Equilibrium state of planar arrays of magnetic dipoles in the presence of exchange interaction

Anatolij M. Shutyi*

Technological Research Institute of Ulyanovsk State University, office 317, campus 4, Universitetskaya Nabereghnaya Str., 432700 Ulyanovsk, Russian Federation

Svetlana V. Eliseeva† and Dmitrij I. Sementsov

Department of High Technology Physics and Engineering, Ulyanovsk State University, Lev Tolstoy 42, 432700 Ulyanovsk, Russian Federation

(Received 20 July 2014; revised manuscript received 30 October 2014; published 22 January 2015)

This article investigates the equilibrium states of square-planar arrays of magnetic dipoles. It has been demonstrated that in the presence of an exchange interaction the main equilibrium states are the configurations of dipoles oriented along the system diagonal, along its side, as well as configurations with vortex structures, which may differ by location of the vortex center and, respectively, by magnitude and direction of the magnetic moment of the system. Also the conditions for transitions in the equilibrium configurations, when influenced by a plane field affecting the whole array, or by a normal local field affecting a part of the system dipoles, were considered. The possibility to control magnetic moment of the dipoles system through transitions between different vortex configurations, including a configuration with zero total magnetic moment, has been shown.

DOI: 10.1103/PhysRevB.91.024421 PACS number(s): 75.75.Jn, 75.40.Mg

I. INTRODUCTION

Magnetic systems and their states are easy to manipulate and, therefore, they make a good subject for the analysis of self-organization processes, as well as for the study of collective effects and phase transitions. Interest in the ensembles of nanoparticles has acquired special significance due to the advances in the area of information technology. In addition, stationary structures formed by a small number of elements became of special importance as well, given the need to record information using various magnetic mediums. In recent years there has been a systematic study and practical application of dipole superstructures of a magnetic type created by nanotechnology [1]. Among such structures, two-dimensional structures in the form of square arrays of nanoparticles of a near circular [2] shape present a particular interest. Specifically, dipole magnetic-ordering lattices can be formed by nanolithography from the nanoparticles of the atoms of the ferromagnetic metals [3]. Such nanoparticles may consist of up to 100 atoms, which ensures their spherical form of about 10 nm and a magnetic moment of several Bohr magnetons [4]. Modern technologies allow producing an ensemble of nanoparticles in which the variation in size does not exceed 5% [5]. The main contribution to the interaction of the magnetic moments in these systems is made by the exchange and dipole-dipole interactions [3,6]. There is a lot of research dedicated to the investigation of spin vortex structures in magnetically ordered media [7–11]. In particular, the vortex state in the nanoparticles [12], as well as the spectrum of spin-wave modes under the influence of exchange and dipole interactions, are also well researched [13–15]. The vortex states of cylindrical magnetic samples of different sizes with a complex structure of the vortex core based on the exchange and dipole-dipole interactions and the magnetic field at an arbitrary ratio of the coupling constants were considered [16]. Searching for the systems with vortices of an extremely small size is both of fundamental and theoretical interest. However, these states appear only at a sufficiently large dipole-dipole interaction. For crystalline magnets with rare earth ions the energy ratio of dipole-dipole and exchange interactions is about 0.1–0.3 [17,18]. For composite materials, it may reach 1 [16]. These magnetic objects differ significantly from the low-dimensional systems, which include the lattice of nanoparticles. Particularities of the lattice properties of nanostructures are due to their discrete and finite features. Practical use of such structures is due to the possibility to widely change parameters of a single nanoparticle, and it is also possible to change the type and energy of the interaction between nanoparticles. This allows, in turn, to control the equilibrium configurations of the magnetic moments of the lattices and to form different vortex configurations using an external field. In particular, storage devices made on the basis of an array of magnetic dipoles are among the most promising types of memory devices. Therefore, the research of an impact of external homogeneous and local static magnetic fields on the dipole array states has practical importance. In [19] orientational transitions in lattices of nanoparticles having a magnetic moment and the associated dipole-dipole interaction were considered. In this paper we investigate the basic equilibrium configuration of the magnetic moments, which are installed in a plane square lattice at the presence of not just the dipole-type but also the exchange-type interaction between the array elements. In the arrays of magnetic nanoparticles the exchange interaction is not as strong as in magnetic crystals, where the distance between the atoms is equal to the parameter of the crystal lattice, and, therefore, the exchange energy is much higher than the energy of the dipole interaction [20,21]. However, this interaction can have a significant influence on the behavior of the magnetic subsystem in nanostructured objects, in particular, in the arrays of magnetic nanoparticles. Moreover, in such systems the exchange and dipole-dipole interaction can be compared, resulting in the realization of new states and properties of lattices. This paper identifies the conditions of transitions between the lattices equilibrium states

*http://www.niti.ulsu.ru/en/

†eliseeva-sv@yandex.ru

under external homogeneous fields and local fields affecting only a part of the magnetic dipoles of the system. Much attention is paid to the vortex orientation configurations that due to the vortex center motion under the influence of an external magnetic field allows the control over magnitude and direction of the systems magnetic moment. Transitions between different vortex configurations, characterized by a certain total magnetic moment, including central-oriented configurations in which the magnetic moment of the array vanishes, have been considered. It allows an easy transition of the system from a state with zero magnetic moment to a configuration with other various values.

II. BASIC EQUATIONS

When analyzing the behavior of a planar square lattice of nanoparticles we assume that the value of their magnetic moments is the same and they are connected by dipole-dipole and exchange interactions. The position of the nanoparticles' mass centers in the system is considered a constant, the nanoparticles themselves as uniform and spherical [22], and the objects material as a magnetically hard material. Each of the nanoparticles can be rotated around its center of mass. Dynamic equations for the system of magnetic dipoles can be represented as follows [23–25]:

$$J_i \frac{d\boldsymbol{\omega}_i}{dt} + \alpha_i \boldsymbol{\omega}_i = \mathbf{p}_i \times \mathbf{H}_i, \quad \frac{d\mathbf{p}_i}{dt} = \boldsymbol{\omega}_i \times \mathbf{p}_i, \tag{1}$$

where $\mathbf{p}_i$ and $\boldsymbol{\omega}_i = d\boldsymbol{\varphi}_i/dt$—magnetic dipole moment and angular velocity of the ith dipole ($\varphi_{i,j}$—angle of dipole rotation around the axis $j = x,y,z$), J_i—moment of inertia, and α_i—dissipation parameter. Effective field generated at the position of the ith dipole by other dipoles and by the external magnetic field $\mathbf{h}$ is determined by the expression

$$\mathbf{H}_i = \mathbf{h} + \sum_{n\neq i}\left[\frac{3(\mathbf{p}_n\mathbf{r}_{\rm in})\mathbf{r}_{\rm in} - \mathbf{p}_n r_{\rm in}^2}{r_{\rm in}^5} + \Lambda \mathbf{p}_n \exp(-\sigma r_{\rm in})\right], \tag{2}$$

where $\mathbf{r}_{\rm in}$ and $r_{\rm in}$ are the radius vector and distance between the centers of the ith and nth dipoles, and Λ and σ^{-1} the constant and the characteristic length of the exchange interaction. The energy of the interparticle interaction is defined by

$$W_{\rm in} = \frac{(\mathbf{p}_i\mathbf{p}_n)r_{\rm in}^2 - 3(\mathbf{p}_i\mathbf{r}_{\rm in})(\mathbf{p}_n\mathbf{r}_{\rm in})}{r_{\rm in}^5} - \Lambda(\mathbf{p}_i\mathbf{p}_n)\exp(-\sigma r_{\rm in}). \tag{3}$$

On the basis of this expression, we can obtain a criterion for the strong and weak exchange interaction. To do this, we introduce a relation of dipole and exchange energies of magnetic moments of neighboring nanoparticles

$$\eta = \frac{W^{\rm exch}}{W^{\rm dip}} = \frac{a^2\Lambda_1}{2},$$

where a is the lattice parameter (distance between the centers of the neighboring dipoles), and $\Lambda_1 = \Lambda\exp(-\sigma a)$. Thus, at $\eta > 1$, there is the exchange interaction, and at $\eta < 1$ the dipole interaction dominates the system.

Next, we assume that the magnetic dipoles in the lattice are identical: $|\mathbf{p}_i| = p$, $J_i = J$, and $\alpha_i = \alpha$. Then we move on to the following dimensionless parameters [23]: $\mathbf{e}_{\rm in} = \mathbf{r}_{\rm in}/r_{\rm in}$, $l_{\rm in} = r_{\rm in}/a$, $\kappa = \sigma a$, $\boldsymbol{\rho}_j = \mathbf{p}_j/p$, $\beta = \alpha/(\nu J)$, $\boldsymbol{\Omega}_i = d\boldsymbol{\varphi}_i/d\tau$, $\tau = \nu t$, and $\nu = \sqrt{p^2/Ja^3}$. External field in this case is converted to the form $\mathbf{f} = \mathbf{h}a^3/p$ (for $p \approx 3\mu_B$, $a \approx 5$ nm, and $h \approx 0.1f$ Oe) and the exchange interaction constant is converted to $\lambda = a^3\Lambda$. In dimensionless parameters the equations (1) can be written as

$$\frac{d\boldsymbol{\Omega}_i}{d\tau} = -\beta\boldsymbol{\Omega}_i + \boldsymbol{\rho}_i \times \left(\mathbf{f} + \sum_{n\neq i}\left[\frac{3\mathbf{e}_{\rm in}(\boldsymbol{\rho}_n\mathbf{e}_{\rm in}) - \boldsymbol{\rho}_n}{l_{\rm in}^3} + \lambda\boldsymbol{\rho}_n\exp(-\kappa l_{\rm in})\right]\right),$$
$$\frac{d\boldsymbol{\rho}_i}{d\tau} = \boldsymbol{\Omega}_i \times \boldsymbol{\rho}_i. \tag{4}$$

III. EQUILIBRIUM CONFIGURATION OF DIPOLES ARRAY

Let's consider the equilibrium configurations of square-planar arrays of dipoles with different values of the exchange interaction of the ferromagnetic connection type, i.e., for ($\lambda > 0$). Figure 1 and Fig. 2 show the diagrams of dependence of the total magnetic moment with value $C = \sum c_i$ of dipole lattices 2×2–7×7 [(a)–(c)] on the exchange parameter λ in the case of equilibrium configuration and in the absence of an external magnetic field. The branches of the diagram correspond to various equilibrium states of the systems. Namely, two branches exist for the systems 2×2, one of which corresponds to a configuration with zero total magnetic moment, and the second one is with $\mathbf{P} \approx N$, where N is the number of dipoles in the system. In the case of arrays 3×3 there are three types of equilibrium configurations, which correspond to three branches on the diagram. For such systems, the orientation of the magnetic moments of individual dipoles is shown in the insets in Figs. 1(a) and 1(b) (the equilibrium configurations type for the array 4×4 is given in Sec. V C, where a detailed discussion of these systems is provided). Typical equilibrium configurations of arrays 5×5 and 6×6, which can be realized with $\lambda = 0.3$ and $\lambda = 1$, are given, respectively, in Fig. 3 and Fig. 4 [numbering of the configurations on these figures corresponds to the numbering of branches in Figs. 2(a) and 2(c)]. Calculations made for other arrays, up to systems 14×14, indicate that their equilibrium configurations are similar to those shown.

Numerical analysis was performed by a program built using the Runge-Kutta method of fourth order, that allows one to take into account all system elements connections to each other. Arbitrary initial states of the system were set to find the equilibrium configurations, after which the system would come to a static state according to the equations of motion considered above (in the absence of external fields). To obtain reliable results, only large time intervals sufficient to establish equilibrium (i.e., when the extension of the time period did not lead to a change in the system within the range of given numerical calculations) were considered. Also the initial conditions from which the system would come to the same equilibrium configurations differed. To identify stable states, we used an additive represented by a noise signal disturbing the system and used in some cases in the calculations. In the

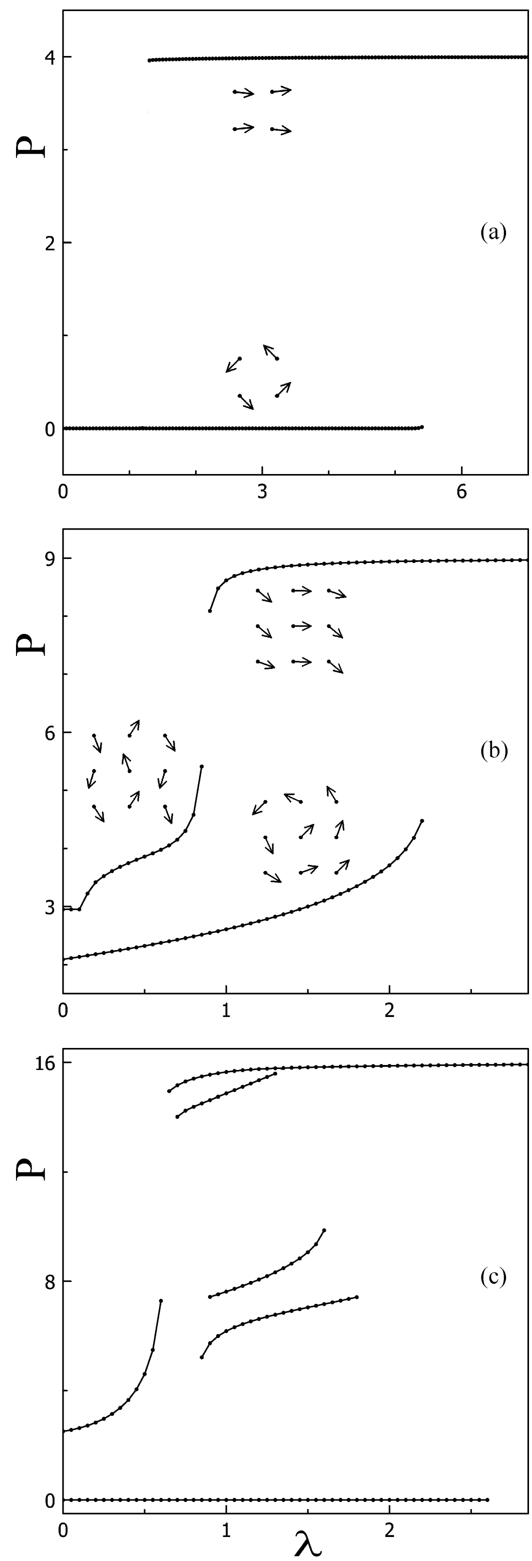

FIG. 1. Dependence of the total magnetic moment **P** of arrays $2 \times 2 - 4 \times 4$ [(a)–(c), respectively] on the exchange parameter λ in the equilibrium configurations (different branches of the diagram correspond to different equilibrium states); $\kappa = 1$. In the inset: the equilibrium configurations of arrays corresponding to different branches of the diagram.

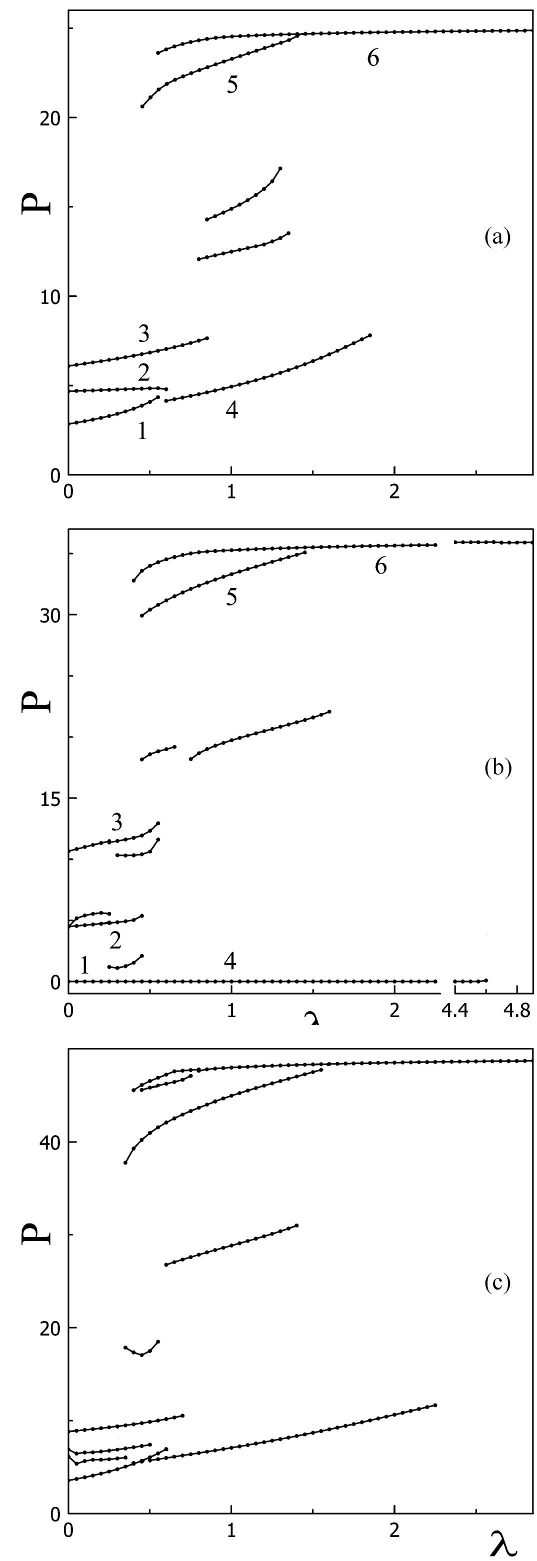

FIG. 2. Dependence $\mathbf{P}(\lambda)$ for arrays $5 \times 5 - 7 \times 7$ [(a)–(c), respectively] in the equilibrium configurations.

case of a stable configuration the addition of this disturbance does not lead to a change in the system.

The dependencies and configurations considered above show that a weak exchange interaction ($\lambda \leqslant 0.5$) contributes to the establishment of equilibrium states with a "saddle" reciprocal orientation of the magnetic moments of separate dipoles (configurations 1 and 3 in Fig. 3), as well as with an area

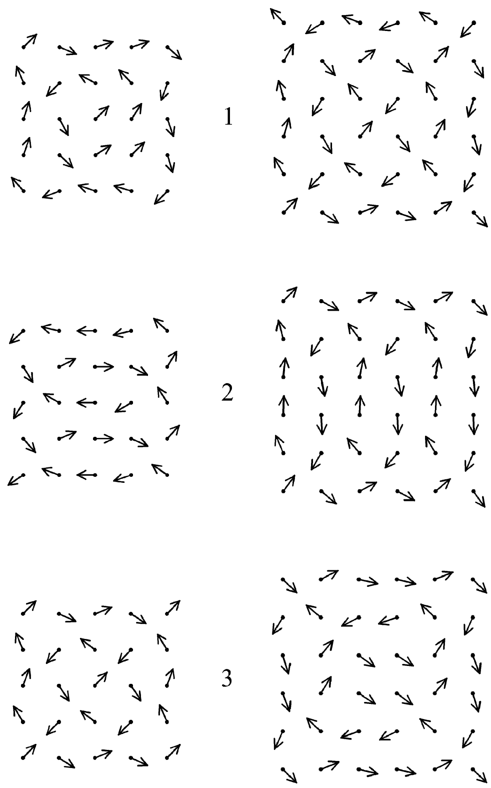

FIG. 3. Equilibrium configurations of arrays 5×5 and 6×6 at $\lambda = 0.3$ (1–3); configurations numbering corresponds to the numbering of branches in Figs. 2(a) and 2(b).

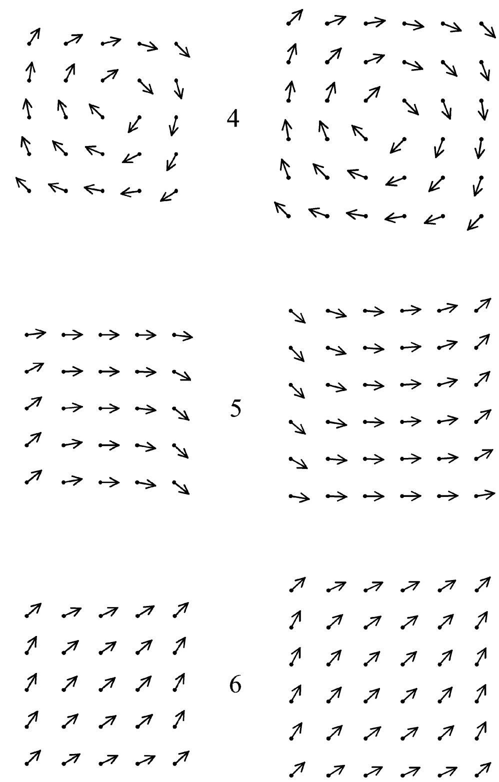

FIG. 4. Equilibrium configurations of arrays 5×5 and 6×6 at $\lambda = 1$ (4–6); configurations numbering corresponds to the numbering of branches in Figs. 2(a) and 2(b).

of oppositely oriented dipoles in adjacent rows (configuration 2 in Fig. 3). In the structures 2×2, a configuration with a zero total magnetic moment is an equilibrium one [see the bottom inset in Fig. 1(a)], and in the structures 3×3 there are two equilibrium configurations—with alternating directions of the magnetic moments in adjacent rows and a "circular" orientation of eight dipoles [see insets in Fig. 1(b)]. We will not consider these configurations further since they are similar to the configurations of the systems with the absence of an exchange interaction [26].

In the case of a strong exchange interaction ($\lambda \geqslant 2 \div 6$—depending on the system), only one equilibrium configuration is established with dipoles oriented along one of the diagonals of the square array (configuration 6 in Fig. 4), and for smaller systems 2×2 and 3×3 they orient along the sides of the arrays. The total magnetic moment is $\mathbf{P} \approx N$. In addition to the configuration above, an equilibrium state with a minimal total magnetic moment is also established at a smaller value of the exchange parameter. For an array with an even number of dipoles $\mathbf{P} = 0$ in this configuration and the dipole moments are oriented as vortices (configurations 4 in Fig. 4). In addition to these two equilibrium configurations, at a weaker exchange interaction there is also a configuration with dipoles oriented primarily along the side of the array (configurations 5 in Fig. 4). In Fig. 2, the relevant branch is located near the branch corresponding to the dipoles' orientation along the diagonal of the arrays, as in this case the value $\mathbf{P}$ is also close to N. In this area of parameter λ, as can be seen in Fig. 2, there are also the equilibrium states with an intermediate value (respective to the minimum and the maximum values) of the total magnetic moment of the array. The relevant mutual orientation of individual dipole moments of the systems in these cases is of a "vortex type" (similar to the configurations 4 in Fig. 4), but with a different offset of the vortex center from the center of the dipoles' arrays. Some of these configurations will be considered below. In the next paragraph we take a closer look at this area of the exchange parameter λ and consider magnetization reversal of dipole systems and transitions within equilibrium configurations.

These curves are plotted at $\kappa = 1$. Modification of the normalized length of the exchange interaction leads to the same equilibrium configurations. Figure 5 shows the diagram of dependence of the total dipole moment on κ at the exchange parameter $\lambda = 5$ for the lattice 6×6. It can be seen that by increasing the return length of the exchange interaction κ, the diagram of equilibrium states is similar to the diagram corresponding to the reduction of λ (see Fig. 2). In this figure the branches corresponding to diagonal (branch 4) and vortex configurations with zero and nonzero values of the total magnetic moment (branches 1 and 2) are clearly seen, as well as configurations with the dipoles' orientation along the sides of the lattice (branch 3).

IV. TRANSITIONS BETWEEN THE EQUILIBRIUM STATES

Let's consider the establishment of different equilibrium states and remagnetization of the arrays when the values of exchange parameter are close to $\lambda = 1$. Under these conditions

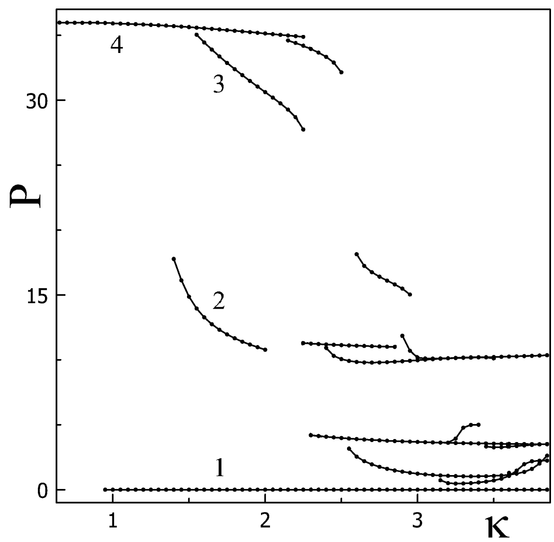

FIG. 5. Dependence of the total moment **P** of the exchange interaction for array 6×6 in equilibrium configurations at $\lambda = 5$ on the reciprocal length.

the equilibrium configurations are with a vortex orientation of the dipoles, an orientation primarily along the diagonal and along the side of the array (see Fig. 4). Implementation of various equilibrium states in numeric modeling could be achieved by repeatedly setting as a reference a random orientation of each of the dipoles of the system, and the consequent solution of the resulting dynamic equations. In practice, however, this method cannot be realized, since the original state is not an arbitrary one, but one of the equilibrium states. Therefore, possible transitions between different equilibrium configurations, or remagnetization of the system without configuration modifications, i.e., the change of the direction of the total magnetic moment of the array while maintaining its value, should be investigated.

The analysis will be carried out for the array 6×6. For other arrays, transitions between equilibrium configurations are similar. Numerical analysis showed that the following transitions were possible when using the uniform remagnetizing field lying in the plane of the array. From a centrally oriented vortex configuration, the transitions can be made to the configuration with a displaced vortex center or to the configuration with the dipoles oriented along the side or along the diagonal of the array. The only transition that can be made from the configuration with the dipoles oriented along the side of the system is to the dipoles oriented along the diagonal. The system does not get remagnetized in this case, and the only thing possible is the turn of the total magnetic moment with transition of the dipoles' orientation along the diagonal. In the case of configuration with a diagonal orientation, only remagnetization of the arrays is possible (preserving diagonal configuration), but transitions to other configurations (the vortex ones or the ones oriented along the side of the system) do not take place. For transitions between equilibrium states, let's consider the time dependence of the total normalized binding energy of the system

$$W_0 = \sum_i \sum_k \frac{a^3}{2p^2}(W_{\text{in}} - W_{\text{in}}^{'}), \tag{5}$$

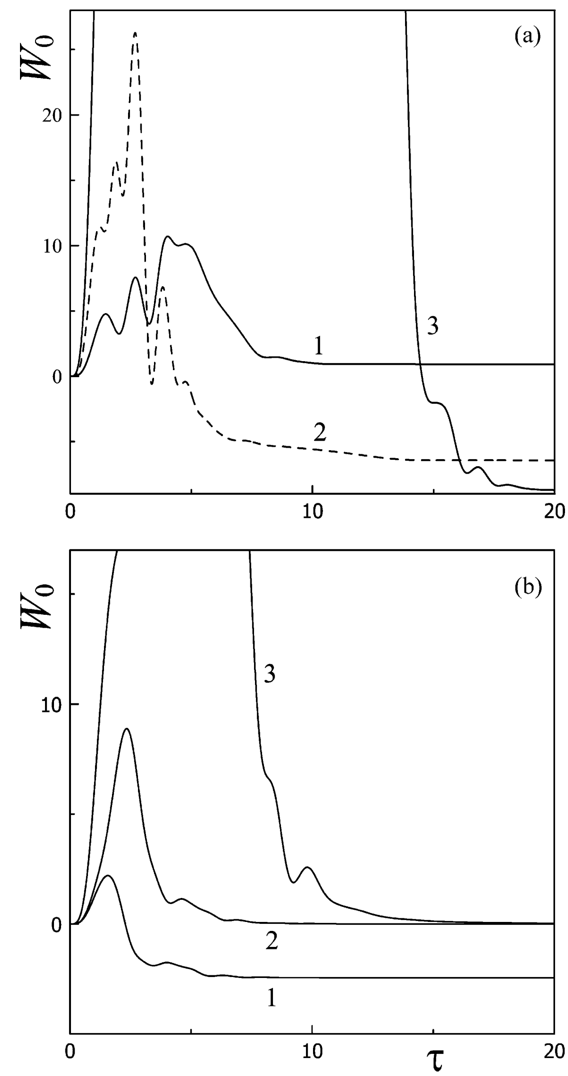

FIG. 6. Time dependence of normalized binding energy systems during transitions between different equilibrium configurations of the array 6×6 under the influence of a planar magnetic field.

where W_{in} is the interaction energy of ith and nth dipoles (3), and $W_{\text{in}}^{'}$ is the interaction energy in the initial state of the array. Figure 6(a) shows the dependence $W_0(\tau)$ for the transition from a vortex configuration with $\mathbf{P} = 0$ to the vortex configuration with a displaced center and, respectively, with $\mathbf{P} \neq 0$ (curve 1), to the configuration with the dipoles' orientation predominantly along the side of the array (curve 2) and to a diagonal configuration (curve 3). In all cases, up to the point of time $\tau_1 = 5$ the whole dipole system experienced a uniform planar field of the value $f = 1,2,3$ (curves 1–3), directed along the side of the array for the first two cases, and along the array diagonal for the third case; after this point of time the external field was turned off and the dipoles' lattice transited to a new equilibrium state. Figure 6(b) shows the dependence of $W_0(\tau)$ for the transition from the configuration with orientation along the side of the lattice to the diagonal configuration (curve 1) and for remagnetization of the diagonal configuration (to another diagonal configuration) with turning moment of the whole lattice at angles $\pi/2$ and π (curves 2 and 3). The external planar magnetic field is also attached to the whole system until the point of time $\tau_1 = 5$, its value $f = 1$ (curves 1 and 2) and $f = 3$ (curve 3), and the angle between the field direction and the initial total dipole moment direction of the system $\pi/4$, $\pi/2$, and π (curves 1–3). As it can be seen from these dependencies, application of an external field increases the binding energy of the system, which indicates the stability of the equilibrium configurations of the considered systems. Furthermore, it can be seen that the diagonal configuration has minimum binding energy, so this equilibrium state is the most stable. The greatest binding energy belongs to the case of a vortex configuration; notably, at configuration with $\mathbf{P} \neq 0$ the binding energy is higher than at configuration with $\mathbf{P} = 0$.

When exposed to the whole array of dipoles, the vortex equilibrium configuration in some cases can be obtained from the configuration with the orientation of the dipoles along the side of the array in the direction of the external field normal to the plane of the dipole system. However, the transition between the equilibrium configurations in this case is of a rare and random nature. The vortex equilibrium configuration with a diagonal configuration baseline or configuration with a magnetic moment oriented along the side of the array (as well as the latest from the diagonal configuration) can be obtained through a normally oriented magnetizing field, which acts only on the part of the system. Figure 7 shows transformations between the equilibrium configurations, using a normal system magnetizing field. The starting point is a diagonal configuration, below which the configurations before turning off the field with $f = 15$ acting during $\tau = 15$ to the dipoles are shown as points in the figure (since their magnetic moments are directed perpendicular to the plane of the array). The following are two finite transformation configurations (longitudinal configuration—with a magnetic moment, oriented mainly on the side of the array—and the vortex one with a shifted center), which are subsequently converted into a vortex configuration $\mathbf{P} = 0$. The figure shows that, acting on the 16 dipoles of system 6×6, it is possible to translate the diagonal configuration to the vortex configuration with shifted center of (transition 1), which can then be translated into a vortex configuration with $\mathbf{P} = 0$ (transition 3). Impact of the normal field on the half of the diagonal configuration translates it into a configuration with a magnetic moment oriented along the side of the array (transition 2). The last configuration can be transformed into a vortex one both with $\mathbf{P} = 0$ (transition 4) and with a shifted center, as shown in Fig. 8. Vortex configuration with a displaced center can also be translated into configuration with $\mathbf{P} = 0$ under the influence of a field on four central dipoles of the system. Time dependence of the total dipole moment for these transformations is shown in Fig. 9 (the curves 1–4 correspond to the respective transformations in Fig. 7; the curve 5 corresponds to the transformation in Fig. 8). Action of the field perpendicular to the plane of the lattice does not lead to an equilibrium state with perpendicular magnetic moment in these arrays of magnetic moments. In addition to the above configuration, an equilibrium state with a minimal total magnetic moment is realized at a smaller value of the exchange parameter. At the considered parameters of the structure after we turn the external field off the dipole-dipole interaction puts the magnetic moments of the array elements in the plane of

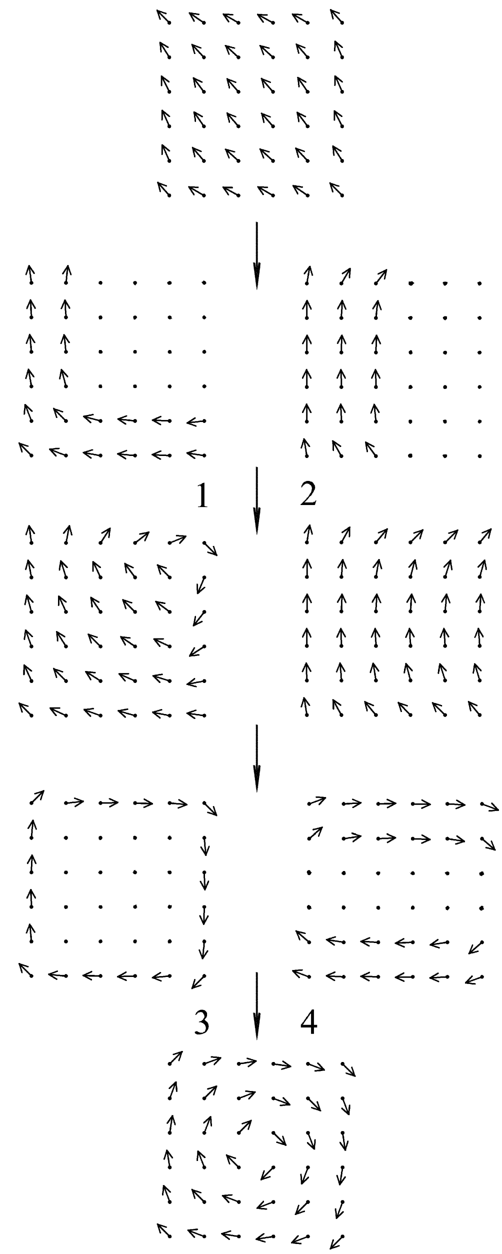

FIG. 7. Transformations within the equilibrium configurations by applying a normal in relation to the system-magnetizing field acting on the part of the dipole array. From top to bottom: the initial diagonal configuration → configuration before shutting down the perpendicular field ($f = 15$, $\tau = 15$) → the final configuration of the corresponding transitions (initial for the next transition) → configuration before shutting down the field → final vortex configuration.

the system. The perpendicular field acting on the part of the array, which is in the vortex configuration, may lead to the displacement of the vortex center. It is caused by reorientation of the magnetic moments lying in the plane of the system under conditions when a part of the magnetic moments of the array is directed perpendicular to the system by the field. After switching off the external field the specified reorientation leads to the fact that after laying all the magnetic moments to the plane of system, a vortex configuration with another location of

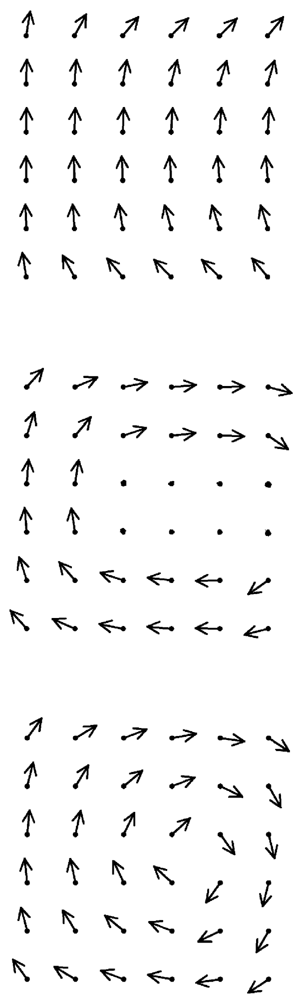

FIG. 8. Transition from longitudinal to vortex configuration using perpendicular magnetic field to the system with $f = 15$, acting during the $\tau = 15$ for eight elements of the array.

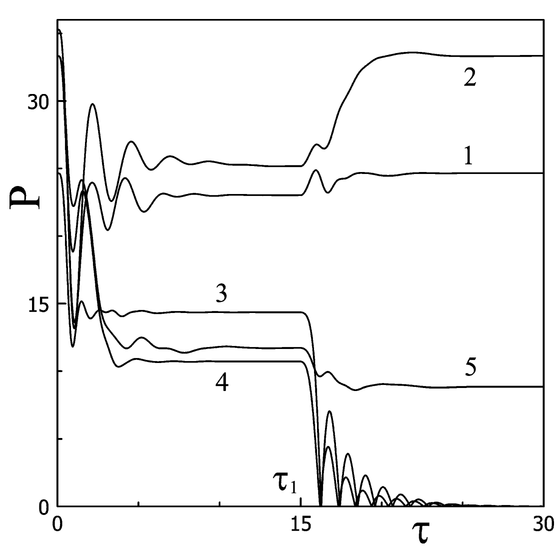

FIG. 9. Time dependence of total dipole moment for the transformation presented in Fig. 6 (the curves 1–4 correspond to the respective transformation in Fig. 7; the curve 5 to transformation on Fig. 8).

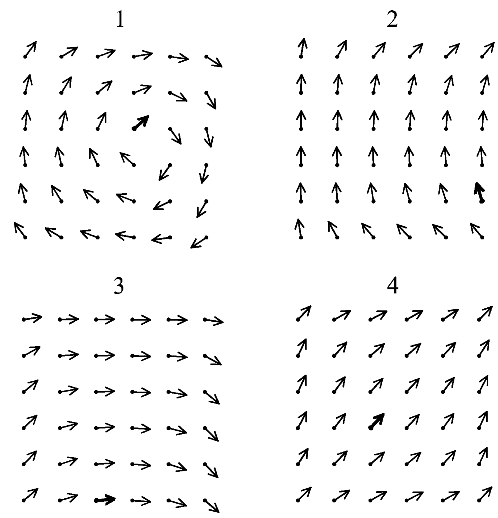

FIG. 10. Equilibrium state in which the vortex configuration with $\mathbf{P} = 0$ (1–3) and the configuration with a longitudinal moment (4) transform under the influence of the field on the dipole moments of bold arrows (field is oriented along the diagonal of the array).

the vortex center appears, i.e., as a result, a shift of the vortex center in relation to the initial configuration is observed. In the case of a vortex configuration and configuration with a magnetic moment oriented along the side of the array, the transformation caused by exposure to only one dipole of system are also carried out. Figure 10 shows a configuration in which the vortex configuration with $\mathbf{P} = 0$ transforms (cases 1–3) the configuration with a longitudinal magnetic moment of the system (case 4) under the influence of a field on the dipole moments of the bold arrows. In all cases the field with $f = 15$ is valid for $\tau = 15$ and is oriented along the diagonal lattice. The figure shows that the choice of the dipole, which is made on the effect of the field, can establish configurations with different magnitude and direction of the total dipole moment of the system. Direction of the established total dipole moment is defined by the orientation of the applied magnetic field. It is worth noting that at the initial vortex structure and due to the influence on various dipoles of the array, the center of the vortex structure can be shifted in different directions, which leads to remagnetization of the system maintaining the vortex character of its configuration.

V. CONTROL OF THE MAGNETIC MOMENT OF THE LATTICE WITH A VORTEX STRUCTURE

A. Displacement of vortex structure center

Equilibrium position of the vortex center in the lattice dipoles is ensured by the achievement in these configurations of the minimum binding energy of the systems' dipoles. When the center of the vortex is removed from the center of the array, the magnetic moment of the system increases. Thus, in the case of a transition between two equilibrium vortex structures, the remagnetization of the array is carried out. Displacement of the center of the vortex structure is carried out both by the aforementioned local action on the system, and by means of acting on the whole lattice of a planar field; the direction of displacement is perpendicular

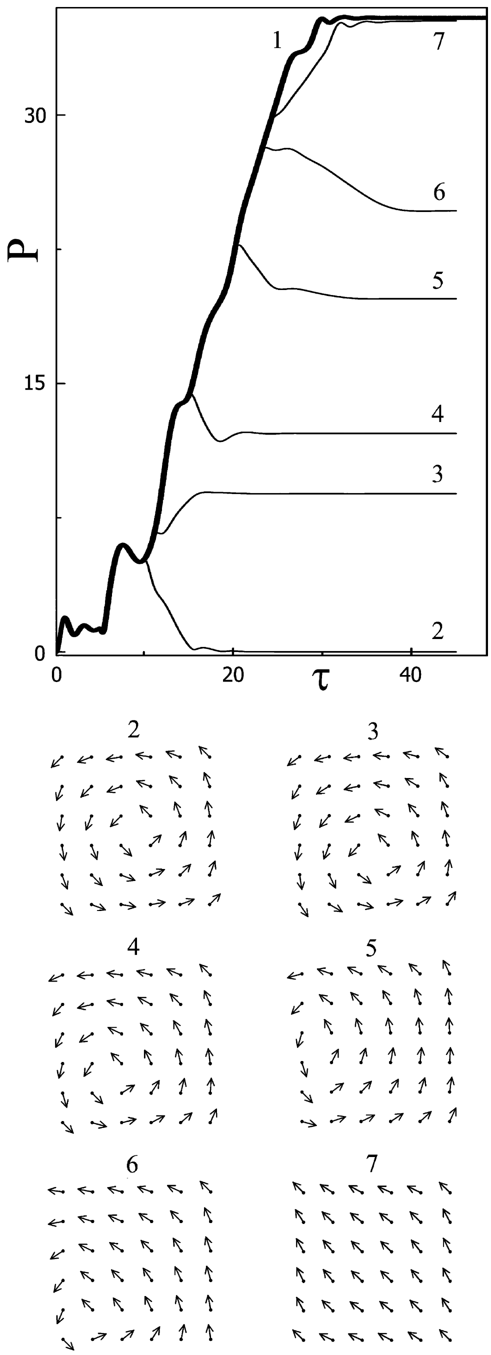

FIG. 11. Time dependence of the dipole moment of the system 6×6 for the initial vortex state with $\mathbf{P} = 0$ at the transition to the represented equilibrium configuration by the field with $f = 1.0$, directed along the diagonal of the array; the curve 1 corresponds to the continued inclusion of the field; in the case of the curves 2–7 the field was turned off at different times.

to the direction of the applied magnetic field. Let's consider remagnetization of an array under the influence of a constant magnetic planar field affecting the entire system. Figure 11 shows time dependence of the total dipole moment of the system 6×6 at the initial central vortex configuration, i.e., with $\mathbf{P} = 0$, in the case of an impact on the system of an external field of value $f = 1.0$ and oriented at an angle of $\psi = 3\pi/4$ (ψ is measured from axis x, which coincides with the horizontal side of a square lattice). Curve 1 corresponds to the influence of the field during the entire time considered in the experiment. Otherwise, the field was switched off (at different points in time), and the system would return to the equilibrium state, with the corresponding configurations also shown in the figure. Curve 2 corresponds to the system that was reset to the initial central vortex configuration, since the time of exposure to the external field was insufficient to move the lattice to another equilibrium state. Curves 3–6 correspond to the remagnetization of the system and its transition into the vortex configurations with a varying distance between the vortex center and the center of the system, and thus with a varying total magnetic moment of the lattice. These transitions are reversible: when changing the direction of the field, the system returns to the initial vortex configuration. The latter curve 7 corresponds to a transition to the diagonal configuration, from which it is impossible to bring the system back into the vortex configuration by using a planar field influencing the whole lattice. Let's consider analogous dependences for arrays with a large number of dipoles, which result in systems with a higher value of total magnetic moment. Figure 12 shows the dependence $\mathbf{P}(\tau)$ for a 10×10 array at the initial configuration that corresponds to the vortex structure with a highest magnetic moment, i.e., at a maximum distance of the vortex center from the center of the array. The dependence corresponds to the influence of a planar field where $f = 0.5$ and orientation angle $\psi = 3\pi/4$. The highlighted curve 1 corresponds to a continuously operating remagnetizing field. The magnetic moment in this system is first reduced, which corresponds to the approximation of the vortex structure center to the center of the array. Next, when it is removed from the center of the array and gets closer to its opposite corner, the growth of the total magnetic moment (with a change of its direction to the opposite one) takes place. Towards the end of the given time interval a dramatic increase in total moment $\mathbf{P}$ from its initial value occurs. This corresponds to the disappearance of the vortex configuration and the establishment of a structure with moments of nanoparticles oriented primarily along the diagonal of the array (see inset configuration 8). The other curves correspond to cases when the remagnetizing field is turned off at various times. The center of the vortex structure is shifted to one of its stable positions (which may require a sufficiently long transition process), and the magnetic moment of the system becomes one of several possible values. Dotted and solid curves correspond to the same value of the magnetic moment but to different yet symmetrical (relative to the center of an array) equilibrium configurations. For the array of these specified parameters the value of total magnetic moment can be either zero (curve 2)—corresponding to the central vortex structure, or value $\mathbf{P} \approx N$, where N is the number of dipoles in the system (line 8)—corresponding to the diagonal (or longitudinal) structure, or one of five intermediate values (excluding opposite directions of vector $\mathbf{P}$)—corresponding to different shifts of the center of the vortex structure. For example, curve 7 corresponds to the establishment of a configuration symmetric to the initial one, i.e., where the magnetic moment of the array has the same value but an opposite direction. The figure also shows the equilibrium vortex configuration with minimum and maximum values of the magnetic moment for this array, the

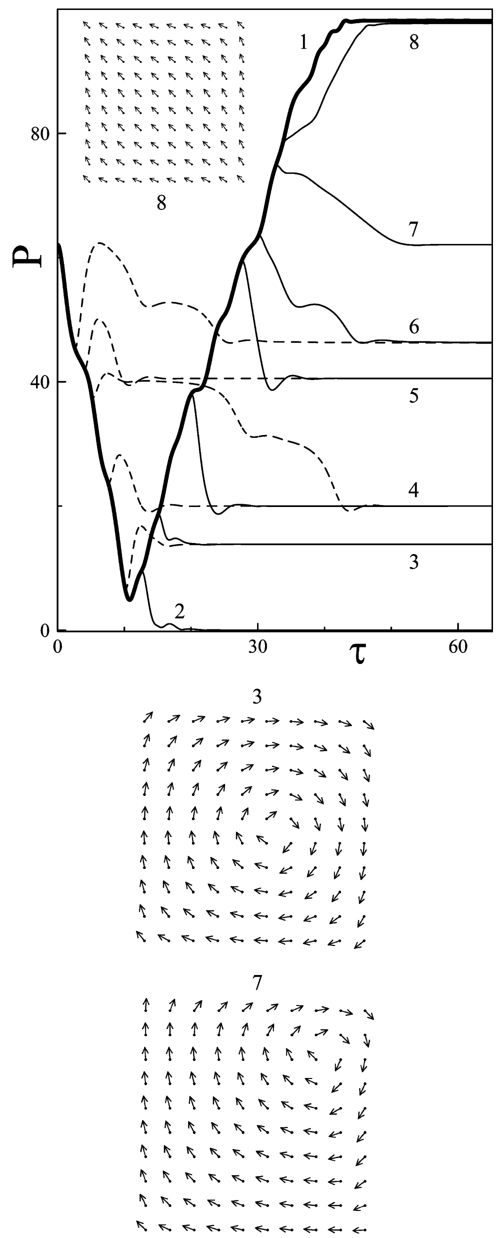

FIG. 12. Time dependence of the dipole moment of the system 10×10 for the initial vortex configuration (with the highest $\mathbf{P} = 0$) for permanent planar field with $f = 0.5$ (bold curve 1) and when it is turned off at different times. The curve 2 is the transition to the center of the vortex configuration; the curve 7 is the transition to configuration symmetrical to the original; the curve 8 is the transition to a diagonal configuration; the curves 4–6 are the transition to vortex configurations with different locations of the center of the vortex.

establishment of which corresponds to curves 3 and 7. The dashed curves 3 and 7 correspond to the establishment of the configurations symmetric to the provided ones. In case of configurations corresponding to curves 4–6, the vortex center is located between its positions in configurations 3 and 7, which can be obtained from the relevant equilibrium values of the total magnetic moment of the system. During the transition from one equilibrium vortex configuration to another, there is at first an increase of the binding energy.

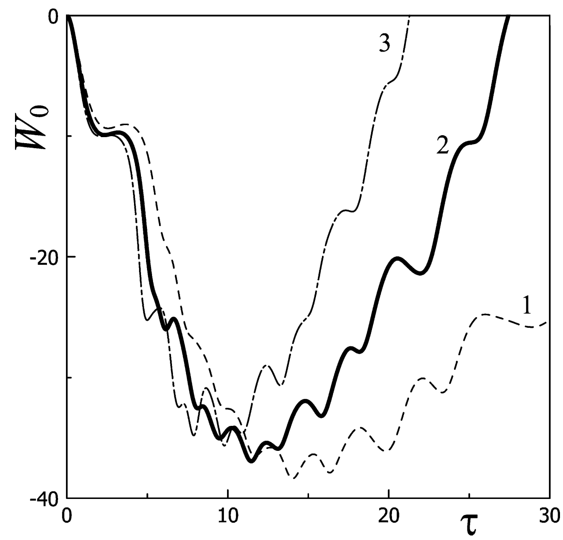

FIG. 13. Time dependence of normalized binding energy systems in the center of the vortex motion under the influence of external fields with $f = 0.4,0.5,0.6$ (curves 1–3).

At approaching another equilibrium configuration the binding energy decreases. Figure 13 shows the dependence $W_0(\tau)$ during the motion of the vortex center influenced by the field with $f = 0.4,0.5,0.6$ (curves 1–3). Orientation of the field and the initial configuration are the same as in the previous figure. The figure shows that during the movement of the vortex, the system passes through several local minima of the binding energy. However, it should be noted that these energy minima do not correspond to energy equilibrium states themselves, as they are revealed in the process of vortex motion under the influence of an external field. This explains the difference between the three dependences obtained for different values of the parameter f, and the absence of symmetry of these dependences, when the vortex approaches the center of the array and moves away from it.

B. Transitions between equilibrium configurations at different exchange interaction

Stable positions of the vortex structure and the number of possible stationary values of the magnetic moment of the system depend, in particular, on the parameters of the exchange interaction. With a relatively weak exchange interaction only $\mathbf{P} = 0$ configuration can occur and, with a strong exchange interaction, $\mathbf{P} = 0$, $\mathbf{P} \approx N$, or $\mathbf{P} \approx N$ configurations occur. Figure 14 shows the time dependence of the magnetic moment of the system with parameter $\lambda = 0.2,0.7,0.8$ [curves 1–3 (a)] and $\lambda = 2.0$ (b) at remagnetized field with $f = 0.5$ acting in the direction of the side of the array ($\psi = 0$) for the initial configuration with $\mathbf{P} = 0$ (the center of the vortex is located in the center of the array). In the case shown in Fig. 14(a) the remagnetizing field was turned off once the saturation of the magnetic moment occurred, i.e., at reaching of the maximum value of $\mathbf{P}$ for a given f. At this, the first two curves correspond to the maximum (for a given field) displacement of the vortex center from the central position and its return to the center of the array when the external

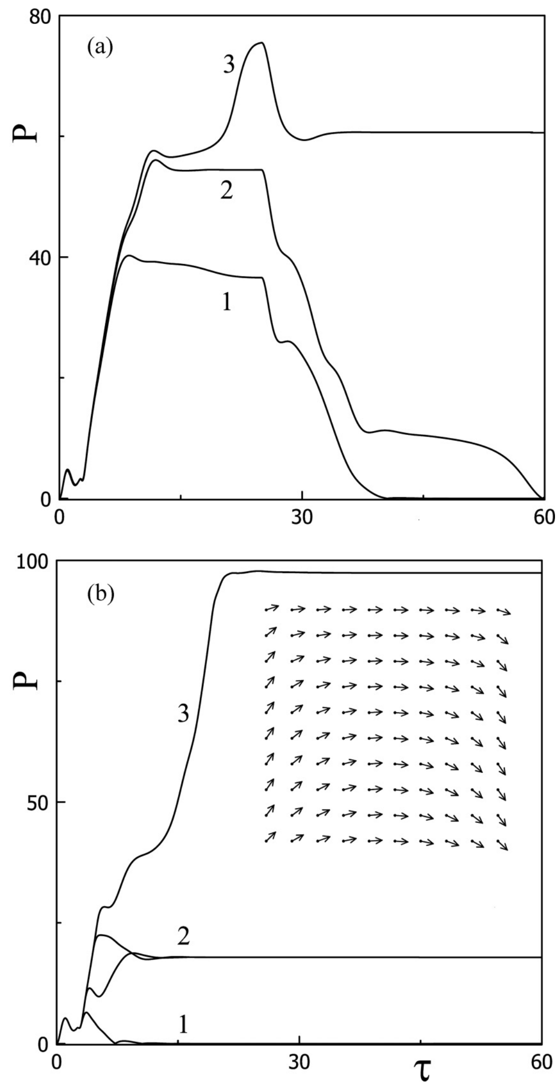

FIG. 14. Time dependence of magnetic moment of the system with the exchange parameter $\lambda = 0.2, 0.7, 0.8$ [curves 1–3 (a)], and $\lambda = 2.0$ (b) at a field with $f = 0.5$, acting along the side of the array, is the initial configuration with $\mathbf{P} = 0$. The field was turned off at the maximum value $\mathbf{P}$ (a) or at different moments of time (b). In the inset the longitudinal configuration of the dipoles is presented.

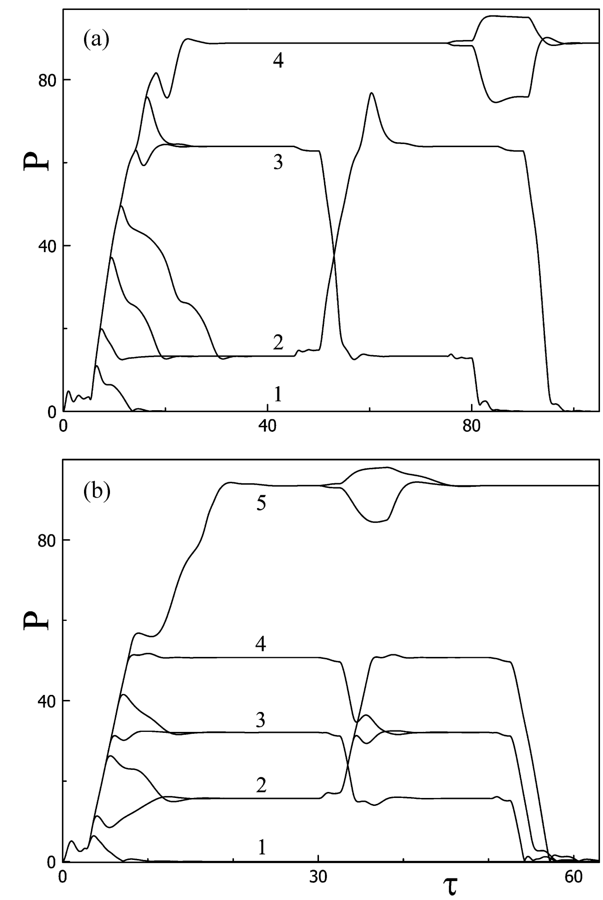

FIG. 15. Transitions due to the planar magnetic field between the different vortex configurations, characterized by the corresponding values of the total magnetic moment; the curve 4(a) and the curve 5(b) correspond to the transition to the longitudinal equilibrium configuration; exchange parameter $\lambda = 1.0, 1.5$ (a),(b).

field is turned off. In the case of curve 3, i.e., at $\lambda = 0.8$, only one equilibrium configuration with a displaced vortex center is possible. The curves in Fig. 14(b) correspond to various time intervals of the influence of an external field. The figure shows that at the exchange parameter $\lambda = 2.0$, the configurations with a central location of the vortex structure with orientation of the dipoles mainly along the side of the array (see inset) and one displaced vortex configuration with a small value $\mathbf{P} \neq 0$ (curves 1, 3, and 2 correspond respectively to the transitions to these states) become stable ones. Figure 15 shows the time dependence of the magnetic moment of the system with exchange parameter $\lambda = 1.0, 1.5$ [(a),(b)] under the influence of an external magnetic field with $f = 0.5$ and the orientation angle $\psi = 0$ in the case of the initial central vortex configuration, i.e., with $\mathbf{P} = 0$. In the beginning the magnetic field was turned off at different times, resulting in either the system returning to its initial state (curve 1), or to the longitudinal configuration [curve 4 (a); curve 5 (b)], or to one of the vortex configurations with a displaced center of the vortex and $\mathbf{P} \neq 0$. When $\lambda = 1.0$, there are two equilibrium configurations with a displaced center of the vortex (excluding symmetric configuration), and with $\lambda = 1.5$ there are three configurations. Next, taking the established configuration as the initial one, an external magnetic field was turned on again (in the same or in the opposite direction), which put the system into another vortex configuration, i.e., a configuration with another location of the vortex center. In the end, the field was turned on which transitioned the system to the initial vortex configuration with $\mathbf{P} = 0$. It should be noted that the latter process is not possible if a longitudinal (or diagonal) equilibrium configuration is established in the system because a planar homogeneous magnetic field cannot transition the system into any of the vortex configurations. The examples in curves 4(a) and 5(b) show how the influence of the magnetic field with orientation angles $\psi = 0$ or $\psi = \pi$ changes the value of $\mathbf{P}$ of longitudinal configuration, but after turning off the field, the configuration is restored. Figure 16 shows the equilibrium vortex configurations 24 which correspond to the

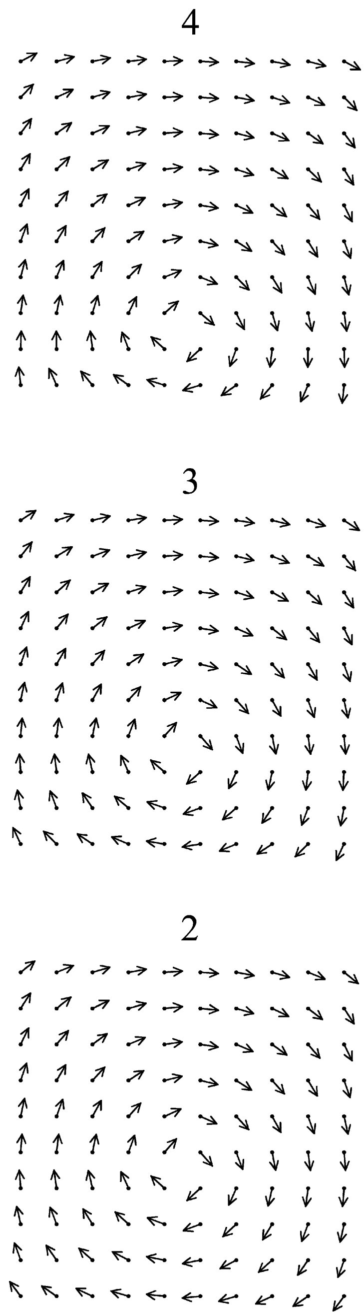

FIG. 16. Equilibrium configurations corresponding to the respective stationary values of the magnetic moment **P** (2–4) in Fig. 15(b).

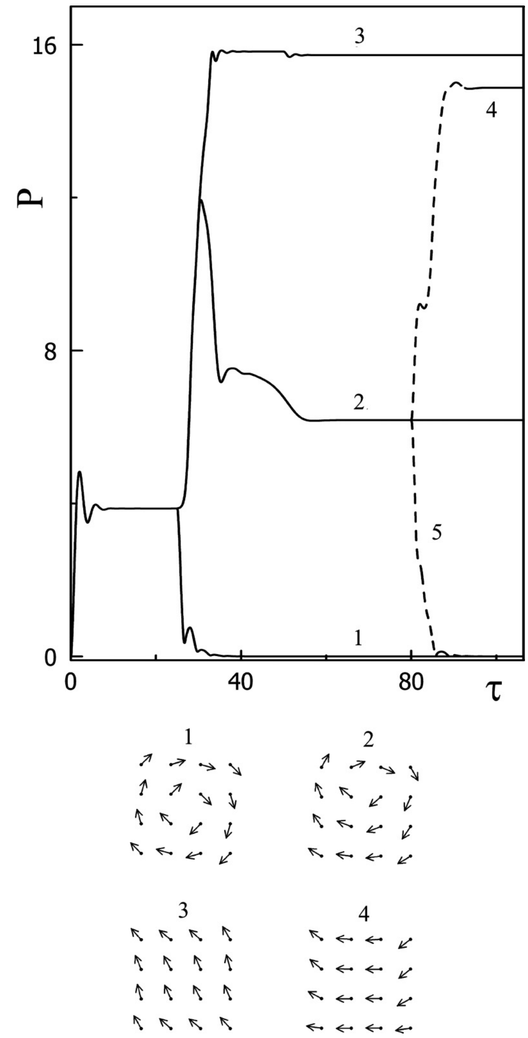

FIG. 17. Transitions between the equilibrium configurations of the system 4×4 under the influence of the field with $f = 1.2$ and the orientation angle $\psi = 3\pi/4$ (solid curves), $\psi = \pi$ (line 4), and $\psi = 0$ (curve 5). Curves 1–3 correspond to different exposure time field on the system.

respective stationary values of the magnetic moment of system of **P** (2–4) in Fig. 15(b).

C. Minimal system with a controlled vortex configuration

The array 4×4 is a minimal system representing a vortex configuration with a possibility to control the magnetic moment and the implementation of its zero value. Figure 17 shows the time dependence of the magnetic moment of the system under the influence of the external field of value $f = 1.2$. In the case of solid curves, the initial configuration is a vortex one 1 ($\mathbf{P} = 0$), with the field oriented at an angle of $\psi = 3\pi/4$. In the case of dashed curves the initial configuration is a vortex configuration 2 ($\mathbf{P} \neq 0$), angle $\psi = \pi$ (curve 4), and $\psi = 0$ (curve 5). Curves 1–3 correspond to the various lengths of the dipoles' systems exposure to the field. The figure shows that if the length of exposure to the field is insufficient (or the field is weak), the system returns to its initial state (curve 1). Otherwise, the system goes into a new vortex state with nonzero magnetic moment (transition 2), from which it can return to its previous state with $\mathbf{P} = 0$ when changing the direction of the external field (transition 5). When exposure to an external field is sufficiently long, the system goes from any vortex configuration into a longitudinal (transition 4) or diagonal (transition 3) configuration, from which it can no longer be returned to the vortex configuration by using a planar field influencing the whole array. As seen in the previous figure in the array 4×4, with the exclusion of the central vortex configuration, only vortex states with one value of the total dipole moment ($\mathbf{P} \approx 6$) are established.

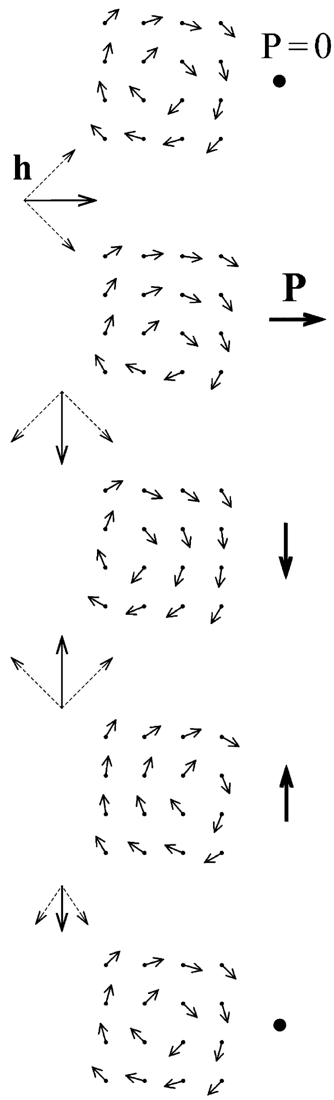

FIG. 18. Switching of the magnetic moment of the system 4 × 4 under the influence of a magnetic field differently oriented.

There are four possible destinations of vector **P** along each of the edges of the array depending on the orientation of the vortex structure. This allows one to obtain a switch of the magnetic moment with the possibility of reversible transition to the state where $\mathbf{P} = 0$. Figure 18 shows switching of the magnetic moment of the system with the help of a variously oriented external field. As shown in the figure, to switch the magnetic moment **P** in the direction of $\varphi = 0, \pm\pi/2, \pi$ the orientation angle of the external field must be in the interval of $\varphi - \pi/4 < \psi < \varphi + \pi/4$. In the case when the system goes into a state with $\mathbf{P} = 0$, the orientation angle of the field must be $-\varphi - \pi/6 \leqslant \psi \leqslant -\varphi + \pi/6$ (i.e., the direction of the field should be close to the direction opposite to the vector **P**). At this the field value should be approximately half the value of the field required to establish a new direction of a nonzero magnetic moment of the system. So, for the establishment of the state with $\mathbf{P} \neq 0$ or for rotation of a magnetic moment, the field with $f \geqslant 0.2$ is sufficient. To return the system to a central vortex state ($\mathbf{P} = 0$) the value $f \geqslant 0.1$ of the field is required, given that the exposure time of the field is sufficient for both the first and the second transitions.

VI. CONCLUSION

(1) Investigation of square arrays of the dipoles showed that in the case of a weak exchange interaction between objects within the system, the establishment of equilibrium configurations with a saddle orientation of the dipole moments takes place, as well as the establishment of the configurations in which the central areas are composed of the dipole moments forming pairs of rows going in opposite directions. These equilibrium states are similar to equilibrium states in dipole systems without exchange interaction.

In the case of a large exchange interaction the only equilibrium configuration is the one in which the dipole moments are aligned predominantly along the diagonal of the array. At an intermediate value of the exchange interaction, the systems can establish both diagonal equilibrium configurations and the configurations with orientation of the magnetic moments mainly along the side of the array, as well as the configurations with a vortex orientation. Vortex equilibrium states may differ by location in the array structure of the vortex center and, consequently, by the magnitude of the total magnetic moment of the system.

When the vortex center is located in the center of an array with an even number of dipoles, the total magnetic moment of the system is equal to zero; at the displacement of the vortex to the edge of an array its dipole moment increases. The greatest magnetic moment for these equilibrium states is possible in the case of a diagonal configuration.

(2) Consideration of the binding energy showed that the most stable configuration is the one with a diagonal orientation of the dipoles, and the least stable is the vortex configuration. Using a planar magnetic field acting on the whole system, transitions from a vortex configuration to a diagonal one or to the configuration with orientation of the dipole moments along the side of the system can be implemented, as well as from the latter one to the diagonal configuration. In the case of a diagonal configuration, the homogeneous planar field can only cause remagnetization of the system changing the orientation of the magnetic moment of the array while preserving the diagonal structure. The transition from a diagonal configuration to a vortex configuration or to a configuration with the orientation along the side of the array of dipoles (and from the latter to the vortex configuration) can be realized with a magnetic field directed perpendicular to the plane of the system and acting only on the part of the dipoles in the array. At the same time the planar field created by the rest of the dipoles of the array is changed in a certain way. As a result, the system proceeds to another equilibrium configuration, after switching off the external field. There is also a possibility of transitions within the equilibrium configurations, changing both the magnitude and direction of the total magnetic moment of the system, under the influence of planar fields, which was shown on one dipole of array.

In the case of a vortex configuration under the influence of a magnetic plane field on the system, the motion of the vortex structures center is carried out in the direction perpendicular to the direction of the external field. When the magnetic field is turned off, the vortex center approaches one of the equilibrium positions, and the relevant value of the magnetic moment of the system is established.

(3) The number of equilibrium states of a vortex configuration, and the resulting number of possible values of the total magnetic moment is determined by the parameters of the system and, in particular, by the parameters of exchange interaction. With a weak exchange interaction (for an array 10×10 of $\lambda \leqslant 0.5$) only the central vortex configuration (**P**) is stable, and the displacement of the vortex center by an external field and the consequent turning off of the field make the vortex return to its initial central position. With a strong exchange interaction ($\lambda > 2.0$) only diagonal configuration is stable, when the dipoles are oriented along the diagonal lattice. In other cases, as a rule, several vortex equilibrium states with $\mathbf{P} \neq 0$ can take place (excluding symmetrical configurations with the opposite direction of the magnetic moment of the system).

Using an external field, a reversible transition between different vortex configurations takes place, including the central-oriented configuration with $\mathbf{P} = 0$. Therefore, the control of both magnitude and direction of the total magnetic moment of the dipoles' system are carried out.

(4) A minimal system, where the vortex states control the total magnetic moment with the help of a plane magnetic field influencing the entire array, is a 4×4 system. In this array, apart from a center-oriented vortex structure with zero magnetic moment, only four symmetric to each other equilibrium configurations can occur and display the same magnitude but a different direction of the total magnetic moment. As a result, a transition of the systems' magnetic moment between its four positions takes place by using a planar field the direction of which is included in one of the quadrants of the plane of the array. Transition to the state of a zero magnetic moment of dipoles is also possible.

These results are general and may apply to different systems of objects with a dipole magnetic moment.

ACKNOWLEDGMENTS

This work was partially supported by the Ministry of Education and Science of the Russian Federation (Contract No. 3.175.214K) and the Russian Foundation for Basic Research (Grant No. 13-02-97026 p_povolzhe_a).

[1] R. Skomski, J. Phys.: Condens. Matter **15**, R841 (2003).
[2] A. Y. Galkin and B. Ivanov, JETP Lett. **83**, 383 (2006).
[3] S. Gusev, Y. N. Nozdrin, M. Sapozhnikov, and A. A. Fraerman, Phys.-Usp. **43**, 288 (2000).
[4] I. M. L. Billas, J. A. Becker, A. Châtelain, and W. A. de Heer, Phys. Rev. Lett. **71**, 4067 (1993).
[5] S. Gubin and Y. A. Koksharov, Inorg. Mater. **38**, 1085 (2002).
[6] I. Karetnikova, I. Nefedov, M. Sapozhnikov, A. Fraerman, and I. Shereshevskii, Phys. Solid State **43**, 2115 (2001).
[7] E. Tartakovskaya, J. Tucker, and B. Ivanov, J. Appl. Phys. **89**, 8348 (2001).
[8] J. K. Ha, R. Hertel, and J. Kirschner, Phys. Rev. B **67**, 224432 (2003).
[9] H. Zhang, Y. Liu, M. Yan, and R. Hertel, IEEE Trans. Magn. **46**, 1675 (2010).
[10] O. V. Sukhostavets, J. M. Gonzalez, and K. Y. Guslienko, Appl. Phys. Express **4**, 065003 (2011).
[11] K. Y. Guslienko, O. V. Sukhostavets, and D. V. Berkov, Nanoscale Res. Lett. **9**, 386 (2014).
[12] V. Novosad, K. Y. Guslienko, H. Shima, Y. Otani, S. G. Kim, K. Fukamichi, N. Kikuchi, O. Kitakami, and Y. Shimada, Phys. Rev. B **65**, 060402 (2002).
[13] C. E. Zaspel, B. A. Ivanov, J. P. Park, and P. A. Crowell, Phys. Rev. B **72**, 024427 (2005).
[14] M. Buess, T. P. J. Knowles, R. Höllinger, T. Haug, U. Krey, D. Weiss, D. Pescia, M. R. Scheinfein, and C. H. Back, Phys. Rev. B **71**, 104415 (2005).
[15] R. Zivieri and F. Nizzoli, Phys. Rev. B **78**, 064418 (2008).
[16] V. Kireev and B. Ivanov, JETP Lett. **94**, 306 (2011).
[17] S. S. Sosin, L. A. Prozorova, P. Bonville, and M. E. Zhitomirsky, Phys. Rev. B **79**, 014419 (2009).
[18] W. Söllinger, W. Heiss, R. T. Lechner, K. Rumpf, P. Granitzer, H. Krenn, and G. Springholz, Phys. Rev. B **81**, 155213 (2010).
[19] A. M. Shutyi, Phys. Met. Metallogr. **115**, 1179 (2014).
[20] O. Kasyutich, R. D. Desautels, B. W. Southern, and J. van Lierop, Phys. Rev. Lett. **104**, 127205 (2010).
[21] V. N. Krivoruchko, M. A. Marchenko, and Y. Melikhov, Phys. Rev. B **82**, 064419 (2010).
[22] S. P. Gubin, Y. A. Koksharov, G. Khomutov, and G. Y. Yurkov, Russian Chem. Rev. **74**, 489 (2005).
[23] F. Lisovskii and O. Polyakov, JETP Lett. **73**, 483 (2001).
[24] A. Shutyi, JETP **108**, 880 (2009).
[25] A. Shutyĭ, JETP **110**, 243 (2010).
[26] A. M. Shutyi, JETP Lett. **97**, 520 (2013).