 mK. This gap is a factor of 2 lower than the gap we found. Since the sample densities are different, the difference in the gaps may be due to a nontrivial spin behavior.

N.S. and G.A.C. acknowledge funds from NSF Grant No. DMR-1207375. The work at Princeton was funded by the Gordon and Betty Moore Foundation through Grant No. GBMF 4420, and by the National Science Foundation MRSEC at the Princeton Center for Complex Materials. We thank J. Jain for useful discussions.

-
- [1] K. von Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* **45**, 494 (1980).
- [2] M. Z. Hassan and C. L. Kane, *Rev. Mod. Phys.* **82**, 3045 (2010).
- [3] X.-G. Wen, *Adv. Phys.* **44**, 405 (1995).
- [4] D. C. Tsui, H. L. Stormer, and A. C. Gossard, *Phys. Rev. Lett.* **48**, 1559 (1982).
- [5] R. B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983).
- [6] J. K. Jain, *Phys. Rev. B* **40**, 8079 (1989).
- [7] R. Willett, J. P. Eisenstein, H. L. Stormer, D. C. Tsui, A. C. Gossard, and J. H. English, *Phys. Rev. Lett.* **59**, 1776 (1987).
- [8] X. G. Wen, *Phys. Rev. Lett.* **66**, 802 (1991).
- [9] G. Moore and N. Read, *Nucl. Phys. B* **360**, 362 (1991).
- [10] C. Nayak, S. Simon, A. Stern, M. Freedman, and S. Das Sarma, *Rev. Mod. Phys.* **80**, 1083 (2008).
- [11] W. Pan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, *Phys. Rev. Lett.* **90**, 016801 (2003).
- [12] P. Sitko, K.-S. Yi, and J. J. Quinn, *Phys. Rev. B* **56**, 12417 (1997).
- [13] K. Park and J. K. Jain, *Phys. Rev. B* **62**, R13274(R) (2000).
- [14] C.-C. Chang, S. S. Mandal, and J. K. Jain, *Phys. Rev. B* **67**, 121305 (2003).
- [15] M. O. Goerbig, P. Lederer, and C. M. Smith, *Phys. Rev. Lett.* **93**, 216802 (2004).
- [16] C.-C. Chang and J. K. Jain, *Phys. Rev. Lett.* **92**, 196806 (2004).
- [17] S.-Y. Lee, V. W. Scarola, and J. K. Jain, *Phys. Rev. Lett.* **87**, 256803 (2001).
- [18] S.-Y. Lee, V. W. Scarola, and J. K. Jain, *Phys. Rev. B* **66**, 085336 (2002).
- [19] N. Shibata and D. Yoshioka, *J. Phys. Soc. Jpn.* **73**, 1 (2004).
- [20] N. Shibata and D. Yoshioka, *J. Phys. Soc. Jpn.* **73**, 2169 (2004).
- [21] A. Wójs, K.-S. Yi, and J. J. Quinn, *Phys. Rev. B* **69**, 205322 (2004).
- [22] A. Wójs, D. Wodziński, and J. J. Quinn, *Phys. Rev. B* **71**, 245331 (2005).
- [23] A. Wójs, D. Wodziński, and J. J. Quinn, *Phys. Rev. B* **74**, 035315 (2006).
- [24] A. Wójs, G. Simion, and J. J. Quinn, *Phys. Rev. B* **75**, 155318 (2007).
- [25] A. Wójs, *Phys. Rev. B* **79**, 041304 (2009).
- [26] S. Mukherjee, S. S. Mandal, Y.-H. Wu, A. Wójs, and J. K. Jain, *Phys. Rev. Lett.* **112**, 016801 (2014).
- [27] F. D. M. Haldane, *Phys. Rev. Lett.* **51**, 605 (1983).
- [28] X. G. Wen and A. Zee, *Phys. Rev. B* **46**, 2290 (1992).
- [29] V. J. Goldman, M. Shayegan, and D. C. Tsui, *Phys. Rev. Lett.* **61**, 881 (1988).
- [30] W. Pan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, *Phys. Rev. Lett.* **88**, 176802 (2002).
- [31] J. S. Xia, W. Pan, C. L. Vicente, E. D. Adams, N. S. Sullivan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, *Phys. Rev. Lett.* **93**, 176809 (2004).
- [32] N. Samkharadze, A. Kumar, M. J. Manfra, L. N. Pfeiffer, K. W. West, and G. A. Csáthy, *Rev. Sci. Instrum.* **82**, 053902 (2011).
- [33] R. H. Morf and N. d'Ambrumenil, *Phys. Rev. B* **68**, 113309 (2003).
- [34] N. Samkharadze, J. D. Watson, G. Gardner, M. J. Manfra, L. N. Pfeiffer, K. W. West, and G. A. Csáthy, *Phys. Rev. B* **84**, 121305 (2011).
- [35] S. Mukherjee, S. S. Mandal, Y.-H. Wu, A. Wójs, and J. K. Jain, *Phys. Rev. Lett.* **112**, 199902(E) (2014).
- [36] H. C. Choi, W. Kang, S. Das Sarma, L. N. Pfeiffer, and K. W. West, *Phys. Rev. B* **77**, 081301 (2008).
- [37] A. Kumar, G. A. Csáthy, M. J. Manfra, L. N. Pfeiffer, and K. W. West, *Phys. Rev. Lett.* **105**, 246808 (2010).
- [38] Nianpei Deng, J. D. Watson, L. P. Rokhinson, M. J. Manfra, and G. A. Csáthy, *Phys. Rev. B* **86**, 201301 (2012).
- [39] Chi Zhang, Chao Huan, J. S. Xia, N. S. Sullivan, W. Pan, K. W. Baldwin, K. W. West, L. N. Pfeiffer, and D. C. Tsui, *Phys. Rev. B* **85**, 241302 (2012).
- [40] Nianpei Deng, G. C. Gardner, S. Mondal, E. Kleinbaum, M. J. Manfra, and G. A. Csáthy, *Phys. Rev. Lett.* **112**, 116804 (2014).
- [41] C. Töke, C. Shi, and J. K. Jain, *Phys. Rev. B* **77**, 245305 (2008).
- [42] V. W. Scarola, J. K. Jain, and E. H. Rezayi, *Phys. Rev. Lett.* **88**, 216804 (2002).
- [43] S. Mukherjee, S. S. Mandal, A. Wójs, and J. K. Jain, *Phys. Rev. Lett.* **109**, 256801 (2012).
- [44] R. R. Du, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, *Phys. Rev. Lett.* **70**, 2944 (1993).
- [45] W. Kang, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, *Phys. Rev. Lett.* **71**, 3850 (1993).
- [46] W. Pan, K. W. Baldwin, K. W. West, L. N. Pfeiffer, and D. C. Tsui, *Phys. Rev. B* **91**, 041301 (2015).