

Ray optics behavior of flux avalanche propagation in superconducting films

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Experimental evidence of wave properties of dendritic flux avalanches in superconducting films is reported. Using magneto-optical imaging the propagation of dendrites across boundaries between a bare NbN film and areas coated by a Cu layer was visualized, and it was found that the propagation is refracted in full quantitative agreement with Snell's law. For the studied film of 170 nm thickness and a 0.9 μm thick metal layer, the refractive index was close to $n = 1.4$. The origin of the refraction is believed to be caused by the dendrites propagating as an electromagnetic shock wave, similar to damped modes considered previously for normal metals. The analogy is justified by the large dissipation during the avalanches raising the local temperature significantly. Additional time-resolved measurements of voltage pulses generated by segments of the dendrites traversing an electrode confirm the consistency of the adopted physical picture.

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In thin-film superconductors placed in a gradually increasing or decreasing transverse magnetic field, the smooth propagation of the flux front can be violated by sudden bursts of flux penetration. These dramatic events occur due to a thermomagnetic instability [1–4] causing large amounts of flux to rush in from seemingly arbitrary positions along the edge. Magneto-optical imaging (MOI) of the flux penetration in films of many superconducting materials [5–17] has shown that it is a generic feature of these avalanches that they form nonrepetitive branching structures; see Ref. [18] for a review. It is also found experimentally that the propagation speed of such avalanches can be amazingly high, up to 160 km/s in their very early stage [19,20].

In recent works [21,22], new insight into the origin of such high velocities was obtained by numerical simulations based on a set of coupled equations for the electrodynamics and thermal behavior of a superconducting film on a substrate. The results show that due to the very large electrical fields and dissipation caused by rapid flux motion, the local temperature during such an avalanche typically rises above the superconducting transition temperature. One may therefore expect that in many superconducting compounds a propagating flux dendrite will show physical similarities with electromagnetic modes in normal-metal films [23].

In the present study of NbN films we have searched for wavelike behavior of propagating flux dendrites. In particular, samples were designed so that invading dendrites will cross boundaries between two different superconducting media, represented by the bare NbN film and areas of the same film coated with a metal layer. It is well known that a metal layer causes inductive braking of the avalanche propagation [22,24], and hence such boundaries should display refraction of dendrites provided they have a traveling wave nature. Indeed, previous work by Albrecht *et al.* [25] showed that the propagation of flux dendrites crossing borders between regions of different material properties depends on the incidence angle of the avalanche. The present Rapid Communication gives direct experimental evidence that the electromagnetic modes excited in the dendritic avalanches in NbN cause systematic

refraction at boundaries between different media. Moreover, the quantitative refraction follows Snell's law, in striking resemblance with geometrical optics of light.

Films of NbN were grown on MgO(001) single crystal substrate to a thickness of 170 nm using pulsed laser deposition. By electron beam lithography and reactive ion etching with $\text{CF}_4 + \text{O}_2$, one film was shaped into a $3.0 \times 1.5 \text{ mm}^2$ rectangle. Then, a 900 nm thick Cu layer was deposited on the film and patterned as shown in Fig. 1. Here, the two long horizontal strips of metal define areas where flux avalanches starting from the lower film edge will experience magnetic braking. The metal coating along the upper edge has the purpose of preventing avalanches from starting from that sample side.

In addition to MOI observations using a Bi-substituted ferrite garnet indicator [26,27], the present work also makes use of the recent finding [28] that a flux avalanche propagating in a metal-coated part of a superconductor film generates a sizable voltage pulse. To measure such pulses, contact pads were placed at the lower corners of the sample, where the left pad contacts the two long metal strips. With this geometry, if two subsequent pulses are detected they provide information about the speed of the avalanche front. Moreover, the fine structure of each pulse tells about the number of flux branches passing the electrodes and the points in time they enter and exit. As voltmeter, a Tektronix TDS 210 oscilloscope was used, and set to record the voltage with sampling interval of a few nanoseconds. The measurement was triggered by the pulse itself when the instantaneous voltage exceeded a preset threshold value. When triggered, the voltmeter stores also data measured in a preceding time interval, thus allowing the full profile of the voltage signal during an avalanche to be recorded.

Shown in the upper panel of Fig. 2 is a magneto-optical image of the flux distribution after a typical avalanche occurred in the NbN film at 3.7 K in descending applied magnetic field. Prior to the field descent, the film was filled with flux by applying a perpendicular field of 17 mT, which removed essentially all the flux trapped from previous experiments, and created an overall flux distribution corresponding to a critical

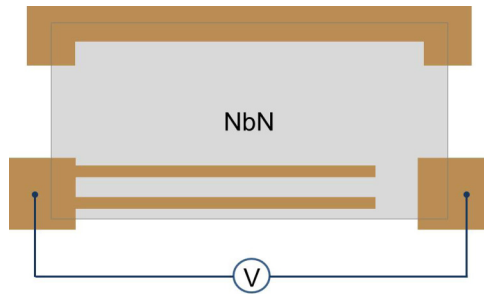


FIG. 1. (Color online) Schematics of the rectangular NbN superconducting film covered by a Cu-layer patterned as seen in the figure. Shown is also the voltage pulse measurement circuit, which allows time-resolved observation of the avalanches starting from the lower film edge.

state. Then, during the subsequent field descent, when the field reached 14 mT, a large-scale avalanche started from a location near the center of the lower sample edge. The dark dendritic structure shows the paths followed by antflux as it abruptly invaded the sample.

Note that in this transverse geometry the local magnetic field around the film edge is oriented oppositely to the descending applied field [29]. Hence, during the field descent the edge region becomes penetrated by antivortices, which

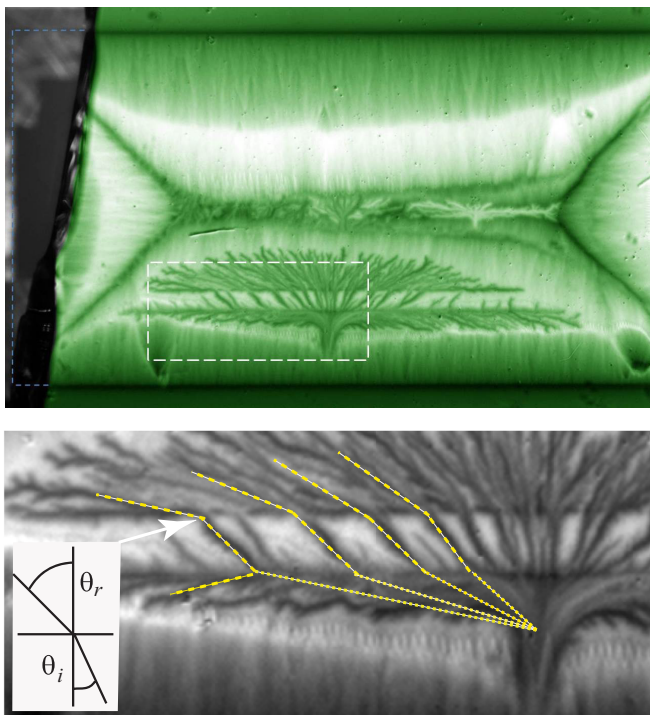


FIG. 2. (Color online) Magneto-optical image of a flux avalanche occurring at 3.7 K in the metal-coated NbN film. The image covers the lower central part of the film, and was recorded in the remanent state after the field was first raised to 17 mT. The horizontal bright strip permeated by dark line segments is the metal-coated strip located nearest to the sample center. The strip near the edge is invisible, as the avalanche crossed this region through a single channel perpendicular to the edge.

under unstable conditions triggers an avalanche of penetrating antflux.

Whereas the overall structure of this avalanche is quite complex, one can see that it consists of many branches, or rays, behaving with considerable regularity. Note from the image that the inner metal-coated strip is seen here as the distinct bright region where only a few rays are crossing. Note also that as long as the ray propagation takes place in the same medium, i.e., either the bare superconductor or the metal-coated area, the rays are often quite straight. Moreover, when the rays traverse an interface between the two media, their propagation direction is changed displaying a clear refraction effect.

To see this in more detail, a magnified view of the flux distribution inside the rectangular area marked in Fig. 2 (upper) is shown in the lower panel. In the metal strip area the rays, indicated by dashed yellow lines, traverse the strip at various angles denoted θ_i ; see the inset for definitions. As the rays cross the interface they continue into the bare superconductor at a different angle θ_r . This refraction angle is consistently larger than the incident angle, θ_i , and it is interesting to compare the two angles quantitatively in relation to Snell's law,

$$\sin \theta_r / \sin \theta_i = n.$$

Here n is the relative index of refraction of the metal-coated and bare areas of the superconductor. From the examples of refraction indicated by the dashed lines in Fig. 2 (lower) one finds $n = 1.37, 1.37, 1.44$, and 1.34 , which are remarkably similar values. Note that the metal strip nearest the edge does not lead to refraction. This is fully consistent with Snell's law since the avalanche here enters the strip at normal incidence.

These observations give strong indications that the avalanche dynamics is governed by oscillatory electromagnetic modes, and that these modes have different propagation velocities in the bare superconductor and metal-coated film. Denoting these two velocities v_s and v_c , respectively, the suggested physical picture then demands that their ratio be equal to the index of refraction, $v_s/v_c = n$. This relation was tested by analyzing additional experimental data.

During the avalanche event seen in Fig. 2, also the voltage signal it generated between the electrodes was recorded, and the result is shown in the upper panel of Fig. 3. The avalanche created a pulse of 200 ns duration and maximum magnitude of 0.14 V. From the profile it is evident that the signal can be considered as composed of a set of shorter subsequent pulses. As shown in previous work [28], such a pulse is observed when a flux dendrite propagates across the electrode area. Decoding of the measured signal is therefore possible using the additional information provided by the magneto-optical image in the lower panel.

The first peak in the voltage signal was generated when the lower metal strip (not clearly seen in the magneto-optical image) was traversed by the full avalanche front. The first peak has a maximal value of 0.11 V, and a width close to 20 ns. All the subsequent peaks are caused by rays traversing the upper strip electrode. It is here reasonable to assume that the rays nearest the main trunk, i.e., those marked in the lower panel by "2", were the first to cross the upper electrode, and hence give rise to the peak marked "2" in the voltage signal. Marked in both panels by increasing numbers is our suggested correspondence between subsequent peaks and ray segments

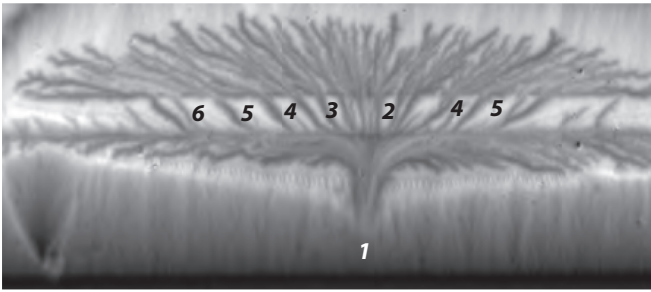
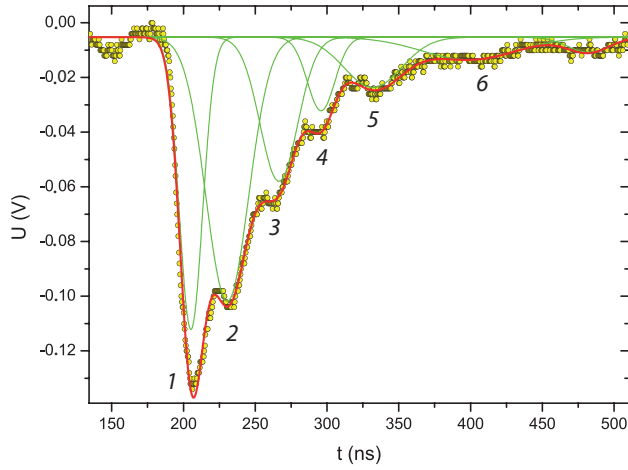


FIG. 3. (Color online) Voltage pulse generated by the flux avalanche seen in the magneto-optical image below. The signal is decomposed into a sequence of shorter pulses (green curves) numbered from 1 to 6, which give rise to corresponding peaks in the overall curve. See the text for the numbers assigned to the peaks and dendrite segments in the image.

crossing the upper electrode. This is not a detailed one-to-one correspondence since several rays may contribute to the same peak.

Let therefore the quantitative analysis take into account only the first two peaks. The time delay between them is $\Delta t = 25$ ns. During this interval the avalanche propagates a distance close to the sum of two strip electrode half widths, each $w = 0.045$ mm, plus the separation between them, $s = 0.16$ mm. The delay time can therefore be expressed as

$$\Delta t = 2 \frac{w}{v_c} + \frac{s}{v_s}.$$

Combining this with $v_s/v_c = n$ where the ray refraction observations gave as average $n = 1.38$, we find that the velocity of ray propagation in the metal-coated part of the NbN film is $v_c = 8.2$ km/s. In the bare superconducting film the velocity is $v_s = 11.5$ km/s. This quantitative result for the propagation speed at an intermediate stage of an avalanche is fully consistent with previous measurements near the final stage of an avalanche in a similar film, which gave a value of 5 km/s [28].

The surprising observation that branches of a flux avalanche propagating across boundaries between two superconducting media show quantitative agreement with Snell's refraction law leads us to conclude that the branches propagate as electromagnetic modes with well-defined speed. Such modes

propagating in a film of resistivity ρ were considered in Ref. [23], where it was found that their speed can be written as

$$v_{em} = \alpha \rho / \mu_0 d. \quad (1)$$

Here, d is the sample thickness, $\alpha \simeq 1$ is a numerical factor depending on the sample geometry and type of mode, and μ_0 is the vacuum magnetic permeability.

As shown by numerical simulations [21,22], Eq. (1) properly describes the propagation velocity of the dendrite's trunk, which is heated to a temperature close to T_c . Coating by a normal film decreases local resistivity, and therefore decreases the trunk velocity. This is the physical reason for the avalanche refraction.

For a superconducting film of thickness d_s and resistivity ρ_s , coated by a metal layer of thickness d_m and resistivity ρ_m , one can define an effective resistivity ρ_c . If there is no exchange of electrical charge between the two layers, the resistivity of the coated film is given by

$$\rho_c = (d_s + d_m) \left(\frac{d_s}{\rho_s} + \frac{d_m}{\rho_m} \right)^{-1}. \quad (2)$$

From Eq. (1) it then follows that the propagation velocity in the bare superconducting film, v_s , and the velocity in the coated film, v_c , are related by

$$\frac{v_s}{v_c} = 1 + \frac{\rho_s d_m}{\rho_m d_s}. \quad (3)$$

Thus, from Snell's law, the relative refractive index for ray propagation between coated and bare areas of a superconducting film is given by the right-hand side of Eq. (3). The ratio $(\rho_s d_m)/(\rho_m d_s) \equiv S$ was introduced recently [22] as a parameter to quantify how efficiently a metal coating will suppress flux avalanches in an adjacent superconductor. Using again $n = 1.38$, we find for the present system that $S = 0.38$. Compared with the case considered in Ref. [22], where $S \gg 1$ and the metal coating caused rapid decay of the avalanches, the present S value represents weak damping, which evidently is a prerequisite for refraction of the branches to be observed.

With the values for d_s and d_m in the present sample, one finds $\rho_s \approx 0.07 \rho_m$. From this it follows that the instantaneous temperature at the front of a propagating avalanche is not far from the superconductor's critical temperature. Also this is consistent with the assumption that the front propagation can be considered analogous to that of the modes introduced in Ref. [23].

To visualize the refraction taking place at the lower edge of the strip, we show in Fig. 2, lower panel, a set of straight dotted lines drawn parallel to the refracted rays in the bare superconductor region above the strip. The construction presumes that Snell's law with same index of refraction applies also at the lower edge, and it turns out that all lines meet in one point. This strongly suggests that the rays originate from one single event at an intermediate stage of the avalanche. In the same panel one can make another interesting observation, namely a clearly visible example of dendrite reflection. The event takes place at the lower edge of the strip, and the reflected ray is drawn as a dashed line at an angle equal to that of the incident ray.

Finally, we mention that deflection of dendrites was observed also during ascending field where the flux flows into a virgin state sample. In contrast to the descending field situation, this process is not accompanied by flux-antiflux annihilation and additional energy release. Experimentally, we find that in ascending field the magneto-optical images and the voltage pulses provide slightly less clear evidence for Snell's law behavior.

In summary, the present work has shown that the propagation of thermomagnetic avalanches in the form of distinct branches observed by MOI has significant similarities with the propagation of optical rays through interfaces between two media. By quantitative comparisons it was demonstrated that the branches undergo refraction in full agreement with Snell's law. These findings support an interpretation of the flux front dynamics at its fast initial stage as propagation of a damped shock electromagnetic wave of the kink type; see, e.g., [30]. We ascribe these waves to damped electromagnetic modes similar to those considered in Ref. [23]. Usually, shock waves

contain many Fourier components. If the medium is dispersive, i.e., the velocities of different component are essentially different, the refracted ray gets smeared. Within the present resolution, this is not observed here, and we conclude that the velocities of the different Fourier components form a narrow distribution.

To observe raylike refraction of flux dendrites one needs superconducting films where the propagation velocities in the different parts of the device is not too different. For a partly metal-coated film the braking parameter S should be not too large; otherwise the damping will dominate. We leave it for future work to identify more precisely the boundaries in parameter space of the interesting low-damping regime reported in this Rapid Communication.

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