Hidden order as a source of interface superconductivity

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Interfacial superconductivity is observed in a variety of heterostructures composed of different materials including superconducting and nonsuperconducting (at appropriate doping and temperatures) cuprates and iron-based pnictides. The origin of this superconductivity remains in many cases unclear. Here, we propose a general mechanism of interfacial superconductivity for systems with competing order parameters. We assume that parameters characterizing the material allow formation of another order like charge- or spin-density wave competing and prevailing superconductivity in the bulk (hidden superconductivity). Diffusive electron scattering on the interface results in a suppression of this order and releasing the superconductivity. Our theory is based on the use of Ginzburg-Landau equations applicable to a broad class of systems. We demonstrate that the local superconductivity appears in the vicinity of the interface and the spatial dependence of the superconducting order parameter $\Delta(x)$ is described by the Gross-Pitaevskii equation. Solving this equation we obtain quantized values of temperature and doping levels at which $\Delta(x)$ appears. Remarkably, the local superconductivity shows up even in the case when the rival order is only slightly suppressed and may arise also on the surface of the sample (surface superconductivity).

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I. INTRODUCTION

Interesting phenomena have been discovered a few years ago in the study of superconductivity in different materials, especially in high-T_c superconductors—cuprates and Fe-based pnictides. It turned out that the critical temperature of the superconducting transition T_c in heterostructures, e.g., in bilayers, is higher than the critical temperature T_c of bare films that can be even nonsuperconducting [1–13]. The authors of Ref. [14] (see also Ref. [9]) studied bilayers consisting of two cuprates—overdoped ($La_{2-x}Sr_xCuO_4$ with $x = 0.35$) and underdoped ($0.1 < x < 0.12$) cuprate films. The largest increase of T_c was about 11 K—from $T_c = 21$ K in bare underdoped films to $T_c = 32$ K in bilayers. Moreover, superconductivity has been observed in bilayers composed of nonsuperconducting materials, $La_{2-x}Sr_xCuO_4$ with $x = 0$ (an insulator) and $La_{2-x}Sr_xCuO_4$ with $x = 0.45$ (normal metal) [2]. The critical temperature reached 50 K. Further experimental studies showed the independence of the critical transition temperature in bilayers $La_{2-x}Sr_xCuO_4/La_2SrCuO_4$ on x in a rather wide doping interval ($0.15 < x < 0.47$) [15].

A slight increase of T_c has been measured also in another high-T_c superconductor—$YBa_2Cu_3O_7$ films covered by a thin Ag film [16]. A few decades ago, a similar effect has been observed in bilayers composed of conventional low-T_c superconductors and normal metals [17–19]. These latter experiments were motivated by original Ginzburg's ideas on the possibility to get a surface superconductivity with a high transition temperature [20,21].

Other high-T_c superconductors where an enhancement of superconductivity at the interface has been established are the so-called iron-based pnictides discovered in 2008 [22]. In this type of superconductor competing order parameters (OP), magnetic and superconducting, may coexist. To be more exact, the spin density wave (SDW) and the superconducting OP may arise in these materials (see reviews in Refs. [23–27]) with the amplitudes W and Δ depending on temperature and doping. The interfacial superconductivity in one of the Fe-based pnictides (CaFeAs doped with La, Ce, Pr, or Nd) was observed by Wei *et al.* [28]. Whereas the bulk critical temperature was equal to about 30 K, a small fraction of samples had a $T_c \simeq 49$ K. Even higher $T_c \simeq 77$ K was achieved by a Chinese experimental group in another Fe-based material (single unit-cell FeSe films on $SrTiO_3$) [29].

Very encouraging for understanding the very nature of high-T_c superconductivity seems to be the effect of apparent enhancement of superconducting transition temperature at the interface between an iron-based chalcogenide superconductor (FeSe) and $SrTiO_3$ used as substrate [30,31]. This discovery questioned the role of phonons in bulk iron-based superconductors [30] and confirmed the presence of the magnetic order (spin-density wave) as a key ingredient for high-T_c superconductivity in iron-based superconductors [31].

Rather actively the interface superconductivity is studied in heterostructures $LaAlO_3/SrTiO_3$ [1]. It is assumed that a two-dimensional electron gas is formed at the interface. The effect of an electric field has been employed to explore the phase diagram of $LaAlO_3/SrTiO_3$ interface [3,4]. Aside from superconductivity, the phenomenon of ferromagnetism induced at the interface of an oxide heterostructure has been observed recently [32,33]. The review in Ref. [34] provides an excellent overview over the possible symmetries and degrees of freedom of correlated electrons that can evolve at oxide interfaces. It includes interalia, superconductivity, magnetism, ferroelectricity, and charge and spin orders as well. Moreover, it has been found that, at the interfaces between $LaAlO_3$ and $SrTiO_3$, superconductivity coexists with ferromagnetism [33,35–37], a surprising result offering a potential for exotic superconducting phenomena due to highly broken inversion symmetry of the interface and a ferromagnetic background [36].

Several theories have been suggested to explain the phenomenon of interface superconductivity. Some of them consider a nonuniform charge distribution near the interface and use a phenomenological relation of T_c to this distribution [7,10,38]. Different ideas have been used in other theories [39], where bilayers composed of superconductors with different ratio of the T_c and pairing strength were considered and it was assumed that T_c is suppressed by phase fluctuations. In the vicinity of the interface the role of these fluctuations is not important as compared to the bulk due to suppression of fluctuations by the proximity effect. However, this suggestion cannot explain the observed independence of T_c on the doping level x. Perhaps, there is no single mechanism responsible for the interface superconductivity because it was observed in quite different materials under various conditions. The important ingredient of this effect is the presence of an interface or some sort of nonhomogeneity.

In this paper we propose a mechanism for the interface superconductivity. The proposed mechanism is very robust and general being independent of the microscopic details of considered materials. It is applicable to any materials where, alongside the superconducting order parameter (OP), another OP exists. We do not pretend to apply our theory to any system where the interface superconductivity occurs, but we show that it can be used for materials in which two OPs may potentially exist. As is well known, in high-T_c superconductors, an important role is played by the charge or spin ordering [23–27]. In cuprates, a charge density wave (CDW) has been observed recently in numerous experiments [40–50] and discussed in Refs. [51–55]. It may exist alongside superconductivity, whereas in Fe-based superconductors, the spin density wave is more important. The presence of the nonsuperconducting OP W (CDW or SDW) changes such characteristics of superconducting state as London penetration depth [56–58], heat capacity [59,60], etc.

We show that the presence of a "hidden" OP Δ in a heterostructure with $W \neq 0$ may lead to the appearance of local superconductivity at the interface at temperatures $T > T_c(W)$, where $T_c(W)$ is the critical temperature of the superconducting transition of bare films composing the heterostructure which depends on the amplitude W. We consider a heterostructure with an interface where W is locally suppressed. This suppression may be caused by an enhanced impurity scattering and doping level in the vicinity of the interface. The enhanced impurity scattering can be caused by the interdiffusion of atoms and/or roughness of the interface. Both factors suppress the CDW or SDW. In this case, in a vicinity of the interface where W is suppressed, local superconductivity arises with $\Delta(x)$ decaying on a characteristic scale of the order of the superconducting coherence length ξ_s. Interestingly, the local superconductivity occurs at "quantized" temperatures because the OP $\Delta(x)$ is described by the linearized Gross-Pitaevskii equation, i.e., by the Schrödinger equation with a one-dimensional potential well which always has discrete energy levels (or only one level). In the one-dimensional case (flat interface) the local superconductivity arises even at a rather small suppression of W.

Note that stimulation of the bulk superconductivity by impurities in materials with two OPs was considered by one of the authors a long time ago [61]. Recently, the effect of superconductivity stimulation by impurities in the bulk of Fe-based pnictides with two OPs has been analyzed by Fernnades *et al.* in Ref. [62].

In Sec. II, the general nonhomogeneous Ginzburg-Landau equations describing two coupled order parameters are introduced. In the homogeneous case, we derive conditions for the coefficients in the Ginzburg-Landau equations that should be fulfilled for obtaining one of the possible three phases: (1) pure superconducting, (2) pure charge- or spin-density wave, and (3) a mixed state. In the nonhomogeneous case, a solution of the Ginzburg-Landau equations for the order parameters is obtained, whereby detailed calculations are shifted to the Appendixes. In Sec. III, we discuss applicability of the theory to the case when the constituents of the heterostructure are high-T_c cuprates or iron-based pnictides. We propose also an experimental setup suitable for testing our predictions. Concluding, we discuss our results in Sec. IV.

II. GENERAL CONSIDERATIONS

A. Free energy and self-consistency equations

We start with the expression for the free energy $\mathcal{F}$ for a system with two order parameters (OPs), Δ and W. In the one-dimensional case, i.e., in the case of a preferred direction provided by the interface, the free energy is given by

$$\mathcal{F} = \frac{1}{2}\int dx\bigg[\xi_s^2(\Delta')^2 - a_s\Delta^2 + \frac{b_s}{2}\Delta^4 + \gamma\Delta^2W^2 + \xi_w^2(W')^2 - a_wW^2 + \frac{b_w}{2}W^4\bigg], \tag{1}$$

where Δ' and W' denote the spatial derivatives of corresponding order parameters, and the coefficients $\xi_{s,w}$, $a_{s,w}$, $b_{s,w}$, and γ are in general not independent and show a complicated dependence on doping and/or temperature, and on the mean free path. These coefficients are presented in Sec. III for the case of cuprates and iron-based pnictides.

The free energy is written in the Ginzburg-Landau form of Eq. (1) in the vicinity of a critical temperature. However, the critical temperatures $T_{dw,s}$ of the transitions into a state with a finite W, respectively, Δ may be quite different. We assume that the doping level described by the parameter μ is chosen in such a way ($\mu = \mu_c$) that the critical temperature T_{dw} for the nonsuperconducting OP W coincides with the critical temperature of the superconducting transition T_s. This is possible because T_{dw} depends on μ, whereas T_s does not. Thus the coefficients $a_{s,w}$ and $b_{s,w}$ depend on the differences $\eta = (1 - T/T_s)$, $\delta[\mu^2] \equiv \mu^2 - \mu_c^2$, and on impurity concentration n_{imp}.

The variation of $\mathcal{F}$ with respect to Δ and W yields the self-consistency (or the Ginzburg-Landau) equations

$$-\xi_s^2\Delta'' + \Delta[-a_s + b_s\Delta^2 + \gamma W^2] = 0, \tag{2}$$

$$-\xi_w^2 W'' + W[-a_w + b_wW^2 + \gamma\Delta^2] = 0, \tag{3}$$

which represent the foundation of our considerations.

B. Homogeneous case

Equations (2) and (3) without the spatial derivatives yield different uniform solutions, i.e., three different points on the plane of order parameters (Δ, W). We denote these points by, respectively, Γ_Δ (where $\Delta \neq 0$, $W = 0$), Γ_W (where $\Delta = 0$, $W \neq 0$), and $\Gamma_{W\Delta}$ (where $\Delta \neq 0$, $W \neq 0$, i.e., both OPs coexist), each corresponding to an extremum of the free energy functional $\mathcal{F}(\Delta, W)$. Analyzing these points we determine the conditions for a particular point to correspond to a minimum as follows.

(1) Γ_Δ, where $\Delta = \sqrt{a_s/b_s}$, corresponds to a minimum if the second derivatives of $\mathcal{F}$ with respect to Δ and W are positive, implying

$$b_s > 0, \quad \gamma a_s - a_w b_s > 0. \tag{4}$$

In particular, the coefficient a_s must be positive.

(2) Γ_W, where $W = \sqrt{a_w/b_w}$, corresponds to a minimum if the conditions

$$b_w > 0, \quad \gamma a_w - a_s b_w > 0 \tag{5}$$

are fulfilled. In particular, the coefficient a_w must be positive.

(3) $\Gamma_{W\Delta}$, where $\Delta = \sqrt{(a_s b_w - \gamma a_w)/D}$ and $W = \sqrt{(a_w b_s - \gamma a_s)/D}$ with $D = b_s b_w - \gamma^2$, corresponds to a minimum provided the conditions

$$b_s > 0, \quad b_w > 0, \quad b_s b_w - \gamma^2 > 0 \tag{6}$$

are satisfied. It follows from the definition of D and the expressions for Δ and W that the conditions

$$a_s b_w - \gamma a_w > 0, \quad a_w b_s - \gamma a_s > 0 \tag{7}$$

should be fulfilled.

Clearly, the latter inequalities (7) are incompatible with those in (4) and (5). This fact is evident from topological arguments, namely, if the point $\Gamma_{W\Delta}$ is a minimum of $\mathcal{F}(\Delta, W)$, then the points Γ_Δ and Γ_W can only correspond to a maximum or a saddle point of the free energy functional.

C. Nonhomogeneous case

In the case of heterostructures where the OPs depend on the coordinate x, the most interesting nontrivial solution of the system of equations (2) and (3) corresponds to those where the OP W goes to a finite value $W_\infty = W_{-\infty}$ (an asymmetric case $W_\infty \neq W_{-\infty}$ can be considered analogously) while Δ vanishes at distances from the interface exceeding ξ_w. In other words, we consider the case when the system is at the point Γ_W far away from the interface. We assume that the OP $W(x)$ is suppressed near the interface, e.g., due to an enhanced impurity scattering in the vicinity of the interface or diffusive scattering on the interface. The diffusive scattering may be caused either by interdiffusion or interface roughnesses. As is known (see Ref. [63] and references within), the critical temperature T_{dw} is suppressed by impurity scattering while the critical temperature T_s of the superconducting transition is only weakly affected. The most essential dependence of the coefficients $a_{s,w}$ and $b_{s,w}$ on impurity scattering is the one of the coefficient a_w.

Doping level near the interface also may be changed due to interdiffusion of atoms. Another reason for a change of the coefficients in the Ginzburg-Landau equations is a different crystal symmetry at the interface. Such a mechanism of enhanced superconductivity at twin boundaries has been considered for conventional low-T_c superconductors in Ref. [64]. A change in the energy spectrum of cuprates near the surface has been calculated in Ref. [65]. This change can also lead to a modification of coefficients in the Ginzburg-Landau equations or even to a surface superconductivity. It is worth noting that the properties of surface superconductivity in systems with one (superconducting) OP in magnetic field have been studied theoretically in Refs. [66–69].

The interface behavior of W can be modeled via the spatial dependence of the coefficient a_w, i.e.,

$$a_w(x) = \begin{cases} -a_0, & |x| < L, \\ a_w, & |x| > L, \end{cases} \tag{8}$$

where L is a characteristic width of the region where W is suppressed. The expression for a_0 is presented in Sec. III for a particular case. Formula for $a_w(x)$ at $|x| > L$ implies that the amplitude $W_{\pm\infty} = \sqrt{a_w/b_w}$ is the same at $x \to \pm\infty$, having a lower value at the interface $x = 0$; see Fig. 1(a). We will show that, under these circumstances, a superconducting OP Δ arises at the interface decaying to zero as $x \to \infty$.

Note that in our previous publication [70] we analyzed nonhomogeneous solutions for the Ginzburg-Landau equations (topological defects) assuming that all the coefficients in these equations are constant. The found solutions may correspond to metastable states with energies higher than that for a uniform solution. In the case considered here, a nonuniform solution

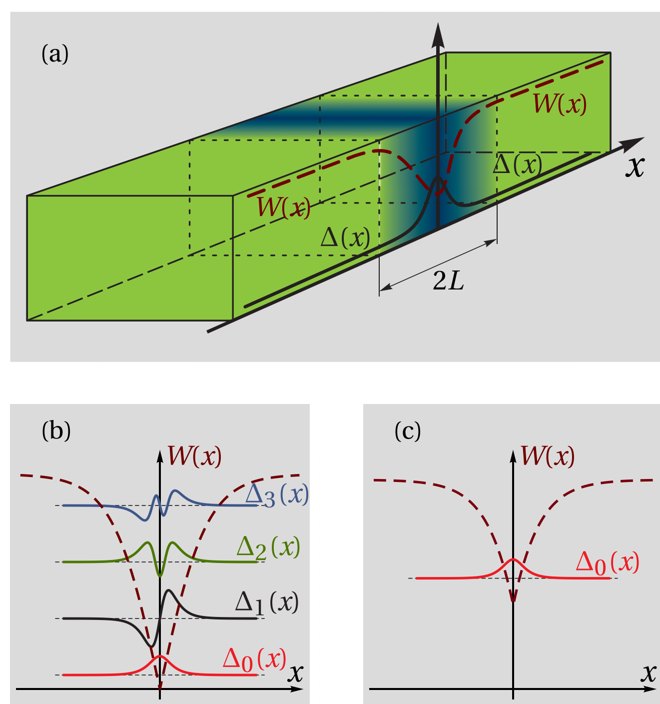

FIG. 1. (Color online) (a) Sketch of considered system. The CDW (or SDW) order parameter W is suppressed near an interface between two materials in which superconducting order parameter Δ may exist alongside W. This suppression leads to the appearance of interfacial superconductivity, which can be thus regarded as "hidden." (b) The case of strong suppression of W with solutions of $\Delta_n(x)$ given by hypergeometric functions. Note that $\Delta_0(x)$ has the shape of a soliton, whereas other solutions have nodes. (c) In the case of a weak suppression of W, one has a shallow "potential" in the "Schrödinger" equation and there exists only one "energy" level given by Eq. (16).

for $\Delta(x)$ is enforced by a built-in defect described by a nonconstant coefficient $a_s(x)$.

In order to find the spatial dependence of $W(x)$ and $\Delta(x)$, we assume that the superconducting OP Δ is small. Then, in the main approximation the equation for W acquires the form

$$-\xi_w^2 W'' + W[-a_w(x) + b_w W^2] = 0, \tag{9}$$

where $a_w(x)$ is given by Eq. (8). This equation can be solved exactly, but for simplicity we restrict ourselves to the simplest case of a narrow region of suppression, i.e., $L \ll \xi_w/\sqrt{a_0}$, thus obtaining the solution

$$W(x) = W_\infty \tanh[(|x| + x_0)/\sqrt{2}\xi_w], \tag{10}$$

where the integration constant x_0 obeys the equation

$$\sinh(2x_0\kappa_w) = 4\xi_w^2\kappa_w/a_0 L \equiv r, \tag{11}$$

with $\kappa_w = \xi_w^{-1}\sqrt{a_w/2}$.

Next, we consider separately the cases of a strong [$r \ll 1$; cf. Fig. 1(b)] and, respectively, weak [$r \gg 1$; cf. Fig. 1(c)] suppression of W at the interface.

Strong suppression, $r \ll 1$. In this case, the product $2x_0\kappa_w \simeq r$ is small and the quantity x_0 in Eq. (10) can be neglected. Substituting $W(x)$ in this approximation into Eq. (2) one obtains

$$\tilde{\xi}_s^2\Delta'' + [\mathcal{E} + \mathcal{U}\cosh^{-2}(\kappa_w x)]\Delta = g\Delta^3, \tag{12}$$

where $\tilde{\xi}_s = \xi_s\sqrt{b_w}$, $\mathcal{E} = a_s b_w - \gamma a_w$, $\mathcal{U} = \gamma a_w$, and $g = b_s b_w$. Equation (12) has a form of the Gross-Pitaevskii equation [71,72]. Solving this equation one can determine the spatial dependence of the superconducting OP Δ. The calculations in this case formally coincide with those carried out in Ref. [70] if the interchange $\Delta \leftrightarrow W$ is done. For completeness we repeat the steps one has to perform seeking for a solution $\Delta(x)$. Assuming that Δ is small, the right-hand side of Eq. (12) can be neglected and we are left with the linearized Gross-Pitaevskii equation, i.e., the "Schrödinger" equation, which can be solved considering the eigenvalue problem

$$\hat{\mathcal{L}}\Delta_n = \mathcal{E}_n\Delta_n, \tag{13}$$

where the operator $\hat{\mathcal{L}} = -\tilde{\xi}_s^2\partial_{xx}^2 - \mathcal{U}\cosh^{-2}(\kappa_w x)$, and Δ_n and $\mathcal{E}_n$ are the eigenfunctions and eigenvalues of $\hat{\mathcal{L}}$. The solutions Δ_n corresponding to a discrete spectrum of $\mathcal{E}_n$ can be expressed in terms of hypergeometric functions and the "energy" levels ($\mathcal{E} < 0$) being given by [73]

$$\mathcal{E}_n = -\frac{\tilde{\xi}_s^2\kappa_w^2}{4}\left[-(1+2n) + \sqrt{1 + \frac{4\mathcal{U}}{\tilde{\xi}_s^2\kappa_w^2}}\right]^2. \tag{14}$$

Provided the inequality $4\mathcal{U}/\tilde{\xi}_s^2\kappa_w^2 < 8$ is fulfilled, there is only one "energy" level with $n = 0$ and the corresponding solution $\Delta_0(x)$ has a form of a soliton. Otherwise there are several solutions decaying far away from the interface and corresponding to $\mathcal{E}_n$.

Representing the OP Δ close to a certain "energy" level $\mathcal{E}_n$ as $\Delta(x) = c_n\Delta_n(x) + \delta\Delta_n(x)$ with a small correction $\delta\Delta_n(x)$ orthogonal to $\Delta_n(x)$, one obtains the coefficients c_n,

$$c_n^2 = \frac{\mathcal{E} - \mathcal{E}_n}{g\langle\langle\Delta_n^4(x)\rangle\rangle}, \tag{15}$$

where $\langle\langle f(x)\rangle\rangle = \int_{-\infty}^{\infty} dx\, f(x)$ (double angle brackets are used to distinguish the notation from the averaging over momenta directions introduced in Appendix A).

Note an important point. The condition $\mathcal{E} < 0$ that determines the appearance of the interface superconductivity coincides with the condition (5) that provides the stability of the state with $W \neq 0$ and $\Delta = 0$ in the bulk. This means that if a nonsuperconducting state in the bulk is characterized by a nonzero OP W, any suppression of W leads to the appearance of local superconductivity. We demonstrate this considering the case of a small suppression of W.

Weak suppression, $r \gg 1$. In this case, one obtains from Eq. (11) $x_0\kappa_w \simeq \ln\sqrt{2r}$ and the spatial dependence of $\Delta(x)$ is determined by the Gross-Pitaevskii equation (12) with the "potential" $\mathcal{U}\cosh^{-2}(\kappa_w x) \to \mathcal{V}(x) = 1 - \tanh^2[\kappa_w(|x| + x_0)]$. In other words, the function $\Delta(x)$ is determined by the "Schrödinger" equation that provides the "energy" levels in a shallow "potential" well $\mathcal{V}(x)$. As is well known [73], there always exists a single "energy" level

$$\mathcal{E}_0 = -J^2/(2\tilde{\xi}_s)^2, \tag{16}$$

where $J = \langle\langle\mathcal{V}(x)\rangle\rangle = 4\kappa_w^{-1}\exp(-2\kappa_w x_0)$. The amplitude c_0 is given again by Eq. (15) with $n = 0$. This means that a superconducting condensate with a small amplitude $\Delta \simeq T_s\mathcal{E}_0$, where T_s is the superconducting transition temperature in the bulk, necessarily arises at the interface as soon as the competing OP is arbitrarily weakly suppressed.

Note that, in the case of a two-dimensional point defect instead of the interface, $|\mathcal{E}_0|$ is an exponentially small [73] quantity and, therefore, the radius of the decay of the condensate is exponentially large.

III. APPLICATION TO CUPRATES AND PNICTIDES

A. Relation of coefficients to microscopic parameters

Here, we present the expressions for the coefficients in the Ginzburg-Landau expansion for the case of cuprates and iron-based pnictides.

As has been shown in Ref. [74], the model that has been developed in detail in Refs. [75–77] for Fe-based pnictides (generally, to two-band superconductors with an SDW) is applicable to quasi-one-dimensional superconductors with a CDW, and, after certain modification, also to cuprates.

First, we consider the region $|x| > L$ and assume that the impurity concentration in this region is small, i.e., the mean free path $l \gg \xi_{s,w}$. In the model of Refs. [75–77], the coefficients are related to the microscopic parameters of the model via $a_s = \eta$, $b_s \simeq 1.05$, $a_w = \eta(1 - \beta_1) - \langle\beta_2\delta[\mu^2]\rangle$, $b_w = s_{3m}$, and $\gamma = s_{2m}$, where $\eta = 1 - T/T_s$ and $\delta[\mu^2] = \mu^2 - \mu_c^2$ with μ being a function that describes the curvature of the Fermi sheets of the quasi-one-dimensional superconductor or a deviation of the average Fermi surface from the perfect circle in iron-based superconductors, where partial nesting between the elliptical electron bands and the circular hole bands leads to the formation of the spin-density wave. The functions s_{2m}, s_{3m}, and $\beta_{1,2}$ (see Appendix A) depend on the dimensionless critical curvature $m = \mu_c/\pi T_s$ defined in such a way that the critical temperature T_{dw} of the formation of the OP W equals T_s, where

$T_{\rm s,dw}$ are the critical temperatures for the transition into the, correspondingly, superconducting and CDW or SDW state in the absence of the competing order and doping. The critical $\mu_{\rm c}$ is determined by the equation (see Appendix A)

$$\langle 2\mu_{\rm c}^2 s_{1m}(\mu_{\rm c})\rangle = \ln(T_{\rm dw}/T_{\rm s}), \tag{17}$$

where the critical temperature $T_{\rm dw}$ depends, generally speaking, on impurity concentration which is assumed to be small far from the interface.

Next, consider the region $|x| < L$, where the impurity scattering is assumed to be stronger. In this case, the temperature $T_{\rm dw} \simeq T_{\rm dw0}[1 - (4\pi T_{\rm dw0}\tau)^{-1}]$, where $T_{\rm dw0}$ is the critical temperature of the transition into a state with $W \neq 0$ in the absence of impurities and superconductivity, $\tau = l/v$ is the momentum relaxation time for intraband scattering, and the mean free path l is assumed to be larger than $v/T_{\rm dw0}$. One can easily show that, in this case, $a_0 = a_{\rm w} - (4\pi T_{\rm dw0}\tau)^{-1}$, where $a_{\rm w}$ is given by the expression above. Provided the condition $a_{\rm w} \ll (4\pi T_{\rm dw0}\tau)^{-1} \ll 1$ is fulfilled, then the coefficient a_0 in Eq. (8) is positive (meaning that $-a_0 < 0$) and considerably exceeds $a_{\rm w}$.

B. Temperature and doping dependence

Considering the expression (15) for the coefficients c_n, one can see that at small difference $|\mathcal{E} - \mathcal{E}_n|$, the OP Δ is also small. It turns to zero at certain temperatures or doping levels, where $\mathcal{E}(T_n,\mu_m) = \mathcal{E}_n(T_n,\mu_n)$ holds, with $\mathcal{E}_n(T_n,\mu_n)$ given by Eq. (14) with coefficients expressed through the microscopic parameters and the temperature. In Fig. 2 we plot the temperature dependence of the amplitudes Δ_n corresponding to different eigenvalues $\mathcal{E}_n$. Clearly, when the temperature T becomes lower than the temperature T_0 determined by Eq. (14), the superconducting OP $\Delta_0(x)$ arises at the interface with the amplitude increasing when T is lowered. The temperature T_0 is lower than $T_{\rm s}$ ($\eta > 0$), but, under certain conditions, higher than the temperature $T_{\rm b}$ at which the superconducting state becomes more favorable in the bulk. At $T < T_1$, a new branch $\Delta_1(T)$ appears, etc.

The minimal temperature T_n (or maximal $n_{\max}$) at which the branch $\Delta_{n_{\max}}(T)$ appears is determined by the condition $2n_{\max} \leqslant \sqrt{1 + 4\mathcal{U}\tilde{\xi}_{\rm s}^{-2}\kappa_{\rm w}^{-2}} - 1$. It can be shown that at a given temperature T, the state with the largest Δ_n, i.e., the state Δ_0, corresponds to a minimum of the free energy. However, the transitions between different Δ_n are possible analogous to transitions between an overcooled state and equilibrium.

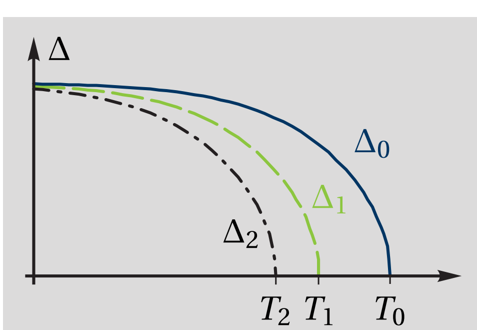

FIG. 2. (Color online) Sketch of temperature dependence of the amplitudes Δ_n corresponding to different eigenvalues $\mathcal{E}_n$; see Eq. (14). At a given temperature T, the state with the largest Δ_n, i.e., the state Δ_0, corresponds to a minimum of the free energy. However, the transitions between different Δ_n are possible.

C. Interface superconducting transition temperature

Our considerations concerned the case when the system is at the point Γ_W far away from the interface, i.e., the conditions (5) are valid while the conditions (4) are violated. When applied to the case of quasi-one-dimensional materials with the CDW or to material like iron-based pnictides with an SDW, these conditions can be presented in the form

$$A_2\eta + B_2\delta[\mu^2] < 0, \quad A_1\eta + B_1\delta[\mu^2] < 0, \quad s_{3m} > 0, \tag{18}$$

where $A_1 = s_{3m} - s_{2m}(1-\beta_1)$, $B_1 = s_{2m}\beta_2$, $A_2 = s_{2m} - s_3(1-\beta_1)$, and $B_2 = 1.05\beta_2$ if expressed via the microscopic parameters of the model. The coefficients $A_{1,2}$ and $B_{1,2}$ depend on $\mu(\mu_0,\mu_\varphi)$ and may attain positive or negative values (only B_2 is a positive quantity). If these coefficients are all positive, then these conditions can be fulfilled provided $\delta[\mu^2] < 0$. Thus we can rewrite them as $B_2|\delta[\mu^2]| > A_2\eta$ and $B_1|\delta[\mu^2]| > A_1\eta$. In terms of microscopic parameters the latter can be written as

$$\eta < c_\mu B_1/A_1 \equiv 1 - \tilde{T}_W, \quad \eta < c_\mu B_2/A_2 \equiv 1 - \tilde{T}_\Delta, \tag{19}$$

where $c_\mu = |\delta[\mu^2]|$ and we introduced the "critical" temperatures $\tilde{T}_{W,\Delta} = T_{W,\Delta}/(\pi T_{\rm dw,s})$.

One can distinguish from the physical point of view two different cases as follows.

(a) The case $T_\Delta < T_W$ is realized if the quantity $D \equiv A_1B_2 - A_2B_1 = s_3s_{3m} - s_{2m}^2 > 0$ provided that A_1 and A_2 are positive. In this case, the pure superconducting state (minimum of the free energy at Γ_Δ) exists at temperatures $T < T_\Delta$ with $\Delta_{\rm un}^2 = a_{\rm s}/b_{\rm s}$. In the temperature range $T_\Delta < T < T_W$, a mixed state (the state of coexistence) with $\Delta \neq 0$ and $W \neq 0$ given by expressions just before Eq. (6) takes place. At $T > T_0$, a pure CDW state or, more generally, a W state occurs with $W_{\rm un}^2 = a_{\rm w}/b_{\rm w}$. In the interval $T_W < T < T_0$, one has a surface (or interface superconductivity), where T_0 is the temperature determined by Eq. (14) corresponding to $n = 0$ (the ground state). At $T_0 < T < T_{\rm dw}$, the system is nonsuperconducting with the OP $W \neq 0$. In Fig. 3(a) we show schematically the temperature dependence of Δ and W and also the temperature range in which the local superconductivity exists. At temperatures $T_{W,\Delta}$, second-order phase transitions occur and the OPs Δ and W arise continuously [see expressions for Δ and W before Eq. (6) where $\Delta \sim \sqrt{T_W - T}$ and $W \sim \sqrt{T - T_\Delta}$]. Note an obvious analogy with conventional second-type superconductors in a magnetic field H [78,79]. The quantities $T_{\Delta,W}$ are analogous to the critical fields $H_{\rm c1,c2}$ [cf. Fig. 3(c)] so that at $T < T_W$ one has a purely superconducting state (full expulsion of the magnetic field), a mixed state in the interval $T_W < T < T_\Delta$ (correspondingly, the Abrikosov's vortex state) and a surface or interface superconductivity in the range $T_\Delta < T < T_0$ (correspondingly, in the range $H_{\rm c2} < H < H_{\rm c3}$). At last, at $T > T_0$, one has a pure W state which corresponds to the normal state in conventional superconductors. Note that same

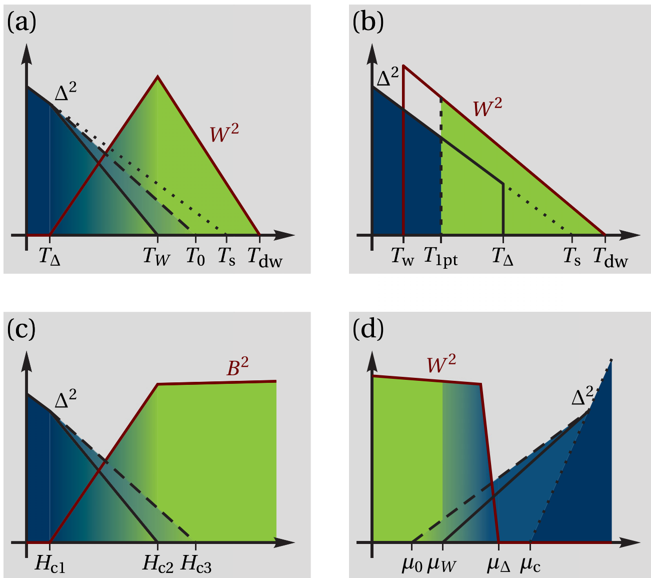

FIG. 3. (Color online) (a) In the case $T_\Delta < T_W$, three ranges of temperature. For $T < T_\Delta$, the system is in a pure superconducting state, whereas in the temperature range $T_\Delta < T < T_W$, a mixed state (the state of coexistence) with $\Delta \neq 0$ and $W \neq 0$ is realized. Thus the bulk superconducting transition temperature corresponds to T_W. The temperature range of the coexistence state is widened (enhancement of superconductivity) on appearance of the interface superconductivity with the highest transition temperature T_0 (there may be other transition temperatures T_n). Finally, for $T > T_0$, the system is in a pure W state (CDW or SDW) up to its transition into the normal state at T_{dw}. (b) In case $T_\Delta > T_W$, the system may be in either the pure superconducting or in the pure W state. The transition from one into another is of the first order at a temperature T_{1pt}. If T_0 falls into the region $T_{1pt} < T_s$, which is possible if the difference $T_s - T_{1pt}$ is positive and sufficiently large, then, at the interface, superconductivity is induced. (c) An analogy with conventional second-type superconductors in a magnetic field H is sketched. The quantities $T_{\Delta,W}$ and T_0 are analogous to the critical fields $H_{c1,c2}$ and H_{c3}, respectively, i.e., up to H_{c1} the system is in a purely superconducting state (full expulsion of the magnetic field), in a mixed state in the interval $H_{c1} < H < H_{c2}$, and in the Abrikosov's vortex state for $H_{c2} < H < H_{c3}$ losing the superconducting properties for $H > H_{c3}$. (d) If, instead of temperature, the doping (or curvature) μ is considered, the situation is similar to the temperature dependence.

analysis can be performed if, instead of temperature, the doping (or curvature) given by c_μ in Eq. (19) is considered and one obtains the corresponding situation as depicted in Fig. 3(d).

(b) The case $T_\Delta > T_W$ is realized if the quantity $D \equiv A_1 B_2 - A_2 B_1 = s_3 s_{3m} - s_{2m}^2 < 0$ provided that A_1 and A_2 are positive. In this case, the pure superconducting state (minimum of the free energy at Γ_Δ) exists again at temperatures $T < T_\Delta$, but this minimum is global only at $T < T_{1pt}$, where the critical temperature T_{1pt} for the first-order phase transition is determined by the equation $\mathcal{F}(a_w/b_w) = \mathcal{F}(a_s/b_s)$. At $T > T_{dw}$, a uniform solution for $W = \sqrt{a_w/b_w}$ arises, but it corresponds to a global minimum at $T > T_{1pt}$. In this case, no region of coexistence exists, and at $T = T_{1pt}$ a first-order phase transition from the superconducting state to a state with $W \neq 0$ takes place with increasing temperature; see Fig. 3(b). Now, if T_0 as determined by Eq. (14) corresponding to $n = 0$ (the ground state) falls into the region $T_{1pt} < T_s$, which is possible if the difference $T_s - T_{1pt}$ is positive and sufficiently large, then, at the interface, superconductivity is induced.

D. Experiments

The obtained appearance of superconductivity (or enhancement of the critical transition temperature) at an interface between two materials in which, alongside superconductivity, another order exists and is energetically more favorable may be realized in two prominent examples of such systems. One of them, the cuprates, show a charge-density wave order alongside superconductivity [40–50]. In these materials, an enhancement of superconducting transition temperature has been found in a bilayer constructed of $La_{1.65}Sr_{0.35}CuO_4$ and $La_{1.875}Ba_{0.125}CuO_4$ [9,14].

Superconductivity accompanied by a spin-density wave is known to exist in the iron-based pnictides, where the interface superconductivity is proposed to be the driving effect behind the almost doubling of superconducting transition temperature in $CaFe_2As_2$ [28,80].

Unfortunately, there are no data on spatial dependence of the order parameter accompanying superconductivity in these experiments; neither has the spatial dependence of the superconducting order parameters been investigated in these experiments. Such a measurement would provide a test of our theory, if charge- or spin-density wave would have been suppressed near the interface.

Another interesting effect also serving as a test of our predictions is related to the fact that there might appear a hysteretic behavior stemming from the presence of different "energy" levels [see Eq. (14)]. This results in a sequence of temperatures T_n at which Δ formally vanishes but the temperature dependence of Δ is determined by the highest temperature of them all, i.e., by T_0 since the minimum of the free energy is deepest here. Interestingly, at a temperature T_n a local minimum of the free energy is present and adjusting Δ by means, e.g., of an external field, one can let it follow the temperature dependence of Δ_n after the relaxation of external constraints, so, indeed, "there's a lot of room for new combinations" [80].

IV. DISCUSSION

We have studied a system with two competing OPs one of which is the superconducting OP Δ and another, W, can be the amplitude of the charge- or spin-density wave. On the basis of Ginzburg-Landau equations we have shown that if the temperature and doping are chosen in such a way that the state with $W \neq 0$ and $\Delta = 0$ is favorable, i.e., it corresponds to a minimum of the free energy in the bulk of the sample, an arbitrarily small suppression of W at an interface or a defect leads to the appearance of local superconductivity. This mechanism of local superconductivity in a system with two OPs may be responsible for the interface superconductivity observed in many materials including cuprates and iron-based pnictides. As is firmly established, in cuprates and iron-based pnictides, CDW (or quadrupole charge order [81]) or SDW can exist alongside superconductivity.

We found that in case of a strong suppression of W at some point, there are several solutions for the superconducting OP $\Delta(x)$ which are localized on the scale of the coherence length ξ_s. These solutions are found from the nonlinear Schrödinger equation (or Gross-Pitaevskii equation) and correspond to different "energy" levels. Each solution arises at certain values of temperatures T_n (or doping level μ_n) and has a different form changing from a solitonlike one to an oscillatory function decaying at infinity. However, only the solitonlike solution $\Delta_0(x)$ corresponds to a minimum of the free energy. Other solutions $\Delta_n(x)$ (with $n \neq 0$) with nodes have higher energies. They correspond to metastable states. If the corresponding potential $\mathcal{V}(x)$ in the Schrödinger equation is not deep enough, there is only one "energy" level and only one solitonlike solution for $\Delta(x)$.

In the case of an asymmetric potential $\mathcal{V}(x)$, the localized solution for $\Delta(x)$ exists provided that the potential well is deep enough [73]. If for some reason the OP W is suppressed at the surface, the solutions for $\Delta(x)$ have the same form as in the case of the symmetric $\mathcal{V}(x)$ and surface superconductivity arises in the sample. Note an important point. The local superconductivity may arise in one- or two-dimensional cases (a flat interface or a point defect) if the suppression of the OP W is small. This means that even if the minimal value of $W(x)$ corresponds to a minimum of the free energy in a uniform case, in a nonuniform case local superconductivity would arise at the interface or at the surface.

The developed theory is able to explain the emergent or enhanced interface superconductivity in some cuprate or iron-based pnictide heterostructures. The predictions can be tested in further experiments.

Remarkably, the used approach based on the consideration of Ginzburg-Landau equations for two coupled order parameters does not depend on the nature of order described by those. The obtained results are also applicable to description of an arbitrary "hidden" order evolving at the interface, e.g., the ferromagnetism induced at the interface of an oxide heterostructure observed recently [32].

Note that localized superconductivity may arise also in a homogeneous sample if a nonuniform $W(x)$ state (for example, stripes) is energetically favorable due to an internal mechanism (e.g., the Larkin-Ovchinnikov-Fulde-Ferrel mechanism) [82,83]. Analysis of this state with two competing order parameters deserves a separate consideration [84].

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APPENDIX A: DOPING DEPENDENCE OF THE COEFFICIENTS IN GINZBURG-LANDAU EQUATIONS

In the notation of Refs. [75–77], the Ginzburg-Landau equations have the form

$$-\xi_s^2\nabla^2\Delta + \Delta\big[W^2 s_{2m} + \Delta^2 s_3 - \ln(T_s/T)\big] = 0, \quad \text{(A1)}$$

$$-\xi_w^2\nabla^2 W + W\big[\langle 2\mu^2 s_{1m}\rangle + W^2 s_{3m} + \Delta^2 s_{2m} - \ln(T_{dw}/T)\big] = 0, \quad \text{(A2)}$$

where $\xi_{s,w}$ are the coherence lengths (at low temperatures) for Δ and W, respectively, and $T_{s,dw}$ are, respectively, the critical temperatures for the transition into the pure superconducting state or into a state with a CDW or an SDW only. In other words, T_{dw} is the critical temperature for the transition into the charge-ordered state in absence of Δ and μ, while T_s is the superconducting transition temperature in absence of W. The angle brackets mean the angle averaging (in Fe-based pnictides) or integration along the sheets of the Fermi surfaces in quasi-one-dimensional superconductors. The functions s_{1m}, s_{2m}, etc. are functions of the normalized curvature $m = \mu/(\pi T_s)$, where $\mu = \mu_0 + \mu_\varphi \cos[(p_y^2 + p_z^2)^{1/2}a]$ is a curvature in quasi-one-dimensional superconductors with a doping-dependent value of μ_0. It is assumed that the Fermi surface of these superconductors consists of two slightly curved sheets which are perpendicular to the x axis [74]. In the case of Fe-based pnictides, $\mu = \mu_0 + \mu_\varphi \cos(2\varphi)$ is a quantity that describes elliptic ($\mu_\varphi \neq 0$) and circular ($\mu_\varphi = 0$) Fermi surfaces of electron and hole bands [75–77]. All quantities – Δ, W, and μ – are measured in units of πT_s. The expressions for the coefficients in the GL expansion with account for impurity scattering have been calculated in Ref. [85].

Replacing the derivative $\nabla \to \nabla - i2\pi A/\Phi_0$, one can use Eqs. (A1) and (A2) to describe vortices in superconductors with a CDW [86], where Φ_0 is the magnetic flux quantum.

As it is seen from Eq. (A2), the critical temperature T_{dw} depends on doping, i.e., on the parameter μ. We choose this parameter $\mu = \mu_c$ in such a way that $T_{dw}(\mu_c) = T_s$. This means that at $T = T_s$, the quantities $\Delta = W = 0$, and, thus, μ_c obeys the equation

$$\big\langle 2\mu_c^2 s_{1m}(\mu_c)\big\rangle = \ln(T_{dw}/T_s), \quad \text{(A3)}$$

where μ_c is a function of two parameters, i.e., $\mu_c = \mu_c(\mu_0,\mu_\varphi)$.

Then, we expand the function $s_{1m}(\mu,T)$ in the deviations $\delta[\mu^2] = \mu^2 - \mu_c^2$ and $\delta T = T_s - T$, thus obtaining $s_{1m}(\mu,T) = s_{1m}(\mu_c,T_s) + \beta_1\delta T + \langle\beta_2\delta[\mu^2]\rangle$, and use Eq. (A3) to obtain equations in a general standard form (assuming that all the functions depend only on one coordinate x),

$$-\xi_s^2\Delta'' + \Delta[-a_s + b_s\Delta^2 + \gamma W^2] = 0, \quad \text{(A4)}$$

$$-\xi_w^2 W'' + W[-a_w + b_w W^2 + \gamma\Delta^2] = 0, \quad \text{(A5)}$$

with Δ' and W' as well as Δ'' and W'' denoting the first and second derivatives with respect to x, respectively. These equations determine extrema of the free energy functional [cf. Eq. (1)]

$$\mathcal{F} = \frac{1}{2}\int dx\bigg[\xi_{\rm s}^2(\Delta')^2 - a_{\rm s}\Delta^2 + \frac{b_{\rm s}}{2}\Delta^4 + \gamma\Delta^2 W^2 + \xi_{\rm w}^2(W')^2 - a_{\rm w}W^2 + \frac{b_{\rm w}}{2}W^4\bigg], \quad (A6)$$

with respect to Δ and W, and the corresponding coefficients of the GL expansion are related to variables in Eqs. (A1) and (A2) via $a_{\rm s} = \eta$, $b_{\rm s} = s_3 \simeq 1.05$, $a_{\rm w} = \eta(1-\beta_1) - \langle\beta_2\delta[\mu^2]\rangle$, $b_{\rm w} = s_{3m}$, and $\gamma = s_{2m}$, where $\eta = 1 - T/T_{\rm s}$. The expressions for the coefficients in terms of the microscopic parameters of the model for cuprates and iron-based pnictides are given as follows:

$$s_3 = \sum_{n=0}^{\infty}(2n+1)^{-3}, \quad (A7)$$

$$s_{1m} = \sum_{n=0}^{\infty}(2n+1)^{-1}[(2n+1)^2t^2 + m^2]^{-1}, \quad (A8)$$

$$s_{2m} = \sum_{n=0}^{\infty}\langle[(2n+1)^2 - m^2](2n+1)^{-1}[(2n+1)^2 + m^2]^{-2}\rangle, \quad (A9)$$

$$s_{3m} = \sum_{n=0}^{\infty}\langle(2n+1)[(2n+1)^2 - 3m^2][(2n+1)^2 + m^2]^{-3}\rangle, \quad (A10)$$

$$\beta_1 = \sum_{n=0}^{\infty}\langle 4m^2(2n+1)[(2n+1)^2 + m^2]^{-2}\rangle, \quad (A11)$$

$$\beta_2 = \sum_{n=0}^{\infty}2(2n+1)^{-1}[(2n+1)^2 + m^2]^{-1}, \quad (A12)$$

where $t = T/T_{\rm s}$, and the angle brackets $\langle\cdots\rangle$ denote the angle averaging (in iron-based pnictides) or integration along the sheets of the Fermi surfaces (in quasi-one-dimensional superconductors or cuprates).

APPENDIX B: DETAILS ON SOLUTION OF THE GROSS-PITAEVSKII EQUATION

Here we sketch the solution of the Gross-Pitaevskii equation for Δ. In zero-order approximation we obtain for Δ_0 from Eq. (12)

$$\tilde{\xi}_\Delta^2\Delta_0'' + \Delta_0[\mathcal{E} + \mathcal{U}\cosh^{-2}(\kappa_{\rm w}x)] = 0. \quad (B1)$$

This equation is integrable and its solutions ψ_n corresponding to a discrete spectrum of $\mathcal{E}_n$ are expressed in terms of hypergeometric functions [73]. In our notations, the "energy" levels of discrete spectrum are given by [73]

$$\mathcal{E}_n = -\frac{\tilde{\xi}_{\rm s}^2\kappa_{\rm w}^2}{4}\bigg[-(1+2n) + \sqrt{1 + \frac{4\mathcal{U}}{\tilde{\xi}_{\rm s}^2\kappa_{\rm w}^2}}\bigg]^2 \quad (B2)$$

and their maximal number $n_{\rm max}$ is determined by $2n_{\rm max} \leqslant \sqrt{1 + 4\mathcal{U}\tilde{\xi}_{\rm s}^{-2}\kappa_{\rm w}^{-2}} - 1$.

We expand the correction $\delta\Delta$ to the zero-order solution Δ_0 in terms of the normalized eigenfunctions $\Delta_n \equiv \psi_n$ of the operator $\hat{\mathcal{L}} = -\tilde{\xi}_{\rm s}^2\partial_{xx}^2 - \mathcal{U}\cosh^{-2}(\kappa_{\rm w}x)$. These functions obey the equation

$$\hat{\mathcal{L}}\psi_n = \mathcal{E}_n\psi_n. \quad (B3)$$

Solutions of Eq. (12) can be written explicitly if the quantity $\mathcal{E} = \mathcal{E}(\eta,\delta[\mu^2])$ is close to a certain "energy" level $\mathcal{E}_n$, say to $\mathcal{E}_N$, such that $\mathcal{E} \simeq \mathcal{E}_N = \mathcal{E}(\eta_N,\delta[\mu_N^2])$ (if considering the model for cuprates or iron-based pnictides, the "temperature" η or doping $\delta[\mu^2]$ should be chosen properly). We write Eq. (12) in the form

$$\hat{\mathcal{L}}\Delta = \mathcal{E}_N\Delta + R(\Delta), \quad (B4)$$

with $R = g\Delta^3 + (\mathcal{E} - \mathcal{E}_N)\Delta$ and represent Δ as $\Delta(x) = c_N\psi_N(x) + \delta\Delta_N(x)$, where $\delta\Delta_N(x) = \sum_n' c_{N,n}\psi_n(x)$, and the summation runs over all n except the term $n = N$. We substitute this $\Delta(x)$ into Eq. (B4) and multiply this equation first by ψ_N and then by ψ_n with $n \neq N$, then integrating the obtained result each time over x. Thus, taking into account the orthogonality of different eigenfunctions, we find the coefficients c_n,

$$c_N^2 = \frac{\mathcal{E} - \mathcal{E}_N}{g\langle\langle\psi_N^4\rangle\rangle}, \quad (B5)$$

$$c_{N,n} = gc_N^3\frac{\langle\langle\psi_N^3\psi_n\rangle\rangle}{E_n - E_N} \quad \text{with } n \neq N, \quad (B6)$$

where $\langle\langle f(x)\rangle\rangle = \int_{-\infty}^{\infty}dx\, f(x)$. Obviously, in Eq. (B6), ψ_n and ψ_N have to have same parity (both even or both odd).

[1] N. Reyren, S. Thiel, A. D. Caviglia, L. F. Kourkoutis, G. Hammerl, C. Richter, C. W. Schneider, T. Kopp, A.-S. Rüetschi, D. Jaccard, M. Gabay, D. A. Muller, J.-M. Triscone, and J. Mannhart, Science **317**, 1196 (2007).
[2] A. Gozar, G. Logvenov, L. F. Kourkoutis, A. T. Bollinger, L. A. Giannuzzi, D. A. Muller, and I. Bozovic, Nature (London) **455**, 782 (2008).
[3] A. D. Caviglia, S. Gariglio, N. Reyren, D. Jaccard, T. Schneider, M. Gabay, S. Thiel, G. Hammerl, J. Mannhart, and J.-M. Triscone, Nature (London) **456**, 624 (2008).
[4] C. Bell, S. Harashima, Y. Kozuka, M. Kim, B. G. Kim, Y. Hikita, and H. Y. Hwang, Phys. Rev. Lett. **103**, 226802 (2009).
[5] Y. Kozuka, M. Kim, C. Bell, B. G. Kim, Y. Hikita, and H. Y. Hwang, Nature (London) **462**, 487 (2009).
[6] J. Biscaras, N. Bergeal, A. Kushwaha, T. Wolf, A. Rastogi, R. Budhani, and J. Lesueur, Nat. Commun. **1**, 89 (2010).
[7] G. Logvenov, A. Gozar, and I. Bozovic, Science **326**, 699 (2009).
[8] S. Smadici, J. C. T. Lee, S. Wang, P. Abbamonte, G. Logvenov, A. Gozar, C. Deville Cavellin, and I. Bozovic, Phys. Rev. Lett. **102**, 107004 (2009).

[9] G. Koren and O. Millo, Phys. Rev. B **81**, 134516 (2010).
[10] V. M. Loktev and Y. G. Pogorelov, Phys. Rev. B **78**, 180501 (2008).
[11] V. Y. Butko, G. Logvenov, N. Bozovic, Z. Radovic, and I. Bozovic, Adv. Mater. **21**, 3644 (2009).
[12] D. L. Bergman and T. Pereg-Barnea, Materials **4**, 1835 (2011).
[13] J. Pereiro, A. Petrovic, C. Panagopoulos, and I. Bozovic, Phys. Express **1**, 208 (2011).
[14] O. Yuli, I. Asulin, O. Millo, D. Orgad, L. Iomin, and G. Koren, Phys. Rev. Lett. **101**, 057005 (2008).
[15] J. Wu, O. Pelleg, G. Logvenov, A. T. Bollinger, G. S. Sun, Y.-J. Boebinger, M. Vanevic, Z. Radovic, and I. Bozovic, Nat. Mater. **12**, 877 (2013).
[16] S. Tinchev, Physica C: Superconductivity **470**, 626 (2010).
[17] W. Rühl, Z. Phys. **186**, 190 (1965).
[18] D. Naugle, Phys. Lett. A **25**, 688 (1967).
[19] M. Strongin, O. F. Kammerer, J. E. Crow, R. D. Parks, D. H. Douglass, and M. A. Jensen, Phys. Rev. Lett. **21**, 1320 (1968).
[20] V. Ginzburg, Phys. Lett. **13**, 101 (1964).
[21] D. Allender, J. Bray, and J. Bardeen, Phys. Rev. B **7**, 1020 (1973).
[22] Y. Kamihara, T. Watanabe, M. Hirano, and H. Hosono, J. Am. Chem. Soc. **130**, 3296 (2008).
[23] G. R. Stewart, Rev. Mod. Phys. **83**, 1589 (2011).
[24] H.-H. Wen and S. Li, Annu. Rev. Condens. Matter Phys. **2**, 121 (2011).
[25] P. J. Hirschfeld, M. M. Korshunov, and I. I. Mazin, Rep. Prog. Phys. **74**, 124508 (2011).
[26] M. R. Norman, Physics **1**, 21 (2008).
[27] A. Chubukov, Annu. Rev. Condens. Matter Phys. **3**, 57 (2012).
[28] F. Y. Wei, B. Lv, L. Z. Deng, J. K. Meen, Y. Y. Xue, and C. W. Chu, arXiv:1309.0034 [cond-mat.supr-con].
[29] W. Qing-Yan, L. Zhi, Z. Wen-Hao, Z. Zuo-Cheng, Z. Jin-Song, L. Wei, D. Hao, O. Yun-Bo, D. Peng, C. Kai, W. Jing, S. Can-Li, H. Ke, J. Jin-Feng, J. Shuai-Hua, W. Ya-Yu, W. Li-Li, C. Xi, M. Xu-Cun, and X. Qi-Kun, Chin. Phys. Lett. **29**, 037402 (2012).
[30] Y.-Y. Xiang, F. Wang, D. Wang, Q.-H. Wang, and D.-H. Lee, Phys. Rev. B **86**, 134508 (2012).
[31] S. Tan, Y. Zhang, M. Xia, Z. Ye, F. Chen, X. Xie, R. Peng, D. Xu, Q. Fan, H. Xu, J. Jiang, T. Zhang, X. Lai, T. Xiang, J. Hu, B. Xie, and D. Feng, Nat. Mater. **12**, 634 (2013).
[32] M. Salluzzo, S. Gariglio, D. Stornaiuolo, V. Sessi, S. Rusponi, C. Piamonteze, G. M. De Luca, M. Minola, D. Marré, A. Gadaleta, H. Brune, F. Nolting, N. B. Brookes, and G. Ghiringhelli, Phys. Rev. Lett. **111**, 087204 (2013).
[33] L. Li, C. Richter, J. Mannhart, and R. C. Ashoori, Nat. Phys. **7**, 762 (2011).
[34] H. Y. Hwang, Y. Iwasa, M. Kawasaki, B. Keimer, N. Nagaosa, and Y. Tokura, Nat. Mater. **11**, 103 (2012).
[35] D. A. Dikin, M. Mehta, C. W. Bark, C. M. Folkman, C. B. Eom, and V. Chandrasekhar, Phys. Rev. Lett. **107**, 056802 (2011).
[36] J. A. Bert, B. Kalisky, C. Bell, M. Kim, Y. Hikita, H. Y. Hwang, and K. A. Moler, Nat. Phys. **7**, 767 (2011).
[37] L. Fidkowski, H.-C. Jiang, R. M. Lutchyn, and C. Nayak, Phys. Rev. B **87**, 014436 (2013).
[38] V. M. Loktev and Y. G. Pogorelov, Phys. Rev. B **83**, 132505 (2011).
[39] E. Berg, D. Orgad, and S. A. Kivelson, Phys. Rev. B **78**, 094509 (2008).
[40] M. J. Lawler, K. Fujita, J. Lee, A. R. Schmidt, Y. Kohsaka, C. K. Kim, H. Eisaki, J. C. Uchida, S. Davis, J. P. Sethna, and E.-A. Kim, Nature (London) **466**, 347 (2010).
[41] C. V. Parker, P. Aynajian, E. H. da Silva Neto, A. Pushp, S. Ono, J. Wen, Z. Xu, G. Gu, and A. Yazdani, Nature (London) **468**, 677 (2010).
[42] T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W. N. Hardy, R. Liang, D. A. Bonn, and M.-H. Julien, Nature (London) **477**, 191 (2011).
[43] T. Wu, H. Mayaffre, S. Krämer, M. Horvatic, C. Berthier, P. L. Kuhns, A. P. Reyes, R. Liang, W. N. Hardy, D. A. Bonn, and M.-H. Julien, Nat. Commun. **4**, 2113 (2013).
[44] G. Ghiringhelli, M. Le Tacon, M. Minola, S. Blanco-Canosa, C. Mazzoli, N. B. Brookes, G. M. De Luca, A. Frano, D. G. Hawthorn, F. He, T. Loew, M. M. Sala, D. C. Peets, M. Salluzzo, E. Schierle, R. Sutarto, G. A. Sawatzky, E. Weschke, B. Keimer, and L. Braicovich, Science **337**, 821 (2012).
[45] J. Chang, E. Blackburn, A. T. Holmes, N. B. Christensen, J. Larsen, J. Mesot, R. Liang, D. A. Bonn, W. N. Hardy, A. Watenphul, M. v. Zimmermann, E. M. Forgan, and S. M. Hayden, Nat. Phys. **8**, 871 (2012).
[46] A. J. Achkar, R. Sutarto, X. Mao, F. He, A. Frano, S. Blanco-Canosa, M. Le Tacon, G. Ghiringhelli, L. Braicovich, M. Minola, M. Moretti Sala, C. Mazzoli, R. Liang, D. A. Bonn, W. N. Hardy, B. Keimer, G. A. Sawatzky, and D. G. Hawthorn, Phys. Rev. Lett. **109**, 167001 (2012).
[47] D. LeBoeuf, S. Kramer, W. N. Hardy, R. Liang, D. A. Bonn, and C. Proust, Nat. Phys. **9**, 79 (2012).
[48] E. Blackburn, J. Chang, M. Hücker, A. Holmes, N. Christensen, R. Liang, D. Bonn, W. Hardy, U. Rütt, O. Gutowski, M. Zimmermann, E. Forgan, and S. Hayden, Phys. Rev. Lett. **110**, 137004 (2013).
[49] R. Comin, A. Frano, M. M. Yee, Y. Yoshida, H. Eisaki, E. Schierle, E. Weschke, R. Sutarto, F. He, A. Soumyanarayanan, Y. He, M. Le Tacon, I. S. Elfimov, J. E. Hoffman, G. A. Sawatzky, B. Keimer, and A. Damascelli, Science **343**, 390 (2014).
[50] K. Fujita, M. H. Hamidian, S. D. Edkins, C. K. Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S.-i. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. S. Davis, Proc. Natl. Acad. Sci. USA **111**, E3026 (2014).
[51] M. Vojta and O. Rösch, Phys. Rev. B **77**, 094504 (2008).
[52] M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
[53] S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013).
[54] H. Meier, C. Pépin, M. Einenkel, and K. B. Efetov, Phys. Rev. B **89**, 195115 (2014).
[55] Y. Wang and A. Chubukov, Phys. Rev. B **90**, 035149 (2014).
[56] K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, Science **336**, 1554 (2012).
[57] A. Levchenko, M. G. Vavilov, M. Khodas, and A. V. Chubukov, Phys. Rev. Lett. **110**, 177003 (2013).
[58] D. Chowdhury, B. Swingle, E. Berg, and S. Sachdev, Phys. Rev. Lett. **111**, 157004 (2013).
[59] P. Walmsley, C. Putzke, L. Malone, I. Guillamón, D. Vignolles, C. Proust, S. Badoux, A. I. Coldea, M. D. Watson, S. Kasahara, Y. Mizukami, T. Shibauchi, Y. Matsuda, and A. Carrington, Phys. Rev. Lett. **110**, 257002 (2013).

[60] D. Kuzmanovski, A. Levchenko, M. Khodas, and M. G. Vavilov, Phys. Rev. B **89**, 144503 (2014).
[61] K. B. Efetov, Sov. Phys. JETP **54**, 1198 (1981).
[62] R. M. Fernandes, M. G. Vavilov, and A. V. Chubukov, Phys. Rev. B **85**, 140512 (2012).
[63] L. N. Bulaevskii, Sov. Phys. Usp. **18**, 131 (1975).
[64] I. Khlyustikov and A. Buzdin, Adv. Phys. **36**, 271 (1987).
[65] H. Eschrig, A. Lankau, and K. Koepernik, Phys. Rev. B **81**, 155447 (2010).
[66] V. Barzykin and L. P. Gor'kov, Phys. Rev. Lett. **89**, 227002 (2002).
[67] D. F. Agterberg, in *Proceedings of the 3rd Polish-US Workshop on Superconductivity and Magnetism of Advanced Materials* [Physica C: Superconductivity **387**, 13 (2003)].
[68] O. Dimitrova and M. V. Feigel'man, Phys. Rev. B **76**, 014522 (2007).
[69] D. F. Agterberg, E. Babaev, and J. Garaud, Phys. Rev. B **90**, 064509 (2014).
[70] A. Moor, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **90**, 224512 (2014).
[71] E. Gross, Nuovo Cimento Ser. 10 **20**, 454 (1961).
[72] L. P. Pitaevskii, Sov. Phys. JETP **13**, 451 (1961).
[73] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, 3rd ed., Course of Theoretical Physics, Vol. III (Butterworth-Heinemann, Oxford, 1977).
[74] A. Moor, P. A. Volkov, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **90**, 024511 (2014).
[75] M. G. Vavilov, A. V. Chubukov, and A. B. Vorontsov, Supercond. Sci. Technol. **23**, 054011 (2010).
[76] A. B. Vorontsov, M. G. Vavilov, and A. V. Chubukov, Phys. Rev. B **81**, 174538 (2010).
[77] R. M. Fernandes and J. Schmalian, Phys. Rev. B **82**, 014521 (2010).
[78] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).
[79] P. G. De Gennes, *Superconductivity of Metals and Alloys*, Advanced Books Classics Series (Westview Press, Boulder, CO, 1999).
[80] E. S. Reich, Nature (London) **501**, 474 (2013).
[81] K. B. Efetov, H. Meier, and C. Pépin, Nat. Phys. **9**, 442 (2013).
[82] A. I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965).
[83] P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).
[84] The possibility of such a state with two coexisting order parameters was noted in Refs. [75,77,87,88].
[85] M. Hoyer, S. V. Syzranov, and J. Schmalian, Phys. Rev. B **89**, 214504 (2014).
[86] H. Meier, M. Einenkel, C. Pépin, and K. B. Efetov, Phys. Rev. B **88**, 020506(R) (2013).
[87] D. F. Agterberg and H. Tsunetsugu, Nat. Phys. **4**, 639 (2008).
[88] L. P. Gor'kov and G. B. Teitel'baum, Phys. Rev. B **87**, 024504 (2013).