Vortex chains due to nonpairwise interactions and field-induced phase transitions between states with different broken symmetry in superconductors with competing order parameters

Julien Garaud* and Egor Babaev

Department of Theoretical Physics, KTH-Royal Institute of Technology, Stockholm SE-10691, Sweden

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We study superconductors with two order components and phase separation driven by intercomponent density-density interaction, focusing on the phase where only one condensate has nonzero ground-state density and a competing order parameter exists only in vortex cores. We demonstrate there that multibody intervortex interactions can be strongly nonpairwise, leading to some unusual vortex patterns in an external field, such as vortex pairs and vortex chains. We demonstrate that in an external magnetic field such a system undergoes a field-driven phase transition from (broken) $U(1)$ to (broken) $U(1)\times U(1)$ symmetries when a subdominant order parameter in the vortex cores acquires global coherence. Observation of these characteristic ordering patterns in surface probes may signal the presence of a subdominant condensate in the vortex core.

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I. INTRODUCTION

The unusual magnetic response that originates in multiscale intervortex interactions recently attracted substantial interest in the framework of multicomponent superconductivity. The interest was sparked by the observations of vortex aggregates in the two-band superconductor MgB_2 [1–5], multiband iron pnictides $Ba(Fe_{1-x}Co_x)_2As_2$ [6,7] and $Ba_{1-x}K_xFe_2As_2$ [8], as well as in spin triplet Sr_2RuO_4 [9,10]. There the existence of multiple coherence lengths may lead to multiscale physics that can account for observation of vortex aggregates. On the other hand, models of multicomponent superconductivity featuring biquadratic density-density interaction are currently discussed in the context of superconductors with pair density wave order [11,12], and most recently in the context of interface superconductors such as $SrTiO_3/LaAlO_3$ [13]. Here we investigate the properties of topological defects in an immiscible phase of a two component model, where there is strong biquadratic interaction that penalizes coexistence of both superconducting condensates. We show that it features unusual multiscale physics of the vortex matter where nonpairwise interactions are important. This is modeled by a theory of two complex fields that have a $U(1)\times U(1)$ symmetry. In the phase-separated regime, that occurs for strong biquadratic interaction, the ground-state spontaneously breaks only one of the $U(1)$ of the symmetry of the theory.

In two-component superconductors, when both condensates have nonzero ground-state density, nonmonotonic interactions can occur, due to competing intervortex interactions with different length scales [14–16]. This typically leads to formation of vortex clusters surrounded by macroscopic regions of the Meissner state [17]. Because it features properties of both type-1 and type-2 superconductors, this regime is termed type-1.5. It is a subject of ongoing studies, both experimental on MgB_2 [1,2,4,5] and more recently in Sr_2RuO_4 [10] and theoretical studies of Ginzburg-Landau [15,16,18], microscopic [19], and effective point-particle [20,21] models.

Here we show that unusual multiscale interaction arises in models of two-component superconductors with strong intercomponent biquadratic coupling that is repulsive. The biquadratic interaction penalizes coexistence of both condensates and above a given critical coupling they cannot coexist, so that one is completely suppressed. However, in the cores of vortices, this interaction is effectively much weaker and the suppressed component can locally condense. We demonstrate that the condensation in vortex cores leads to new unusual multiscale, nonmonotonic interactions between vortex matter, where nonpairwise forces are important [22]. Because it originates in multiple condensates with a particular hierarchy of the physical length scales, it is somewhat akin to the type-1.5 regime, but with the substantial difference here that only one condensate has nonzero ground-state density.

Below we study the two-component Ginzburg-Landau model where intercomponent density-density interaction can be strong enough to completely suppress one of the condensates, in the ground-state. We characterize the different possible ground-state phases of that model and the associated length scales. Finally, we numerically investigate the properties of vortices within the phase above a critical density-density coupling, where both components cannot coexist. There we demonstrate the existence of the above mentioned regime where intervortex interactions are nonmonotonic, and where multibody forces are important. Unlike the type-1.5 regime where vortices typically aggregate into clusters [15,16,18], vortices here tend to form chains and irregular structures. Unlike chains forming in multiscale systems with long-range repulsive interaction [26–30], chains here originate in nonpairwise intervortex forces.

II. THE MODEL

The Ginzburg-Landau model we consider here is a theory two complex fields ψ_1 and ψ_2 standing for two superconducting condensates. They interact together by their coupling to the vector potential of the magnetic field $\boldsymbol{B}=\nabla\times\boldsymbol{A}$, through the kinetic term $\boldsymbol{D}\equiv\nabla+ie\boldsymbol{A}$:

$$\mathcal{F}=\frac{\boldsymbol{B}^2}{2}+\sum_{a=1,2}\left\{\frac{1}{2}|\boldsymbol{D}\psi_a|^2+\alpha_a|\psi_a|^2+\frac{1}{2}\beta_a|\psi_a|^4\right\} +\gamma|\psi_1|^2|\psi_2|^2. \tag{1}$$

*garaud.phys@gmail.com

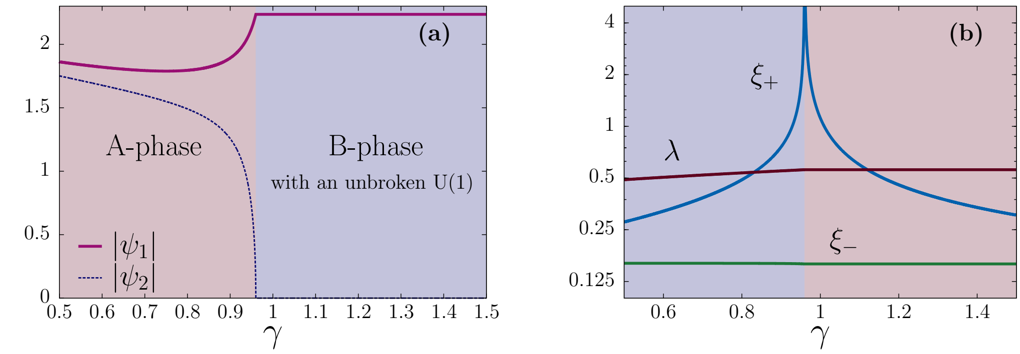

FIG. 1. (Color online) Ground-state properties of the model. (a) and (b) Display ground-state densities and length scales [computed from the eigenvalues of the Hessian matrix (3)] when the intracondensate couplings are not equal: $\alpha_1 = -5$, $\alpha_2 = -4.8$. $\beta_1 = \beta_2 = 1$ and $e = 0.8$. Depending on the strength of the biquadratic coupling γ, the ground-state corresponds to either the A phase or the B phase, as defined in Eqs. (4) and (5). One of the length scales ξ_+ diverges at the critical value $\gamma_\star$ that separates both phases, while the other one ξ_- is always finite.

Moreover, the condensates are directly coupled together by a biquadratic (density-density) interaction potential term when $\gamma \neq 0$ and because the biquadratic interaction is repulsive, $\gamma > 0$. For generic values of the parameters of the potential, α, β and γ, the theory has a $U(1) \times U(1)$ symmetry [31].

Depending on the relation between the parameters of the potential, two qualitatively different superconducting phases can be identified. These are determined by the ground-state properties of the theory. Since the potential depends on the fields moduli only, the ground-state is the state with constant densities of the superconducting condensates $|\psi_a| = u_a$ and where the vector potential is a pure gauge ($\boldsymbol{A} = \nabla\chi$ for arbitrary χ) that can consistently chosen to be zero. The extrema of the potential are given by $\partial V/\partial|\psi_a| = 0$ and the ground-state densities u_a satisfy

$$\begin{aligned} &2\left(\alpha_1 + \beta_1 u_1^2 + \gamma u_2^2\right)u_1 = 0, \\ &2\left(\alpha_2 + \beta_2 u_2^2 + \gamma u_1^2\right)u_2 = 0. \end{aligned} \tag{2}$$

For the extrema to be stable (minima), the eigenvalues of the Hessian matrix $\mathcal{H} = \partial^2 V/\partial|\psi_a|\partial|\psi_b|$ must be positive. Here the Hessian matrix reads

$$\mathcal{H} = 2\begin{pmatrix} \alpha_1 + 3\beta_1 u_1^2 + \gamma u_2^2 & 2\gamma u_1 u_2 \\ 2\gamma u_1 u_2 & \alpha_2 + 3\beta_2 u_2^2 + \gamma u_1^2 \end{pmatrix}. \tag{3}$$

Apart from the normal state ($u_1 = u_2 = 0$), there are two qualitatively different solutions of (2): The A phase (miscible) for which both condensates have nonzero ground-state density ($u_1,u_2 \neq 0$), and the B phase (immiscible) for which only one condensate has nonzero ground-state density: either $u_1 \neq 0$ and $u_2 = 0$ or $u_1 = 0$ and $u_2 \neq 0$. Assuming that $\alpha_a < 0$ and $\beta_a > 0$, the qualitatively different stable phases determined by (2) and (3) are

$$\textbf{A phase}:\ \left(u_1^2,u_2^2\right) = \left(\frac{\alpha_2\gamma - \alpha_1\beta_2}{\beta_1\beta_2 - \gamma^2}\,,\frac{\alpha_1\gamma - \alpha_2\beta_1}{\beta_1\beta_2 - \gamma^2}\right) \tag{4}$$

if $\beta_1\beta_2 > \gamma^2$,$\alpha_2\gamma - \alpha_1\beta_2 > 0$, and $\alpha_1\gamma - \alpha_2\beta_1 > 0$.

$$\textbf{B phase}:\ \left(u_1^2,u_2^2\right) = \left(\frac{-\alpha_1}{\beta_1}\,,0\right) \quad \text{or} \quad \left(0,\frac{-\alpha_2}{\beta_2}\right) \tag{5}$$

if $\alpha_2\beta_1 - \alpha_1\gamma > 0$ or $\alpha_1\beta_2 - \alpha_2\gamma > 0$.

Clearly to understand properties of the B phase it is enough to consider only the first case where $u_1 \neq 0$ and $u_2 = 0$, as the case $u_2 \neq 0$ and $u_1 = 0$ can straightforwardly be obtained from the first one. Note that we disregard the possibility of having one positive α_a. For both $\alpha_a > 0$, the ground-state is the normal state $u_1 = u_2 = 0$. The ground-state in the A phase spontaneously breaks the $U(1) \times U(1)$ symmetry. In the B phase only one of the $U(1)$ is spontaneously broken while the other, associated with the suppressed condensate, remains unbroken.

In this work we are primarily interested in the properties of the B phase (5), in the vicinity of the phase transition between A and B phases. A convenient parametrization to understand this transition is to investigate the role of the biquadratic coupling γ. As shown in Fig. 1, for fixed values of α_a and β_a, the biquadratic coupling γ can be used to parametrize the transition between the two phases. The length scales $\xi_\pm$ are defined from the eigenvalues $m_\pm^2$ of the Hessian (3) as $\xi_\pm = 1/m_\mp$, while the penetration depth is $\lambda = 1/e\sqrt{u_1^2 + u_2^2}$. Here m_+^2 stands for the largest eigenvalue of the Hessian and m_-^2 the smallest. The relation between the Hessian matrix and the length scales can be heuristically understood as follows. The Hessian matrix contains the information about the stability of the ground-state and thus how it recovers from a small perturbation. It is important to understand that $\xi_\pm$ corresponds to hybridized modes and cannot be attributed to a given condensate separately. That is, $m_\pm^2$ are the decay rates of a linear combination of ψ_1 and ψ_2. Long-range intervortex interaction is controlled by the masses of normal modes. The linearized theory yields the following long-range intervortex interaction [16]:

$$V = q_\lambda K_0(r/\lambda) - q_- K_0(r/\xi_-) - q_+ K_0(r/\xi_+), \tag{6}$$

where K_0 is the modified Bessel function of the second kind and the coefficients q_λ and $q_\pm$ are determined by nonlinearities. Here the first term describes the repulsion driven by current-current and magnetic interactions, while the second and third terms describe the density-fields-driven interactions.

Single component superconductors are classified into type-1/type-2 when the penetration depth λ is smaller/larger than the coherence length ξ. From this, the vortex interactions are attractive in type-1 because long-range interaction is mediated by core-core interactions. On the other hand, it is repulsive for type-2, due to current-current interactions that range with λ. In two-component superconductors, such a classification is not directly applicable because of the existence of multiple length scales $\xi_\pm$. In particular, if the penetration depth is an intermediate length scale $\xi_- < \lambda < \xi_+$, it, under certain conditions, leads to nonmonotonic interactions that are long-range attractive and short-range repulsive [14,16]. This can result in the formation of vortex clusters surrounded by macroscopic regions of the Meissner state [17]. This phase is coined type-1.5 and observation of clusters were reported from measurements in clean MgB_2 [1,1,2] and in Sr_2RuO_4 [10] samples.

When increasing γ, toward the critical value $\gamma_\star = \alpha_2\beta_1/\alpha_1$ that separates A and B phases, the disparity in densities becomes more important. This is accompanied with the increase of the largest length scale ξ_+. At $\gamma_\star$ this length scale diverges, while all the other length scales remain finite. In the A phase, where both condensates have nonzero ground-state density, elementary topological excitations are vortices with winding in either condensate. These carry a fraction of the flux quantum, but finiteness of the energy imposes that they form a bound state that has phase winding in both condensates and that carries integer flux quantum. The most simple version of such a bound state is to have vortices in both condensates and that they superimpose. However, solutions where vortices do not coincide can exist and be preferred energetically. It has recently been argued that such topological defects, characterized by an additional topological invariant, could be realized in interface superconductors, such as $SrTiO_3/LaAlO_3$ [13]. If λ is not the smallest length scale (i.e., not a type-1 regime), then there always exists a regime, in the vicinity of $\gamma_\star$, where the penetration depth is an intermediate length scale: $\xi_- < \lambda < \xi_+$. In the A phase this length scale hierarchy is known to be a necessary condition for the nonmonotonic vortex interaction [15]. Clearly this is realized close to $\gamma_\star$, see Fig. 1.

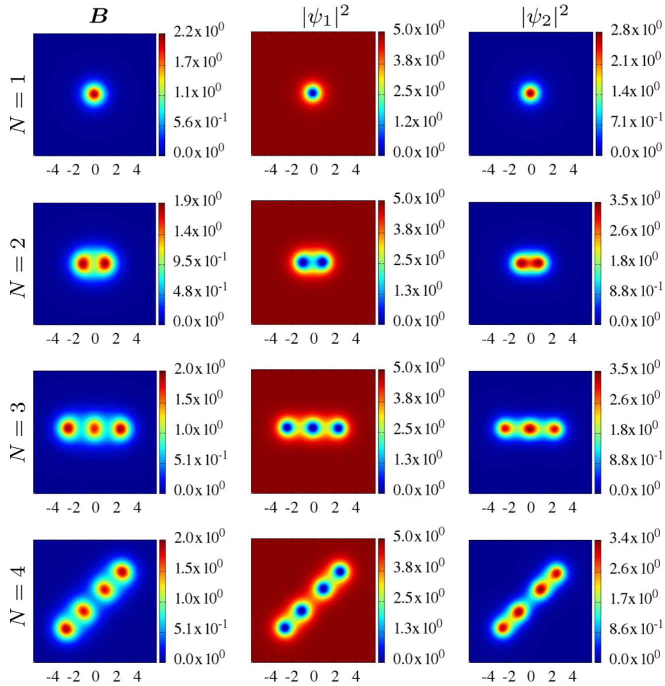

FIG. 2. (Color online) Vortex solutions in the B phase of Fig. 1, for the coupling constant of the biquadratic interaction $\gamma = 1.0$. The first column displays the magnetic field, while the second and third columns show $|\psi_1|^2$ and $|\psi_2|^2$, respectively. The lines show configurations carrying $N = 1$, 2, 3, and 4 flux quanta, respectively. In the B phase, only ψ_1 has nonzero ground-state density, because the biquadratic coupling is too strong to allow coexistence of both condensates. Thus only ψ_1 forms vortices, while ψ_2 is zero everywhere except in vortex cores. As expected from the length scales considerations, intervortex interaction is nonmonotonic and vortices stand at a preferred distance, see second line. For a larger number of flux quanta (third and fourth line), vortices form straight chains. This contrasts with the two-body picture that would predict formation of compact clusters. The chainlike structures thus signal existence of strong nonpairwise forces between vortices. We should remark that the simulations are performed on a domain that is large enough, so that the vortices do not interact with boundaries. The plots show only a small fraction of the numerical grid.

III. EVIDENCES FOR STRONG NONPAIRWISE INTERVORTEX FORCES

Here our main interest are the properties of the B phase, in particular in the vicinity of $\gamma_\star$. In contrast to the above mentioned type-1.5 regime of the A phase, the topological excitations in the B phase are vortices that have core in ψ_1 only. Away from vortex cores, the fields recover their ground-state values and thus only ψ_1 can contribute to the flux quantization.

To investigate the properties of topological excitations and their interactions, we numerically minimize the free energy (1) within a finite element framework [32]. That is, for a given choice of parameters, a starting configuration with desired winding is created and the energy is then minimized with a nonlinear conjugate gradient algorithm. For detailed discussion on the numerical methods, see for example Appendix in Ref. [33]. In the B phase only the condensate ψ_1 has nonzero ground-state density and thus only ψ_1 has vortex excitations. Since the component ψ_1 vanishes at the vortex core, it can be beneficial for the suppressed component ψ_2 to assume nonzero density in the cores of vortices. A similar mechanism of condensation in vortex cores was also discussed in the context of cosmic strings [34]. Minimizing the free energy (1) for an initial configuration carrying a single flux quantum relaxes to such a vortex state, see first line in Fig. 2. The condensate ψ_2 that lives inside the vortex cores is gradually suppressed where the other condensate ψ_1 recovers toward its ground-state density. The rate at which ψ_2 recovers is determined by the fundamental length scales $\xi_\pm$ of the theory. Because the modes are hybridized, the length scales associated with the recovery of ψ_1 and the decay of ψ_2 are not independent.

In the B phase, in the vicinity of $\gamma_\star$, the length scales satisfy the necessary condition for nonmonotonic interactions. Indeed, as shown on the second line of Fig. 2, interactions between two vortices can also be nonmonotonic in the B phase, even if only one condensate has nonzero ground-state density. There, in agreement with the linear theory (6), pairwise interaction between vortices is long-range attractive due to the largest hybridized density mode and short-range repulsive due to current-current interactions. It results in a preferred distance at which vortices minimize their interaction energy by forming a vortex pair. Based on these observations, natural expectation from the two-body interactions is that states with more than two vortices will form compact clusters inside which vortices tend to have triangular arrangement [17]. However, because it is a nonlinear problem, interactions between vortices can become more complicated, beyond the linear approximation. In particular, from studies of point particle effective models [35], it follows that strong nonpairwise interactions can dramatically affect structure formation, resulting in stripe, gossamer, and glass phases.

The configurations for few isolated vortices displayed in Fig. 2 show chain organization of vortices. This indicates that there are nonmonotonic interactions, but also that there are strong multibody forces. Indeed the two-body picture would naively lead us to conclude that many vortices would organize in a compact cluster. Because theory (1) is completely isotropic, the linelike organization can originate only in complicated interactions. This poses the question of the response of the system to an external field. At an elevated external field, vortex matter usually forms lattices (hexagonal, square, etc). Since the low field results indicate strong nonpairwise forces, the question arises if these have a substantial influence at elevated fields. To sort this out, we investigate the response in an external field $\boldsymbol{H} = H_z \boldsymbol{e}_z$, perpendicular to the plane. For this, the Gibbs free energy $\mathcal{G} = \mathcal{F} - \boldsymbol{B} \cdot \boldsymbol{H}$ is minimized, with requiring that $\nabla \times \boldsymbol{A} = \boldsymbol{H}$ on the boundary (see e.g. discussion in Appendix of Ref. [33]). As shown in Fig. 3, the typical response in an external field shows a long-living irregular vortex structure. For example, similar simulations, but in the A phase, show very regular square lattices [36]. We show such a lattice in the Appendix.

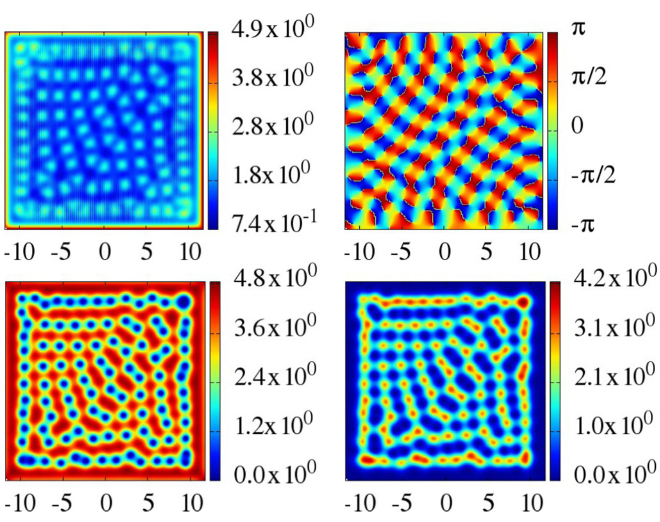

FIG. 3. (Color online) The parameters are the same as in Fig. 2. The panels on the first row display the magnetic field and the phase difference $\varphi_{12} = \varphi_2 - \varphi_1$. The second line shows the densities $|\psi_1|^2$ and $|\psi_2|^2$, respectively. Note that this configuration is not a true ground-state in the external field, but is a very stable state. Here the tendency to form chains competes with finite size effects, resulting in a very irregular pattern for vortices. Due to the presence of multibody forces, obtaining true ground-state in the simulations of magnetization processes for systems of these sizes turns out to be very difficult. For a discussion of glassiness arising from the nonpairwise forces see [35]. This suggests that the shown patterns should also be physically representative for experimental situations in such systems.

There is a tendency here to form chains, but this tendency competes with the increased importance of current-current interactions in the relatively dense vortex matter. Note that the nonpairwise forces, when strong enough, typically promote metastable or long-living disordered states. Also, when minimizing the Gibbs free energy with the condition that $\nabla \times \boldsymbol{A} = \boldsymbol{H}$ on the boundary, the interaction energy between vortices is minimized not independently from the interaction with the Meissner currents on the boundary. Such finite size effects play as well a role in having imperfect lattices.

Observe that it was demonstrated earlier, that in type-1.5 systems multibody forces can aid formation of vortex chains for dynamic and entropic reasons [17]. However, here the nonpairwise forces are clearly much stronger, as chains form as ground-state solutions in low fields, see Fig. 2. Note also that the chains and vortex dimers forming here originate in nonpairwise interactions and not because of pairwise interactions with multiple repulsive length scales [26,27,37]. They should also not be confused with vortex chains predicted for multilayer structures, where they originate in a stray field that lead to long-range repulsive interaction [21,38].

IV. INDUCING STATE WITH DIFFERENT BROKEN SYMMETRY BY APPLIED FIELD

For isolated vortices in ψ_1, the other component ψ_2 develops nonzero amplitude in the vortex core. However, as shown in Fig. 2, ψ_2 is asymptotically suppressed and thus it has has no phase winding. As mentioned in the Introduction, in this state the system breaks only one $U(1)$ symmetry. In a high external field there is a large density of vortices and on average $|\psi_2|$ becomes nonzero. There the areas with nonzero $|\psi_2|$ get interconnected across the whole system and thus the system thus undergoes a phase transition to a state that breaks the $U(1) \times U(1)$ symmetry. By saying that the system breaks $U(1) \times U(1)$ symmetry in an external field we assume a robust vortex structure, we do not consider here vortex liquids. The interconnection of ψ_2 across the whole sample is signaled by a change in the phase winding pattern. If two condensates have nonzero density, phase winding in only one condensate gives a logarithmically divergent contribution to the energy [39]. As a result, it is energetically beneficial for the component ψ_2 to form vortices as well. This is in strong contrast with the results for isolated vortices. The breakdown of the $U(1)$ symmetry associated with the condensate ψ_2, and the corresponding formation of vortices, can be seen from phase difference $\varphi_{12} = \varphi_2 - \varphi_1$ shown in the upper right panel in Fig. 3. There the dipolelike structure of φ_{12} shows the existence of phase winding in both condensates but around different points. This unambiguously signals that both condensates have the same total phase winding and thus it is $U(1) \times U(1)$ a symmetry-broken state.

V. METASTABLE MULTIQUANTA SOLUTIONS

When γ becomes large enough as compared to $\gamma_\star$, condensation of ψ_2 in the vortex core becomes less important. As a result, deeper in the B phase, individual vortices show no condensation of ψ_2 in the core [40]. Moreover, deep into the B-phase, λ becomes the largest length scale,

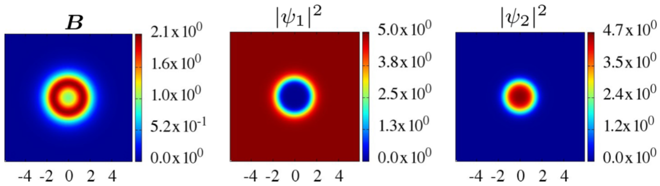

FIG. 4. (Color online) Metastable solution, deep into the B phase. This is a localized configuration that carries four flux quanta, for the same parameters as in Fig. 2 except that $\gamma = 1.2$. This object carries multiple flux quanta, despite not being in a type-1 regime. It is made of a large central region of the condensate ψ_2 where $\psi_1 = 0$, embedded in a domain where $\psi_2 = 0$. The magnetic flux is screened by ψ_1 outside the vortex, while ψ_2 is responsible for screening inside. As a result, the magnetic flux is localized on a cylindrical shell around the vortex and resembles a pipe.

and the interaction between vortices becomes long-range repulsive. Since this follows from asymptotic analysis, this holds sufficiently far from the vortex core. However, it does not preclude more involved interactions at shorter ranges. We computed vortex solutions as in Fig. 2, but deeper in the B phase ($\gamma = 1.2, 1.4, \ldots$). There we find that indeed, isolated vortices are preferred over vortex bound states. Nevertheless, we could find a special kind of metastable bound states of vortices. Namely, we found configuration carrying N flux quanta whose energy $E(N)$ is larger than the one of N isolated vortices: $E(N) > NE(N=1)$. These configurations are thus local minima of the energy functional and, for the parameters which we considered, they differ by less than 5% from isolated vortices. Such a metastable state is shown in Fig. 4. Being obtained through energy minimization, it is stable to small perturbations and depends on the starting configuration. Namely, if the starting configuration is in the attractive basin of the local minimum, it will converge to the local minimum. Typically if the starting configuration consists of dense packing of vortices, then it may lead to the metastable bound state. The metastable state shown in Fig. 4 are lumps where ψ_2 is nonzero, despite that, from an energetic viewpoint it should be suppressed. There the magnetic flux is screened by ψ_1 outside the vortex, while ψ_2 is responsible for screening inside. As a result, the magnetic flux is localized on a cylindrical shell around the vortex and resembles a pipe. In different systems, similar pipelike configurations can actually appear as true stable states for the special case where $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$. This was recently investigated in a separate work [42]. Also pipelike vortices were discussed in the Bogomol'nyi regime of SU(2) theory where additionally $\gamma = \beta_1 = \beta_2$ [43]. There the pipelike solutions feature both properties of vortices and domain walls. The remarkable feature of the pipelike vortices in this regime is that here the model does not have topological domain walls solutions. This makes it distinct from the other models that support metastable bound states of vortices due to existence of a broken $\mathbb{Z}_2$ symmetry [42,44,45].

According to the asymptotics, intervortex interactions are long-range repulsive. The attractive channel is activated only at shorter range. This means that when there are many vortices, relatively close to each other, they may form the bound states similar to the one displayed in Fig. 4, because of the "pressure" of other vortices. Such a situation is likely to occur

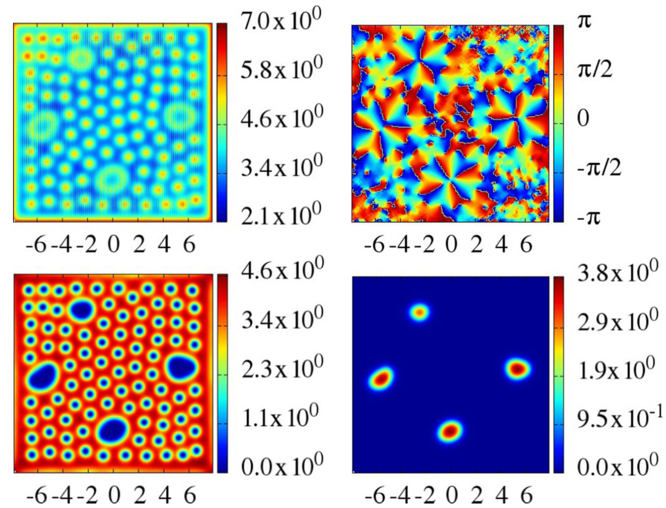

FIG. 5. (Color online) Solution in an external field for the same parameters as in Fig. 3, but stronger biquadratic coupling $\gamma = 1.5$. These parameters for the potential set the system deep into the B phase where the penetration depth is the largest length scale. Thus it should behave as an ordinary type-2 system. In such a regime, preferred solutions are isolated Abrikosov vortices. However, there also exist metastable states as the one shown in Fig. 4. The metastable bound state of vortices appears as an inclusion of a domain where ψ_2 condenses. Because these are surrounded by vortices exerting some pressure, in practice they do not decay into ordinary vortices.

in an external field and it may result in coexistence of single vortices and bound vortices. As shown in Fig. 5, this indeed happens, despite that the parameters are deep into the B phase. Note that the energy difference and the stability of bound vortices depends on all parameters of the free energy. More precisely, when the difference between α_a is important then the metastable solution does not form anymore in our simulations. Thus, the coexistence of bound vortices and usual vortices is not a universal feature and needs both condensates to have parameters with rather similar values.

In our simulation of the model, the creation of the pipelike metastable states was very history dependent. However, if they are created at all, it may be very difficult to destroy them. That is, if isolated, pipelike vortices are only metastable and may be very sensitive to small perturbations that can trigger decay into ordinary vortices. However, when surrounded by vortices, the decay channel may be different. Indeed, because it is type-2, vortices interact repulsively and they exert some pressure on the lump whose decay may thus be more difficult. We show in Fig. 5 that this is indeed the case that in an external field, deep into the B phase, lumps coexist with vortices. Note that because their creation depends on past configurations, slowly ramping up the external field may make these more rare events. Deeper in the B phase, pipelike bound states are unstable and as shown in Fig. 6, there only usual vortices ψ_1 exist and ψ_2 never condenses (up to numerical accuracy).

VI. SUMMARY

In this paper we have investigated the physical properties of two-component Ginzburg-Landau models, with

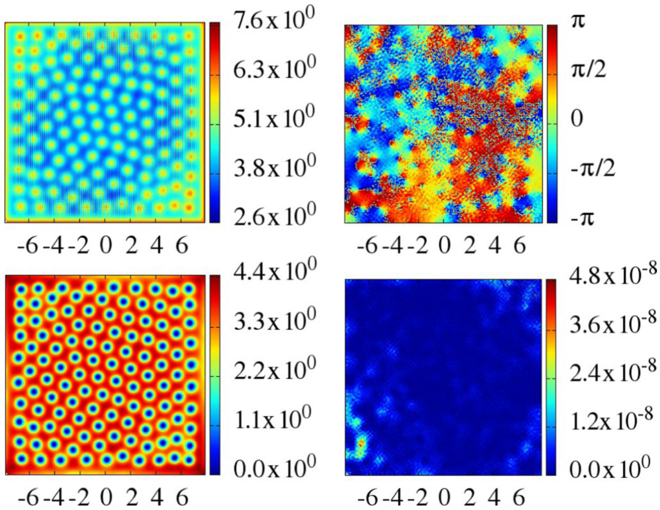

FIG. 6. (Color online) Solution in an external field for the same parameters as in Fig. 3, but stronger biquadratic coupling $\gamma = 1.6$. These parameters set the system deep into the B phase where the penetration depth is no longer an intermediate length scale. Thus, it behaves as an ordinary type-2 system. There vortices have no condensation of ψ_2 inside the core, as can be seen from the last panel. Vortices in ψ_1 behave as regular Abrikosov vortices and try to arrange as a triangular lattice. Finite size effects and interaction with Meissner current deform the lattice, so that it is not really triangular. Note that since ψ_2 is zero (up to numerical precision), the phase difference φ_{12} is reduced to numerical noise.

inequivalent components, where biquadratic interactions penalize coexistence of both condensates. Above a critical coupling $\gamma_\star$, the condensates cannot coexist and only one preferred component can have nonzero ground-state density, thus breaking only one of the $U(1)$ symmetries. We have demonstrated that in a sufficiently strong magnetic field the second component nevertheless appears resulting in a phase transition where the (second) $U(1)$ symmetry is also broken. This kind of phase transition is by no means restricted to systems with $U(1)$ symmetry. It should also exist in other systems where different order parameters are localized at the core of topological defects. Also we shown that under certain conditions such systems may form metastable states carrying multiple flux quanta distributed in a cylinder around the vortex that resembles a pipe.

Near the critical coupling $\gamma_\star$ one of the coherence lengths becomes the largest length scale. On the $U(1) \times U(1)$ side this results in the situation where the system cannot be a type-2 superconductor but be either of type-1 or type-1.5. In the later case one coherence length is larger and another is smaller than the magnetic field's penetration depth and the system vortices form clusters.

Our main results pertain to the $U(1)$ ground-state, where both condensates are phase separated. There the simple picture from the two-body interactions fails to account for the structure of vortex bound states. Indeed, instead of forming vortex clusters as suggests the two-body picture, vortex chains are formed. Because the theory is fully isotropic, this can be interpreted as the hallmark of strong nonpairwise forces. These also affect the response in an external field, where there is a clear tendency to form vortex chains. In a finite sample it results in rather irregular (metastable) vortex patterns with vortex dimers and vortex chains, as shown in Fig. 3. The result should hold for a variety of multicomponent models with competing order parameters. Thus observation of such vortex patterns may serve as an experimental hint for the presence of competing phases condensing in vortex cores. Interestingly the rather disordered vortex patterns are quite similar to those observed experimentally in iron-based superconductors [6,7,9]. The richness of static and dynamic phases which can form in systems with strong multibody forces [35,46] calls for further investigation of vortex states in these models. In samples with disorder the pattern formation will be affected by pinning which also calls for the investigation of its role. However, one can still expect substantial presence of vortex pairs, in the presence of disorder.

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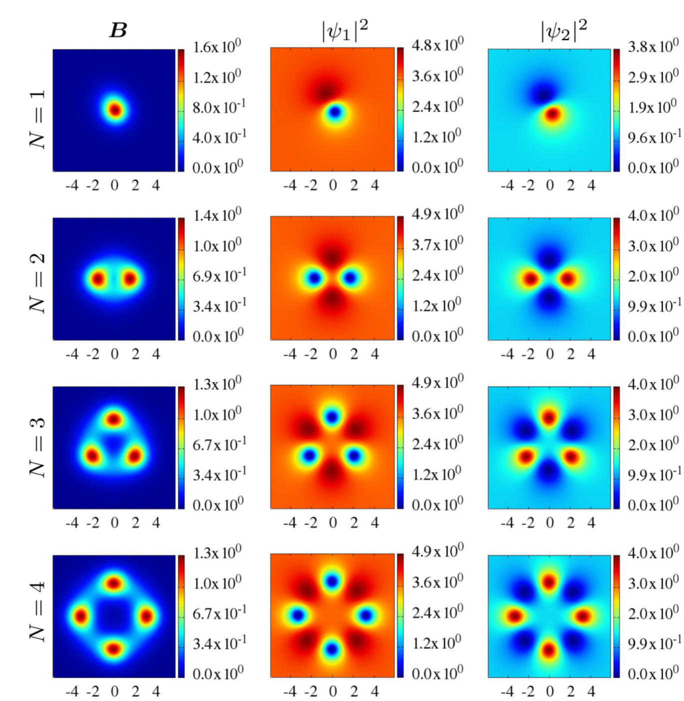

FIG. 7. (Color online) Vortex solutions in the A phase of the phase diagram Fig. 1. There the coupling constant of the biquadratic interaction is $\gamma = 0.92$. Displayed quantities are the same as in Fig. 2. In the A phase, where both components have nonzero ground-state densities, the biquadratic coupling makes it beneficial to split cores. This induces long-range interaction between flux carrying defects through dipole interactions. This interaction is responsible for the binding of vortices.

for Computing (SNIC) at National Supercomputer Center at Linköping, Sweden.

APPENDIX: VORTEX MATTER IN THE A PHASE

In the main body of the paper we focus on vortex matter in the B phase where the biquadratic interactions are strong enough to segregate condensates. For completeness, in this Appendix we provide additional materials that show the behavior of vortex matter in the A phase for the model with these parameters (although it is not directly related to the main topic of the paper).

In the A phase, both condensates have nonzero ground-state density. Thus, in order to have finite energy solutions both components must wind the same number of times. However, the cores do not necessarily have to overlap. Because of the biquadratic interaction, if the penetration depth is large enough, it is beneficial to split cores. As shown in Fig. 7, the cores in ψ_2 do not superimpose with those in ψ_1. Core splitting in single vortices induces a dipolar interaction through the phase difference mode, that is long range. As can be seen in Fig. 7, the long-range dipolar forces heavily affect multiple vortex structure. This was discussed in slightly different models in Refs. [13,36].

The long-range dipolar forces also heavily affect the magnetization process and the lattice solutions that are formed in high fields. Indeed, in the external field, vortices form a checkerboard pattern of two interlaced square lattices, as shown in Fig. 8.

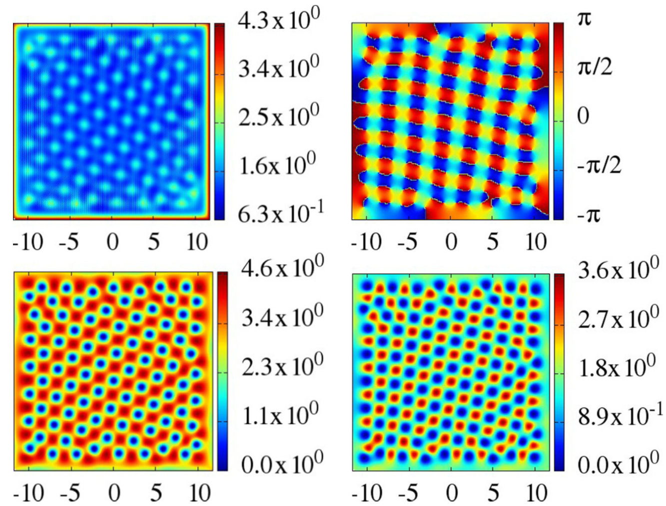

FIG. 8. (Color online) Solution in an external field, for the applied field corresponding to 301 flux quanta going through the sample's area. The parameters are the same as in Fig. 7 and displayed quantities are the same as in Fig. 3. Vortices in each condensate form square lattices that are translated from each other because of the biquadratic interaction. This results in a checkerboard pattern. Because of the disparity on ground-state densities, vortices in ψ_2 carry less flux than vortices in ψ_1. As a result the "brighter spots" of the magnetic field correspond to the vortices in ψ_1. Note that the lattices are not perfect because of finite-size effects due to the interaction with Meissner currents and vortex entries at the boundaries.

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