Orbital-cooperative spin fluctuation and orbital-dependent transport in ruthenates

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Unusual transport properties deviating from the Fermi liquid are observed in ruthenates near a magnetic quantum-critical point (QCP). To understand the electronic properties of the ruthenates near and away from an antiferromagnetic (AF) QCP, I study the electronic structure and magnetic and transport properties for the t_{2g}-orbital Hubbard model on a square lattice in fluctuation-exchange approximation including Maki-Thompson (MT) current vertex correction (CVC). The results away from the AF QCP reproduce several experimental results of Sr_2RuO_4 qualitatively and provide new mechanisms about the enhancement of spin fluctuation at $Q_{IC-AF} \approx (0.66\pi, 0.66\pi)$, larger mass enhancement of the d_{xy} orbital than that of the $d_{xz/yz}$ orbital, and nonmonotonic temperature dependence of the Hall coefficient. Also, the results near the AF QCP explain the T-linear inplane resistivity in $Sr_2Ru_{0.075}Ti_{0.025}O_4$ and give an experimental test on the obtained temperature dependence of the Hall coefficient. I reveal spatial correlation including the self-energy of electrons beyond mean-field approximations is essential to determine the electronic properties of the ruthenates. I also show several ubiquitous transport properties near an AF QCP and characteristic transport properties of a multiorbital system by comparison with results of a single-orbital system near an AF QCP.

Many-body effects cause unusual transport properties deviating from the Fermi liquid (FL) [1]. For example, the T-linear inplane resistivity, ρ_{ab}, and Curie-Weiss-like T dependence of the Hall coefficient, R_H, are observed in a quasi-2D single-orbital system near an antiferromagnetic (AF) quantum-critical point (QCP) [2]. Also, unusual transport properties are observed in ruthenates (i.e., Ru oxides), quasi-2D t_{2g}-orbital systems: $Sr_2Ru_{0.075}Ti_{0.025}O_4$, located near an AF QCP, shows the T-linear ρ_{ab} [3]; $Ca_{2-x}Sr_xRuO_4$ around $x = 0.5$, located near a ferromagnetic QCP, shows the $T^{1.4}$ dependence of ρ_{ab} and Curie-Weiss-like T dependence of R_H [4]. Note that Sr_2RuO_4 shows the FL behaviors [5,6].

The origins of these unusual transport properties of the ruthenates are unclear, although its understanding leads to a deeper understanding of roles of electron correlation and each orbital in transport properties.

To clarify these origins, we should understand roles of electron correlation and each t_{2g} orbital. In particular, it is necessary to reveal effects of the self-energy of electrons and electron-hole four-point vertex function due to electron correlation. These will give considerable effects in multiorbital systems since these play important roles in the single-orbital Hubbard model on a square lattice near an AF QCP [7] (referred to as the single-orbital case); the characteristic T and k dependence of quasiparticle (QP) damping causes the T-linear ρ_{ab}, and the characteristic T and k dependence of Maki-Thompson (MT) current vertex correction (CVC) due to MT four-point vertex function [8] causes the Curie-Weiss-like T dependence of R_H; these characteristic dependence arise from the Curie-Weiss-like T dependence of the spin susceptibility at $k = (\pi, \pi)$.

In this paper, I reveal the roles of electron correlation and each t_{2g} orbital in several electronic properties of the ruthenates near and away from the AF QCP and achieve

qualitative agreement with experiments [3,5,6]. I show the importance of spatial correlation including the self-energy of electrons beyond mean-field approximations (MFAs). Also, I show several similarities and differences between the transport properties of the present case and the single-orbital case [7] and propose the emergence of the orbital-dependent transport in other systems.

To describe the electronic structure of the ruthenates, I use the t_{2g}-orbital Hubbard model on a square lattice,

$$\hat{H} = \sum_{k} \sum_{a,b=1}^{3} \sum_{s=\uparrow,\downarrow} \epsilon_{ab}(k)\hat{c}_{kas}^{\dagger}\hat{c}_{kbs} + U \sum_{j} \sum_{a} \hat{n}_{ja\uparrow}\hat{n}_{ja\downarrow}$$
$$+ U' \sum_{j} \sum_{a>b} \hat{n}_{ja}\hat{n}_{jb} + J' \sum_{j} \sum_{a>b} \hat{c}_{ja\uparrow}^{\dagger}\hat{c}_{ja\downarrow}^{\dagger}\hat{c}_{jb\downarrow}\hat{c}_{jb\uparrow}$$
$$- J_H \sum_{j} \sum_{a>b} \left(2\hat{s}_{ja} \cdot \hat{s}_{jb} + \frac{1}{2}\hat{n}_{ja}\hat{n}_{jb} \right), \tag{1}$$

with $\epsilon_{11/22}(k) = -\frac{\Delta_{t_{2g}}}{3} - 2t_1 \cos k_{x/y} - 2t_2 \cos k_{y/x} - \mu$, $\epsilon_{12/21}(k) = 4t' \sin k_x \sin k_y$, $\epsilon_{33}(k) = \frac{2\Delta_{t_{2g}}}{3} - 2t_3(\cos k_x + \cos k_y) - 4t_4 \cos k_x \cos k_y - \mu$, $\epsilon_{13/23/31/32}(k) = 0$, $J' = J_H$, and $U' = U - 2J_H$. Hereafter, I label the d_{xz}, d_{yz}, and d_{xy} orbitals 1, 2, and 3, respectively, fix the energy unit at eV, and set $\hbar = c = e = \mu_B = k_B = 1$.

The parameters in $\epsilon_{ab}(k)$ are chosen so as to reproduce the electronic structure of Sr_2RuO_4 obtained in local-density approximation (LDA) [9]: I set $(t_1, t_2, t_3, t_4, t', \Delta_{t_{2g}}) = (0.675, 0.09, 0.45, 0.18, 0.03, 0.13)$ and choose μ so that the total occupation number is four. In this choice, the total band-width is about 4, being twice as large as the experimentally estimated value of U [10], and the occupation numbers of the $d_{xz/yz}$ and d_{xy} orbitals are $n_{xz/yz} = 1.38$ and $n_{xy} = 1.25$. The inconsistency of $n_{xz/yz}$ and n_{xy} with the experimental values [11] ($n_{xz/yz} = n_{xy} = 1.33$) arises from the quantitative
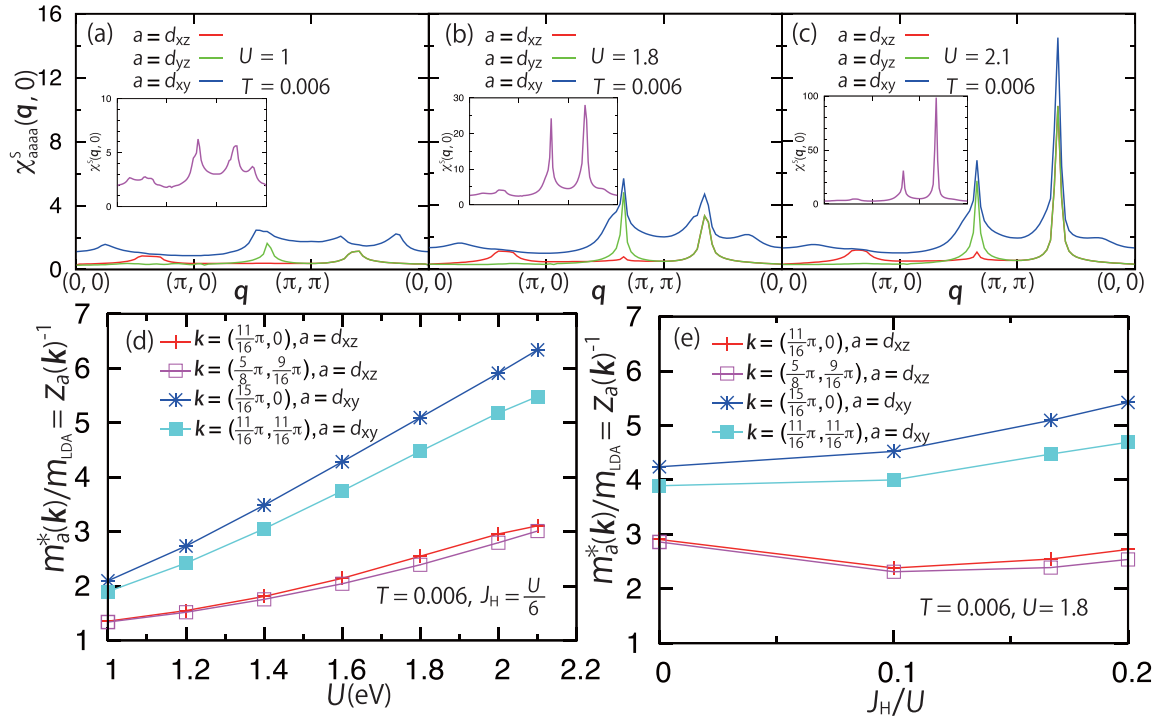
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FIG. 1. (Color online) (a)–(c) $\chi^S_{aaaa}(\boldsymbol{q},0)$ for several U with insets showing the static spin susceptibility, $\chi^S(\boldsymbol{q},0) = \sum_{a,b} \chi^S_{aabb}(\boldsymbol{q},0)$, and (d) U or (e) J_H/U dependence of mass enhancement factor, $z_a(\boldsymbol{k})^{-1} = 1 - \frac{\partial \Sigma^{(R)}_{aa}(\boldsymbol{k},\omega)}{\partial \omega}|_{\omega \to 0}$.

difference that the Fermi surface (FS) of the d_{xy} orbital in the LDA [9] is closer to the inner sheet in $k_x = k_y$ line.

The interaction term is treated by fluctuation-exchange (FLEX) approximation [12,13] that bubble and ladder diagrams only for electron-hole scattering processes are considered. This is suitable for describing electronic properties for moderately strong interaction at low T since this is a perturbation theory beyond MFAs and can treat spatial correlation appropriately [12]. By using the procedure [13] for a paramagnetic phase and taking 64^2 meshes of the Brillouin zone and 2048 Matsubara frequencies, I solve the self-consistent equations by iteration until the relative error of the self-energy is less than 10^{-4}.

The magnetic property and electronic structure of Sr_2RuO_4 can be well described in the FLEX approximation. First, enhancing the spin susceptibility at $\boldsymbol{Q}_{\text{IC-AF}} = (\frac{21}{32}\pi, \frac{21}{32}\pi) \approx (0.66\pi, 0.66\pi)$ [Figs. 1(b) and 1(c)] agrees with the experiment in Ref. [14]; in contrast to MFAs [9,15], its main orbital comes from the d_{xy} orbital. This enhancement arises from the combination of the self-energy of electrons beyond MFAs and orbital-cooperative spin fluctuation: the self-energy causes merging of the nesting vectors for the $d_{xz/yz}$ and d_{xy} orbitals around $\boldsymbol{Q}_{\text{IC-AF}}$ due to the FS deformation for the d_{xy} orbital and mode-mode coupling for spin fluctuations [Figs. 1(a)–1(c)]; this merging leads to enhancing the nondiagonal term of spin fluctuation at $\boldsymbol{Q}_{\text{IC-AF}}$ between these orbitals; this and diagonal terms cause the orbital-cooperative enhancement of spin fluctuation at $\boldsymbol{Q}_{\text{IC-AF}}$. Second, the larger mass enhancement [11] of the d_{xy} orbital than that of the $d_{xz/yz}$ orbital is naturally reproduced due to the stronger (nonlocal) spin fluctuation of the d_{xy} orbital [Figs. 1(d) and 1(e)]. The agreement with

experiment is better than that in dynamical-mean-field theory (DMFT) [16]. Third, the values of $n_{xz/yz}$ and n_{xy} are improved in comparison to the LDA values [9]; e.g., at $(T,U,J_H) = (0.006, 1.8, 0.3)$, these are $(n_{xz/yz}, n_{xy}) = (1.36, 1.28)$. This improvement is similar to that of the DMFT [16].

Then, I derive ρ_{ab} and R_H in the weak-field limit by using the Kubo formulas and considering only the most divergent terms [17] with respect to the QP lifetime [18]. This treatment is correct in the FL and remains reasonable in the metallic phases where a perturbation theory works [19]. In this treatment, $\rho_{ab} = \sigma_{xx}^{-1}$ and $R_H = \sigma_{xy}/H\sigma_{xx}^2$ ($\sigma_{yy} = \sigma_{xx}$ is used) are determined by

$$\sigma_{xx} = \frac{2}{N} \sum_{\boldsymbol{k}} \sum_{\{a\}=1}^{3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \left(-\frac{\partial f(\epsilon)}{\partial \epsilon} \right) \Lambda^{(0)}_{x;ba}(k) \Lambda_{x;dc}(k)$$
$$\times G^{(R)}_{ad}(k) G^{(A)}_{cb}(k), \tag{2}$$

and

$$\frac{\sigma_{xy}}{H} = \frac{1}{N} \sum_{\boldsymbol{k}} \sum_{\{a\}=1}^{3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \left(-\frac{\partial f(\epsilon)}{\partial \epsilon} \right)$$
$$\times \left[\Lambda_{x;ba}(k) \overset{\leftrightarrow}{\frac{\partial}{\partial k_y}} \Lambda_{y;dc}(k) \right] \text{Im} \left[G^{(R)}_{ad}(k) \overset{\leftrightarrow}{\frac{\partial}{\partial k_x}} G^{(A)}_{cb}(k) \right]. \tag{3}$$

Here I use $\sum_{\{a\}} \equiv \sum_{a,b,c,d}$, $k \equiv (\boldsymbol{k}, \epsilon)$, and $[g(x) \overset{\leftrightarrow}{\frac{\partial}{\partial x}} h(x)] \equiv g(x) \frac{\partial h(x)}{\partial x} - \frac{\partial g(x)}{\partial x} h(x)$, $G^{(R \text{ or } A)}_{ab}(k)$ is retarded or advanced Green's function, $f(\epsilon)$ is Fermi function, $\Lambda^{(0)}_{v;ab}(k)$ is
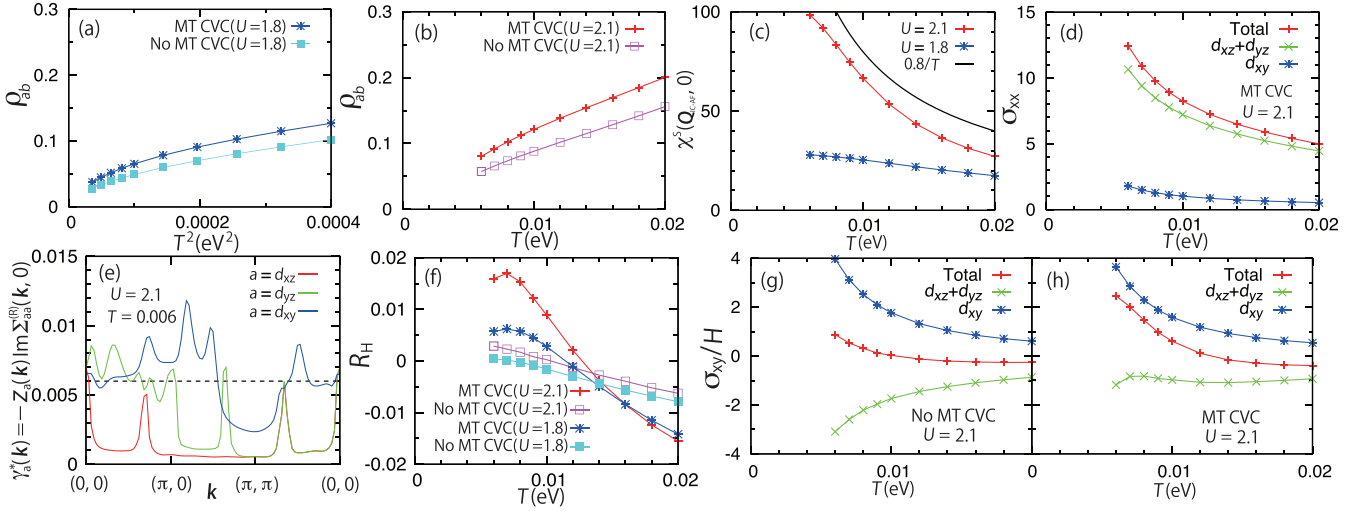
FIG. 2. (Color online) (a) ρ_{ab} against T^2 at $U = 1.8$, (b) ρ_{ab} against T at $U = 2.1$, (c) $\chi^S(\boldsymbol{Q}_{\text{IC-AF}},0)$ against T at $U = 2.1$ and 1.8, (d) σ_{xx} and orbital components with the MT CVC against T at $U = 2.1$, (e) the QP damping against $\boldsymbol{k}$ at $(T,U) = (0.006, 2.1)$, (f) R_H against T at $U = 2.1$ and 1.8, and σ_{xy}/H and orbital components (g) without or (h) with the MT CVC against T at $U = 2.1$. The dashed line in panel (e) corresponds to $T = 0.006$.

renormalized group velocity,

$$\Lambda^{(0)}_{v;ab}(k) = \frac{\partial \epsilon_{ab}(\boldsymbol{k})}{\partial k_v} + \frac{\partial \text{Re}\Sigma^{(R)}_{ab}(k)}{\partial k_v}, \quad (4)$$

where $\Sigma^{(R)}_{ab}(k)$ is the retarded self-energy, and $\Lambda_{v;dc}(k)$ is renormalized current,

$$\Lambda_{v;dc}(k) = \Lambda^{(0)}_{v;dc}(k) + \Delta\Lambda^{(\text{CVC})}_{v;dc}(k), \quad (5)$$

with

$$\Delta\Lambda^{(\text{CVC})}_{v;dc}(k) = \frac{1}{N}\sum_{\boldsymbol{k}'}\sum_{\{A\}=1}^{3}\int_{-\infty}^{\infty}\frac{d\epsilon'}{4\pi i}\mathcal{J}^{(0)}_{dcCD}(k,k')G^{(R)}_{CA}(k')$$
$$\times G^{(A)}_{BD}(k')\Lambda_{v;AB}(k'), \quad (6)$$

where $\mathcal{J}^{(0)}_{dcCD}(k,k')$ is electron-hole four-point vertex function being irreducible with respect to a pair of the retarded and advanced Green's functions. $\Delta\Lambda^{(\text{CVC})}_{v;dc}(k)$ is vital to satisfy conservation laws [20] since it plays the similar role to the backflow correction.

To calculate $\Lambda_{v;dc}(k)$, I use MT four-point vertex function in the FLEX approximation,

$$\mathcal{J}^{(0)}_{abcd}(k,k') = 2i\left(\coth\frac{\epsilon-\epsilon'}{2T} + \tanh\frac{\epsilon'}{2T}\right)\text{Im}V^{(R)}_{acbd}(k-k'), \quad (7)$$

where $V^{(R)}_{acbd}(q)$ is retarded effective interaction in this approximation [13]. This treatment will be sufficient for a qualitative discussion since the neglected terms [21], being of higher order with respect to the QP damping, are much smaller than the MT term in the single-orbital case [7] and the similar result will hold in the present case. Thus, I believe the FLEX approximation including the MT CVC is suitable to analyze the transport properties of the metallic phases not far away from the AF QCP.

We turn to results of ρ_{ab} and R_H. Several quantities as a function of ϵ are calculated by the Padé approximation [22]

using the data for the lowest four Matsubara frequencies. The ϵ and ϵ' integrations are done by discretizing the interval 0.0025 and replacing the upper and lower values by 0.1 and -0.1. $\Lambda_{v;dc}(k)$ is calculated by iteration until its relative error is less than 10^{-4}; the singularity of the principal integral for the term containing $\coth\frac{\epsilon-\epsilon'}{2T}$ is removed by the ϵ' derivatives of its numerator and denominator by using $\text{Im}V^{(R)}_{dCcD}(\boldsymbol{q},0) = 0$.

We first compare ρ_{ab} at $U = 1.8$ and 2.1 in Figs. 2(a) and 2(b); hereafter, I consider $U = 2.1$ ($U = 1.8$) case near (away from) the AF QCP since $\chi^S(\boldsymbol{Q}_{\text{IC-AF}},0)$ shows the Curie-Weiss-like (Pauli paramagnetic) T dependence [Fig. 2(c)]. ρ_{ab} with or without the MT CVC is roughly proportional to T^2 at $U = 1.8$ and to T at $U = 2.1$. Thus, the power of the T dependence of ρ_{ab} is determined by the self-energy and becomes one near the AF QCP.

To reveal the role of each t_{2g} orbital in ρ_{ab}, orbital components of σ_{xx} with the MT CVC at $U = 2.1$ are shown in Fig. 2(d); the component of the d_{xz} and d_{yz} orbitals or the d_{xy} orbital is calculated from the equation that $\sum_{\{a\}=1}^{3}$ in Eq. (2) is replaced by $\sum_{\{a\}=1}^{2}$ or $\sum_{\{a\}=3}$, respectively. The main contribution to σ_{xx} (σ_{yy}) comes from the d_{xz} (d_{yz}) orbital in contrast to that of the spin fluctuation. This result arises from the smaller QP damping and larger renormalized group velocity of the $d_{xz/yz}$ orbital than those of the d_{xy} orbital. Note that similar results are obtained at $U = 1.8$ (not shown).

In addition, the QP damping of the d_{xz} orbital around $\boldsymbol{k} = \boldsymbol{Q}_{\text{IC-AF}}$ becomes a hot spot at $U = 2.1$, although that around $\boldsymbol{k} = (\frac{23}{32}\pi,0) \approx (0.72\pi,0)$ remains a cold spot [Fig. 2(e)]. [At the cold (hot) spot, the QP damping is (is not) much smaller than temperature considered.] Thus, the origin of the T-linear ρ_{ab} at $U = 2.1$ is the hot-spot structure of the QP damping of the $d_{xz/yz}$ orbital around $\boldsymbol{k} = \boldsymbol{Q}_{\text{IC-AF}}$. I emphasize that this T-linear ρ_{ab} is not due to a breakdown of perturbation theory.

We next compare R_H at $U = 2.1$ and 1.8 in Fig. 2(f). There are two main and four secondary results. The main results are, first, that the peak of R_H at $T = 0.007$ is induced by
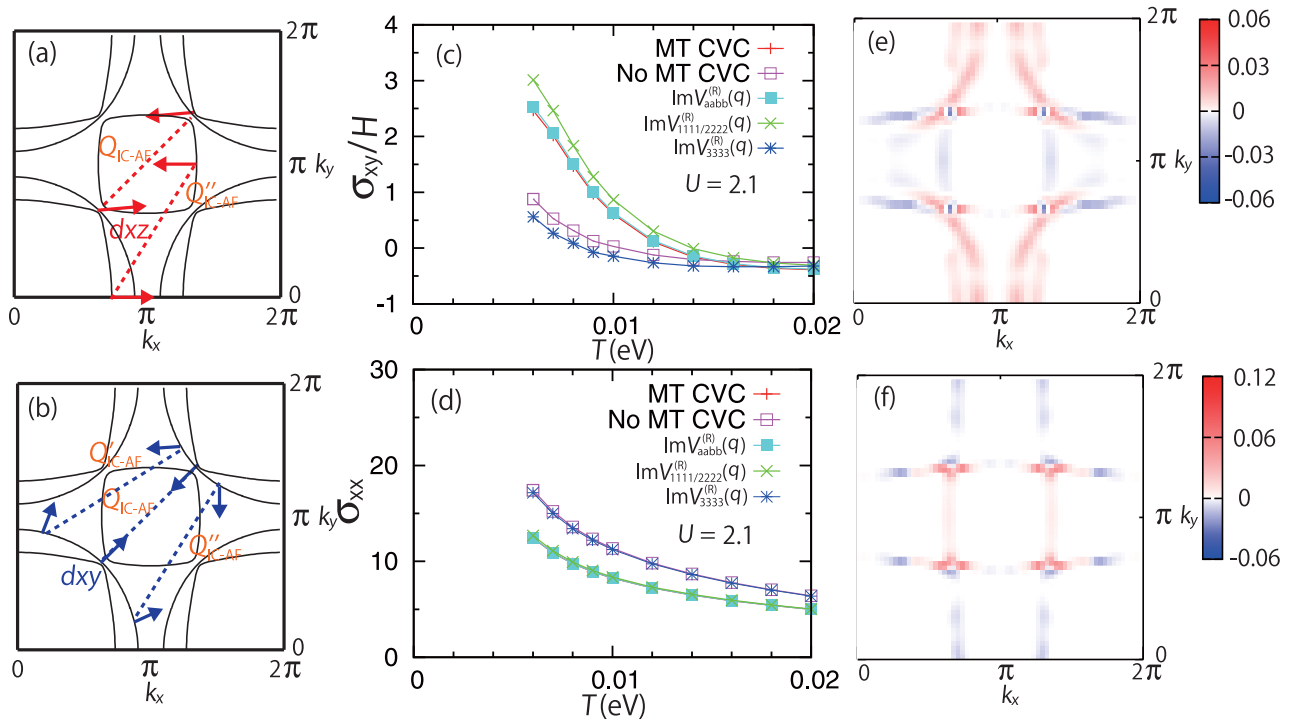
FIG. 3. (Color online) Schematic pictures of the currents of (a) the d_{xz} and (b) the d_{xy} orbital connected by the MT CVC, (c) σ_{xy}/H and (d) σ_{xx} against T at $U = 2.1$ for several special cases whose data are obtained by using part of Im$V_{acbd}^{(R)}(q)$ as the CVC, (e) $\sigma_{xy}(\boldsymbol{k})/H$ with the MT CVC against $\boldsymbol{k}$ at $(T,U) = (0.006,2.1)$, and (f) the difference between the $d_{xz} + d_{yz}$ components of $\sigma_{xy}(\boldsymbol{k})/H$ with and without the MT CVC against $\boldsymbol{k}$ at $(T,U) = (0.006,2.1)$.

the MT CVC at $U = 2.1$ and 1.8; second, that the Curie-Weiss-like T dependence of R_{H} is absent at $U = 2.1$, although $\chi^{\mathrm{S}}(\boldsymbol{Q}_{\mathrm{IC\text{-}AF}},0)$ shows the Curie-Weiss-like behavior. The first secondary result is that the difference between R_{H} without the MT CVC at $U = 2.1$ and 1.8 is small, although the QP dampings are different. This arises from the small effects of the QP damping since its effects on σ_{xy}/H and σ_{xx}^2 are nearly canceled out. The second is that the values of these R_{H} are nearly zero. The third is that at $U = 2.1$ and 1.8, the MT CVC causes the positive enhancement of R_{H} in the range of $0.006 \leqslant T \leqslant 0.012$ and the negative enhancement of R_{H} in the range of $0.014 \leqslant T \leqslant 0.02$. The fourth is that the positive enhancement at $U = 2.1$ is larger than that at $U = 1.8$, while the negative enhancement at $U = 2.1$ is of the same order of magnitude as that at $U = 1.8$.

To understand the two main and last three secondary results, I present orbital components of σ_{xy}/H, calculated in a similar way to σ_{xx}, without or with the MT CVC at $U = 2.1$ in Fig. 2(g) or 2(h); the following results (i)–(iv) remain qualitatively the same at $U = 1.8$ (not shown). (i) The sign of the component of the d_{xz} and d_{yz} orbitals is minus, and that of the d_{xy} orbital is plus. (ii) The components of the d_{xz} and d_{yz} orbitals and the d_{xy} orbital without the MT CVC are nearly the same in magnitude. Thus, the nearly zero R_{H} without the MT CVC arises from the comparable and opposite-sign components of these orbitals. (iii) The magnitude decrease for the $d_{xz/yz}$ orbital due to the MT CVC is larger than that for the d_{xy} orbital in the range of $0.006 \leqslant T \leqslant 0.012$, while the magnitude decrease for these t_{2g} orbitals are very small in the higher-T region. Combining

this with the effect of the MT CVC on σ_{xx}, we find that the positive enhancement of R_{H} in the low-T region arises from the combination of the decrease of σ_{xx}^2 and positive enhancement of σ_{xy}/H due to the MT CVC, and that the negative enhancement of R_{H} in the high-T region arises from the combination of the decrease of σ_{xx}^2 due to the MT CVC and the negative sign of σ_{xy}/H with the MT CVC. In addition, the larger positive enhancement of R_{H} at $U = 2.1$ than at $U = 1.8$ arises from the larger reduction of σ_{xx}^2 due to the MT CVC, and the small magnitude difference between the negative enhancement at $U = 2.1$ and 1.8 arises from the small effects of the MT CVC on σ_{xx} and σ_{xy}/H at high T. (iv) The component of the $d_{xz/yz}$ orbital with the MT CVC shows the similar peak to that of R_{H}, although such peak does not appear in the total component. [Note that no peak in σ_{xy}/H does not contradict with the peak in R_{H} since R_{H} is $(\sigma_{xy}/H) \times \sigma_{xx}^{-2}$.] This result implies the peak and the absence of the Curie-Weiss-like enhancement of R_{H} are related to orbital dependence of the MT CVC.

Then, I analyze how the MT CVC affects the current of each orbital. Combining Eqs. (4)–(7) with the facts in the present model that $\Lambda_{v;aa}^{(0)}(k)$ are much larger than $\Lambda_{v;ab(\neq a)}^{(0)}(k)$ due to the larger intraorbital hopping integrals and that the dominant terms of Im$V_{acbd}^{(R)}(q)$ are Im$V_{aabb}^{(R)}(q)$ due to stronger spin fluctuation than other fluctuations, we find the dominant effects of the MT CVC in the present model are the connections between the intraorbital terms of the currents at $\boldsymbol{k}$ and $\boldsymbol{k}'$ near the Fermi level. In particular, since the main terms of Im$V_{aabb}^{(R)}(q)$ are the low-ω terms at $\boldsymbol{q} = \boldsymbol{Q}_{\mathrm{IC\text{-}AF}}$ and the secondary are the low-ω terms at $\boldsymbol{q} = \boldsymbol{Q}'_{\mathrm{IC\text{-}AF}} = (\pi,\frac{21}{32}\pi) \approx$

$(\pi, 0.66\pi)$ or $\boldsymbol{Q}''_{\text{IC-AF}} = (\frac{21}{32}\pi, \pi) \approx (0.66\pi, \pi)$ (not shown), we see from Figs. 3(a) and 3(b), first, that the main effects are the magnitude decreases of the currents of the $d_{xz/yz}$ and d_{xy} orbitals at $\boldsymbol{k} = \boldsymbol{Q}_{\text{IC-AF}}$ near the Fermi level, arising from the low-ω terms of $\text{Im}V^{(\text{R})}_{1111/2222}(\boldsymbol{Q}_{\text{IC-AF}}, \omega)$ and $\text{Im}V^{(\text{R})}_{3333}(\boldsymbol{Q}_{\text{IC-AF}}, \omega)$, respectively; second, that the secondary effects are the magnitude decrease of the current of the d_{xz} [d_{yz}] orbital at $\boldsymbol{k} = (\frac{23}{32}\pi, 0) \approx (0.72\pi, 0)$ [$(0, \frac{23}{32}\pi) \approx (0, 0.72\pi)$] due to the low-$\omega$ terms of $\text{Im}V^{(\text{R})}_{1111}(\boldsymbol{Q}''_{\text{IC-AF}}, \omega)$ [$\text{Im}V^{(\text{R})}_{2222}(\boldsymbol{Q}'_{\text{IC-AF}}, \omega)$] and the angle changes of the current of the d_{xy} orbital at $\boldsymbol{k} = (\frac{7}{8}\pi, 0) \approx (0.88\pi, 0)$ and $(0, \frac{7}{8}\pi) \approx (0, 0.88\pi)$ due to the low-ω terms of $\text{Im}V^{(\text{R})}_{3333}(\boldsymbol{q}, \omega)$ at $\boldsymbol{q} = \boldsymbol{Q}''_{\text{IC-AF}}$ and $\boldsymbol{Q}'_{\text{IC-AF}}$, respectively. In addition to these main and secondary effects, the MT CVCs arising from the low-ω terms of $\text{Im}V^{(\text{R})}_{aaaa}(\boldsymbol{q})$ whose $\boldsymbol{q}$ slightly differs from $\boldsymbol{Q}_{\text{IC-AF}}$ or $\boldsymbol{Q}'_{\text{IC-AF}}$ or $\boldsymbol{Q}''_{\text{IC-AF}}$ cause the angle changes of the corresponding currents near the Fermi level.

Among these effects, the most important effect on R_{H} arises from the magnitude decrease of the current of the $d_{xz/yz}$ orbital around $\boldsymbol{Q}_{\text{IC-AF}}$ near the Fermi level. One of the facts is that the T dependence of σ_{xy}/H and σ_{xx} with the MT CVC are almost reproduced by using the MT CVC arising from $\text{Im}V^{(\text{R})}_{1111/2222}(\boldsymbol{q})$ [Figs. 3(c) and 3(d)]. This orbital dependence arises mainly from the smaller QP damping of the $d_{xz/yz}$ orbital than that of the d_{xy} orbital since the kernel of the MT CVC for the $d_{xz/yz}$ [d_{xy}] orbital is inversely proportional to the QP damping of the $d_{xz/yz}$ [d_{xy}] orbital and proportional to $\text{Im}V^{(\text{R})}_{1111/2222}(\boldsymbol{q})$ [$\text{Im}V^{(\text{R})}_{3333}(\boldsymbol{q})$]. The other is that the most drastic effect of the MT CVC on σ_{xy}/H is the positive enhancement of the $d_{xz} + d_{yz}$ component of $\sigma_{xy}(\boldsymbol{k})/H$ around $\boldsymbol{k} = \boldsymbol{Q}_{\text{IC-AF}}$, while the secondary is the negative enhancement of that around $\boldsymbol{k} = (\frac{11}{16}\pi, \frac{\pi}{4}) \approx (0.69\pi, 0.25\pi)$ and $(\frac{\pi}{4}, \frac{11}{16}\pi) \approx (0.25\pi, 0.69\pi)$ [Figs. 3(e) and 3(f)].

From those results of the effects of the MT CVC, we find the peak of R_{H} with the MT CVC arises from the peak of the $d_{xz} + d_{yz}$ component of σ_{xy}/H as a result of the competition between the positive enhancement around $\boldsymbol{k} = \boldsymbol{Q}_{\text{IC-AF}}$ and negative enhancement around $\boldsymbol{k} \approx (0.69\pi, 0.25\pi)$ and $(0.25\pi, 0.69\pi)$ due to the MT CVC arising from spin fluctuations of the $d_{xz/yz}$ orbital.

In addition, combining the orbital dependence of the MT CVC with the equations of the dependence of σ_{xx} and σ_{xy}/H on the leading order of the angle change, $\Delta\varphi_{ab}(k) = \varphi_{ab}(k) - \varphi^{(0)}_{ab}(k)$, which are, respectively,

$$\left|\Lambda^{(0)}_{ba}(k)\right| \cos\varphi^{(0)}_{ba}(k) |\Lambda_{dc}(k)| \cos\varphi^{(0)}_{dc}(k) \left[1 - \frac{\Delta\varphi_{dc}(k)^2}{2}\right] \quad (8)$$

and

$$|\Lambda_{ba}(k)| \cos\varphi^{(0)}_{ba}(k) |\Lambda_{dc}(k)| \cos\varphi^{(0)}_{dc}(k) \frac{\partial\varphi_{dc}(k)}{\partial k_y}$$
$$+ |\Lambda_{ba}(k)| \sin\varphi^{(0)}_{ba}(k) \frac{\partial\varphi_{ba}(k)}{\partial k_y} |\Lambda_{dc}(k)| \sin\varphi^{(0)}_{dc}(k), \quad (9)$$

we find the absence of the Curie-Weiss-like enhancement of R_{H} near the AF QCP arises from the absence of the angle change of the current due to the main term of the MT CVC. Note that although the Curie-Weiss-like T-dependent spin fluctuation leads to the Curie-Weiss-like T dependence

of the magnitude and angle changes of the current through $\text{Im}V^{(\text{R})}_{aabb}(\boldsymbol{q})$ in the MT CVC, the effects of its T dependence of the magnitude change on σ_{xy}/H and σ^2_{xx} are nearly canceled out, while the Curie-Weiss-like T dependence of the angle change appearing in σ_{xy}/H causes the Curie-Weiss-like enhancement of R_{H}.

Before comparison with experiment, I remark on main similarities and differences between the present case and the single-orbital case [7] and propose the realization of the similar transport properties in other systems.

For ρ_{ab}, the similarity is the T-linear dependence near the AF QCP, and the difference is the difference between the main orbitals for ρ_{ab} and spin fluctuation. This difference arises from the facts that σ_{xx} is inversely proportional to the QP damping within the leading order, and that the strong spin fluctuation enhances the QP damping. Since these facts hold in metallic phases of other multiorbital systems, this orbital-dependent transport is realized in other systems. It should be noted that due to this difference in the main orbital, the criticality of ρ_{ab} (i.e., the power of its T dependence) is not always connected with the criticality of fluctuation (i.e., the kind of the QCP) in multiorbital systems. In the present case, these criticalities become the same due to orbital-cooperative enhancement of spin fluctuation at $\boldsymbol{Q}_{\text{IC-AF}}$.

For R_{H}, the similarity is the considerable effects of the MT CVC on its low-T values, and the differences are the absence of the Curie-Weiss-like T dependence and the peak without the peak of the T dependence of the spin susceptibility. Since the former difference is related to the $\boldsymbol{k}$ dependence of the main term of the MT CVC, as explained, this finding gives another ubiquitous mechanism for the T dependence of R_{H} near an AF QCP: the Curie-Weiss-like T-dependent spin fluctuation, characterizing the AF QCP, does not cause the Curie-Weiss-like T dependence of R_{H} if the directions of the currents connected by the MT CVC arising from this spin fluctuation are antiparallel. This will be realized in some single-orbital or multiorbital systems near an AF QCP. In addition, the peak of R_{H} will be realized in some metallic phases satisfying four conditions (e.g., some transition metal oxides and organic conductors): electron correlation is strong; quasi-1D orbitals form the conducting bands; there are opposite-sign components of $\sigma_{xy}(\boldsymbol{k})/H$ of these orbitals; there are at least two nesting vectors for these orbitals, each of which affects each component of $\sigma_{xy}(\boldsymbol{k})/H$ through the MT CVC arising from the corresponding spin fluctuation. These conditions are necessary for the competition between the opposite-sign enhancement of these opposite-sign components of $\sigma_{xy}(\boldsymbol{k})/H$ of the quasi-1D orbitals due to the MT CVC arising from spin fluctuations of these orbitals.

Finally, we compare the results with experiment. The results with the MT CVC at $U = 1.8$ reproduce experimental results [5,6] of Sr_2RuO_4, the T-square ρ_{ab}, monotonic increase of R_{H} in $0.007 \leqslant T \leqslant 0.02$, crossing of R_{H} over zero, and peak of R_{H} at $T \sim 0.007$. (Although those [5,6] are reproduced in relaxation-time approximation [23], neglecting all the CVCs, by choosing some parameters of the QP damping, I do not use any such parameters.) Since the small quantitative difference in the value of T where R_{H} crosses over zero (which is 0.014 in an experiment [6]) is related to the difference in the occupation numbers, an analysis by the model having

the same occupation numbers is a future work. Then, the results with the MT CVC at $U = 2.1$ can explain the T-linear ρ_{ab} [3] in $Sr_2Ru_{0.075}Ti_{0.025}O_4$. Since the measurement of R_H in $Sr_2Ru_{0.075}Ti_{0.025}O_4$ has been restricted to a low-T value [24], the T dependence of R_H obtained near the AF QCP can be tested in further measurement if the main effect of Ti substitution can be assumed to make the system near the AF QCP compared with Sr_2RuO_4.

In summary, I have studied several electronic properties of the ruthenates near and away from the AF QCP in the FLEX approximation including the MT CVC. I have found, first, that the enhancement [14] of spin fluctuation at $\boldsymbol{Q}_{\text{IC-AF}}$ arises from the combination of the self-energy of electrons beyond MFAs and orbital-cooperative spin fluctuation; second, that the larger mass enhancement [11] of the d_{xy} orbital arises from the stronger spatial correlation of that orbital; third, that the nonmonotonic T dependence of R_H [6] arises from the competition between the opposite-sign enhancement of $\sigma_{xy}(\boldsymbol{k})/H$ of the d_{xz} and d_{yz} orbitals around $\boldsymbol{k} = \boldsymbol{Q}_{\text{IC-AF}}$ and $\boldsymbol{k} \approx (0.69\pi, 0.25\pi)$ and $(0.25\pi, 0.69\pi)$ due to the MT CVCs arising from spin fluctuations of these orbitals. Also, I have explained that the T-linear ρ_{ab} of $Sr_2Ru_{0.075}Ti_{0.025}O_4$ [3] can

be understood as the hot-spot structure of the QP damping of the $d_{xz/yz}$ orbital around $\boldsymbol{k} = \boldsymbol{Q}_{\text{IC-AF}}$. I have proposed, first, that the T dependence of R_H near the AF QCP can be experimentally tested in $Sr_2Ru_{0.075}Ti_{0.025}O_4$ if the main effect of Ti substitution can be assumed to tune the system to the vicinity of the AF QCP; second, that multiorbital systems in a metallic phase show the inplane transport whose main orbital differs from that for spin fluctuation; third, that some strongly correlated electron systems having quasi-1D orbitals show the peak of R_H at low T without the peak of the T dependence of the spin susceptibility; fourth, that the absence of the Curie-Weiss-like enhancement of R_H near an AF QCP is realized in some single-orbital or multiorbital systems where the angle change of the current due to the main term of the MT CVC is absent.

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