Phase slips in a current-biased narrow superconducting strip

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The theory of current transport in a narrow superconducting strip is revisited taking the effect of thermal fluctuations into account. The value of voltage drop across the sample is found as a function of temperature (close to the transition temperature, $T - T_c \ll T_c$) and bias current $J < J_c$ (J_c is the critical current calculated in the framework of the BCS approximation, neglecting thermal fluctuations). It is shown that careful analysis of vortices crossing the strip results in considerable increase of the activation energy.

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I. INTRODUCTION

The fundamental property of currents flowing dissipationless through superconducting components is the underlying principle for the operation of numerous nanoelectronic devices. One component of particular interest is a narrow superconducting strip (NSS), in which thermal and quantum fluctuations can result in a resistive state of the system. Understanding the role of such fluctuations is a problem of great importance. Various models have been proposed to explain the appearance of nonzero resistances in NSSs and its temperature dependence in the region of low temperatures (for a review see Refs. [1,2]).

The role of thermal fluctuations responsible for energy dissipation, when currents flow through a one-dimensional superconductor, was considered for the first time in the paper by Langer and Ambegaokar [3] almost 50 years ago. The publication of this paper has strongly influenced all subsequent studies in this field, becoming part of multiple monographs and handbooks on superconductivity [4–6].

It is necessary to mention that a "one-dimensional superconductor" is *de facto* often a narrow strip with finite width L, much less than the Ginzburg-Landau coherence length $\xi_{\rm GL}(T) = \sqrt{\pi \mathcal{D}/16T\tau}$ ($\tau = 1 - T/T_c$ is the reduced temperature and $\mathcal{D}$ is the electron diffusion coefficient [7]). The energy dissipation in this system is related to phase-slip processes, i.e., the process of vortices or flux quanta crossing the strip. It is clear that such events cannot be realized in the framework of a purely one-dimensional model. Indeed, the solution found in Ref. [3] shows that even when the current density flowing through the one-dimensional superconductor reaches its critical value J_c the minimal value of the order parameter is $(2/3)^{1/2}\Delta_{\rm BCS}$, while in order to perform a phase-slip event it should become zero at least in one point.

In this work we will resolve the mentioned paradox, describing the true mechanism of phase-slip events in NSS and determining the corresponding value of the activation energy. We will demonstrate that the saddle-point solution of the Ginzburg-Landau (GL) equation for the order parameter $\widetilde{\Delta}$ in the presence of a fixed current J, possessing at least one vortex, exists only for weak enough currents $J < J_{c1} = \eta\,(L/\xi_{\rm GL})\,J_c$ (J_c is the critical current of the strip, and $\eta = 0.0312$ is a small number which will be found below). Under the expression "saddle-point solution" $\widetilde{\Delta}(x,y,J,r_1,\ldots,r_i)$ we understand the solution of the GL equation, which depends not only on the coordinates x,y, and current J but also on some set of parameters $\{\mathbf{r}_i\}$ satisfying the extremal conditions for the GL functional $\mathcal{F}_s$:

$$\frac{\partial}{\partial \mathbf{r}_i}\mathcal{F}_s[\widetilde{\Delta}(x,y,J,\mathbf{r}_1,\ldots,\mathbf{r}_i),J] = 0. \tag{1}$$

In the case under consideration, when one or several vortices penetrate the system through its edge, those parameters can be chosen as the vortex center coordinates (zeros of the order-parameter function).

When the current J exceeds the value J_{c1} the saddle-point solutions Eq. (1) leading to phase-slip events cease to exist and the scenario described above does not hold anymore. In that case another mechanism comes into play. In order to explain this, let us recall that the minimum of the GL free energy is reached for the ground state, corresponding to a solution with spatially independent modulus $|\Delta_{\rm GS}(x,y)| = \Delta_0$. When $J < J_{c1}$ the saddle-point solutions of the GL equations, including vortices, exist with energies higher than the one of the ground state. The transition from the ground-state to the saddle-point solution can be imagined as the motion of the order-parameter "vector" in Hilbert space, accompanied by the motion of the "points" $\{\mathbf{r}_i\}$ in the finite-dimensional space of those parameters.

As we already stated, in the case of "strong currents" $J_{c1} < J < J_c$ the saddle-point solutions of the GL equations, possessing vortices, do not exist anymore. In this interval the minimal activation energy is reached at some function $\Delta_{\rm v}(x,y,J,\mathbf{r}_1)$ corresponding to the state with a single vortex. We choose such a gauge (i.e., the form of vector potential A) where the phase of the order parameter is determined by the vortex position only and the boundary conditions at the strip edges. The modulus of $\Delta_{\rm v}(x,y,J,\mathbf{r}_1)$ is an even function of the longitudinal coordinate y in that case.

In order to determine the order parameter in the state with vortices and subsequently to calculate the corresponding value

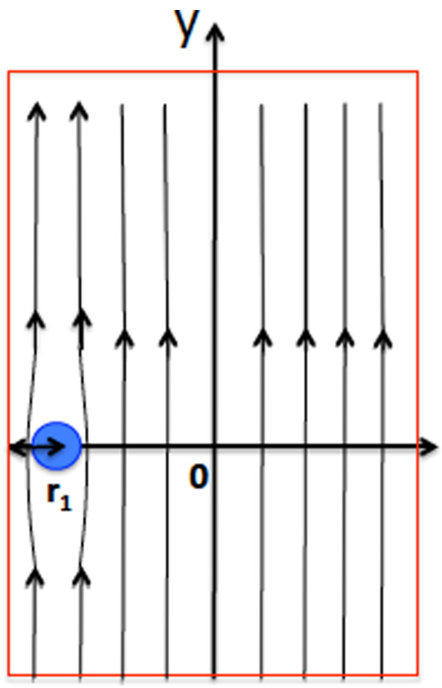

FIG. 1. (Color online) Distribution of the current flowing in the strip in the presence of a vortex located in distance r_1 from the edge of the strip.

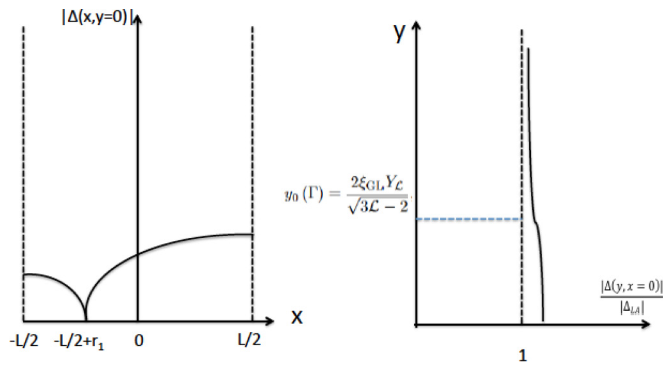

FIG. 2. (Color online) Distribution of the modulus of the order parameter in the strip with flowing current disturbed in the presence of a vortex: (a) transversal distribution at $y = 0$ and (b) longitudinal distribution at $x = 0$.

of free energy, we will use the variational principle with respect to several free parameters in the following. One of them is the distance r_1 from the edge of the strip to the center of a vortex (i.e., the coordinates of the vortex center are $x_1 = -L/2 + r_1$ and $y_1 = 0$, see Fig. 1). We will look for the maximum value r_1^{ext} for which the conditional extremum of the free-energy functional (i.e., the extremum at a given value of the parameter r_1) still exists. If the vortex penetrates further into the system, i.e., for $r_1 > r_1^{\text{ext}}$, such an extremum ceases to exist. Let us recall that the order parameter of the current-biased one-dimensional superconducting channel, which corresponds to the saddle-point solution of the GL equation, does not have zeros at all [3]. When we consider a strip with finite width instead of such a channel a lateral penetration of a vortex is possible. This allows us to suppress the modulus of the order parameter to zero at some point, allowing a phase-slip event at this location. It is clear that such a deformation of the order parameter on a small scale requires some excess energy. The system partially compensates this energy loss by means of a deformation of the order-parameter distribution in relatively large distances from the vortex along the strip with respect to the corresponding one-dimensional solution [3]. This deformation is accounted for by means of the variational parameter y_0. We derive the equations which allow us to determine the value of the parameter y_0 maximizing r_1 (i.e., r_1^{ext}) below. It is essential that the value of such penetration depth r_1^{ext} itself does not appear explicitly in the expression for the free energy of the system with the intruded vortex, which is again accounted for by means of y_0. It becomes of the order of L for weak enough currents and of the order of the effective coherence length $\xi_{\text{GL}}(J)$ [see Eq. (25)] for the strong currents (see Fig.2).

II. GENERALITIES

In order to calculate the value of the activation energy δF for the current-biased NSS we start with the free-energy functional $\mathcal{F}_s$ including both GL and the current-field interaction terms (see Ref. [7]):

$$\mathcal{F}_s = \nu \int d^3\mathbf{r}\Big[-\tau|\Delta(\mathbf{r})|^2 + \frac{\pi\mathcal{D}}{8T}|\partial_-\Delta(\mathbf{r})|^2 + \frac{7\zeta(3)}{16\pi^2T^2}|\Delta(\mathbf{r})|^4\Big] + \frac{1}{c}\int d^3\mathbf{r}\Big(\mathbf{A}(\mathbf{r}) - \frac{c}{2e}\nabla\varphi\Big)\cdot\mathbf{j}_\infty. \tag{2}$$

Here $\Delta(\mathbf{r})$ is the order parameter, $\mathbf{A}(\mathbf{r})$ is the vector potential, $\nu = mp_F/(2\pi^2)$ is the density of states (p_F is the electron Fermi momentum), $\partial_- = \partial/\partial\mathbf{r} - 2ie\mathbf{A}/c$, $\zeta(x)$ is the Riemann zeta function, $j_\infty = J/S$, S is the cross section of the strip, c is the speed of light, and φ is the phase of the order parameter. We use the system of units where $k_B = 1$ and $\hbar = 1$. This functional allows us to write down the equations both for order-parameter and vector-potential coordinate dependencies.

A. Close to zero current value

Let us start with the simplest case of zero current, $j_\infty = 0$. In this case an infinite number of saddle-point solutions exist. If the saddle-point solution has only one zero corresponding to a single vortex state, symmetry considerations show that the center of this vortex is located at the central line of the strip (see Fig. 3). Choosing the latter as the center of coordinates one can find that the phase and modulus of the order parameter are determined as

$$\left[\frac{\partial\varphi(x,y)}{\partial\mathbf{r}}\right]^2 = \frac{\pi^2}{L^2}\frac{1}{\sin^2(\pi x/L) + \sinh^2(\pi y/L)}, \tag{3}$$

$$|\Delta(x,y,y_0)| = \frac{\pi\Delta_0}{L\cosh\frac{\pi y}{L}}\frac{1}{\sqrt{(\partial\varphi/\partial\mathbf{r})^2}}\tanh\frac{\sqrt{y^2+y_0^2}}{2\xi_{\text{GL}}}, \tag{4}$$

where L is the width of the strip and y_0 is the free parameter discussed at the end of the introduction (see also Fig. 2). The phase of the order parameter φ is the solution of the two-dimensional Laplace equation $\Delta\varphi = 0$ with boundary

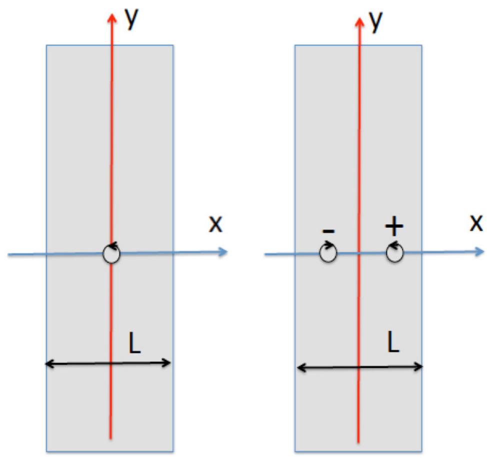

FIG. 3. (Color online) Positions of the zeros of the saddle-point solutions with one (a), two (b), etc., vortices. Here by means of $\{+,-\}$ are denoted the vorticities (phase factor changes by $\pm 2\pi$ for anticlockwise or clockwise circulation of the order-parameter zeros).

conditions $\partial\varphi/\partial x|_{x=\pm L/2} = 0$. The expression (4), obtained by means of the variational procedure, coincides with the corresponding solution of the GL equation in the limit $|y| \gg L$.

Substitution of Eqs. (3) and (4) to Eq. (2) gives the value of the activation energy versus y_0:

$$\delta F^{(1)}(y_0) = 4\nu\Delta_0^2\tau S\xi_{\mathrm{GL}}\left\{\frac{2}{3} - \frac{\pi y_0}{8\xi_{\mathrm{GL}}} + \frac{L}{4\pi\xi_{\mathrm{GL}}}\left[\frac{8\pi^2(y_0/L)^2 + 4\zeta(2) - 1}{6} + \frac{\pi^2 y_0^2}{2L^2} I_0\left(\frac{\pi y_0}{L}\right)\right]\right\}, \tag{5}$$

with

$$I_0(\alpha) = \int_0^\infty \frac{dx}{\cosh^2 x}\frac{1}{x^2+\alpha^2}.$$

The details of the derivation of Eq. (5) are given in Appendix A.

Minimization of Eq. (5) over y_0 gives the value $y_0 = 0.47L/\pi$; the corresponding value for the one-vortex configuration activation energy is

$$\delta F^{(1)} = 4\nu\Delta_0^2\tau S\xi_{\mathrm{GL}}\left(\frac{2}{3} + 0.058\frac{L}{\xi_{\mathrm{GL}}}\right). \tag{6}$$

An analogous consideration of the two vortices' configuration gives for $\delta F^{(2)}$ an answer similar to Eq. (6) with the second term in brackets being twice as small (see Appendix A). Further increase of the number of zeros in the order parameter results in the decrease of the second term in $\delta F^{(n)}$ by the factor n with respect to $\delta F^{(1)}$. In the limit $n \to \infty$ the latter reaches the value

$$\delta F^{(\infty)} = 8\nu\Delta_0^2\tau S\xi_{\mathrm{GL}}/3 \tag{7}$$

first obtained in Ref. [3] in the frameworks of the one-dimensional model.

The flow of any finite current through the strip results in the finiteness of the number of the saddle-point solutions. This number rapidly decreases with the current growth, and already at so small a current as $J_{c1} = \eta(L/\xi_{\mathrm{GL}})J_c$ only the saddle-point solution with one vortex remains. At higher currents the saddle-point solutions do not exist anymore; the critical points appear instead of them.

One can see that at zero current the solution of the GL equation found in Ref. [3] actually is the limiting one for the multiple-vortex solutions obtained above. As a result we can state that the point $J = 0$ is a singular point in the current dependence of the activation energy. Therefore the dependence of $\delta F^{(\infty)}(J)$, obtained in Ref. [3] for small currents as the linear one, in fact turns out to be substantially more complicated. It is worth mentioning that the multiplicity of saddle-point solutions in the domain of weak currents results in an increase of the possibilities of the phase-slip events, i.e., a noticeable increase of the pre-exponential factor.

III. "WEAK" CURRENTS

Now we consider the most simple one-vortex state in the region of weak currents $J \leqslant J_{c1}$. The vortex, corresponding to the saddle-point solution, now slightly shifts with respect to the central axis of the strip. Denoting the distance between the axis ($x = 0$) and vortex center as $\delta(\delta = L/2 - r_1)$, we look for the solution of the Ginzburg-Landau equation in the form

$$|\Delta| = \Delta_0(\Gamma)Z^{1/2}\left(\sqrt{y^2+y_0^2}\right)\Phi(x,y,\delta), \tag{8}$$

with the functions

$$\Phi^2(x,y,\delta) = \frac{\sinh^2(\pi y/L) + \sin^2(\pi x/L) + \sin^2(\pi\delta/L) - 2\cosh(\pi y/L)\sin(\pi x/L)\sin(\pi\delta/L)}{\cosh^2(\pi y/L)} \tag{9}$$

and

$$Z(y,\Gamma) = 1 - \frac{3\mathcal{L}-2}{\mathcal{L}}\left[\cosh\left(\frac{y\sqrt{3\mathcal{L}-2}}{2\xi_{\mathrm{GL}}}\right)\right]^{-2}. \tag{10}$$

Here $\Gamma = J/J_c$, while $\Delta_0(\Gamma)$ is the order parameter of the homogeneous ground state of the NSS carrying on the current J, i.e., the asymptotic form of our $\Delta(x,y,\Gamma,y_0,\delta)$ far from the vortex, at $y \to \pm\infty$. The latter can be related to the BCS value $\Delta_{00}(\tau) = \{8\pi^2T^2\tau/[7\zeta(3)]\}^{1/2}$ of the order parameter in the absence of current by means of the relation

$$\Delta_0^2(\Gamma) = \Delta_{00}^2\mathcal{L}(\Gamma). \tag{11}$$

The choice of the two former multipliers in the ansatz Eq. (8) is based on the Langer-Ambegaokar solution of the GL equation for a current-biased one-dimensional channel distorted by the vortex presence (and accounting for its evenness in y). The latter multiplier [see Eq. (9)] accounts for the appearance in the case under consideration of asymmetry of the order-parameter dependence on the transversal coordinate, and in the case of $\delta = 0$ it leads to the coincidence of Eq. (8) and Eqs. (4) and (3). Substitution of the ansatz Eqs. (8)–(10) to the GL equations gives the explicit value for $\mathcal{L}(\Gamma)$:

$$\mathcal{L}(\Gamma) = \frac{1}{3} + \frac{2}{3}\sin\left[\frac{\pi}{6} + \frac{2}{3}\arcsin\sqrt{1-\Gamma^2}\right]. \tag{12}$$

The quantity $y_0 \sim L$, and hence for $|y| \gg L$ the modulus of the order parameter $|\Delta|$ [see Eq. (8)] should obey the exact GL equations with the corresponding boundary conditions at $x = \pm L/2$. The function Φ is related to the order-parameter phase φ which satisfies the equation

$$\left(\frac{\partial\varphi(x,y)}{\partial \mathbf{r}}\right)^2 = \left(\frac{\pi}{L}\right)^2 \frac{\cos^2\frac{\pi\delta}{L}}{\cosh^2\left(\frac{\pi y}{L}\right)\Phi^2(x,y,\delta)}. \tag{13}$$

Substitution of Eqs. (8), (9), and (13) into Eq. (2) leads to the expression for free energy:

$$\delta F_\delta(y_0) = 4\nu\Delta_0^2\tau S\xi_{\mathrm{GL}}\left\{\frac{2}{3} - \frac{\pi y_0}{8\xi_{\mathrm{GL}}} - \frac{16\pi\xi_{\mathrm{GL}}\Gamma^2}{27L}\left(\frac{\pi}{4a} + \frac{L}{8y_0} + I_1(a)\right) + \frac{L}{4\pi\xi_{\mathrm{GL}}}\left[\frac{4a^2}{3} + \frac{2}{3}\zeta(2) - \frac{1}{6} + \frac{a^2}{2}I_0(a)\right]\right.$$
$$\left. + \frac{\pi\Gamma}{3}\sqrt{\frac{2}{3}}\sin(\pi\delta/L) - \frac{L}{4\pi\xi_{\mathrm{GL}}} biggleft[\frac{2a^2}{3} + \frac{1}{3}\zeta(2) + a^2 I_0(a)\right]\sin^2(\pi\delta/L)\right\}, \tag{14}$$

where

$$a^2 = \frac{\pi^2 y_0^2}{L^2} + \frac{32\pi^2\Gamma^2}{27L^2}\xi_{\mathrm{GL}}^2$$

and

$$I_1(a) = \int_0^\infty \frac{dx}{\cosh x}\frac{1}{x^2+4a^2}.$$

The details of transition from Eq. (2) to Eq. (14) are presented in Appendix A.

Let us recall that the quantities $\{y_0,\delta\}$ still remain indefinite: their values one can determine from the conditions of the GL functional $\delta F_\delta(y_0)$ extremum:

$$\frac{\partial \delta F_\delta(y_0)}{\partial y_0} = 0, \tag{15}$$

$$\frac{\partial \delta F_\delta(y_0)}{\partial \delta} = 0. \tag{16}$$

In result of the solution of Eq. (16) the value δ can be presented as the function of Γ:

$$\sin\frac{\pi\delta}{L} = \left(\frac{2}{3}\right)^{3/2}\frac{\pi^2\xi_{\mathrm{GL}}\Gamma}{L[2a^2/3 + \zeta(2)/3 + a^2 I_0(a)]}. \tag{17}$$

What concerns the value y_0 is determined by Eqs. (15) and (17). The corresponding equation is very cumbersome and we do not present it here. Importantly, it has the solution only in the very narrow currents interval:

$$\Gamma^2 \leqslant \Gamma_{c1}^2 = 0.009\frac{L^2}{\xi_{\mathrm{GL}}^2}.$$

Finally, the value of the free energy δF_δ in the critical point $J = J_{c1}$ is

$$\delta F_\delta(\tau, J_{c1}) = 4\nu\Delta_0^2(\Gamma)\tau S\xi_{\mathrm{GL}}\left(\frac{2}{3} + 0.054\frac{L}{\xi_{\mathrm{GL}}}\right). \tag{18}$$

Comparing Eqs. (18) and (6) one can see that the state with $J = J_{c1}$, when only the saddle-point solution with one vortex remains, energetically differs from that one with $J = 0$ by a very small quantity $0.004(\frac{L}{\xi_{\mathrm{GL}}})4\nu\Delta_0^2(\Gamma)\tau S\xi_{\mathrm{GL}}$.

IV. "STRONG" CURRENTS

Let us pass to discussion of the mechanism of energy dissipation in the wide range of currents $J_{c1} \ll J < J_c$, when the GL equations no longer have any saddle-point solution. Let us suppose that through the edge of the strip penetrates a single vortex and assume that its center is located at some small distance $r_1 (r_1 \ll L)$ from the edge; i.e., the vortex center coordinates are $(-L/2 + r_1, 0)$. Our goal is to obtain the maximal possible value of the "penetration length" r_1 at which the requirement of existence of the conditional extremum of the functional (2) is still satisfied. In order to do this we look for the phase and the modulus of the order parameter in the form containing three free parameters $r_1, y_0,$ and γ:

$$\left(\frac{\partial\varphi(x,y,r_1)}{\partial \mathbf{r}}\right)^2 = \left(\frac{\pi}{L}\right)^2\frac{\sin^2\frac{\pi r_1}{L}}{\cosh^4\left(\frac{\pi y}{2L}\right)}[Q(x,y)]^{-1}, \tag{19}$$

and

$$|\Delta(x,y,r_1,y_0,\gamma)| = \Delta_0(\Gamma)\frac{\ln\left[1 + \frac{\gamma L^2}{r_1^2}Q^{1/2}(x,y)\right]}{\ln\left(\frac{2\gamma L^2}{r_1^2} + 1\right)}Z^{\frac{1}{2}}(\Gamma, y + y_0). \tag{20}$$

The function

$$Q(x,y,r_1) = \left\{4\sinh^2\left(\frac{\pi y}{2L}\right)\left[\cosh^2\frac{\pi y}{2L} + \cos\frac{\pi r_1}{L}\sin\frac{\pi x}{L}\right] + \left[\sin\frac{\pi x}{L} + \cos\frac{\pi r_1}{L}\right]^2\right\}\left[\cosh\left(\frac{\pi y}{2L}\right)\right]^{-4} \tag{21}$$

is the result of direct calculation of the phase φ in the one-vortex state of the strip [see Eq. (19)]. The function $|\Delta(x,y)|$ approaches the solution of the GL equation in the range $|y| \gg L$. Both Eqs. (19) and (20) satisfy the boundary conditions for the order parameter and its derivatives at the edge of the strip and at infinity. What concerns the variational parameter γ can be found from the condition

$$\frac{\partial \delta F(r_1, y_0, \gamma)}{\partial \gamma} = 0. \tag{22}$$

This determines the shape of the order parameter and, correspondingly, the contribution to the free energy from the domain close to the vortex $|y| \lesssim L$. Its introduction allows us to improve the variational approximation in this region. The corresponding expression turns out to be of the order of 1 and does not appear explicitly in the final expression for the free energy, and this is why we do not present it here.

The current conservation law leads to the next expression for the essential part of the vector potential $\mathbf{A}$:

$$A_y^{(0)}(y) = \frac{A_\infty \Delta_0^2(\Gamma)}{\langle |\Delta(x,y)|^2 \rangle_x}, \tag{23}$$

where $\langle \cdots \rangle_x$ means the averaging over the transverse coordinate. The value of the vector potential at infinity A_∞ is determined by the current density:

$$\frac{J}{S} = -\frac{\pi \nu e^2 \mathcal{D}}{T} \Delta_0^2(\Gamma) A_\infty. \tag{24}$$

Substituting the expressions (19), (20), and (23) into the GL functional (2) and calculating the integral one can find the value of excess free energy δF_s of the strip with the penetrated vortex with respect to its ground state with the fixed current. The requirement of existence of the conditional extremum determines the value y_0:

$$y_0(\Gamma) = \frac{2\xi_{\mathrm{GL}} Y_{\mathcal{L}}}{\sqrt{3\mathcal{L} - 2}},$$

$$\tanh Y_{\mathcal{L}}(\Gamma) = 2\left[\frac{(1-\mathcal{L})}{4 - 3\mathcal{L} + \sqrt{\mathcal{L}(16 - 15\mathcal{L})}}\right]^{1/2}. \tag{25}$$

The main steps of this calculus are presented in Appendix B. The obtained results allow us to write down the expression for the activation energy in the whole region of "strong currents" $J_{c1} \ll J < J_c$:

$$\delta F(\tau, J) = 4\nu \Delta_0^2(\Gamma)\tau S \xi_{\mathrm{GL}} \sqrt{(3\mathcal{L} - 2)} \left\{ \frac{1 - \tanh Y_{\mathcal{L}}}{6\mathcal{L}} [4 + (3\mathcal{L} - 2)\tanh Y_{\mathcal{L}}(1 + \tanh Y_{\mathcal{L}})] - \sqrt{\frac{2(1-\mathcal{L})}{3\mathcal{L} - 2}} \left[\arctan \sqrt{\frac{3\mathcal{L} - 2}{2(1-\mathcal{L})}} - \arctan\left(\sqrt{\frac{3\mathcal{L} - 2}{2(1-\mathcal{L})}} \tanh Y_{\mathcal{L}} \right) \right] \right\}. \tag{26}$$

It is seen that the difference between Eq. (26) and the expression for the activation energy of the one-dimensional superconducting channel carrying current J (the main result of Ref. [3]) consists of the contribution occurring due to the nonzero value of the parameter $Y_{\mathcal{L}}$, i.e., due to the existence of the conditional extremum of the free-energy functional at the distance $y_0 \neq 0$. Let us stress that the activation energy δF depends on the geometry of a sample, which here is assumed as a strip. The increase of the energy barrier in the Arrhenius law with respect to the result of Ref. [3] is related to the necessity of the vortex penetration in a sample at the moment of the phase-slip event.

The expression for activation energy $\delta F^{(LA)}(\tau, J)$ of the one-dimensional superconducting channel found in Ref. [3] can be easily reproduced from Eq. (26) simply setting $Y_{\mathcal{L}} = 0$ [what follows from Eq. (25)]. One can compare the result of our careful consideration of the vortex penetration mechanisms [Eq. (26)] with the latter:

$$\frac{\delta F(\tau, J) - \delta F^{(LA)}(\tau, J)}{\delta F^{(LA)}(\tau, J)} = \frac{\left[\frac{3\mathcal{L}-2}{6\mathcal{L}} \frac{\tanh Y_{\mathcal{L}}}{\cosh^2 Y_{\mathcal{L}}} - 2\frac{\tanh Y_{\mathcal{L}}}{3\mathcal{L}} + \sqrt{\frac{2(1-\mathcal{L})}{3\mathcal{L}-2}} \arctan\left(\sqrt{\frac{3\mathcal{L}-2}{2(1-\mathcal{L})}} \tanh Y_{\mathcal{L}}\right)\right]}{\left[\frac{2}{3\mathcal{L}} - \sqrt{\frac{2(1-\mathcal{L})}{3\mathcal{L}-2}} \arctan \sqrt{\frac{3\mathcal{L}-2}{2(1-\mathcal{L})}}\right]}. \tag{27}$$

For the currents larger than J_{c1} but still much smaller than J_c the saddle-point solutions of GL equations considered in Ref. [3] do not exist anymore. Nevertheless one can see that the difference between the free energy of the conditional extremum and that one calculated by Langer and Ambegaokar [see Eq. (27)] turns out to be proportional only to $(J/J_c)^2$; i.e., the result of Ref. [3] remains valid. The situation considerably changes when the current approaches its critical value, $J \to J_c(\Gamma \to 1)$. Here $\tanh Y_{\mathcal{L}} \to 1/\sqrt{3}$; $\cosh^2 Y_{\mathcal{L}} \to 3/2$; $1 + 2^{-1}\cosh^{-2} Y_{\mathcal{L}} - L\tanh^2 Y_{\mathcal{L}}/[2(1-\mathcal{L})] \to 1; 3L - 2 \to 2^{3/2}\sqrt{1-\Gamma}/\sqrt{3}$; and

$$\delta F^{(LA)}(\tau, \Gamma \to 1) = \frac{4}{15}\sqrt{\frac{2}{\sqrt{3}}} \nu\tau \Delta_{00}^2(\tau) S \xi_{\mathrm{GL}} (1 - \Gamma^2)^{5/4}, \tag{28}$$

while

$$\delta F(\tau, \Gamma \to 1) = \frac{2^{3/2}}{3^{9/4}} \nu\tau \Delta_{00}^2(\tau) S \xi_{\mathrm{GL}} (1 - \Gamma^2)^{3/4}. \tag{29}$$

The relative difference of the free energies Eq. (27) in this case diverges:

$$\left.\frac{\delta F(\tau, J) - \delta F^{(LA)}(\tau, J)}{\delta F^{(LA)}(\tau, J)}\right|_{J \to J_c} = \frac{5J_c}{6\sqrt{J_c^2 - J^2}}; \tag{30}$$

i.e., the height of the barrier in Arrhenius law turns out to be parametrically larger than predicted in Ref. [3]. The behavior of Eq. (27) as the function of Γ in the interval [0,0.9] is presented in Fig. 4.

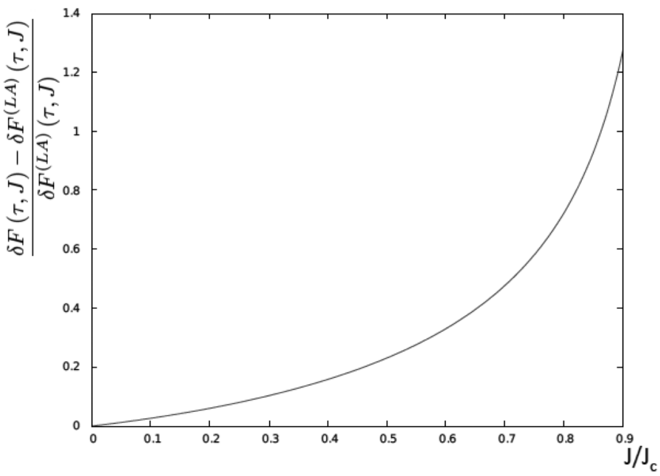

FIG. 4. Excess activation energy related to the account for the true mechanism of the vortices penetration in the strip as the function of flowing current.

V. PRE-EXPONENTIAL FACTOR

In order to obtain the exact value of the pre-exponential factor Ω for phase-slip events one should have in possession the expression for the effective action of superconducting strip containing vortices. In Ref. [8] was proposed a general procedure which, in the regime of thermal fluctuations, is reduced to the solution of the spectral problem for a linear operator corresponding to the action at its saddle point. The difficulty of the problem under consideration consists in the fact that no microscopic action operator is known and no saddle point (for currents $J_{c1} < J$) exists. Nevertheless, the knowledge of action would allow one to get the precise value of Ω at least for weak currents $J < J_{c1}$, while for strong currents one could believe that change of the saddle point to a singular point would not strongly affect the value of the pre-exponential factor. In light of the above the evaluations of Ω in both papers [3,9] seem doubtful: use of the time-dependent GL equation below T_c, as today is well known, cannot be justified, unless in the gapless regime [10].

The main contribution to the average time between two subsequent phase-slip events is related to the existence of the "zero mode" (see Refs. [8,9]). In the case under consideration the size of the vortex is determined by the transversal size L of the strip. The vortices which slip on the distances larger than L can be considered as independent. This is why the main factor determining the pre-exponential one is the ratio of the transversal size L to the strip length $\mathfrak{L}$: $L/\mathfrak{L}$. Another coefficient which forms the pre-exponential factor is the characteristic "crossing time" $\Delta t_{\text{cross}} = L/v_{\text{cross}}$ of the strip by the vortex, moving with the velocity [12]:

$$v_{\text{cross}} = \frac{cJ\sqrt{\tau}}{4SH_{c2}\sigma_n}, \tag{31}$$

where σ_n is the conductivity of the strip in its normal phase. Finally, accounting for the Arrhenius factor, one finds the characteristic time Δt between two phase-slipping events:

$$\Delta t = \frac{L}{\mathfrak{L}}\left(\frac{L}{v_{\text{cross}}}\right)\exp\left(\frac{\delta F}{k_B T}\right). \tag{32}$$

The average voltage V at the strip is related to the average time interval Δt between the voltage jumps by the Josephson relation [11]: $V = \pi\hbar/(e\Delta t)$. The corresponding resistance of the strip is

$$\frac{R}{R_0} = \frac{\pi\hbar c\sqrt{\tau}}{4eH_{c2}L^2}\exp\left[-\frac{\delta F(\tau,\Gamma)}{k_B T}\right], \tag{33}$$

where $R_0 = \mathfrak{L}/(\sigma_n S)$ is the normal resistance of the strip. It is necessary to mention that the above approximation of the independent phase slips is valid only [according to Eq. (32)] when $\delta F \gtrsim k_B T$.

VI. CONCLUSIONS

We have demonstrated that considering only the longitudinal spatial dependence of the order parameter in narrow superconducting strips carrying finite current (see Ref. [3]) is not sufficient to describe the properties of its resistive state correctly. Taking the transversal coordinate into account when calculating the saddle-point solutions of the GL equation turns out to be essential. Namely, the value of the activation energy in the Arrhenius law for the resistance of a narrow superconducting channel starting from a relatively low currents differs from the value obtained by simply using the difference of the free energy of such a saddle and the ground-state energy. The mechanism for phase-slip events turns out to be much more sophisticated than the one described in Ref. [3].

Already at weak currents ($J < J_{c1}$) a sequence of the saddle points appears, which is characterized by the number n of zeros of the order parameter along the transverse coordinate. The energy of such a state equals the one found by Langer and Ambegaokar [3] only in the limit $n \to \infty$, when the system carries no current. One could say that the state of the strip in the current-free case is singular. The number n of saddle points rapidly decreases with the growth of the current. It reaches $n = 1$ when the current has the value $J_{c1} = 0.0312\,(L/\xi)\,J_c$: at this point only a stationary state remains.

When $J > J_{c1}$, stationary solutions of the GL equations with fixed current and a vortex in the strip do not exist. Instead one needs to look for a critical point, corresponding to the existence of a specific conditional extremum of the GL functional. These conditions are that the current J is fixed and that the distance between the vortex center and the strip edge is maximal. The energy of such a state turns out to be larger than the activation energy $\delta F^{(LA)}(\tau,J)$ obtained in Ref. [3]. The normalized difference Eq. (30) increases with growth of the current, and when the latter approaches the critical value J_c the former diverges (see Fig. 4).

Experimentally, the discrepancy between the theoretical prediction of Ref. [3] and the mechanism proposed here can be detected by analyzing the current dependence of the resistance close to J_c. The predicted dependence of $\log R(I)$ with exponent 5/4 in Ref. [3] should transform into a weaker one with exponent 3/4 in the region $J \to J_c$. Some experimental papers indicated an unexpected decrease of the resistance in the regime of strong currents (see Refs. [4,13,14]). This long standing enigma can potentially be resolved by the above analysis. A recent numerical study on narrow superconducting channels using the time-dependent GL equation in the strong current regime also indicated that the critical exponent of the activation energy is of the order of 0.7 for the widths $L \sim \xi_{\rm GL}$ (see Ref. [15]).

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APPENDIX A

In the process of derivation of Eq. (5) we used the following integrals:

$$\frac{1}{L}\int_{-L/2}^{L/2}\frac{dx}{\sin^2\frac{\pi x}{L}+\sinh^2\frac{\pi y}{L}}=\frac{2}{\sinh\left(\frac{2\pi y}{L}\right)}, \tag{A1}$$

$$\frac{1}{L}\int_{-L/2}^{L/2}\frac{\sin^2(\pi x/L)\,dx}{\sin^2\frac{\pi x}{L}+\sinh^2\frac{\pi y}{L}}=\frac{\exp\left(\frac{-\pi y}{L}\right)}{\cosh\frac{\pi y}{L}}, \tag{A2}$$

$$\frac{1}{L}\int_{-L/2}^{L/2}\frac{\sin^2(\pi x/L)\,dx}{\left[\sin^2\frac{\pi x}{L}+\sinh^2\frac{\pi y}{L}\right]^2}=\frac{1}{\sinh\left(\frac{2\pi y}{L}\right)\cosh^2\left(\frac{\pi y}{L}\right)},$$

$$\frac{1}{L}\int_{-L/2}^{L/2}\frac{\sin^4(\pi x/L)\,dx}{\left[\sin^2\frac{\pi x}{L}+\sinh^2\frac{\pi y}{L}\right]^3}=\frac{3}{8}\frac{1}{\sinh\left(\frac{\pi y}{L}\right)\cosh^5\left(\frac{\pi y}{L}\right)},$$

for $y>0$.

Using these relations in Eqs. (3) and (4) one finds

$$\begin{aligned}\left\langle\frac{\partial|\Delta(x,y)|^2}{\partial\mathbf{r}}\right\rangle_x=\Delta_0^2\Bigg\{&\left(\frac{\pi}{2L}\right)^2\tanh^2\frac{\sqrt{y^2+y_0^2}}{2\xi_{\rm GL}}\frac{1-\exp(-4\pi y/L)}{\sinh\frac{2\pi y}{L}\cosh^2\frac{\pi y}{L}}+\frac{y^2}{8\left(y^2+y_0^2\right)}\frac{1+\tanh^2(\pi y/L)}{\xi_{\rm GL}^2\cosh^4\frac{\sqrt{y^2+y_0^2}}{2\xi_{\rm GL}}}\\&+\left(\frac{\pi}{2L}\right)^2\tanh^2\left(\frac{\sqrt{y^2+y_0^2}}{2\xi_{\rm GL}}\right)\frac{\tanh^2(\pi y/L)}{\sinh(2\pi y/L)\cosh^2(\pi y/L)}[3+4\exp(-2\pi y/L)+\exp(-4\pi y/L)]\\&+\frac{\pi}{2\xi_{\rm GL}L}\frac{y}{\sqrt{y^2+y_0^2}}\tanh\left(\frac{\sqrt{y^2+y_0^2}}{2\xi_{\rm GL}}\right)\frac{\tanh(\pi y/L)}{\cosh^2(\pi y/L)\cosh^2\left[\sqrt{y^2+y_0^2}/(2\xi_{\rm GL})\right]}\Bigg\},\end{aligned}$$

for $y>0$. Here we introduced the symbol of averaging over the transverse coordinate:

$$\langle(\cdots)\rangle_x=\frac{1}{L}\int_{-L/2}^{L/2}dx(\cdots).$$

Next we present the explicit integrals of the type of Eqs. (A1) and ((A2)) over y:

$$\int_0^\infty\frac{\left(y^2+y_0^2\right)\tanh^2\frac{\pi y}{L}}{\sinh\frac{2\pi y}{L}\cosh^2\frac{\pi y}{L}}\left[3+4\exp\left(-\frac{2\pi y}{L}\right)+\exp\left(-\frac{4\pi y}{L}\right)\right]dy=\frac{2L}{\pi}\left[y_0^2\left(\frac{5}{3}-2\ln 2\right)+\left(\frac{L}{\pi}\right)^2\left(\frac{5}{6}\zeta(2)-\frac{5}{4}\zeta(3)\right)-\frac{1}{3}\right],$$

$$\int_0^\infty\frac{dy}{\cosh^4\frac{\sqrt{y^2+y_0^2}}{2\xi_{\rm GL}}}\frac{y^2}{y^2+y_0^2}=\frac{4}{3}\xi_{\rm GL}-\frac{\pi}{2}y_0,$$

$y_0\ll\xi_{\rm GL}$.

In the two vortex state with zero current, instead of Eqs. (3) and (4) one finds

$$\left[\frac{\partial\varphi(x,y)}{\partial\mathbf{r}}\right]^2=\frac{4\pi^2}{L^2}\frac{1}{\cos^2\frac{2\pi x}{L}+\sinh^2\frac{2\pi y}{L}},\quad|\Delta|=\Delta_0\tanh\frac{\sqrt{y^2+y_0^2}}{2\xi_{\rm GL}}\phi,\quad\phi=\frac{1}{\cosh\frac{2\pi y}{L}}\left[\cos^2\frac{2\pi x}{L}+\sinh^2\frac{2\pi y}{L}\right]^{1/2}.$$

All following considerations are similar to those in a single vortex state. In the domain of weak currents the current conservation law gives in the main approximation the expression for the vector potential:

$$A = \left(0, \frac{A_\infty \Delta_0^2(\Gamma)}{\langle |\Delta|^2 \rangle_x}, 0\right), \quad \Gamma = \frac{J}{J_c} = -3\sqrt{6}|e|A_\infty \xi_{\mathrm{GL}}. \tag{A3}$$

Next is the calculus of the integrals of the type $\langle \Phi^{-2}(x,y)\rangle_x$. One finds

$$\begin{aligned} I_b &= \frac{1}{L}\int_{-L/2}^{L/2} \frac{dx}{\sinh^2\frac{\pi y}{L} + \sin^2\frac{\pi x}{L} + \sin^2\frac{\pi\delta}{L} - 2\sin\frac{\pi x}{L}\sin\frac{\pi\delta}{L}\cosh\frac{\pi y}{L}} \\ &= -4\frac{z_1 z_2\left[(z_3+z_4) - 2\cosh\frac{2\pi y}{L} - 4\sin^2\frac{\pi\delta}{L}\right] + (z_1+z_2)\left[(1-z_3z_4) + 4z_3z_4\sin^2\frac{\pi\delta}{L} + 2z_3z_4\cosh\frac{2\pi y}{L} - (z_3+z_4)\right]}{(z_1-z_3)(z_2-z_3)(z_1-z_4)(z_2-z_4)}, \end{aligned} \tag{A4}$$

where

$$z_{1,2} = \exp\left(\frac{2\pi y}{L}\right)\left[1 \pm 2i\sin\frac{\pi\delta}{L} - 2\sin^2\frac{\pi\delta}{L}\right], \quad z_{3,4} = \exp\left(-\frac{2\pi y}{L}\right)\left[1 \pm 2i\sin\frac{\pi\delta}{L} - 2\sin^2\frac{\pi\delta}{L}\right].$$

The direct and cumbersome integration of Eq. (A4) results in

$$I_b = \frac{2}{\sinh\frac{2\pi y}{L}}\left[1 + \frac{4\sin^2\frac{\pi\delta}{L}\sinh^2\frac{\pi y}{L}}{\sinh^2\frac{2\pi y}{L} + 4\sin^2\frac{\pi\delta}{L}}\right].$$

Now, using Eqs. (A3) and (A4), and the definition (10), one can find the necessary values:

$$\left\langle \left[\frac{\partial\varphi(x,y)}{\partial\mathbf{r}}\right]^2 |\Delta|^2 \right\rangle_x = \frac{\pi^2}{4L^2\xi_{\mathrm{GL}}^2}\frac{\cos^2\frac{\pi\delta}{L}}{\cosh^2\frac{\pi y}{L}}\left(y^2 + y_0^2 + \frac{32}{27}\Gamma^2\xi_{\mathrm{GL}}^2\right), \tag{A5}$$

$$\left\langle \frac{\partial|\Delta|^2}{\partial x}\right\rangle_x = \left(\frac{\pi}{2L}\right)^2 \frac{\Delta_0^2(\Gamma)}{\xi_{\mathrm{GL}}^2\cosh^2\frac{\pi y}{L}}\left(y^2 + y_0^2 + \frac{32}{27}\Gamma^2\xi_{\mathrm{GL}}^2\right)\left[\frac{1}{2}\exp\left(-\frac{2\pi y}{L}\right) + \sin^2\left(\frac{\pi\delta}{L}\right)\sinh\left(\frac{\pi y}{L}\right)\exp\left(-\frac{\pi y}{L}\right)\right], \tag{A6}$$

$$\begin{aligned} \left\langle \frac{\partial|\Delta|^2}{\partial y}\right\rangle_x &= \frac{Z(y,\Gamma)\,\Delta_0^2(\Gamma)}{2L^2}\left\{\frac{L^2}{4Z^2}\left(\frac{\partial Z}{\partial y}\right)^2\left[1 + \tanh^2\frac{\pi y}{L}\right] + \frac{\pi L}{Z}\left(\frac{\partial Z}{\partial y}\right)\frac{\tanh\frac{\pi y}{L}}{\cosh^2\frac{\pi y}{L}}\right. \\ &\left.+ \pi^2\frac{\tanh\left(\frac{\pi y}{L}\right)}{\cosh^4\left(\frac{\pi y}{L}\right)}\left[\frac{3}{4} + \exp\left(-\frac{2\pi y}{L}\right) + \frac{1}{4}\exp\left(-\frac{4\pi y}{L}\right)\right]\right\} \; y > 0. \end{aligned} \tag{A7}$$

In order to obtain the value of the activation energy from Eq. (2) one has to learn how to integrate Eqs. (A5) and (A6) over y. We demonstrate here some of them:

$$\int_0^\infty \frac{dy}{Z(y,\Gamma)}\left(\frac{\partial Z}{\partial y}\right)^2 = \frac{4}{3\xi_{\mathrm{GL}}} - \frac{\pi}{2\xi_{\mathrm{GL}}^2}\sqrt{y_0^2 + \frac{32}{27}\Gamma^2\xi_{\mathrm{GL}}^2}, \tag{A8}$$

$$\int_0^\infty \frac{dy}{Z(y,\Gamma)\cosh^2\frac{\pi y}{L}}\left(\frac{\partial Z}{\partial y}\right)^2 = \frac{L}{\pi\xi_{\mathrm{GL}}^2}\left[1 - \frac{\pi^2}{L^2}\left(y_0^2 + \frac{32}{27}\Gamma^2\xi_{\mathrm{GL}}^2\right) I_2\left(\frac{\pi}{2L}\sqrt{y_0^2 + \frac{32}{27}\Gamma^2\xi_{\mathrm{GL}}^2}\right)\right],$$

$$\begin{aligned} \int_0^\infty \left\langle \frac{\partial|\Delta|^2}{\partial x}\right\rangle_x dy &= \frac{\Delta_0^2(\Gamma)}{8}\frac{L}{\pi\xi_{\mathrm{GL}}^2}\left\{\frac{3}{4}\zeta(3) - \frac{1}{2}\zeta(2) + \frac{\pi^2}{L^2}(2\ln 2 - 1)\left(y_0^2 + \frac{32}{27}\Gamma^2\xi_{\mathrm{GL}}^2\right)\right. \\ &\left.+ \sin^2(\pi\delta/L)\left[\zeta(2) - \frac{3}{4}\zeta(3) + \frac{2\pi^2}{L^2}(1 - \ln 2)\left(y_0^2 + \frac{32}{27}\Gamma^2\xi_{\mathrm{GL}}^2\right)\right]\right\}, \end{aligned}$$

$$I_1(\alpha) = \int_0^\infty \frac{dx}{\cosh x}\frac{1}{x^2 + 4\alpha^2} = \frac{\pi}{2}\left[\frac{\pi}{2\alpha\cos 2\alpha} + 2\sum_{n=0}^\infty \frac{(-1)^n}{4\alpha^2 - \pi^2\left(n+\frac{1}{2}\right)^2}\right],$$

$$I_2(\alpha) = \int_0^\infty \frac{dx}{\cosh^2 x}\frac{1}{(x^2 + 4\alpha^2)} = \frac{1}{2}\left\{\frac{\pi}{2\alpha\cos^2 2\alpha} - \sum_{n=0}^\infty \frac{4\pi^2\left(n+\frac{1}{2}\right)}{4\alpha^2 - \pi^2\left(n+\frac{1}{2}\right)^2}\right\},$$

$\alpha > 0$.

APPENDIX B

Let us notice that, if the function $\widetilde{\Delta}(x,y)$ is that one for which the conditional extremum of the GL functional Eq. (2) is reached, the value of free energy in this state takes an especially simple form:

$$F_s = -\nu \int d^3\mathbf{r} \left[\frac{7\zeta(3)}{16\pi^2 T^2} |\Delta(\mathbf{r})|^4 + \frac{1}{c}\left(\mathbf{A} - \frac{c}{2e}\nabla\varphi\right)\cdot \mathbf{j}_\infty \right]. \quad \text{(B1)}$$

This expression enables us to determine the value of parameter r_1. In order to do this we calculate the value of the GL free energy Eq. (B1) using Eqs. (19) and (20). In result one finds the equation

$$\ln^2\left(\frac{2\gamma L^2}{r_1^2}\right) \frac{\tanh Y}{\cosh^2 Y - (3\mathcal{L} - 2)/\mathcal{L}} = \text{const}, \quad \text{(B2)}$$

where $Y = y_0\sqrt{3\mathcal{L} - 2}/(2\xi_{\text{GL}})$ and the value of the constant is independent on y_0. The maximal value of r_1 is reached when Y satisfies the condition of the extremum:

$$\left\{ \frac{\partial}{\partial Y}\left[\frac{\tanh Y}{\cosh^2 Y - (3\mathcal{L}-2)/\mathcal{L}} \right] \right\}_{Y=Y_{\mathcal{L}}} = 0. \quad \text{(B3)}$$

Equation (B3) can be solved:

$$\tanh Y_{\mathcal{L}} = 2\left[\frac{1-\mathcal{L}}{4 - 3\mathcal{L} + \sqrt{\mathcal{L}(16 - 15\mathcal{L})}} \right]^{1/2}. \quad \text{(B4)}$$

One can see that our assumption that in Eq. (B2) the value of the constant is independent on y_0 is confirmed (the found value $Y_{\mathcal{L}}$ is independent on it). Now we can use the found value

$$y_0 = \frac{2\xi_{\text{GL}}}{\sqrt{3\mathcal{L} - 2}} \operatorname{arctanh}\left[\frac{4(1-\mathcal{L})}{4 - 3\mathcal{L} + \sqrt{\mathcal{L}(16 - 15\mathcal{L})}} \right]^{1/2}$$

to perform the final integration in Eq. (B1), which results in

$$\delta F_s = 4\nu \Delta_0^2(\Gamma)\tau S \xi_{\text{GL}} \sqrt{3\mathcal{L} - 2} \left\{ \frac{1 - \tanh Y_{\mathcal{L}}}{6\mathcal{L}} [4 + (3\mathcal{L} - 2)\tanh Y_{\mathcal{L}}(1 + \tanh Y_{\mathcal{L}})] \right.$$
$$\left. - \sqrt{\frac{2(1-\mathcal{L})}{3\mathcal{L}-2}} \left[\arctan\sqrt{\frac{3\mathcal{L}-2}{2(1-\mathcal{L})}} - \arctan\left(\sqrt{\frac{3\mathcal{L}-2}{2(1-\mathcal{L})}} \tanh Y_{\mathcal{L}}\right) \right] \right\}.$$

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