Dissipation due to pure spin-current generated by spin pumping

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Based on spin-dependent transport theory and thermodynamics, we develop a generalized theory of the Joule heating in the presence of a spin current. Along with the conventional Joule heating consisting of an electric current and electrochemical potential, it is found that the spin current and spin accumulation give an additional dissipation because the spin-dependent scatterings inside bulk and ferromagnetic/nonmagnetic interface lead to a change of entropy. The theory is applied to investigate the dissipation due to pure spin-current generated by spin pumping across a ferromagnetic/nonmagnetic/ferromagnetic multilayer. The dissipation arises from an interface because the spin pumping is a transfer of both the spin angular momentum and the energy from the ferromagnet to conduction electrons near the interface. It is found that the dissipation is proportional to the enhancement of the Gilbert damping constant by spin pumping.

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I. INTRODUCTION

Dissipation due to electron transport in a conductor is an important issue for both fundamental and applied physics [1–5]. According to electron transport theory [6], the conductivity of the electron becomes finite because of impurity scattering inside the conductor, which leads to Joule heating $J_e E$, where J_e and E are the electric current density and electric field, respectively. Motivated to reduce power consumption due to Joule heating, as well as because of a fundamental interest in its quantum mechanical nature, the generation of a pure spin-current by spin pumping, spin-Seebeck effect, or spin-Hall effect has been extensively investigated [7–15].

Dissipation is associated with the production of entropy. Spin-flip processes and spin-dependent scatterings within a bulk ferromagnet (F) or nonmagnet (N) and at an F/N interface mix the spin-up and spin-down states, leading to a change of the entropy. Therefore a physical system, such as a F/N metallic multilayer, carrying a pure spin-current, still dissipates energy even in the absence of an electric current. A quantitative evaluation of the dissipation due to pure spin-current therefore is a fundamentally important problem.

In 1987, Johnson and Silsbee [1] studied the surface and bulk transport coefficients for spin conduction, and the associated entropy production rates, without considering the rate of interface heating. More recently, Sears and Saslow [4] used irreversible thermodynamics to study interface heating due to electric current in a magnetic system, and Tulapurkar and Suzuki [5] used the Boltzmann equation to investigate bulk and interface heating for spin conduction. Reference [5] shows that, roughly speaking, the dissipation due to spin current is proportional to the square of the spin polarization of the conduction electrons, indicating that the heating associated with the spin current is much smaller than that due to the electric current. However, these works consider only a collinear alignment of the magnetizations in a F/N multilayer, so only the longitudinal components of the spin current and spin accumulation (i.e., spin chemical potential, proportional to the nonequilibrium spin density) appear. (*Longitudinal* and *transverse* will be used to mean that the direction of the spin polarization is collinear or normal to the local magnetization.) On the other hand, in many physical phenomena, such as spin torque switching [16] and spin pumping [7,8], a noncollinear alignment of the magnetizations generally appears, in which transverse spin current and spin accumulation exist. For example, spin pumping is a generation of the transverse spin current by the transfer of spin angular momentum from the ferromagnetic layer to the conduction electrons [7,8,17–22]. Bulk heating due to spin pumping in a magnetic wire within a domain wall (driven by $\mathbf{m}\times\mathbf{H}$) has also been studied [3], but was not extended to include interface heating. In these works, the main contribution to the dissipation arises from the electric current. The present work develops a unified theory of dissipation which enables the simultaneous evaluation of both bulk and interface heating in a ferromagnetic system, with the spin current having arbitrary alignment of the magnetizations. Also, an evaluation of the dissipation due to a pure spin-current is indispensable for comparison with experiments that determine the rate of heating.

This paper develops a general theory of dissipation in the presence of spin current based on the spin-dependent transport theory and thermodynamics. It is found that, along with the conventional Joule heating, the spin current $\mathbf{I}_s$ (or its density $\mathbf{J}_s$) and spin accumulation $\boldsymbol{\mu}$ contribute to the bulk and interface dissipations, as shown in Eqs. (17) and (18). We apply the theory to evaluate the dissipation due to a pure spin-current generated by spin pumping in the ferromagnetic (F_1)/nonmagnetic (N)/ferromagnetic (F_2) multilayer. Spin pumping provides an interesting example to study the dissipation problem of pure spin-current. In spin pumping, electric current is absent throughout the system. The electron transport is described by a one-dimensional equation, and an external temperature gradient is absent, which makes evaluation of the dissipation simple compared with the spin-Seebeck effect or spin-Hall effect. It is found that the dissipation is proportional to the enhancement of the Gilbert damping by spin pumping. The amount of the dissipation due to the spin pumping is maximized for an orthogonal alignment of the two magnetizations. For the conditions we study, the maximum dissipation is estimated to be two to three orders

of magnitude smaller than the dissipation due to the electric current when there is spin torque switching.

The paper is organized as follows. In Sec. II, the system we consider is illustrated. Section III formulates a theory of dissipation of spin-polarized conduction electrons, using diffusive spin transport theory and thermodynamics. Section IV studies the relationship between the dissipation due to spin pumping and the equation developed in the previous section. Section V quantitatively evaluates the dissipation due to spin pumping. Section VI compares the spin pumping dissipation with the dissipation in the case of spin torque switching. Section VII provides our conclusions.

II. SPIN PUMPING IN F/N/F SYSTEM

Figure 1 shows a schematic view of the $F_1/N/F_2$ ferromagnetic multilayer system, where $\mathbf{m}_1$ and $\mathbf{m}_2$ are unit vectors pointing along the magnetizations of the F_1 and F_2 layers, respectively. Where needed, subscripts $k = 1,2$ denote the F_k layer. The thickness of the F_k layer is denoted by d_k. The F_1 and F_2 layers lie in the regions $-d_1 \leqslant x \leqslant 0$ and $0 \leqslant x \leqslant d_2$, respectively. We assume that the spin current is conserved in the N layer, and thus consider its zero-thickness limit because a typical value for the spin diffusion length of an N layer is much greater than its thickness: for example, the spin diffusion length for Cu is on the order of 100 nm, whereas experimental thicknesses are less than 5 nm [7,8,23].

Steady precession of $\mathbf{m}_1$ with the cone angle θ can be excited by microwave radiation of the angular velocity ω for ferromagnetic resonance (FMR) in the F_1 layer. Then, the F_1 layer pumps the pure spin-current

$$\mathbf{I}_s^{\text{pump}} = \frac{\hbar}{4\pi}\left(g_{\text{r(F}_1)}^{\uparrow\downarrow}\mathbf{m}_1\times\frac{d\mathbf{m}_1}{dt} + g_{\text{i(F}_1)}^{\uparrow\downarrow}\frac{d\mathbf{m}_1}{dt}\right), \tag{1}$$

where the real and imaginary parts of the mixing conductance are denoted by $g_{\text{r}}^{\uparrow\downarrow}$ and $g_{\text{i}}^{\uparrow\downarrow}$, respectively [24,25]. The pumped spin current creates spin accumulations in the ferromagnetic ($\boldsymbol{\mu}_{\text{F}}$) and nonmagnetic ($\boldsymbol{\mu}_{\text{N}}$) layers, which induce backflow

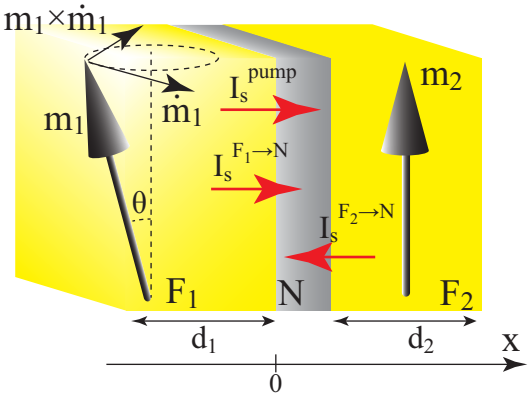

FIG. 1. (Color online) Schematic view of the $F_1/N/F_2$ ferromagnetic multilayer system. The directions of $\dot{\mathbf{m}}_1$ and $\mathbf{m}_1\times\dot{\mathbf{m}}_1$ are indicated by arrows.

spin current (into N) [20,24–26], given by

$$\begin{aligned}\mathbf{I}_s^{\text{F}\to\text{N}} = \frac{1}{4\pi}\Big[&\frac{(1-\gamma^2)g}{2}\mathbf{m}\cdot(\boldsymbol{\mu}_{\text{F}}-\boldsymbol{\mu}_{\text{N}})\mathbf{m}\\ &- g_{\text{r}}^{\uparrow\downarrow}\mathbf{m}\times(\boldsymbol{\mu}_{\text{N}}\times\mathbf{m}) - g_{\text{i}}^{\uparrow\downarrow}\boldsymbol{\mu}_{\text{N}}\times\mathbf{m}\\ &+ t_{\text{r}}^{\uparrow\downarrow}\mathbf{m}\times(\boldsymbol{\mu}_{\text{F}}\times\mathbf{m}) + t_{\text{i}}^{\uparrow\downarrow}\boldsymbol{\mu}_{\text{F}}\times\mathbf{m}\Big].\end{aligned} \tag{2}$$

The total interface conductance $g = g^{\uparrow\uparrow} + g^{\downarrow\downarrow}$ and the spin polarization of the interface conductance $\gamma = (g^{\uparrow\uparrow} - g^{\downarrow\downarrow})/(g^{\uparrow\uparrow} + g^{\downarrow\downarrow})$ are defined from the interface resistance of the spin-ν ($\nu = \uparrow, \downarrow$) electrons $r^{\nu\nu} = (h/e^2)S/g^{\nu\nu}$, where S is the cross section area. The real and imaginary parts of the transmission mixing conductance at the F/N interface are denoted by $t_{\text{r(i)}}^{\uparrow\downarrow}$. The condition that the spin current is conserved in the N layer can be expressed as

$$\mathbf{I}_s^{\text{pump}} + \mathbf{I}_s^{\text{F}_1\to\text{N}} + \mathbf{I}_s^{\text{F}_2\to\text{N}} = \mathbf{0}. \tag{3}$$

III. DISSIPATION FORMULAS

To obtain the dissipation due to spin pumping, it is necessary to investigate how the spin accumulation relaxes inside the F layers and at the F/N interfaces. For generality we include the terms related to the electric current and field, although these are absent in the spin-pumped system. The spin accumulation in the ferromagnetic layer relates to the distribution function $\hat{F} = (f_0 + \mathbf{f}\cdot\boldsymbol{\sigma})/2$, which is a 2×2 matrix in spin space and satisfies the Boltzmann equation [5,26–33], via [34] $\boldsymbol{\mu} = \int_{\varepsilon_{\text{F}}} \text{Tr}[\boldsymbol{\sigma}\hat{F}]d\varepsilon$, $\boldsymbol{\sigma}$ being the Pauli matrices. The charge and spin distributions are denoted by f_0 and $\mathbf{f}$, respectively. The distributions for spin parallel, $f_\uparrow = (f_0 + \mathbf{m}\cdot\mathbf{f})/2$, or antiparallel, $f_\downarrow = (f_0 - \mathbf{m}\cdot\mathbf{f})/2$, to the local spin, give the longitudinal spin. On the other hand, the components of $\mathbf{f}$ orthogonal to $\mathbf{m}$ correspond to the transverse spin. Below, we introduce the following notations to distinguish the longitudinal (L) and transverse (T) components of the spin current $\mathbf{I}_s$ and spin accumulation $\boldsymbol{\mu}$:

$$\mathbf{I}_s^{\text{L}} = (\mathbf{m}\cdot\mathbf{I}_s)\mathbf{m}, \tag{4}$$

$$\mathbf{I}_s^{\text{T}} = \mathbf{m}\times(\mathbf{I}_s\times\mathbf{m}), \tag{5}$$

$$\boldsymbol{\mu}^{\text{L}} = (\mathbf{m}\cdot\boldsymbol{\mu})\mathbf{m}, \tag{6}$$

$$\boldsymbol{\mu}^{\text{T}} = \mathbf{m}\times(\boldsymbol{\mu}\times\mathbf{m}), \tag{7}$$

where $\mathbf{I}_s$ equals $\mathbf{I}_s^{\text{pump}} + \mathbf{I}_s^{\text{F}_1\to\text{N}}$ at the F_1/N interface and $-\mathbf{I}_s^{\text{F}_2\to\text{N}}$ at the F_2/N interface, respectively. The spin current density is denoted as $\mathbf{J}_s = \mathbf{I}_s/S$.

We first consider the diffusive transport for the longitudinal spin [27–33]. The longitudinal spin accumulation relates to the electrochemical potential $\bar{\mu}_\nu = \mu_0 + \delta\mu_\nu - eV$ ($\nu = \uparrow,\downarrow$) via $\boldsymbol{\mu}^{\text{L}} = (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)\mathbf{m}$, where μ_0, $\delta\mu_\nu$, and $-eV$ are the chemical potential in equilibrium, its deviation in nonequilibrium, and the electric potential. The longitudinal electron density $n_\nu = \int d^3\mathbf{k}/(2\pi)^3 f_\nu$ and its current density

$j_\nu = \int d^3\mathbf{k}/(2\pi)^3 v_x f_\nu$ satisfy [27]

$$\frac{\partial n_\nu}{\partial t} + \frac{\partial j_\nu}{\partial x} = -\frac{n_\nu}{2\tau_{\rm sf}^\nu} + \frac{n_{-\nu}}{2\tau_{\rm sf}^{-\nu}}, \tag{8}$$

where the spin-flip scattering time from spin state ν to $-\nu$ (up to down or down to up) is denoted by $\tau_{\rm sf}^\nu$. The charge density $n_e = -e(n_\uparrow + n_\downarrow)$ and electric current density $J_e = -e(j_\uparrow + j_\downarrow)$ satisfy the conservation law, $\partial n_e/\partial t + \partial J_e/\partial x = 0$. The electron density n_ν is related to $\delta\mu_\nu$ via $n_\nu \simeq \mathcal{N}_\nu \delta\mu_\nu$, where $\mathcal{N}_\nu$ is the density of states of the spin-ν electron at the Fermi level. In the diffusive metal, j_ν can be expressed as

$$j_\nu = -\frac{\sigma_\nu}{e^2}\frac{\partial \bar{\mu}_\nu}{\partial x}, \tag{9}$$

where the conductivity of the spin-ν electron σ_ν relates to the diffusion constant D_ν and the density of state $\mathcal{N}_\nu$ via the Einstein law $\sigma_\nu = e^2\mathcal{N}_\nu D_\nu$. Detailed balance [35], $\mathcal{N}_\uparrow/\tau_{\rm sf}^\uparrow = \mathcal{N}_\downarrow/\tau_{\rm sf}^\downarrow$, is satisfied in the steady state. The spin polarizations of the conductivity and the diffusion constant are denoted by $\beta = (\sigma_\uparrow - \sigma_\downarrow)/(\sigma_\uparrow + \sigma_\downarrow)$ and $\beta' = (D_\uparrow - D_\downarrow)/(D_\uparrow + D_\downarrow)$. From Eq. (8), the longitudinal spin accumulation in the steady state satisfies the diffusion equation [27]

$$\frac{\partial^2}{\partial x^2}\boldsymbol{\mu}^{\rm L} = \frac{1}{\lambda_{\rm sd(L)}^2}\boldsymbol{\mu}^{\rm L}, \tag{10}$$

where $\lambda_{\rm sd(L)}$ is the longitudinal spin diffusion length defined as $1/\lambda_{\rm sd(L)}^2 = [1/(D_\uparrow\tau_{\rm sf}^\uparrow) + 1/(D_\downarrow\tau_{\rm sf}^\downarrow)]/2$. The longitudinal spin current density can be expressed as

$$\mathbf{J}_s^{\rm L} = -\frac{\hbar}{2e^2}\frac{\partial}{\partial x}(\sigma_\uparrow\bar{\mu}_\uparrow - \sigma_\downarrow\bar{\mu}_\downarrow)\mathbf{m}. \tag{11}$$

The issue of whether transport of the transverse spin in the ferromagnet is ballistic or diffusive has been discussed in [16,25,36] and [29–32]. These two theories are supported by different experiments [26,37–39], and the validity of each theory is still controversial. The present work considers the case of diffusive transport for generality. Ballistic transport corresponds to the limit of $\lambda_J, t_{\rm r(i)}^{\uparrow\downarrow} \to 0$, where λ_J is the spin coherence length introduced below. In the steady state, the transverse spin accumulation $\boldsymbol{\mu}^{\rm T} = \boldsymbol{\mu} - \boldsymbol{\mu}^{\rm L}$ obeys [26,29]

$$\frac{\partial^2}{\partial x^2}\boldsymbol{\mu}^{\rm T} = \frac{1}{\lambda_J^2}\boldsymbol{\mu}^{\rm T}\times\mathbf{m} + \frac{1}{\lambda_{\rm sd(T)}^2}\boldsymbol{\mu}^{\rm T}, \tag{12}$$

where the first term on the right-hand side describes the precession of the spin accumulation around the magnetization due to the exchange coupling. The exchange coupling constant $J_{\rm sd}$ is in relation to the spin coherence length λ_J via $\lambda_J = \sqrt{\hbar(D_\uparrow + D_\downarrow)/(2J_{\rm sd})}$ [28–33]. The spin diffusion length of the transverse spin is $\lambda_{\rm sd(T)}$ [29]. The transverse spin current density is related to the transverse spin accumulation via [26,29]

$$\mathbf{J}_s^{\rm T} = -\frac{\hbar\sigma_{\uparrow\downarrow}}{2e^2}\frac{\partial}{\partial x}\boldsymbol{\mu}^{\rm T}, \tag{13}$$

where $\sigma_{\uparrow\downarrow} = e^2[(\mathcal{N}_\uparrow + \mathcal{N}_\downarrow)/2][(D_\uparrow + D_\downarrow)/2]$. The solutions of the transverse spin accumulation and current are linear combinations of $e^{\pm x/\ell}$ and $e^{\pm x/\ell^*}$ with $1/\ell = \sqrt{(1/\lambda_{\rm sd(T)}^2) - (i/\lambda_J^2)}$.

In the nonmagnetic layer, the distinction between the longitudinal and transverse spin is unnecessary. In fact, in the limit of zero-spin polarization ($\beta = \beta' = 0$) and in the absence of the exchange coupling between the magnetization and electrons' spin ($J_{\rm sd} = 0$), as for the nonmagnet, Eqs. (10) and (12), or Eqs. (11) and (13), become identical.

The relation between the spin accumulation and dissipation is as follows. The heat density of the longitudinal spin-ν electrons dq_ν relates to the energy density $u_\nu = \int d^3\mathbf{k}/(2\pi)^3\varepsilon f_\nu$, chemical potential $\mu_\nu = \mu_0 + \delta\mu_\nu$, and the electron density n_ν via [40,41]

$$dq_\nu = du_\nu - \mu_\nu dn_\nu. \tag{14}$$

The energy density $u^{\rm L} = u_\uparrow + u_\downarrow$ for the longitudinal spin satisfies [6]

$$\frac{\partial u^{\rm L}}{\partial t} + \frac{\partial j_u^{\rm L}}{\partial x} = J_e E, \tag{15}$$

where $j_u^{\rm L} = j_{u,\uparrow} + j_{u,\downarrow}$, and $j_{u,\nu} = \int d^3\mathbf{k}/(2\pi)^3\varepsilon v_x f_\nu$ is the energy current density [6]. Here, the term $J_e E$ is the Joule heating due to the electric current. On the other hand, the energy current of the transverse spin $j_u^{\rm T}$ satisfies $\partial j_u^{\rm T}/\partial x = 0$ in the steady state, where the right-hand side is zero because there is no source of the transverse spin inside the F and N layers. We introduce the heat current density by [34]

$$j_q = j_u^{\rm L} - \sum_{\nu=\uparrow,\downarrow}\mu_\nu j_\nu + j_u^{\rm T} - \boldsymbol{\mu}^{\rm T}\cdot\frac{\mathbf{J}_s^{\rm T}}{\hbar}. \tag{16}$$

In steady state, the heat current is related to the dissipation via [42] $\partial Q_V/\partial t = T[\partial(j_q/T)/\partial x]$, where the temperature T is assumed to be spatially uniform in the following calculations. The subscript V is used to emphasize that this is the dissipation per unit volume per unit time. Then, $\partial Q_V/\partial t$ is

$$\frac{\partial Q_V}{\partial t} = \frac{J_e}{e}\frac{\partial\bar{\mu}}{\partial x} - \frac{\partial}{\partial x}\frac{\mathbf{J}_s}{\hbar}\cdot\boldsymbol{\mu}, \tag{17}$$

where $\bar{\mu} = (\bar{\mu}_\uparrow + \bar{\mu}_\downarrow)/2$ is the electrochemical potential. The interface resistance also gives the dissipation, where the dissipation per unit area per unit time is

$$\frac{\partial Q_A}{\partial t} = \frac{J_e}{e}\delta\bar{\mu} - \frac{\mathbf{J}_s}{\hbar}\cdot\delta\boldsymbol{\mu}, \tag{18}$$

where $\delta\bar{\mu}$ and $\delta\boldsymbol{\mu}$ are the differences of $\bar{\mu}$ and $\boldsymbol{\mu}$ at the F/N interface. The subscript A is used to emphasize that this is the dissipation per unit area per unit time. Equations (17) and (18) are generalized Joule heating formulas in the presence of spin current, and the main results in this section. The total spin current $\mathbf{J}_s$ and spin accumulation $\boldsymbol{\mu}$ include both the longitudinal and transverse components, whereas only the longitudinal components appeared in the previous work [5]. The amount of the dissipation can be evaluated by substituting the solution of the diffusion equation of the spin accumulation into Eqs. (17) and (18) with accurate boundary conditions provided by Eqs. (1) and (2). We call Eqs. (17) and (18) the bulk and interface dissipations, respectively.

IV. DISSIPATION DUE TO SPIN PUMPING

In spin pumping, transverse spin angular momentum is steadily transferred from the magnetic system (F_1 layer) to the conduction electrons near the F_1/N interface. The net spin angular momentum, $d\mathbf{s} = [\mathbf{I}_s^{\text{pump}} + \mathbf{m}_1 \times (\mathbf{I}_s^{F_1 \to N} \times \mathbf{m}_1)]dt$, transferred from the ferromagnet should overcome the potential difference $\boldsymbol{\mu}_N - \boldsymbol{\mu}_{F_1}$ to be pumped steadily from the F_1/N interface to the N layer during the time dt. This means that not only the spin angular momentum but also the energy is transferred from the F_1 layer to the conduction electrons. The transferred energy per unit area per unit time is given by $(\boldsymbol{\mu}_N - \boldsymbol{\mu}_{F_1}) \cdot (d\mathbf{s}/dt)/(\hbar S)$. In terms of the spin current and spin accumulation, this transferred energy is expressed as

$$\frac{\partial Q_A^{\text{SP}}}{\partial t} = \frac{1}{\hbar S}\left[\mathbf{I}_s^{\text{pump}} + \mathbf{m}_1 \times \left(\mathbf{I}_s^{F_1 \to N} \times \mathbf{m}_1\right)\right] \cdot [\boldsymbol{\mu}_N(x=0) - \boldsymbol{\mu}_{F_1}(x=0)]. \tag{19}$$

Comparing Eq. (19) with Eq. (18), we find the relation

$$\left(\frac{\partial Q_A}{\partial t}\right)^{\text{T}}_{F_1/N} = -\frac{\partial Q_A^{\text{SP}}}{\partial t}, \tag{20}$$

where $(\partial Q_A/\partial t)^{\text{T}}_{F_1/N}$ is defined by

$$\left(\frac{\partial Q_A}{\partial t}\right)^{\text{T}}_{F_1/N} = \left(\frac{\partial Q_A}{\partial t}\right)_{F_1/N} - \left(\frac{\partial Q_A}{\partial t}\right)^{\text{L}}_{F_1/N}. \tag{21}$$

Here, $(\partial Q_A/\partial t)_{F_1/N}$ is the F_1/N interface dissipation defined by Eq. (18), whereas

$$\left(\frac{\partial Q_A}{\partial t}\right)^{\text{L}}_{F_1/N} = -\frac{1}{\hbar S}\left(\mathbf{m}_1 \cdot \mathbf{I}_s^{F_1 \to N}\right)\mathbf{m}_1 \cdot [\boldsymbol{\mu}_N(x=0) - \boldsymbol{\mu}_{F_1}(x=0)]. \tag{22}$$

Because Eq. (22) is defined by the longitudinal components of the spin current and spin accumulation in Eq. (18), we call this quantity the longitudinal part of the F_1/N interface dissipation. On the other hand, Eq. (21) is defined by the transverse components of the spin current and spin accumulation at the F_1/N interface. Moreover, using Eqs. (17), (18), and (21), Eq. (19) can be rewritten as

$$\frac{\partial Q_A^{\text{SP}}}{\partial t} = \left(\frac{\partial Q_A}{\partial t}\right)_{F_2/N} + \int_0^{d_2} dx \left(\frac{\partial Q_V}{\partial t}\right)_{F_2} + \left(\frac{\partial Q_A}{\partial t}\right)^{\text{L}}_{F_1/N} + \int_{-d_1}^{0} dx \left(\frac{\partial Q_V}{\partial t}\right)_{F_1}, \tag{23}$$

where the F_2/N interface dissipation, $(\partial Q_A/\partial t)_{F_2/N}$ in Eq. (23), and the F_1 and F_2 bulk dissipations, $(\partial Q_V/\partial t)_{F_1}$ and $(\partial Q_V/\partial t)_{F_2}$, are defined from Eqs. (17) and (18). As discussed below, Eq. (23) describes the energy dissipation process carried by the spin current. Therefore, we define Eq. (23), or equivalently, Eq. (19), the dissipation due to spin pumping.

With the help of Figs. 2(a) and 2(b) we now discuss the physical interpretation of Eq. (23), which schematically show the flows of spin angular momentum and of energy. In spin pumping one usually focuses attention only on the flow of spin angular momentum, i.e., spin current, but because we are also interested in energy dissipation we also show energy flow. When the pumped angular momentum reaches

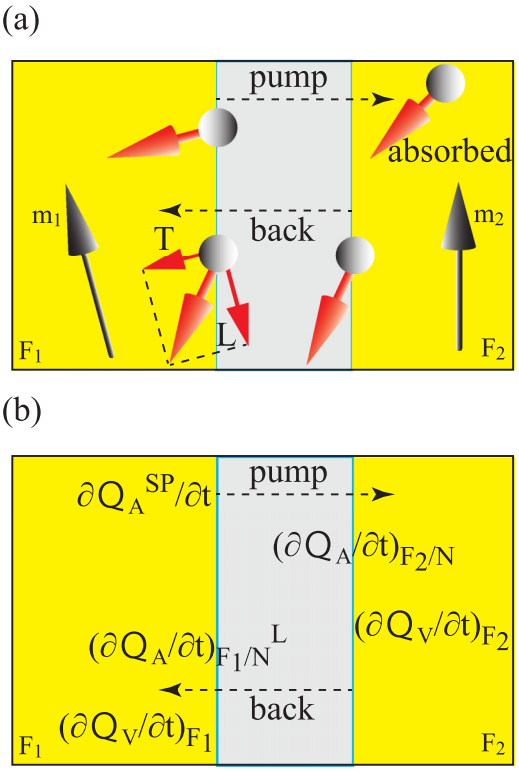

FIG. 2. (Color online) Schematic views of the flows of (a) angular momentum and (b) energy from the microwave to the ferromagnetic multilayer, in which L and T define the longitudinal and transverse components with respect to $\mathbf{m}_1$.

the F_2/N interface, part of it is absorbed in the F_2 layer, and is depolarized by scattering at the F_2/N interface and by spin flip and spin diffusion within the F_2 layer. The remaining part returns to the F_1/N interface, which we call backflow. The backflow to the F_1 layer is relaxed by scattering at the F_1/N interface and by spin flip and spin diffusion within the F_1 layer, where the transverse component of the backflow at the F_1/N interface renormalizes the pumped spin current. In terms of the energy flow shown in Fig. 2(b), spin absorption at the F_2/N interface leads to the interface dissipation $(\partial Q_A/\partial t)_{F_2/N}$ and bulk dissipation $(\partial Q_V/\partial t)_{F_2}$ due to spin depolarization. The backflow at the F_1 layer also gives the interface dissipation $(\partial Q_A/\partial t)^{\text{L}}_{F_1/N}$ and bulk dissipation $(\partial Q_V/\partial t)_{F_1}$. The total dissipation is the sum of these dissipations, as indicated by Eq. (23). In other words, the transferred energy from the F_1 layer to the conduction electrons at the F_1/N interface is not localized, and is dissipated throughout the system. Then, Eq. (23), or equivalently, Eq. (19), can be regarded as the dissipation due to spin pumping. Also, Eq. (21) is regarded as the energy transfer from the F_1 layer to the conduction electrons near the F_1/N interface. Appendix A shows that all terms on the right-hand side of Eq. (23) are positive, thus guaranteeing the second law of thermodynamics.

To conclude this section, it is of interest to compare Eq. (19) with the dissipation due to electric current. Let us assume that an electric current is flowing through a multilayer, driven by a voltage difference across two electrodes. The total dissipation per unit area per unit time is obtained from Eqs. (17) and (18)

as [5]

$$\frac{\partial Q_A^{\rm EC}}{\partial t} = \frac{J_e}{e}[\bar{\mu}(\infty) - \bar{\mu}(-\infty)], \tag{24}$$

where $[\bar{\mu}(\infty) - \bar{\mu}(-\infty)]/e$ is the voltage difference between the electrodes. Comparing Eq. (19) with (24), we notice that the net transverse spin current and the difference in the spin accumulation at the F_1/N interface correspond to the electric current and applied voltage, respectively, and that in spin pumping the F_1/N interface plays the role of the electrode. This is because the angular momentum and the energy transferred from the magnetization of the F_1 layer to the conduction electron are pumped from this interface to the multilayer.

V. EVALUATION OF DISSIPATION

In this section, we quantitatively evaluate the dissipation due to spin pumping, Eq. (19). Substituting the solutions of Eqs. (10) and (12) into Eq. (2), the total spin currents at the F_1/N and F_2/N interfaces are, respectively, expressed as

$$\begin{aligned}\mathbf{I}_s^{\rm pump} + \mathbf{I}_s^{\rm F_1\to N} = {}& \frac{\hbar}{4\pi}\left(\tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow}\mathbf{m}_1\times\frac{d\mathbf{m}_1}{dt} + \tilde{g}_{\rm i(F_1)}^{\uparrow\downarrow}\frac{d\mathbf{m}_1}{dt}\right)\\ &- \frac{1}{4\pi}\big[\tilde{g}_{\rm F_1}^{*}(\mathbf{m}_1\cdot\boldsymbol{\mu}_{\rm N})\mathbf{m}_1 + \tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow}\mathbf{m}_1\\ &\times(\boldsymbol{\mu}_{\rm N}\times\mathbf{m}_1) + \tilde{g}_{\rm i(F_1)}^{\uparrow\downarrow}\boldsymbol{\mu}_{\rm N}\times\mathbf{m}_1\big],\end{aligned} \tag{25}$$

$$\begin{aligned}\mathbf{I}_s^{\rm F_2\to N} = {}& -\frac{1}{4\pi}\big[\tilde{g}_{\rm F_2}^{*}(\mathbf{m}_2\cdot\boldsymbol{\mu}_{\rm N})\mathbf{m}_2 + \tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow}\mathbf{m}_2\times(\boldsymbol{\mu}_{\rm N}\times\mathbf{m}_2)\\ &+ \tilde{g}_{\rm i(F_2)}^{\uparrow\downarrow}\boldsymbol{\mu}_{\rm N}\times\mathbf{m}_2\big].\end{aligned} \tag{26}$$

The renormalized conductances, $\tilde{g}^*$ and $\tilde{g}_{\rm r,i}^{\uparrow\downarrow}$, are defined by the following ways:

$$\frac{1}{\tilde{g}^*} = \frac{2}{(1-\gamma^2)g} + \frac{1}{g_{\rm sd}\tanh(d/\lambda_{\rm sd(L)})}, \tag{27}$$

$$\begin{pmatrix}\tilde{g}_{\rm r}^{\uparrow\downarrow}\\ \tilde{g}_{\rm i}^{\uparrow\downarrow}\end{pmatrix} = \frac{1}{K_1^2+K_2^2}\begin{pmatrix}K_1 & K_2\\ -K_2 & K_1\end{pmatrix}\begin{pmatrix}g_{\rm r}^{\uparrow\downarrow}\\ g_{\rm i}^{\uparrow\downarrow}\end{pmatrix}, \tag{28}$$

where $g_{\rm sd} = h(1-\beta^2)S/(2e^2\rho\lambda_{\rm sd(L)})$, and $\rho = 1/(\sigma^\uparrow + \sigma^\downarrow)$ is the resistivity. The terms K_1 and K_2 are defined as

$$K_1 = 1 + t_{\rm r}^{\uparrow\downarrow}{\rm Re}\left[\frac{1}{g_{\rm t}\tanh(d/\ell)}\right] + t_{\rm i}^{\uparrow\downarrow}{\rm Im}\left[\frac{1}{g_{\rm t}\tanh(d/\ell)}\right], \tag{29}$$

$$K_2 = t_{\rm i}^{\uparrow\downarrow}{\rm Re}\left[\frac{1}{g_{\rm t}\tanh(d/\ell)}\right] - t_{\rm r}^{\uparrow\downarrow}{\rm Im}\left[\frac{1}{g_{\rm t}\tanh(d/\ell)}\right], \tag{30}$$

where $g_{\rm t} = hS\sigma_{\uparrow\downarrow}/(e^2\ell)$. In the ballistic transport limit for the transverse spin, $\tilde{g}^{\uparrow\downarrow}$ equals $g^{\uparrow\downarrow}$. Then, we expand $\boldsymbol{\mu}_{\rm N}$ as $\boldsymbol{\mu}_{\rm N} = \hbar(\omega a\sin\theta\mathbf{m}_1 + b\dot{\mathbf{m}}_1 + c\mathbf{m}_1\times\dot{\mathbf{m}}_1)$, where $y = \delta_y/\Delta$ ($y = a,b,c$) are dimensionless coefficients determined by Eq. (3) with Eqs. (25) and (26). In the limit of $g_{\rm r}^{\uparrow\downarrow} \gg g_{\rm i}^{\uparrow\downarrow}$ [25], $\delta_b = 0$, and Δ, δ_a, and δ_c are given by

$$\begin{aligned}\Delta = {}& \big(\tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow} + \tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow}\big)\big[\big(\tilde{g}_{\rm F_1}^{*} + \tilde{g}_{\rm F_2}^{*}\cos^2\theta + \tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow}\sin^2\theta\big)\\ &\times\big(\tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow} + \tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow}\cos^2\theta + \tilde{g}_{\rm F_2}^{*}\sin^2\theta\big)\\ &- \big(\tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow} - \tilde{g}_{\rm F_2}^{*}\big)^2\sin^2\theta\cos^2\theta\big],\end{aligned} \tag{31}$$

$$\delta_a = \tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow}\big(\tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow} + \tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow}\big)\big(\tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow} - \tilde{g}_{\rm F_2}^{*}\big)\sin\theta\cos\theta, \tag{32}$$

$$\delta_c = \tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow}\big(\tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow} + \tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow}\big)\big(\tilde{g}_{\rm F_1}^{*} + \tilde{g}_{\rm F_2}^{*}\cos^2\theta + \tilde{g}_{\rm r(F_2)}^{\uparrow\downarrow}\sin^2\theta\big). \tag{33}$$

Equation (19) in the limit of $g_{\rm r}^{\uparrow\downarrow} \gg g_{\rm i}^{\uparrow\downarrow}$ is then given by

$$\begin{aligned}\frac{\partial Q_A^{\rm SP}}{\partial t} = {}& \frac{\hbar\omega^2\sin^2\theta\,\tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow}(1-c)}{4\pi S}\\ &\times\left\{c + \tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow}(1-c){\rm Re}\left[\frac{1}{g_{\rm t}\tanh(d_1/\ell)}\right]\right\}.\end{aligned} \tag{34}$$

In the ballistic transport limit of the transverse spin, Eq. (34) is simplified to $\hbar\omega^2 g_{\rm r(F_1)}^{\uparrow\downarrow}(1-c)c/(4\pi S)$. We emphasize that Eq. (34) is proportional to the enhancement of the Gilbert damping by spin pumping [20,26]:

$$\alpha' = \frac{\gamma_0\hbar\tilde{g}_{\rm r(F_1)}^{\uparrow\downarrow}(1-c)}{4\pi MSd_1}, \tag{35}$$

where γ_0 is the gyromagnetic ratio. Here, α' is derived in the following way. According to the conservation law of the total angular momentum, the pumped spin from the F_1/N interface per unit time, $d\mathbf{s}/dt$, should equal the time change of the magnetization in the F_1 layer, i.e., a torque $d\mathbf{m}_1/dt = [(g\mu_{\rm B})/(\hbar MSd)]d\mathbf{s}/dt$ acts on $\mathbf{m}_1$, where $M/(g\mu_{\rm B})$ is the number of the magnetic moments in the F_1 layer, and the Landé g factor satisfies $g\mu_{\rm B} = \gamma_0\hbar$. This torque, $[(g\mu_{\rm B})/(\hbar MSd)]d\mathbf{s}/dt$, with $d\mathbf{s}/dt = \mathbf{I}_s^{\rm pump} + \mathbf{m}_1\times(\mathbf{I}_s^{\rm F_1\to N}\times\mathbf{m}_1)$, can be expressed as $\alpha'\mathbf{m}_1\times(d\mathbf{m}_1/dt)$. Then, α' is identified as the enhancement of the Gilbert damping constant due to the spin pumping. The present result indicating that the dissipation is proportional to α' represents that the pumped spin current at the F_1/N interface carries not only the angular momentum but also the energy from the F_1 to N layer.

We quantitatively evaluate Eq. (34) by using parameters taken from experiments for the NiFe/Cu multilayer with the assumption $\beta = \beta'$ [23,26,29,43]; $(h/e^2)S/[(1-\gamma^2)g] = 0.54\ {\rm k\Omega\ nm^2}$, $\gamma = 0.7$, $g_{\rm r}^{\uparrow\downarrow}/S = 15\ {\rm nm^{-2}}$, $g_{\rm i}^{\uparrow\downarrow}/S = 1\ {\rm nm^{-2}}$, $t_{\rm r}^{\uparrow\downarrow}/S = t_{\rm i}^{\uparrow\downarrow}/S = 4\ {\rm nm^{-2}}$, $\rho = 241\ \Omega\ {\rm nm}$, $\beta = 0.73$, $\lambda_{\rm sd(L)} = 5.5$ nm, $\lambda_{\rm sd(T)} = \lambda_{\rm sd(L)}/\sqrt{1-\beta^2}$, $\lambda_J = 2.8$ nm, $d = 5$ nm, $\gamma_0 = 1.8467\times10^{11}$ rad/(T s), $M = 605\times10^3$ A/m, and $\omega = 2\pi\times9.4\times10^9$ rad/s, respectively, where the parameters of the F_1 and F_2 layers are assumed to be identical, for simplicity. In Fig. 3(a), we show the dissipation due to spin pumping, Eq. (34), for an arbitrary cone angle θ. The damping α', Eq. (35), is also shown in Fig. 3(b). The cone angle θ in typical FMR experiments [7,8] is small. However, the spin pumping affects not only the FMR experiment but also spin torque switching [37], in which θ varies from 0° to 180°.

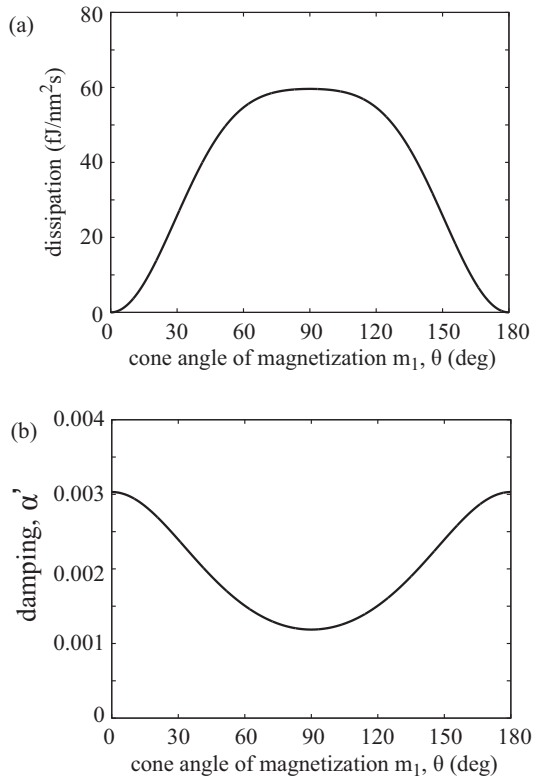

FIG. 3. Dependencies of (a) the dissipation due to pure spin-current, Eq. (19), and (b) the damping α', Eq. (35), on the cone angle θ.

Therefore, we show the dissipation and damping for the whole range of θ in Fig. 3.

The dissipation is zero for $\theta = 0^\circ$ and 180° because $d\mathbf{m}_1/dt = \mathbf{0}$ at these angles. The maximum dissipation is about 60 fJ/(nm^2 s). To understand how large this dissipation is, we compare this value with the dissipation due to spin torque switching current in the same system; we discuss this in the next section.

To conclude this section, we briefly mention that the dissipation due to spin pumping can be evaluated not only from Eq. (19) but also from Eq. (23). Appendix B gives explicit forms for each term on the right-hand side of Eq. (23), from which the dissipation can be calculated.

VI. COMPARISON WITH SPIN TORQUE SWITCHING

Spin pumping occurs not only in FMR experiments but also in spin torque switching experiments. An important issue in the spin torque switching problem is the reduction of power consumption due to heating [44]. Whereas heating has usually meant the dissipation due to electric current, the results of the previous section indicate that spin pumping also contributes to the dissipation. Thus it is of interest to quantitatively evaluate the dissipation due to the electric current, and compare it with that due to spin pumping studied in the previous section, which will clarify the ratio of the contribution of spin pumping to heating in the spin torque switching experiment.

We assume that an electric current I is injected from the F_2 layer to the F_1 layer. Then, a term

$$\mathbf{I}_{s(e)}^{F_k\to N} = \frac{\hbar\gamma}{2e} I^{F_k\to N}\mathbf{m}_k \tag{36}$$

should be added to Eq. (2), which represents a spin current due to the electric current [25]. The current $I^{F_k\to N}$ is the electric current which flows from the F_k layer to the N layer, meaning that $I^{F_1\to N} = -I^{F_2\to N} = -I$. As in the system studied in the previous section, we assume that the spin current is zero at both ends of the ferromagnet. Taking into account Eq. (36), Eqs. (25) and (26) are replaced by

$$\mathbf{I}_s^{\text{pump}} + \mathbf{I}_s^{F_1\to N} = \frac{\hbar}{4\pi}\tilde{g}_r^{\uparrow\downarrow}\mathbf{m}_1\times\frac{d\mathbf{m}_1}{dt} - \frac{1}{4\pi}\left[\tilde{g}^*(\mathbf{m}_1\cdot\boldsymbol{\mu}_N)\mathbf{m}_1 + \frac{h\tilde{g}^* I}{\tilde{g}_e e}\mathbf{m}_1 + \tilde{g}_r^{\uparrow\downarrow}\mathbf{m}_1\times(\boldsymbol{\mu}_N\times\mathbf{m}_1)\right], \tag{37}$$

$$\mathbf{I}_s^{F_2\to N} = -\frac{1}{4\pi}\left[\tilde{g}^*(\mathbf{m}_2\cdot\boldsymbol{\mu}_N)\mathbf{m}_2 - \frac{h\tilde{g}^* I}{\tilde{g}_e e}\mathbf{m}_2 + \tilde{g}_r^{\uparrow\downarrow}\mathbf{m}_2\times(\boldsymbol{\mu}_N\times\mathbf{m}_2)\right], \tag{38}$$

where, as done in the previous section, we assume that the material parameters of two ferromagnets are identical, and thus, omit subscripts F_k from the conductances, for simplicity. We also assume that $g_r^{\uparrow\downarrow} \gg g_i^{\uparrow\downarrow}$. A new conductance $\tilde{g}_e$ is defined as

$$\frac{1}{\tilde{g}_e} = \frac{2\gamma}{(1-\gamma^2)g} + \frac{\beta}{g_{sd}}\tanh\left(\frac{d}{2\lambda_{sd(L)}}\right). \tag{39}$$

A characteristic current of the spin torque switching is the critical current of the magnetization dynamics I_c, which can be defined as the current canceling the Gilbert damping torque of the F_1 layer at the equilibrium state [38]. The equilibrium state in the present study corresponds to $\theta = 0^\circ$. In this limit ($\theta \to 0$), Eq. (35) is replaced by

$$\alpha' = \frac{\gamma_0\hbar\tilde{g}_r^{\uparrow\downarrow}}{4\pi MSd_1}\left(\frac{1}{2} - \frac{\pi\tilde{g}^* I}{e\omega\tilde{g}_r^{\uparrow\downarrow}\tilde{g}_e}\right). \tag{40}$$

We assume that the Gilbert damping purely comes from the spin pumping. Then, the critical current is defined as the current satisfying $\alpha' = 0$; i.e.,

$$I_c = \frac{e\omega\tilde{g}_r^{\uparrow\downarrow}\tilde{g}_e}{2\pi\tilde{g}^*}. \tag{41}$$

Using the same parameter values as in the previous section, the critical current density $J_c = I_c/S$ is estimated as 6.3×10^6 A/cm^2. This value is about the same order of an experimentally observed value [45] ($\sim 6\times10^6$ A/cm^2 on average) of the critical current having a magnetic anisotropy field H_K, whose magnitude (1–3 kOe) is about the same order of the parameter value, $\omega/\gamma_0 \simeq 3.2$ kOe, used here. The dissipation due to this electric current based on the conventional Joule heating formula, $\partial Q_A^{EC}/\partial t = \sum_k[\rho J_c^2 d_k + r_{F_k/N}J_c^2]$, is evaluated as 11.8×10^3 fJ/(nm^2 s), where $r_{F/N} = (h/e^2)S/g$ is the F/N interface resistance. This value of the dissipation is two to three orders of magnitude larger than the dissipation due to the spin pumping studied in the previous section.

We briefly investigate the origins of a large difference between the dissipations due to the spin and electric currents. Let us assume that the bulk and interface spin polarizations (β and γ) are identical, and that the thickness of the ferromagnetic layer is much larger than the spin diffusion length ($d \gg \lambda_{\mathrm{sd(L)}}$), for simplicity, from which the critical current is simplified as $I_{\mathrm{c}} = e\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}/(2\pi\beta)$. Then the ratio between the dissipations due to spin pumping and electric current becomes $(\partial Q_A^{\mathrm{SP}}/\partial t)/(\partial Q_A^{\mathrm{EC}}/\partial t) \sim \beta^2\hbar/[e^2(\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}/S)(\rho d + r)]$. The square of the spin polarization, β^2, is on the order of 10^{-1}. Also, the orders of $[(h/e^2)S/(\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}r)]$ and $r/\rho d$ are 1 and 0.1, respectively. Then, the ratio $(\partial Q_A^{\mathrm{SP}}/\partial t)/(\partial Q_A^{\mathrm{EC}}/\partial t)$ is roughly 10^{-2}, which is roughly consistent with the above evaluation. This consideration implies that a large dissipation due to the electric current comes from the smallness of the spin polarization. Also, a large bulk resistivity (ρ), in addition to the interface resistance (r), also contributes to the large dissipation due to the electric current, whereas only the interface resistance contributes to the spin pumping dissipation because spin pumping is an interface effect.

To conclude this section, we mention that the total dissipation in the FMR consists of that due to spin pumping, Eq. (34), and that due to the intrinsic damping in the F_1 layer. One can consider the possibility that the total dissipation in the FMR might become comparable to or exceed the dissipation due to the electric current (calculated above) when the dissipation due to intrinsic magnetic damping is included, despite the fact the dissipation due to spin pumping is small. However, we found that the intrinsic damping constant α_0 should be at least on the order of 0.1–1 to make the dissipation in the FMR comparable with that due to the electric current; see Appendix C. On the other hand, the experimental value of the intrinsic Gilbert damping constant is on the order of 0.001–0.01 [46]. Therefore, the dissipation in the FMR is still much smaller than that due to the electric current even after the dissipation due to the intrinsic damping is taken into account. The energy supplied by the microwave to the F_1 layer is divided into the power to sustain the magnetization precession and that transferred to the conduction electrons near the F_1/N interface, where their ratio is roughly $\alpha_0 : \alpha'$. The former ($\propto \alpha_0$) is dissipated by the bulk magnetic dissipation whereas the latter ($\propto \alpha'$) is dissipated by the spin-flip processes and spin-dependent scatterings within bulk and at the interface, as shown by Eq. (23).

VII. CONCLUSION

The dissipation and heating due to a pure spin-current generated by spin pumping in a ferromagnetic/nonmagnetic/ferromagnetic multilayer was quantitatively investigated. Using spin-dependent transport theory and thermodynamics we generalized the Joule heating formula in the presence of spin current flowing in a ferromagnetic multilayer. The bulk and interface dissipation formulas are given by Eqs. (17) and (18), respectively. For spin pumping, the transferred energy from the ferromagnet to the conduction electrons is not localized at the interface, and is dissipated throughout the system by the flow of a pure spin-current, as shown by Eq. (23). The dissipation due to the spin pumping, Eq. (34), is proportional to the enhancement of the Gilbert damping by spin pumping, Eq. (35). Using typical values of parameters in a metallic multilayer system, the amount of the dissipation at maximum is estimated to be two to three orders of magnitude smaller than the dissipation due to the electric current for spin torque switching.

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APPENDIX A: NONNEGATIVITY OF BULK AND INTERFACE DISSIPATIONS

In this Appendix, we prove that all terms on the right-hand side of Eq. (23) are positive, which guarantees the second law of thermodynamics; i.e., the dissipation, or rate of the entropy production, is positive [41]. Here, we omit the subscript F_k ($k = 1,2$) from conductances, for simplicity.

First, we prove the nonnegativity of the longitudinal and transverse parts of the bulk dissipation. The longitudinal part of Eq. (17) can be rewritten as

$$\begin{aligned}\left(\frac{\partial Q_V}{\partial t}\right)^{\mathrm{L}} &= \frac{J_e}{e}\frac{\partial\bar{\mu}}{\partial x} - \frac{\partial}{\partial x}\frac{\mathbf{J}_s^{\mathrm{L}}}{\hbar}\cdot\boldsymbol{\mu}^{\mathrm{L}} \\ &= -\sum_{\nu=\uparrow,\downarrow} j_\nu\frac{\partial\bar{\mu}_\nu}{\partial x} - \frac{(\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)}{2}\frac{\partial}{\partial x}(j_\uparrow - j_\downarrow) \\ &= \sum_{\nu=\uparrow,\downarrow}\frac{e^2}{\sigma_\nu}(j_\nu)^2 + \frac{(1-\beta^2)}{4e^2\rho\lambda_{\mathrm{sd(L)}}^2}(\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)^2, \quad \text{(A1)}\end{aligned}$$

which is clearly positive. Here, we use the relation $\partial(j_\uparrow - j_\downarrow)/\partial x = -(1-\beta^2)(\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)/(2e^2\rho\lambda_{\mathrm{sd(L)}}^2)$. Also, we can confirm from Eqs. (12) and (13) that the transverse part,

$$\begin{aligned}\left(\frac{\partial Q_V}{\partial t}\right)^{\mathrm{T}} &= -\frac{\partial}{\partial x}\frac{\mathbf{J}_s^{\mathrm{T}}}{\hbar}\cdot\boldsymbol{\mu}^{\mathrm{T}} \\ &= \frac{2e^2}{\hbar^2\sigma_{\uparrow\downarrow}}\left(\mathbf{J}_s^{\mathrm{T}}\right)^2 + \frac{\sigma_{\uparrow\downarrow}}{2e^2\lambda_{\mathrm{sd(T)}}^2}(\boldsymbol{\mu}^{\mathrm{T}})^2, \quad \text{(A2)}\end{aligned}$$

is positive. Therefore, the bulk dissipation is positive at any x.

Next, let us prove the nonnegativity of the interface dissipation by using the solutions of the spin current and spin accumulation (see also Appendix B). The longitudinal part of the F_1/N interface dissipation can be written as

$$\left(\frac{\partial Q_A}{\partial t}\right)_{\mathrm{F_1/N}}^{\mathrm{L}} = \frac{\tilde{g}^*}{4\pi\hbar S}\left[1 - \frac{\tilde{g}^*}{g_{\mathrm{sd}}\tanh(d_1/\lambda_{\mathrm{sd(L)}})}\right](\mathbf{m}_1\cdot\boldsymbol{\mu}_{\mathrm{N}})^2. \quad \text{(A3)}$$

According to Eq. (27), $1 - \tilde{g}^*/[g_{\mathrm{sd}}\tanh(d_1/\lambda_{\mathrm{sd(L)}})]$ is larger than zero. Therefore, the longitudinal part of the F_1/N interface dissipation is positive. The longitudinal part of the F_2/N

interface dissipation,

$$\left(\frac{\partial Q_A}{\partial t}\right)_{\mathrm{F_2/N}} = \frac{\mathbf{J}_s^{\mathrm{F_2}\to\mathrm{N}}}{\hbar}\cdot(\boldsymbol{\mu}_{\mathrm{F_2}}-\boldsymbol{\mu}_{\mathrm{N}}), \tag{A4}$$

is positive because of the same reason. The transverse part of the F_2/N interface dissipation,

$$\left(\frac{\partial Q_A}{\partial t}\right)^{\mathrm{T}}_{\mathrm{F_2/N}} = \frac{\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}}{4\pi\hbar S}\left\{1-\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}\mathrm{Re}\left[\frac{1}{g_{\mathrm{t}}\tanh(d_2/\ell)}\right]\right\} \times\left[\boldsymbol{\mu}_{\mathrm{N}}^2-(\mathbf{m}_2\cdot\boldsymbol{\mu}_{\mathrm{N}})^2\right], \tag{A5}$$

is also positive due to similar reasons, where we use approximation $\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}\gg\tilde{g}_{\mathrm{i}}^{\uparrow\downarrow}$ used in Sec. V for simplicity.

APPENDIX B: THEORETICAL FORMULAS FOR BULK AND INTERFACE DISSIPATION

In this Appendix, we discuss how to calculate the dissipation due to spin pumping from Eq. (23). To this end, we first show the solutions for the spin current and spin accumulation in the F_1 and F_2 layers because each term on the right-hand side of Eq. (23) consists of spin current and spin accumulation, as shown in Eqs. (17) and (18). The general solution for the spin current and spin accumulation are summarized in our previous work [47]. Here, we use these solutions, and express the spin current and spin accumulation in terms of the coefficients a and c of $\boldsymbol{\mu}_{\mathrm{N}}$ defined in Sec. V with the assumptions $\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}\gg\tilde{g}_{\mathrm{i}}^{\uparrow\downarrow}$.

First, we present the theoretical formulas for the spin current and spin accumulation within the F_1 layer. We introduce two unit vectors $\mathbf{t}_1=\mathbf{m}_1\times\dot{\mathbf{m}}_1/|\mathbf{m}_1\times\dot{\mathbf{m}}_1|$ and $\mathbf{t}_2=-\dot{\mathbf{m}}_1/|\dot{\mathbf{m}}_1|$, which are orthogonal to the magnetization $\mathbf{m}_1$ and satisfy $\mathbf{t}_1\times\mathbf{t}_2=\mathbf{m}_1$, because the transverse components of the spin current and spin accumulation, Eqs. (5) and (7), can be projected to these two directions. Then, the longitudinal and transverse components of the spin current in the F_1 layer are given by

$$\mathbf{m}_1\cdot\mathbf{I}_{s(\mathrm{F_1})} = -\frac{\hbar\omega\tilde{g}^* a\sin\theta}{4\pi}\frac{\sinh[(x+d_1)/\lambda_{\mathrm{sd(L)}}]}{\sinh(d_1/\lambda_{\mathrm{sd(L)}})}, \tag{B1}$$

$$\mathbf{t}_1\cdot\mathbf{I}_{s(\mathrm{F_1})} = \frac{\hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(1-c)\sin\theta}{4\pi}\mathrm{Re}\left[\frac{\sinh[(x+d_1)/\ell]}{\sinh(d_1/\ell)}\right], \tag{B2}$$

$$\mathbf{t}_2\cdot\mathbf{I}_{s(\mathrm{F_1})} = \frac{\hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(1-c)\sin\theta}{4\pi}\mathrm{Im}\left[\frac{\sinh[(x+d_1)/\ell]}{\sinh(d_1/\ell)}\right]. \tag{B3}$$

We can confirm that the sum of these components, $(\mathbf{m}_1\cdot\mathbf{I})\mathbf{m}_1+(\mathbf{t}_1\cdot\mathbf{I}_s)\mathbf{t}_1+(\mathbf{t}_2\cdot\mathbf{I}_s)\mathbf{t}_2$, at $x=0$ is identical to the spin current at the F_1/N interface, $\mathbf{I}_s^{\mathrm{pump}}+\mathbf{I}_s^{\mathrm{F_1}\to\mathrm{N}}$. Similarly, the longitudinal and transverse spin accumulation in the F_1 layer are given by

$$\mathbf{m}_1\cdot\boldsymbol{\mu}_{\mathrm{F_1}} = \frac{\hbar\omega\tilde{g}^* a\sin\theta}{g_{\mathrm{sd}}}\frac{\cosh[(x+d_1)/\lambda_{\mathrm{sd(L)}}]}{\sinh(d_1/\lambda_{\mathrm{sd(L)}})}, \tag{B4}$$

$$\mathbf{t}_1\cdot\boldsymbol{\mu}_{\mathrm{F_1}} = -\hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(1-c)\sin\theta\mathrm{Re}\left[\frac{\cosh[(x+d_1)/\ell]}{g_{\mathrm{t}}\sinh(d_1/\ell)}\right], \tag{B5}$$

$$\mathbf{t}_2\cdot\boldsymbol{\mu}_{\mathrm{F_1}} = -\hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(1-c)\sin\theta\mathrm{Im}\left[\frac{\cosh[(x+d_1)/\ell]}{g_{\mathrm{t}}\sinh(d_1/\ell)}\right]. \tag{B6}$$

Next, we present the explicit forms of the spin current and spin accumulation in the F_2 layer. The magnetization $\mathbf{m}_2$ can be expressed in terms of $(\mathbf{t}_1,\mathbf{t}_2,\mathbf{m}_1)$ as $\mathbf{m}_2=\cos\theta\mathbf{m}_1+\sin\theta\mathbf{t}_1$. We introduce two unit vectors, $\mathbf{u}_1=-\sin\theta\mathbf{m}_1+\cos\theta\mathbf{t}_1$ and $\mathbf{u}_2=\mathbf{t}_2$ satisfying $\mathbf{u}_1\times\mathbf{u}_2=\mathbf{m}_2$, to decompose the transverse component. In terms of $(\mathbf{u}_1,\mathbf{u}_2,\mathbf{m}_2)$, $\boldsymbol{\mu}_{\mathrm{N}}$ can be expressed as $\boldsymbol{\mu}_{\mathrm{N}}=\hbar\omega\sin\theta[(a\cos\theta+c\sin\theta)\mathbf{m}_2+(-a\sin\theta+c\cos\theta)\mathbf{u}_1]$. Then, the longitudinal and transverse spin currents are given by

$$\mathbf{m}_2\cdot\mathbf{I}_{s(\mathrm{F_2})} = -\frac{\hbar\omega\tilde{g}^*(a\sin\theta\cos\theta+c\sin^2\theta)}{4\pi} \times\frac{\sinh[(x-d_2)/\lambda_{\mathrm{sd(L)}}]}{\sinh(d_2/\lambda_{\mathrm{sd(L)}})}, \tag{B7}$$

$$\mathbf{u}_1\cdot\mathbf{I}_{s(\mathrm{F_2})} = -\frac{\hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(-a\sin^2\theta+c\sin\theta\cos\theta)}{4\pi} \times\mathrm{Re}\left[\frac{\sinh[(x-d_2)/\ell]}{\sinh(d_2/\ell)}\right], \tag{B8}$$

$$\mathbf{u}_2\cdot\mathbf{I}_{s(\mathrm{F_2})} = -\frac{\hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(-a\sin^2\theta+c\sin\theta\cos\theta)}{4\pi} \times\mathrm{Im}\left[\frac{\sinh[(x-d_2)/\ell]}{\sinh(d_2/\ell)}\right]. \tag{B9}$$

We can confirm that the sum of these components, $(\mathbf{m}_2\cdot\mathbf{I})\mathbf{m}_2+(\mathbf{u}_1\cdot\mathbf{I}_s)\mathbf{u}_1+(\mathbf{u}_2\cdot\mathbf{I}_s)\mathbf{u}_2$, at $x=0$ is identical to the spin current at the F_2/N interface, $-\mathbf{I}_s^{\mathrm{F_2}\to\mathrm{N}}$. The longitudinal and transverse spin accumulations are given by

$$\mathbf{m}_2\cdot\boldsymbol{\mu}_{\mathrm{F_2}} = \frac{\hbar\omega\tilde{g}^*(a\sin\theta\cos\theta+c\sin^2\theta)}{g_{\mathrm{sd}}} \times\frac{\cosh[(x-d_2)/\lambda_{\mathrm{sd(L)}}]}{\sinh(d_2/\lambda_{\mathrm{sd(L)}})}, \tag{B10}$$

$$\mathbf{u}_1\cdot\boldsymbol{\mu}_{\mathrm{F_2}} = \hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(-a\sin^2\theta+c\sin\theta\cos\theta) \times\mathrm{Re}\left[\frac{\cosh[(x-d_2)/\ell]}{g_{\mathrm{t}}\sinh(d_2/\ell)}\right], \tag{B11}$$

$$\mathbf{u}_2\cdot\boldsymbol{\mu}_{\mathrm{F_2}} = \hbar\omega\tilde{g}_{\mathrm{r}}^{\uparrow\downarrow}(-a\sin^2\theta+c\sin\theta\cos\theta) \times\mathrm{Im}\left[\frac{\cosh[(x-d_2)/\ell]}{g_{\mathrm{t}}\sinh(d_2/\ell)}\right]. \tag{B12}$$

Figures 4(a) and 4(b) show the spatial distributions of the spin current density and spin accumulation, respectively. The spin current density and spin accumulation are decomposed into the longitudinal and transverse directions, where the solid lines correspond to the longitudinal components whereas the dotted ($\parallel\mathbf{t}_1$ or $\mathbf{u}_1$) and dashed ($\parallel\mathbf{t}_2$ or $\mathbf{u}_2$) correspond to the transverse components. The values of the parameters are identical to those used in Sec. V with $\theta=45^\circ$. Because spin pumping occurs at the F_1/N interface, the spin current density and spin accumulation are concentrated near this interface. We emphasize that the spatial directions of the longitudinal

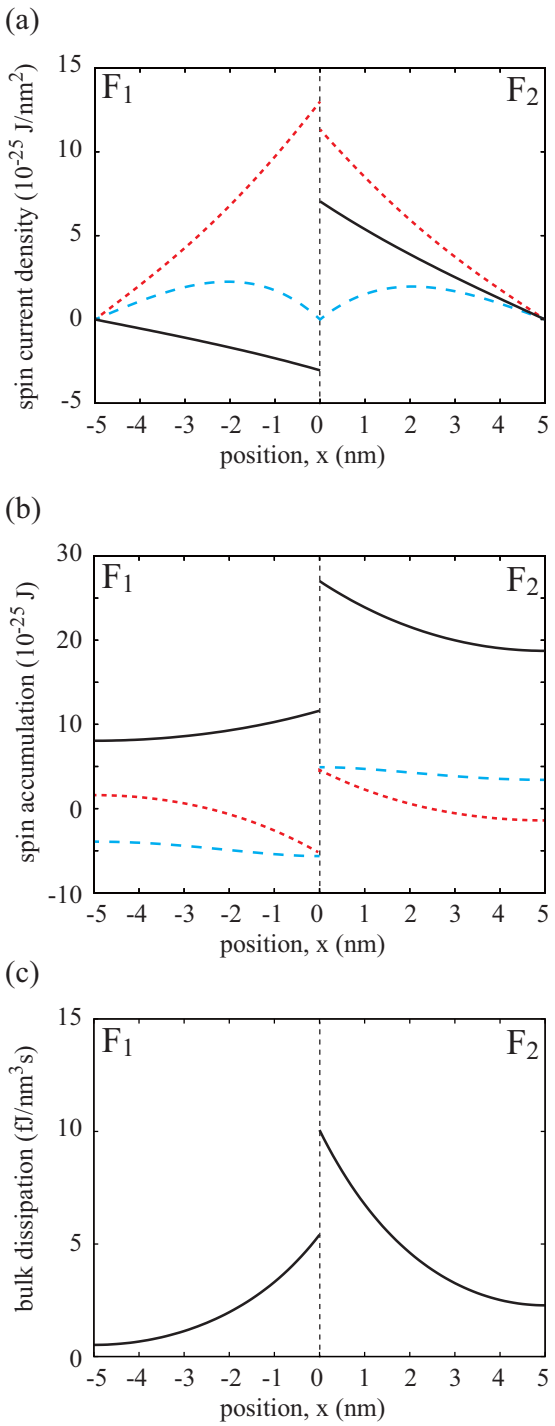

FIG. 4. (Color online) Examples of the distributions of (a) longitudinal (solid) and transverse (dotted and dashed) spin current densities, (b) longitudinal (solid) and transverse (dotted and dashed) spin accumulations, and (c) bulk dissipations for $\theta = 45^\circ$.

and transverse spin are different between the F_1 and F_2 layers when the magnetizations, $\mathbf{m}_1$ and $\mathbf{m}_2$, are noncollinear; as a result the spin current in Fig. 4(a) looks discontinuous at the interface, although Eq. (3) is satisfied.

We now consider the dissipation formulas. The longitudinal and transverse parts of the bulk dissipation in the F_1 layer can be expressed as

$$\left(\frac{\partial Q_V}{\partial t}\right)^{\rm L}_{\rm F_1} = \frac{\hbar\omega^2}{4\pi S}\frac{\tilde{g}^{*2}a^2\sin^2\theta}{g_{\rm sd}\lambda_{\rm sd(L)}\sinh^2(d_1/\lambda_{\rm sd(L)})} \times\cosh\left[\frac{2(x+d_1)}{\lambda_{\rm sd(L)}}\right], \tag{B13}$$

$$\left(\frac{\partial Q_V}{\partial t}\right)^{\rm T}_{\rm F_1} = \frac{\hbar\omega^2\tilde{g}_{\rm r}^{\uparrow\downarrow 2}(1-c)^2\sin^2\theta}{4\pi S^2}\frac{e^2}{h\sigma_{\uparrow\downarrow}} \times\left\{\frac{1}{\lambda^2_{\rm sd(T)}}\left|\frac{\ell\cosh[(x+d_1)/\ell]}{\sinh(d_1/\ell)}\right|^2 + \left|\frac{\sinh[(x+d_1)/\ell]}{\sinh(d_1/\ell)}\right|^2\right\}. \tag{B14}$$

Similarly, the longitudinal and transverse parts of the bulk dissipation in the F_2 layer can be expressed as

$$\left(\frac{\partial Q_V}{\partial t}\right)^{\rm L}_{\rm F_2} = \frac{\hbar\omega^2}{4\pi S}\frac{\tilde{g}^{*2}(a\sin\theta\cos\theta + c\sin^2\theta)^2}{g_{\rm sd}\lambda_{\rm sd(L)}\sinh^2(d_2/\lambda_{\rm sd(L)})} \times\cosh\left[\frac{2(x-d_2)}{\lambda_{\rm sd(L)}}\right], \tag{B15}$$

$$\left(\frac{\partial Q_V}{\partial t}\right)^{\rm T}_{\rm F_2} = \frac{\hbar\omega^2\tilde{g}_{\rm r}^{\uparrow\downarrow 2}(-a\sin^2\theta + c\sin\theta\cos\theta)^2}{4\pi S^2}\frac{e^2}{h\sigma_{\uparrow\downarrow}} \times\left\{\frac{1}{\lambda^2_{\rm sd(T)}}\left|\frac{\ell\cosh[(x-d_2)/\ell]}{\sinh(d_2/\ell)}\right|^2 + \left|\frac{\sinh[(x-d_2)/\ell]}{\sinh(d_2/\ell)}\right|^2\right\}. \tag{B16}$$

Figure 4(c) shows the spatial distribution of the bulk dissipation, which is also concentrated near the interface.

The longitudinal part of the F_1/N interface dissipation and the longitudinal and transverse parts of the F_2/N interface dissipations are given by

$$\left(\frac{\partial Q_A}{\partial t}\right)^{\rm L}_{\rm F_1/N} = \frac{\hbar\omega^2\tilde{g}^*a^2\sin^2\theta}{4\pi S}\left[1-\frac{\tilde{g}^*}{g_{\rm sd}\tanh(d_1/\lambda_{\rm sd(L)})}\right], \tag{B17}$$

$$\left(\frac{\partial Q_A}{\partial t}\right)^{\rm L}_{\rm F_2/N} = \frac{\hbar\omega^2\tilde{g}^*(a\sin\theta\cos\theta + c\sin^2\theta)^2}{4\pi S} \times\left[1-\frac{\tilde{g}^*}{g_{\rm sd}\tanh(d_2/\lambda_{\rm sd(L)})}\right], \tag{B18}$$

$$\left(\frac{\partial Q_A}{\partial t}\right)^{\rm T}_{\rm F_2/N} = \frac{\hbar\omega^2\tilde{g}_{\rm r}^{\uparrow\downarrow}(-a\sin^2\theta + c\sin\theta\cos\theta)^2}{4\pi S} \times\left\{1-\tilde{g}_{\rm r}^{\uparrow\downarrow}{\rm Re}\left[\frac{1}{g_{\rm t}\tanh(d_2/\ell)}\right]\right\}. \tag{B19}$$

For $\theta = 45^\circ$, we quantitatively evaluate that $\int_{-d_1}^{0} dx(\partial Q_V/\partial t)^{\rm L}_{\rm F_1} = 3.34$ fJ/(nm^2 s), $\int_{-d_1}^{0} dx(\partial Q_V/\partial t)^{\rm T}_{\rm F_1} = 6.51$ fJ/(nm^2 s), $\int_{0}^{d_2} dx(\partial Q_V/\partial t)^{\rm L}_{\rm F_2} = 18.15$ fJ/(nm^2 s), and $\int_{0}^{d_2} dx(\partial Q_V/\partial t)^{\rm T}_{\rm F_2} = 4.95$ fJ/(nm^2 s), respectively. Also,

the interface dissipations are quantitatively evaluated as $(\partial Q_A/\partial t)^{\mathrm{L}}_{\mathrm{F_1/N}} = 0.44$ fJ/(nm^2 s), $(\partial Q_A/\partial t)^{\mathrm{L}}_{\mathrm{F_2/N}} = 2.39$ fJ/(nm^2 s), and $(\partial Q_A/\partial t)^{\mathrm{T}}_{\mathrm{F_2/N}} = 8.03$ fJ/(nm^2 s) for $\theta = 45^\circ$, respectively. We can confirm that the value of the dissipation evaluated from these values as Eq. (23) is the same as that evaluated from Eq. (19) with Fig. 3.

APPENDIX C: DISSIPATION DUE TO INTRINSIC DAMPING

In this Appendix, we briefly evaluate the dissipation due to the magnetization precession in the FMR experiment, which arises from the intrinsic Gilbert damping. In the FMR, the energy supplied by the microwave balances with the dissipation due to the damping, and the magnetization precesses practically on the constant-energy curve. The magnetization dynamics with the macrospin assumption is described by the Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{d\mathbf{m}_1}{dt} = -\gamma_0 \mathbf{m}_1 \times \mathbf{H} - \alpha_0 \gamma_0 \mathbf{m}_1 \times (\mathbf{m}_1 \times \mathbf{H}), \tag{C1}$$

where the magnetic field $\mathbf{H}$ relates to the magnetic energy density E via $\mathbf{H} = -\partial E/\partial(M\mathbf{m}_1)$. From Eq. (C1), the change of the energy density averaged on the constant-energy curve is given by

$$\begin{aligned}\overline{\frac{dE}{dt}} &\equiv \frac{1}{\tau}\oint dt \frac{dE}{dt} \\ &= -\frac{\alpha\gamma_0 M}{\tau}\oint dt[\mathbf{H}^2 - (\mathbf{m}_1 \cdot \mathbf{H})^2],\end{aligned} \tag{C2}$$

where $\tau = \oint dt$ is the precession period on a constant-energy curve. Assuming that the ferromagnet has uniaxial anisotropy $\mathbf{H} = (0,0,H_{\mathrm{K}}m_z)$ as done in Sec. VI, Eq. (C2) is given by

$$\overline{\frac{dE}{dt}} = -\alpha_0\gamma_0 M H_{\mathrm{K}}^2 \sin^2\theta\cos^2\theta. \tag{C3}$$

The microwave should supply the energy density $-\overline{dE/dt}$ to sustain the precession. Then, the energy supplied by the microwave per unit area per unit time is $\alpha_0\gamma_0 M H_{\mathrm{K}}^2 d_1 \sin^2\theta\cos^2\theta$, where d_1 is the thickness of the ferromagnet. Comparing this energy with the dissipation due to the spin pumping carried by the spin current, Eq. (34), the ratio of the dissipation between the intrinsic damping and spin pumping is

$$\frac{|\overline{dE/dt}|d_1}{\partial Q_A^{\mathrm{SP}}/\partial t} \sim \frac{\alpha_0}{\alpha'}, \tag{C4}$$

where α' is given by Eq. (35). The dissipation due to the spin pumping ($\propto \alpha'$) is two to three orders of magnitude smaller than the dissipation due to the electric current. Therefore, the intrinsic Gilbert damping constant α_0 giving bulk magnetic dissipation of the same order of magnitude as the dissipation due to the electric current is roughly $10^{2-3} \times \alpha'$. From the value of α' in Fig. 3(b), this gives an α_0 on the order of 0.1–1.

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