Superfluid amplitude fluctuations above T_c in a unitary Fermi gas

Hua Li,[1,*] Jason Jackiewicz,[2] and Kevin S. Bedell[1]

[1]*Physics Department, Boston College, Chestnut Hill, Massachusetts 02467, USA*

[2]*Department of Astronomy, New Mexico State University, Las Cruces, New Mexico 88003, USA*

(Received 29 October 2014; revised manuscript received 21 December 2014; published 10 February 2015)

We study the transport properties of a Fermi gas with strong attractive interactions close to the unitary limit. In particular, we compute the spin diffusion lifetime of the Fermi gas due to superfluid fluctuations above the BCS transition temperature T_c. To calculate the spin diffusion lifetime we need the scattering amplitudes. The scattering amplitudes are dominated by the superfluid fluctuations at temperatures just above T_c. The normal scattering amplitudes are calculated from the Landau parameters. These Landau parameters are obtained from the local version of the induced interaction model for computing Landau parameters. We also calculate the leading order finite temperature correction to the diffusion lifetime. A calculation of the spin diffusion coefficient is presented in the end. Upon choosing a proper value of F_0^a, we are able to present a good match between the theoretical result and the experimental measurement, which indicates the presence of the superfluid fluctuations near T_c.

DOI: 10.1103/PhysRevB.91.075107 PACS number(s): 67.85.Lm, 67.10.Jn, 51.20.+d

I. INTRODUCTION

With the first experimental realization of Bose-Einstein condensation (BEC) in a Bose gas in 1995 [1–3], there has been an enormous amount of experimental and theoretical work carried out to study ultracold atomic physics [4,5]. In addition to Bose gases. there are as well cold Fermi gases, with an interaction strength that can be tuned by the proximity of a Feshbach resonance. At resonance the Fermi gas is said to be at unitarity. The superfluid transition temperature of an ultracold Fermi gas decreases exponentially with decreasing interaction strength in the weakly attracting limit [5] $T_c \approx 0.28 T_F e^{\pi/2k_F a}$, where T_F and k_F are the Fermi temperature and the Fermi wave vector, respectively, and a is the scattering length. At unitarity the Fermi gas is strongly correlated, so one expects a big boost in T_c due to the increasing pairing gap approaching unitarity and thus a significantly larger critical region above T_c compared to a nonunitary dilute Fermi gas with attractive interactions (BCS regime). The superfluid lambda transition which was once difficult to observe in a dilute Fermi gas has also been experimentally realized recently in a unitary Fermi gas [6].

In this paper, we are interested in revealing superfluid fluctuations in the spin diffusion lifetime of a unitary Fermi gas above T_c. The quasiparticle scattering amplitudes near the Fermi surface are essential in calculating the spin diffusion lifetime [7]. At temperatures close to T_c, the scattering amplitudes are greatly affected by the formation of Cooper pairs. Superfluid fluctuations dominate the quasiparticle scattering right above T_c. The spin diffusion lifetime is calculated using Fermi-liquid theory by evaluating the total scattering probability with the superfluid fluctuations included. The Landau parameters needed in calculating the scattering amplitudes are computed using the local induced interaction model [8,9]. Further, the leading order finite temperature correction to the spin diffusion lifetime is calculated to complete the calculation. Finally, the spin diffusion coefficient of a two-component unitary Fermi gas is calculated to compare with the experiment [10].

*huali@bc.edu

II. SUPERFLUID FLUCTUATIONS IN THE SCATTERING AMPLITUDES

Superfluid fluctuations in the transport lifetimes of a unitary Fermi gas are investigated through calculating the quasiparticle scattering amplitudes of the gas near T_c in a similar fashion as an earlier study on zero-sound attenuation in liquid ^{3}He [11]. As the temperature approaches T_c, the virtual formation of Cooper pairs starts to dominate the quasiparticle scattering process. Singularities in the scattering amplitudes are found for small total momentum quasiparticle scattering, leading to diverging scattering amplitudes at T_c for zero total momentum quasiparticle scattering. The exact calculation of superfluid fluctuations in the scattering amplitudes is done by evaluating the temperature vertex function of particle-particle type in the singlet channel. The equation of the temperature vertex function is given by summing over the various "ladder diagrams" of the vertex function [12],

$$\mathcal{T}_s(p_1,p_2;p_3,p_4) = \tilde{\mathcal{T}}_s(p_1,p_2;p_3,p_4) - \frac{T}{2(2\pi)^3} \times \Sigma_{\omega_n} \int \tilde{\mathcal{T}}_s(p_1,p_2;k,q-k)\mathcal{G}(q-k) \times \mathcal{G}(k)\mathcal{T}_s(k,q-k;p_3,p_4)d^3\mathbf{k}, \quad (1)$$

where $\tilde{\mathcal{T}}_s$ is the temperature particle-particle irreducible vertex function, $\mathcal{G}$ is the exact temperature Green's function, and $\omega_n = (2n+1)\pi T$ are the "odd frequencies" for fermions. Here we have introduced the four-vector $p_i = (\mathbf{p}_i,\omega_i)$ to denote the momenta of the incident and scattered particles, and $q = (\mathbf{q},\omega_0)$ stands for the total momentum of the incident particles. $\mathcal{T}_s$ depends only on the total momentum q, $\mathcal{T}_s(p_1,p_2;p_3,p_4) \equiv \mathcal{T}_s(q)$, when $|\mathbf{p}_i| = k_F$ for $i = 1,\ldots,4$ and $|\mathbf{q}| \ll k_F$. Integrating out the second term on the right-hand side of Eq. (1), we have in the small q limit, with $\omega_0 = 0$ the temperature vertex function,

$$\mathcal{T}_s(\mathbf{q},0) = \frac{1}{\frac{mp_f}{4\pi^2}\left[\ln\frac{T_c}{T} - \frac{1}{6}\left(\frac{v_f|\mathbf{q}|}{2\omega_D}\right)^2 - \frac{7\zeta(3)}{3\pi^2}\left(\frac{v_f|\mathbf{q}|}{4T}\right)^2\right]}, \quad (2)$$

where $T_c = \frac{2\gamma\omega_D}{\pi} e^{-4\pi^2/mp_f|\tilde{\Gamma}_s|}$, $\ln\gamma$ is the Euler's constant, p_f is the Fermi momentum, and $\omega_D = 0.244\varepsilon_F$ is the cutoff frequency [13]. Here we have set $\tilde{\mathcal{T}}_s = \tilde{\Gamma}_s$, where $\tilde{\Gamma}_s$ is the zero temperature irreducible particle-particle vertex function. $\tilde{\Gamma}_s$ is equivalent to the normal Fermi-liquid scattering amplitude $\tilde{\Gamma}_s N(0) = A_0^{\text{sing}} = A_0^s - 3A_0^a$, where $N(0) = \frac{m^* p_f}{\pi^2\hbar^3}$ is the density of states at the Fermi surface, $A_0^{s,a} = \frac{F_0^{s,a}}{1+F_0^{s,a}}$, and $F_0^{s,a}$ are the Landau parameters. The total quasiparticle scattering probability is obtained by averaging the scattering amplitudes of different $\mathbf{q}$'s over the phase space [7], $\langle W\rangle \equiv \int \frac{d\Omega}{4\pi}\frac{W(\theta,\phi)}{\cos(\theta/2)}$. Superfluid fluctuations are phase space limited as virtual Cooper pair formation breaks down when the total momentum of the pair exceeds a certain value $\mathbf{q}_{\max}$, where $v_f|\mathbf{q}_{\max}| = \sqrt{6\varpi}$ and $\varpi = 2\omega_D e^{-4\pi^2/mp_f|\tilde{\Gamma}_s|}$ from regular quantum field theory computations [12]. Hence quasiparticle scattering processes with total momentum larger than $\mathbf{q}_{\max}$ are treated by normal Fermi-liquid theory with the scattering amplitudes being the normal Fermi-liquid scattering amplitudes. The phase space average of the scattering amplitudes could then be readily separated into a normal part and a superfluid fluctuation part,

$$\langle W\rangle = \int_0^{\mathbf{q}_{\max}} \frac{d\Omega}{4\pi}\frac{W_f(\theta,\phi)}{\cos(\theta/2)} + \int_{\mathbf{q}_{\max}}^{2P_f} \frac{d\Omega}{4\pi}\frac{W_n(\theta,\phi)}{\cos(\theta/2)}$$
$$= \langle W\rangle_{\text{fluctuations}} + \langle W\rangle_{\text{normal}}, \quad (3)$$

where $\langle W\rangle_{\text{fluctuations}}$ and $\langle W\rangle_{\text{normal}}$ stand for superfluid fluctuations and normal Fermi-liquid scattering amplitudes, respectively.

III. LOCAL INDUCED INTERACTION MODEL

The Landau parameters are calculated using the local induced interaction model. The induced interaction model was first introduced in the 1970's [14] to describe the quasiparticle interaction of liquid ^{3}He. The more general momentum dependent scattering amplitude model was developed in the 1980's [15–17]. Such a theory splits the quasiparticle interaction into two pieces, the direct and the induced, as shown diagrammatically in Fig. 1. The induced term comes from the part of the interactions induced through the exchange of the collective excitations, whereas the direct term is the Fourier transform of a model dependent effective quasiparticle potential. The generalized expressions of the Landau parameters were derived diagrammatically by Ainsworth and Bedell [17]. In the local limit of a Fermi liquid, the quasiparticle interaction is independent of the momentum [8], thus Landau parameters $F_l^{s(a)}$ with $l > 0$ are all zero. The set of equations is reduced [9,18] to

$$F_0^s = D_0^s + \tfrac{1}{2}F_0^s A_0^s + \tfrac{3}{2}F_0^a A_0^a, \quad (4)$$

$$F_0^a = D_0^a + \tfrac{1}{2}F_0^s A_0^s - \tfrac{1}{2}F_0^a A_0^a. \quad (5)$$

Additionally, the scattering amplitudes are related to the Landau parameters as [7]

$$A_l^{s,a} = \frac{F_l^{s,a}}{1 + F_l^{s,a}/(2l+1)}, \quad (6)$$

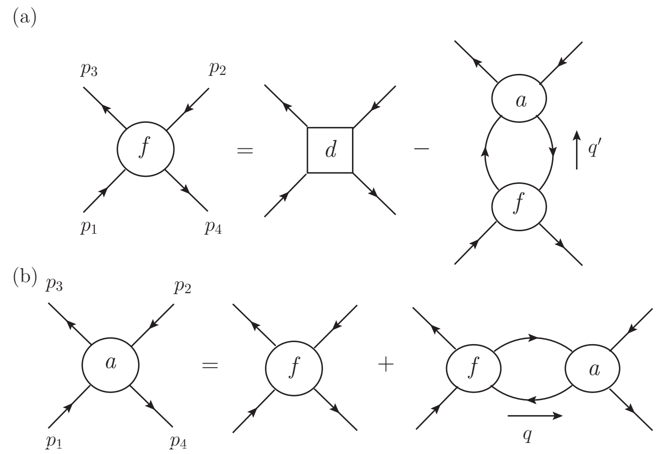

FIG. 1. Diagrammatic representation of the integral equations for the Landau parameters F and the scattering amplitudes a. (a) represents the equation for Landau parameters decomposed into direct and induced terms; (b) sums all the reducible diagrams. It represents the equation relating F to the scattering amplitudes a.

and the forward scattering sum rule [7] $\sum_l (A_l^s + A_l^a) = 0$ is reduced to $A_0^s + A_0^a = 0$. The direct terms are fully antisymmetrized so that $D_0^{\uparrow\uparrow(\downarrow\downarrow)} = 0$. According to Ainsworth and Bedell [17], $D_0^s = \frac{N(0)}{2}(D_0^{\uparrow\uparrow} + D_0^{\uparrow\downarrow}) = \frac{2}{\pi}k_F a_s$ and $D_0^a = -\frac{N(0)}{2}(D_0^{\uparrow\uparrow} + D_0^{\uparrow\downarrow}) = -\frac{2}{\pi}k_F a_s$, where a_s is the quasiparticle s-wave scattering length. We derive from the local induced interaction model the expression for the scattering length as a function of F_0^a,

$$\frac{-1}{k_F a_s} = \frac{8}{\pi}\frac{(1+F_0^a)(1+2F_0^a)}{F_0^a + 3F_0^a(1+2F_0^a)^2}. \quad (7)$$

This relation is depicted in Fig. 2. The quasiparticle interaction strength of a Fermi gas is characterized by the s-wave scattering length a_s. On the BCS side of the BCS-BEC crossover [19], the s-wave scattering length of the Fermi gas is always negative and it goes to negative infinity at unitarity. Therefore, F_0^a of a unitary Fermi gas approaches positive infinity. Utilizing the local induced interaction model, we are

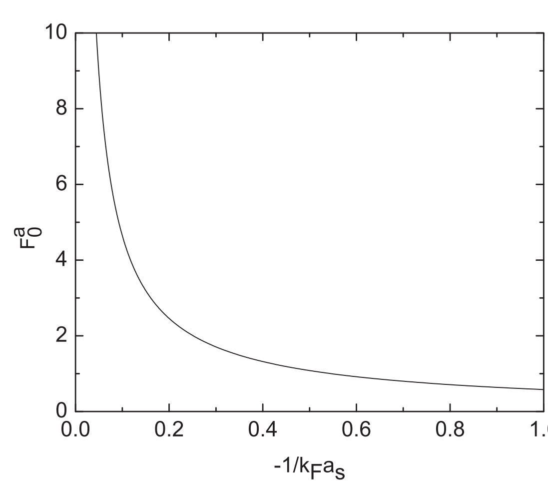

FIG. 2. The Landau parameter F_0^a vs $-(k_F a_s)^{-1}$ curve.

able to calculate the $F_0^{s,a}$ given the quasiparticle interaction strength of the Fermi gas.

Despite its simple structure and easy mathematics, the local induced interaction model does a good job of explaining the universal thermodynamics of a unitary Fermi gas. In a Galilean invariant system the mass renormalization disappears for a local Fermi liquid, $m^* = m$. The Landau parameter F_0^s saturates to -0.5 at unitarity from the local model. Hence the quasiparticle mass and Landau parameter F_0^s are both independent of the particle density n. From the compressibility of a normal gas extrapolated to zero temperature $\kappa = \frac{1}{n^2}\frac{N(0)}{1+F_0^s}$, we derive the relation between chemical potential μ and the Fermi energy E_F of an ideal gas, $\mu(n,0) = (1+F_0^s)\frac{m}{m^*}E_F$. We notice $1+F_0^s$ relates to the hypothetical zero temperature limit [6,20] ξ_n extrapolated from the normal Fermi-liquid chemical potential through $(1+F_0^s) = \frac{m^*}{m}\xi_n$. In the absence of the mass renormalization, $1+F_0^s$ is equivalent to ξ_n and differs from the true zero temperature Berstch parameter [6] ξ for not considering the superfluid condensation energy. The local induced interaction model [9,18] gives the value of $1+F_0^s = 0.5$ at unitarity, in agreement with the Monte Carlo calculations [20,21] $\xi_n = 0.54$. The leading order temperature dependence of several thermodynamic quantities is studied using basic thermodynamic analysis. We introduce the temperature scale $T_s \equiv \mu(n,0)$, and in the absence of spin polarization and mass renormalization, the chemical potential of a Fermi gas is given as [7]

$$\mu(n,T) = \mu(n,0)\left[1 - \frac{\pi^2(1+F_0^s)}{12}\left(\frac{T}{T_s}\right)^2\right]. \tag{8}$$

The chemical potential scales the same in temperature as a free Fermi gas. The total entropy is given as $S/Nk_B = \frac{\pi^2(1+F_0^s)}{2}\frac{T}{T_s}$ from Fermi-liquid theory. Based on thermodynamic relations $\kappa = \frac{1}{n^2}\frac{\partial n}{\partial \mu}$ and $dP = nd\mu + sdT$, we calculate the compressibility to be $\kappa(n,T) = \kappa(n,0)[1 + \frac{\pi^2(1+F_0^s)}{12}(\frac{T}{T_s})^2]^{-1}$ and the pressure to be $P(n,T) = P(n,0)[1 + \frac{5\pi^2(1+F_0^s)}{12}(\frac{T}{T_s})^2]$, where $\kappa(n,0) = \frac{1}{n^2}\frac{N(0)}{1+F_0^s}$ and $P(n,0) = \frac{2}{5}(1+F_0^s)nE_F$. All the thermodynamic quantities calculated above involve universal functions of the Fermi energy E_F and the ratio T/T_s, as expected for a unitary Fermi gas [22]. In addition, an effective s-wave scattering amplitude $\tilde{a}_0$ could be defined as [12]

$$\frac{\tilde{\Gamma}_s}{2} = \frac{4\pi\tilde{a}_0}{m}. \tag{9}$$

Analogous to the two-body scattering problem, we can write down the s-wave phase shift [23]

$$\delta_0 = k_F\tilde{a}_0 = \frac{\pi}{8}A_0^{\mathrm{sing}}. \tag{10}$$

Using the local induced interaction model, we show $\delta_0 = -\frac{\pi}{2}$ on the BCS side of the BCS-BEC crossover and $\delta_0 = \frac{\pi}{2}$ on the BEC side of the crossover. Based on Levinson's theorem, the increase in the phase shift by π indicates the appearance of a bound state on the BEC side of the BCS-BEC crossover, in agreement with the physics of the BCS-BEC crossover. A rough estimate of the molecular binding energy on the BEC side is made using $E_b = -\hbar^2/m\tilde{a}_0^2 \approx -0.8E_F$. We can also calculate the superfluid transition temperature of a unitary Fermi gas. According to the local induced interaction model, $F_0^a \to \infty$, hence $A_0^a = 1$ in the unitary limit. The superfluid transition temperature is then given [13] by $T_c = 0.28T_F e^{-4\pi^2/mp_f|\tilde{\Gamma}_s|} = 0.28T_F e^{-1/|A_0^a|} = 0.102T_F$. The local model predicts a T_c value relatively close to the experimentally measured [6] $T_c = 0.167T_F$. In the later calculations, we introduce a scaling factor $L = 1.64$ in the exponential term of T_c, $e^{-1/|A_0^a|} \to Le^{-1/|A_0^a|}$, to artificially lift T_c at unitarity from the local model prediction to its experimental value.

IV. TRANSPORT LIFETIMES AND SPIN DIFFUSION COEFFICIENT

The calculation of the spin diffusion lifetime τ_D is straightforward. In the low temperature limit $T \ll T_F$, it can be formulated in the language of Landau Fermi-liquid theory [7]. τ_D is proportional to the characteristic relaxation time τ, defined as

$$\tau \equiv \frac{8\pi^4\hbar^6}{m^{*3}\langle W\rangle(k_BT)^2}, \tag{11}$$

where $\langle W\rangle \equiv \int \frac{d\Omega}{4\pi}\frac{W(\theta,\phi)}{\cos(\theta/2)}$ and $W(\theta,\phi) = \frac{1}{2}(\frac{1}{2}W_{\uparrow\uparrow} + W_{\uparrow\downarrow}) = \frac{1}{2}W_{\uparrow\downarrow}(\theta,\phi)$ is the average scattering probability. The triplet scattering amplitude is zero in the local limit. Taking into consideration the superfluid fluctuations, the scattering probabilities $W_f(\theta,\phi)$ and $W_n(\theta,\phi)$ are calculated from their respective scattering amplitudes,

$$W_f(\theta,\phi) = \frac{1}{2}W_{\uparrow\downarrow} = \frac{1}{2}\frac{2\pi}{\hbar}|t_{\uparrow\downarrow}|^2 = \frac{1}{2}\frac{2\pi}{\hbar}\left|\frac{\mathcal{T}_s(\mathbf{q},0)}{2}\right|^2, \tag{12}$$

$$W_n(\theta,\phi) = \frac{1}{2}W_{\uparrow\downarrow} = \frac{1}{2}\frac{2\pi}{\hbar}|t_{\uparrow\downarrow}|^2 = \frac{1}{2}\frac{2\pi}{\hbar}\left|\frac{-2A_0^a}{N(0)}\right|^2. \tag{13}$$

Performing the integrals in Eq. (3), we have the phase space average of the scattering amplitudes,

$$\langle W\rangle_{\mathrm{normal}} = \frac{2\pi}{\hbar}\frac{2}{|N(0)|^2}2\left(1 - \frac{\sqrt{6}\pi}{4\gamma}\frac{T_c}{T_F}\right)|A_0^a|^2, \tag{14}$$

$$\begin{aligned}\langle W\rangle_{\mathrm{fluctuation}} &= \frac{2\pi}{\hbar}\frac{2}{|N(0)|^2} \\ &\times\left[\frac{\frac{\sqrt{6}\pi T_c}{4\gamma T_F}}{\ln\frac{T}{T_c}\left[\ln\frac{T}{T_c} + \left(\frac{\sqrt{6}\pi T_c}{4\gamma T_F}\right)^2\left(11.2 + 0.28\left(\frac{T_F}{T_c}\right)^2\right)\right]}\right. \\ &\left. + \frac{\tan^{-1}\left(\sqrt{\left(\frac{\sqrt{6}\pi T_c}{4\gamma T_F}\right)^2\left(11.2 + 0.28\left(\frac{T_F}{T_c}\right)^2\right)}\Big/\sqrt{\ln\frac{T}{T_c}}\right)}{\left(\ln\frac{T}{T_c}\right)^{3/2}\sqrt{11.2 + 0.28\left(\frac{T_F}{T_c}\right)^2}}\right].\end{aligned} \tag{15}$$

The low temperature expression of the spin diffusion lifetime is readily given by

$$\tau_D^0 = \left(\frac{\tau_D}{\tau}\right)\tau = \frac{0.129 \times 8\pi^4\hbar^6}{m^3\langle W\rangle(k_BT)^2}. \tag{16}$$

The leading order finite temperature correction to τ_D^0 is computed [24],

$$\frac{1}{\tau_D}-\frac{1}{\tau_D^0}=-\frac{3}{2}\pi\zeta(3)\frac{k_BT_F}{\hbar}\left(\frac{T}{T_F}\right)^3 \times\left[-2.95\left(A_0^a\right)^3+1.564\left(A_0^a\right)^2+1.278A_0^aF_0^a\right]. \tag{17}$$

We obtain the full expression of τ_D by solving Eq. (17),

$$\tau_D=\frac{\hbar}{k_BT_F}\left(\frac{T_F}{T}\right)^2\left(\frac{\hbar|N(0)|^2}{0.129\times 16}\langle W\rangle-\frac{3}{2}\pi\zeta(3) \times\left[-2.95\left(A_0^a\right)^3+1.564\left(A_0^a\right)^2+1.278A_0^aF_0^a\right]\frac{T}{T_F}\right)^{-1}. \tag{18}$$

In the end, we calculate the spin diffusion coefficient of a Fermi gas from τ_D to compare with the experiment [10]. The high temperature limit ($T \gg T_F$) of τ_D scales as $\tau \propto \frac{\hbar}{k_BT_F}(\frac{T}{T_F})^{1/2}$ [25]. The numerical factor in front is extrapolated from the experimental data [10] to give $\tau_D \approx 5.84\frac{\hbar}{k_BT_F}(\frac{T}{T_F})^{1/2}$. The spin diffusion coefficient is expressed as

$$D=\begin{cases}\frac{1}{3}v_f^2\left(1+F_0^a\right)\tau_D & \text{for } T\ll T_F,\\ \frac{k_BT}{m}\tau_D & \text{for } T\gg T_F.\end{cases} \tag{19}$$

V. RESULTS

There exists a singularity in τ_D when the temperature increases to a point T^* where the finite temperature correction term becomes comparable to $1/\tau_D^0$, according to Eq. (18). This singularity is an artifact of overextending the correction term in temperature and can be removed by expanding τ_D up to second order in T_F/T in Eq. (18). The low temperature feature of τ_D is well approximated by the expansion for $T \ll T^*$. Hereafter, we use the expansion to describe the low temperature behavior of τ_D. By choosing $F_0^a = 1.7$, we are able to present a good match between the calculated and the measured spin diffusion coefficient [10], as depicted in Fig. 3. The superfluid fluctuations cause the spin diffusion coefficient D to drop drastically when temperature approaches T_c. As the temperature moves away from T_c, D exhibits normal Fermi-liquid-like behavior going as $1/T^2$ for $T_c < T \ll T_F$. Since our theory is based on uniform Fermi systems, instead of introducing a scaling factor to account for the trap effect [26], we make an approximation in treating the trapped gas used in the experiment as a uniform one with an effective average density. We interpret $F_0^a = 1.7$ as the effective Landau parameter for the trapped gas. Although we are unable to make $F_0^a = \infty$ as predicted by the local model for unitarity, a Fermi system with $k_Fa \approx -3.3$ suggested by $F_0^a = 1.7$ is still considered as strongly interacting. In addition, the local model is constructed under zero temperature, therefore it is possible that F_0^a becomes temperature dependent and the theory deviates from its zero temperature version when temperature increases. We also plot τ_D with respect to T/T_F for several different values of F_0^a to see how τ_D evolves with different choices of F_0^a. The result is depicted in Fig. 4. The height of the peak in τ_D decreases as F_0^a increases, which

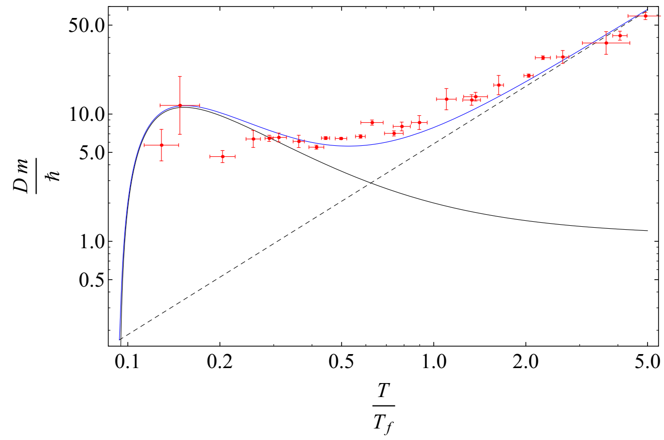

FIG. 3. (Color online) Spin diffusion coefficient curve from low to high temperature. The black solid curve is the low temperature expansion of D at $F_0^a = 1.7$; the dashed line is the classical limit of D; the blue curve represents the summation of the two limits; the red dots with error bars are the experimental data [10].

indicates that the superfluid fluctuations start to dominate at a higher temperature for a bigger F_0^a. This is expected since T_c increases when F_0^a increases. The theory fails to capture the correct feature of τ_D at intermediate temperatures when F_0^a becomes too large, but it succeeds in revealing the superfluid fluctuations above T_c through τ_D, regardless of the choice of F_0^a.

We have introduced the local approximation for the Fermi-liquid description of the cold atom Fermi gases and used the local version of the induced interaction to calculate the Fermi-liquid parameters. This has been done since this provides simple analytic results that provide qualitative and reasonably good quantitative results for the Fermi-liquid parameter F_0^s and the thermodynamic scaling temperature T_s as well as T_c. In earlier publications [18,27] we used the momentum dependent induced interaction which generated Fermi-liquid parameters with $l > 0$. In the unitary limit the induced interaction gives a small mass correction, about 15% above the bare mass,

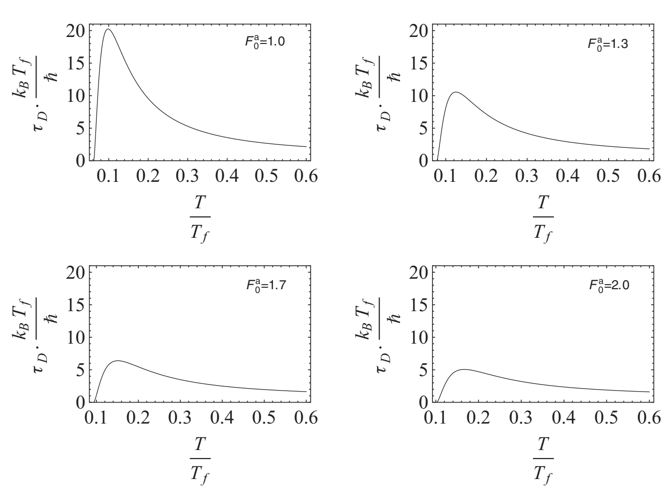

FIG. 4. The calculated spin diffusion lifetime curves with different F_0^a values.

and it gives $F_0^s = -0.6$. These numbers are independent of the density at unitarity so the thermodynamic scaling is just as what we found for the local model, but with a smaller value for T_s. We also found that when we use the $s-p$ approximation [7,27] to construct the scattering amplitude from our Fermi-liquid parameters, we get better fits for some of our calculated properties. These include T_c and E_b, where $T_c \approx 0.14 T_F$ and $E_b \approx -0.3 E_F$. Clearly, we can get better numerical results going beyond the local model, but it would not give us qualitatively different insights into some of the properties of this cold atom system. In particular, this would not qualitatively change the nature of the strong superfluid fluctuation effects in the spin diffusion just above T_c.

VI. CONCLUSION

In conclusion, we have presented a complete formula for calculating the transport lifetime above T_c of a Fermi gas with arbitrary quasiparticle interaction strength through control of F_0^a. Superfluid fluctuations above T_c in a unitary Fermi gas are revealed through calculation of the spin diffusion lifetime. Sudden decreases in τ_D above T_c are found as the evidence of the superfluid fluctuations. Upon choosing a proper value of $F_0^a = 1.7$, we are able to describe the experimental data of the spin diffusion coefficient using our theory. A similar analysis will be performed to the viscous lifetime and thermal diffusion lifetime in a future paper. Further work could also be done by using the $s-p$ approximation with the induced interaction model for calculating the Landau parameters.

ACKNOWLEDGMENTS

We thank J. Engelbrecht, S. Gaudio, B. Mihaila, P. Souders, J. Thomas, E. Timmermans, and M. Zwierlein for valuable and insightful discussions. This work is supported by John H, Rourke, Boston College endowment fund.

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[2] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).
[3] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[4] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[5] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **80**, 1215 (2008).
[6] M. J. H. Ku, A. T. Sommer, L. W. Cheuk, and M. W. Zwierlein, Science **335**, 563 (2012).
[7] G. Baym and C. J. Pethick, *Landau Fermi-Liquid Theory* (Wiley, New York, 1991).
[8] J. R. Engelbrecht and K. S. Bedell, Phys. Rev. Lett. **74**, 4265 (1995).
[9] J. A. Jackiewicz and K. S. Bedell, Philos. Mag. Lett. **85**, 1755 (2005).
[10] A. Sommer, M. Ku, G. Roati, and M. W. Zwierlein, Nature (London) **472**, 201 (2011).
[11] Vi. K. Samalam and J. W. Serene, Phys. Rev. Lett. **41**, 497 (1978).
[12] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1975).
[13] L. P. Gorkov and T. K. Melik-Barkhudarov, Sov. Phys. JETP **13**, 1018 (1961).
[14] S. Babu and G. E. Brown, Ann. Phys. **78**, 1 (1973).
[15] T. L. Ainsworth, K. S. Bedell, G. E. Brown, and K. F. Quader, J. Low Temp. Phys. **50**, 319 (1983).
[16] K. S. Bedell and K. F. Quader, Phys. Rev. B **32**, 3296 (1985).
[17] T. L. Ainsworth and K. S. Bedell, Phys. Rev. B **35**, 8425 (1987).
[18] S. Gaudio, J. Jackiewicz, and K. S. Bedell, Philos. Mag. Lett. **89**, 1823 (2009).
[19] C. J. Pethick and H. Smith, *Bose-Einstein Condensation in Dilute Gases* (Cambridge University Press, New York, 2008).
[20] S. Nascimbene, N. Navon, K. J. Jiang, F. Chevy, and C. Salomon, Nature (London) **463**, 1057 (2010).
[21] J. Carlson, S. Y. Chang, V. R. Pandharipande, and K. E. Schmidt, Phys. Rev. Lett. **91**, 050401 (2003)
[22] T.-L. Ho, Phys. Rev. Lett. **92**, 090402 (2004).
[23] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory* (Elsevier, Amsterdam, 1977).
[24] K. S. Dy and C. J. Pethick, Phys. Rev. **185**, 373 (1969).
[25] G. M. Brunn, New J. Phys. **13**, 035005 (2011).
[26] T. Enss and R. Haussmann, Phys. Rev. Lett. **109**, 195303 (2012).
[27] S. Gaudio, J. Jackiewicz, and K. S. Bedell, Philos. Mag. Lett. **87**, 713 (2007).