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Statistics of energy dissipation in a quantum dot operating in the cotunneling regime

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At Coulomb blockade valleys inelastic cotunneling processes generate particle-hole excitations in quantum dots (QDs), and lead to energy dissipation. We have analyzed the probability distribution function (PDF) of energy dissipated in a QD due to such processes during a given time interval. We obtained analytically the cumulant generating function, and extracted the average, variance, and Fano factor. The latter diverges as $T^3/(eV)^2$ at bias eV smaller than the temperature T, and reaches the value $3eV/5$ in the opposite limit. The PDF is further studied numerically. As expected, the Crooks fluctuation relation is not fulfilled by the PDF. Our results can be verified experimentally utilizing transport measurements of charge.

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Thermal properties of nanostructures are of profound importance, inasmuch as they are manifestations of the dynamics of the particle zoo inside them. The latter includes electrons, phonons, photons, and other (quasi)particles, depending on the system and its surrounding environment. At the same time, understanding thermal characteristics and gaining the ability to manipulate them will facilitate higher control over nanocircuits, which is at the heart of technological advances. Importantly, it may push forward the effort towards finding sustainable energy resources.

As a consequence, recently there has been a growing interest in the thermal aspects of nanostructures [1]. For instance, thermoelectricity in semiconductor nanostructures is investigated in Ref. [2]. The validity of the Wiedemann-Franz law in several mesoscopic systems is studied in Ref. [3]. Experimental investigations of photon and phonon emission from nanostructures is reported in Ref. [4]. The temperature of nanostructures is analyzed in Ref. [5]. Verification of the recently discovered nonequilibrium fluctuation relations [6] in the context of heat is reported in Refs. [7,8]. Energy relaxation in a quantum dot (QD), which is a pillar in the study of nanoelectronic systems, is investigated in Ref. [9]. It was found there that half of the Joule heating produced in transport is due to energy dissipation through the QD. Importantly, there are physical phenomena which are not fully accessible by charge related measurements. As an example, we note the recently observed neutral modes in the fractional quantum Hall regime [10], whose characterization may require thermometry [11].

Here we study the statistical properties of energy dissipated in a QD [12] tuned to be in a Coulomb blockade valley. In this regime sequential tunneling processes are mostly suppressed, and cotunneling processes play a leading role in transport. Cotunneling is a many-body coherent process, where electrons are transferred from one lead to another via a virtual (classically forbidden) state in the QD [13]. We are interested in the "inelastic" contribution, where a "trace" is left on the QD in the form of an electron-hole excited pair with energy ΔE (cf. Fig. 1). Since the QD is practically always in contact with an environment, this energy is dissipated. We focus on the regime where the time needed for equilibration of the QD constitutes the shortest time scale in the problem. The probability distribution function (PDF) $P(E,t)$ of the total energy dissipated in the QD, E, within a given time interval t possesses complete information on the statistics of energy dissipation in the QD. We note other works where the absorbing environment has been modeled specifically by photon or phonon modes [14].

The main goal of our study is to tackle the PDF of energy dissipation in the context of virtual (classically forbidden) many-body states. Specifically, we obtain the following: (i) An analytic result for the cumulant generating function of $P(E,t)$ [cf. Eq. (8)]. This function fully characterizes the statistics of energy dissipation in the QD, and can be utilized to obtain all the cumulants of the distribution. (ii) The PDF $P(E,t)$ in an integral form, which we study numerically. (iii) The first two cumulants of the PDF, average and variance [cf. Eq. (9)], and the Fano factor (cf. Fig. 2). (iv) We have analyzed our results in the context of nonequilibrium fluctuation relations, and have found that the PDF violates the Crooks relation. This is, in fact, expected, since the energy accounted for by the PDF is not the total work performed by the voltage source: It accounts only for the energy gain in the QD but not for

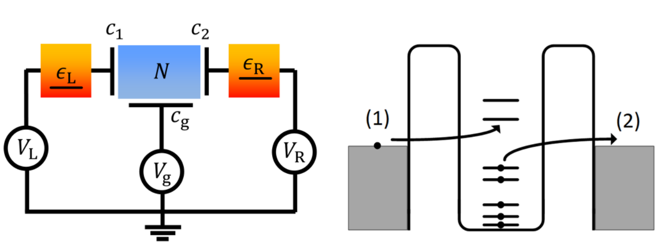

FIG. 1. (Color online) Left: An equivalent circuit representing a quantum dot (QD) (the region bounded by the three capacitors c_1, c_2, and c_g, marked by a blue rectangle), tunnel coupled to two leads with potentials V_L and V_R. The energy levels of the QD can be shifted by an additional capacitively coupled gate V_g. The two other orange rectangles denote energy filters. Right: Schematic illustration of a particlelike inelastic cotunneling process. The numbers denote the order of hopping. In the corresponding holelike process the order is interchanged.

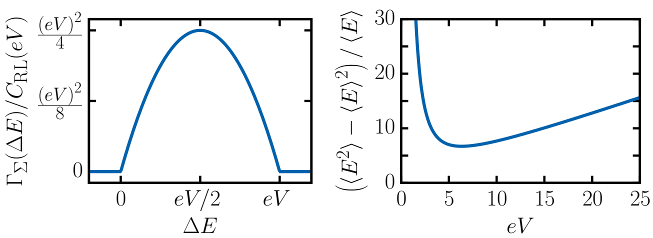

FIG. 2. (Color online) Left: Total rate of energy dissipation in the QD, $\Gamma_\Sigma(\Delta E)$, at temperature $T=0$ [cf. Eq. (5)]. In this limit only inelastic cotunneling processes from left to right contribute to the energy dissipation in the QD, which is confined to the range $0<\Delta E<eV$. Right: Fano factor [cf. Eq. (10c)]. Here the temperature $T=1$. At $eV\gg T$ the Fano factor $\sim 3eV/5$. At $eV\to 0$ the divergence is a manifestation of the fact that on average no energy is dissipated in the QD, while fluctuations around this value are finite [cf. Eqs. (10)].

the energy dissipated in the two leads. (v) We propose an experimental method whereby the statistics of energy can be acquired via charge-transport measurements.

The Hamiltonian of a QD, tunnel coupled to two leads (cf. Fig. 1), is denoted by $H=H_0+V$. The unperturbed Hamiltonian $H_0=H_{\rm L}+H_{\rm R}+H_{\rm D}$ is the Hamiltonian of the three subsystems in the absence of tunneling, where $H_{\rm L}=\sum_k \varepsilon_k c_k^\dagger c_k$, $H_{\rm R}=\sum_q \varepsilon_q c_q^\dagger c_q$, and $H_{\rm D}=\sum_n \varepsilon_n c_n^\dagger c_n + H_N$ are the Hamiltonians of the left lead, right lead, and the QD, respectively. H_N denotes the interactions in the QD in the presence of N electrons. The tunneling Hamiltonian is considered to be the perturbation. It is $V=H_{\rm TL}+H_{\rm TR}$, where $H_{\rm TL}=\sum_{k,n} t_{kn}c_k^\dagger c_n + \text{H.c.}$ and $H_{\rm TR}=\sum_{q,m} t_{qm}c_q^\dagger c_m + \text{H.c.}$ denote dot-left-lead tunneling and dot-right-lead tunneling, respectively.

We employ the second-order version of Fermi's golden rule [15] to calculate the cotunneling rates of electrons from lead to lead [16] that deposit energy ΔE in the QD (which may be positive or negative) in the form of a particle-hole excitation (cf. Fig. 1). For the transition rate per unit energy from the left lead to the right lead ($\text{L}\to\text{R}$) we obtain

$$\begin{aligned}\Gamma_{\rm RL}(\Delta E) = \frac{\gamma^{\rm L}\gamma^{\rm R}}{2\pi}\int_{-\infty}^{+\infty} d\varepsilon_k \int_{-\infty}^{+\infty} d\varepsilon_n \int_{-\infty}^{+\infty} d\varepsilon_m \\ \times \int_{-\infty}^{+\infty} d\varepsilon_q f(\varepsilon_k)[1-f(\varepsilon_n)]\delta(\varepsilon_n-\varepsilon_m-\Delta E) \\ \times f(\varepsilon_m)[1-f(\varepsilon_q)]\delta(\varepsilon_q-\varepsilon_k+\varepsilon_n-\varepsilon_m-eV) \\ \times \left|\frac{1}{\varepsilon_k-\varepsilon_n+eV_{\rm L}-\mu_N}+\frac{1}{\varepsilon_m-\varepsilon_q+\mu_{N-1}-eV_{\rm R}}\right|^2. \end{aligned} \tag{1}$$

Here $\mu_N\equiv e^2/2c_\Sigma + e(eN+Q_{\rm g})/c_\Sigma$ is a charging energy associated with electron processes and μ_{N-1} is a charging energy associated with hole processes; e is the charge of an electron, $c_\Sigma=c_1+c_2+c_{\rm g}$ the total capacitance of the QD to the leads and gate (cf. Fig. 1), $Q_{\rm g}$ the effective charge on the gate, and $eV\equiv eV_{\rm L}-eV_{\rm R}>0$ the bias voltage. It is assumed that the occupation of electronic states in each of the subsystems can be described by a Fermi function $f(\varepsilon)=(e^{\varepsilon/T}+1)^{-1}$ (i.e., fast relaxation time). Although the temperature in the leads and in the QD may differ [5], for simplicity, in what follows we assume that the temperature is uniform across the system. Our results are easily generalizable for the case of a higher steady state temperature in the QD. The constants $\gamma^{\rm L(R)}=2\pi\rho_{\rm L(R)}\rho_{\rm D}|t_{kn(qm)}|^2$, where $\rho_{\rm L(R)}$ is the density of states in the left (right) lead, are assumed to be energy independent. The energies ε_k, ε_n, ε_m, ε_q correspond to levels in the left lead, QD, QD, and right lead, respectively.

Similar expressions can be obtained for the rates of the other cotunneling processes, namely, from the right lead to the left lead, from the left lead to itself, and from the right lead to itself. The total rate is given by $\Gamma_\Sigma(\Delta E)=\sum_{s,s'={\rm L,R}}\Gamma_{ss'}(\Delta E)$, where $\Gamma_{\rm RL}(\Delta E)=\tilde\Gamma_{\rm RL}(\Delta E,eV)$, $\Gamma_{\rm LR}(\Delta E)=\tilde\Gamma_{\rm LR}(\Delta E,-eV)$, $\Gamma_{\rm LL}(\Delta E)=\tilde\Gamma_{\rm LL}(\Delta E,0)$, and $\Gamma_{\rm RR}(\Delta E)=\tilde\Gamma_{\rm RR}(\Delta E,0)$. The rates marked with a tilde are given by

$$\begin{aligned}\tilde\Gamma_{ss'}(\Delta E,eV) \equiv \int_{-\infty}^{\infty} d\varepsilon\, P_{\rm eh}(\varepsilon,eV-\Delta E) \\ \times \int_{-\infty}^{\infty} d\varepsilon'\, P_{\rm eh}(\varepsilon',\Delta E)P_{\rm cot}^{ss'}(\varepsilon,\varepsilon',\Delta E),\end{aligned} \tag{2a}$$

$$P_{\rm eh}(\varepsilon,\Delta E)\equiv f(\varepsilon)[1-f(\varepsilon+\Delta E)], \tag{2b}$$

$$\begin{aligned}P_{\rm cot}^{ss'}(\varepsilon,\varepsilon',\Delta E) \equiv \gamma^s\gamma^{s'}(\mu_N-\mu_{N-1})^2/2\pi \\ \times(\varepsilon'-\varepsilon+\mu_N-eV_{s'}+\Delta E)^{-2} \\ \times(\varepsilon'-\varepsilon+\mu_{N-1}-eV_{s'}+\Delta E)^{-2}.\end{aligned} \tag{2c}$$

The quantity $P_{\rm eh}(\varepsilon,\Delta E)$ represents the probability for electron-hole excitations, and $P_{\rm cot}^{ss'}(\varepsilon,\varepsilon',\Delta E)$ has the meaning of a probability of a cotunneling process which leaves energy ΔE in the QD.

For temperatures and voltages that are small relative to the charging energy of the QD, this analysis can be further pursued analytically. We expand the integrands up to first order with respect to the kinetic energies over the charging energies, and evaluate the integrals. The result is

$$\begin{aligned}\tilde\Gamma_{ss'}(\Delta E,eV) \simeq C_{ss'}(eV)b(-\Delta E)b(\Delta E-eV) \\ \times \Delta E(eV-\Delta E),\end{aligned} \tag{3}$$

where $b(\varepsilon)=(e^{\varepsilon/T}-1)^{-1}$ is the Bose function, and

$$\begin{aligned}C_{ss'}(eV)\equiv \frac{\gamma^s\gamma^{s'}}{2\pi}\left(\frac{1}{\mu_{N-1}-eV_{s'}}-\frac{1}{\mu_N-eV_{s'}}\right)^2 \\ \times\left[1-\left(\frac{1}{\mu_{N-1}-eV_{s'}}+\frac{1}{\mu_N-eV_{s'}}\right)eV\right].\end{aligned} \tag{4}$$

In order to obtain some physical intuition, we look now at the limit of zero temperature. Equations (2) readily show that in this limit all rates vanish besides $\Gamma_{\rm RL}(\Delta E)$, due to the presence of the Fermi functions. Furthermore, $0<\Delta E<eV$. This is expected, since at zero temperature the only way the QD can be excited is when an energetic electron starts at the left lead and passes to the right lead while depositing some energy in the QD. All other transitions are impossible, due to the filled Fermi seas in the left lead, right lead, and QD.

Equation (3) then yields, at $T=0$,

$$\Gamma_\Sigma(\Delta E) \simeq \begin{cases} C_{\rm RL}(eV)\Delta E(eV-\Delta E), & 0<\Delta E<eV, \\ 0, & \text{elsewhere.} \end{cases} \tag{5}$$

This is depicted in Fig. 2.

We turn now to the calculation of $P(E,t)$, which denotes the PDF of the QD to absorb an excessive amount of energy E during the time interval t due to inelastic cotunneling processes. It is assumed that any amount of energy transferred to the QD due to a cotunneling electron immediately dissipates to the environment, namely, that the relaxation time of the QD to an equilibrium state is the shortest time scale in the problem. $P(E,t)$ fulfills the following master equation,

$$\frac{\partial P(E,t)}{\partial t} = -\Gamma_\Sigma P(E,t) + \int_{-\infty}^{\infty} d(\Delta E)\Gamma_\Sigma(\Delta E)P(E-\Delta E,t), \tag{6}$$

where $\Gamma_\Sigma \equiv \int_{-\infty}^{\infty} d(\Delta E)\Gamma_\Sigma(\Delta E)$ is the sum of rates of inelastic cotunneling at all energies. Taking the Fourier transform of Eq. (6) with respect to E (τ will designate the variable conjugate to E) and solving the resulting differential equation, one obtains

$$P(\tau,t) = P(\tau,0)\exp\{2\pi\,[\Gamma_\Sigma(\tau)-\Gamma_\Sigma(\tau=0)]\,t\}, \tag{7a}$$

$$P(E,t) = \int_{-\infty}^{\infty} d\tau\, P(\tau,t)e^{iE\tau}. \tag{7b}$$

Here $\Gamma_\Sigma(\tau)=(2\pi)^{-1}\int_{-\infty}^{\infty} d(\Delta E)\Gamma_\Sigma(\Delta E)e^{-i\Delta E\tau}$. Normalization gives $\int_{-\infty}^{\infty} dE\,P(E,t) = 2\pi P(\tau=0,t=0) = \int_{-\infty}^{\infty} dE\,P(E,t=0)$, namely, the PDF evolves in time such that the total probability is conserved, as it should. To facilitate the numerical evaluation of Eq. (7b) (see below), we choose the initial condition $P(E,t=0)=\exp(-E^2/2\sigma^2)/\sqrt{2\pi\sigma^2}$. Physically it may reflect some initial uncertainty in the energy counter [17].

The cumulant generating function is given by $\ln\langle e^{iE\tau}\rangle = 2\pi[\Gamma_\Sigma(-\tau)-\Gamma_\Sigma(\tau=0)]t$. Consequently, the nth cumulant is given by $2\pi t\, i^n \partial_\tau^n \Gamma_\Sigma(\tau)|_{\tau=0}$, where

$$\Gamma_\Sigma(\tau) = \frac{\pi T^3\,[C_{\rm LL}(0)+C_{\rm RR}(0)]}{\sinh^3(\pi T\tau)}\,[\pi T\tau\cosh(\pi T\tau)-\sinh(\pi T\tau)] + \frac{\pi^2T^3\left[C_{\rm RL}(eV)e^{\frac{eV}{2}\left(\frac{1}{T}-i\tau\right)} + C_{\rm LR}(-eV)e^{-\frac{eV}{2}\left(\frac{1}{T}-i\tau\right)}\right]}{\sinh\left(\frac{eV}{2T}\right)\sinh^3(\pi T\tau)} \times\left[\sin\left(\frac{eV\tau}{2}\right)\cosh(\pi T\tau) - \frac{eV}{2\pi T}\cos\left(\frac{eV\tau}{2}\right)\sinh(\pi T\tau)\right]. \tag{8}$$

This function, which provides complete information on the statistics of energy dissipation in the QD upon differentiation, is the central result of our Rapid Communication. As a consistency check we obtain the standard inelastic charge current [13] from these results, which, for $C_{\rm RL}(eV)\simeq C_{\rm LR}(-eV)$, reads $I=2\pi e[\Gamma_{\rm RL}(\tau=0)-\Gamma_{\rm LR}(\tau=0)]\propto eV[(2\pi T)^2+(eV)^2]$.

The first two cumulants of $P(E,t)$—the mean value and the variance—are given by

$$\frac{\langle E\rangle}{t} = \frac{C_{\rm RL}(eV)e^{eV/2T}-C_{\rm LR}(-eV)e^{-eV/2T}}{24\sinh(eV/2T)} \times(eV)^2[(eV)^2+(2\pi T)^2], \tag{9a}$$

$$\frac{\langle E^2\rangle-\langle E\rangle^2}{t} = \frac{1}{30}[C_{\rm LL}(0)+C_{\rm RR}(0)](2\pi T)^4T + \frac{C_{\rm RL}(eV)e^{eV/2T}+C_{\rm LR}(-eV)e^{-eV/2T}}{120\sinh(eV/2T)}eV \times[(eV)^2+(2\pi T)^2][3(eV)^2+2(2\pi T)^2]. \tag{9b}$$

It is noted that $\langle E\rangle/t = IV/2$ (cf. Ref. [9]). Similarly, it is possible to evaluate higher-order cumulants of $P(E,t)$.

In the symmetric case where $\gamma^{\rm L}=\gamma^{\rm R}\equiv\gamma$, and for values of $eV_{\rm L}$ and $eV_{\rm R}$ which are small relative to the charging energies, one has $C_{ss'}(eV)\simeq(\mu_{N-1}^{-1}-\mu_N^{-1})^2\gamma^2/2\pi\equiv C$. It follows that

$$\frac{\langle E\rangle}{t} = \frac{C}{12}(eV)^2[(eV)^2+(2\pi T)^2], \tag{10a}$$

$$\frac{\langle E^2\rangle-\langle E\rangle^2}{t} = \frac{C}{60}\left\{(2\pi T)^4 4T + \coth\left(\frac{eV}{2T}\right)eV \times[(eV)^2+(2\pi T)^2][3(eV)^2+2(2\pi T)^2]\right\}, \tag{10b}$$

$$\frac{\langle E^2\rangle-\langle E\rangle^2}{\langle E\rangle} = \coth\left(\frac{eV}{2T}\right)\frac{3\,(eV)^2+2\,(2\pi T)^2}{5eV} + \frac{4T(2\pi T)^4}{5\,(eV)^2\,[(2\pi T)^2+(eV)^2]}. \tag{10c}$$

The information on the average and variance is encapsulated in the Fano factor, which is the ratio between them; it is shown in Fig. 2.

In the high bias regime, $eV\gg T$, one observes the following. The average $\langle E\rangle/t\propto(eV)^4$, implying that the QD is more probable to absorb energy than to emit energy. The fluctuations (i.e., standard deviation) $\propto(eV)^{5/2}$. The Fano factor in this limit $\simeq 3eV/5$, expressing a corresponding "effective energy charge."

The results in the linear response regime, $eV\ll T$, are quite different. The average $\langle E\rangle/t\propto(eV)^2T^2$, and the fluctuations $\propto T^{5/2}$. This is reflected in the divergence of the Fano factor, which in this limit $\simeq 32\pi^2T^3/5(eV)^2$ [see Fig. 2 (right)].

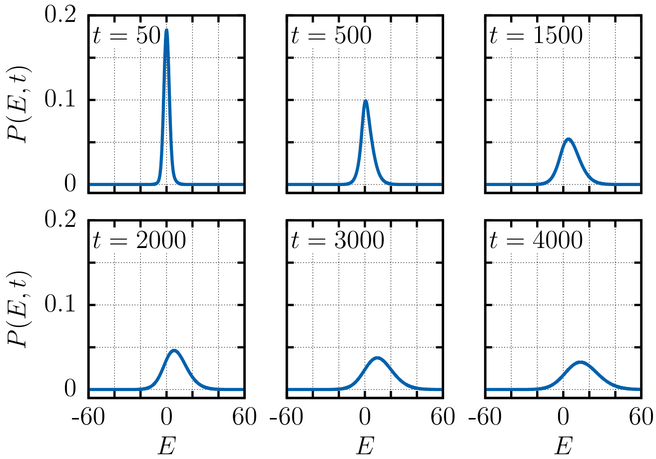

FIG. 3. (Color online) The time evolution of the probability distribution of energy dissipated in the QD, $P(E,t)$ [cf. Eqs. (7) and (8)]. The time intervals are indicated in the panels. $P(E,t)$ is obtained by numerical integration with $T = 1$, $eV = 3$, $C = 10^{-4}$, and $\sigma = 2$. The typical time scale associated with the evolution of $P(E,t)$ is given by $\Gamma_\Sigma^{-1}(\tau = 0) \simeq 1570$.

The Crooks fluctuation relation is not fulfilled by $P(E,t)$. In the present context the Crooks relation reads $P(E,t) = P(-E,t)e^{E/T}$, which upon Fourier transform yields $P(\tau,t) = P(-\tau - i/T,t)$. The latter relation is generically violated by $P(\tau,t)$ given in Eq. (7a). This can be understood by recalling that the Crooks relation applies to the total energy (work) gained by a system, while here E denotes only the energy gained by the QD (and not the energy dissipated in the left and right leads). As a consequence of the symmetry of the problem, $P(E,t)$ is unchanged with respect to a simultaneous interchange of $V_L \rightleftharpoons V_R$ and $\gamma^L \rightleftharpoons \gamma^R$.

It is possible to evaluate $P(E,t)$ by performing the Fourier transform in Eq. (7b) numerically. The evolution of $P(E,t)$ for a case where $eV > T$ is shown in Fig. 3. $P(E,t)$ is seen to propagate and widen, where the typical time scale of its evolution is given by $\Gamma_\Sigma^{-1}(\tau = 0)$.

Experimental considerations. One route to measure $P(E,t)$ is with sensitive thermometry [1]. However, issues concerning "back action" due to the measurement device may then arise [8]. In what follows we propose another method, which is based on a transport measurement of charge. We first conceive ideal energy filters deployed in the left and right leads (see Fig. 1). These filters will allow only electrons with certain energies, say, ϵ_L and ϵ_R, to pass through. We define the rates of charge transfer at these energies, $\Gamma_{RL}(\epsilon_L,\epsilon_R)$ and $\Gamma_{LR}(\epsilon_L,\epsilon_R)$. A measurement of the current and noise, which are proportional to the difference and the sum of these rates, respectively, suffices for determining each of them separately [18]. Change of variables $\epsilon_L,\epsilon_R \to \epsilon_L + \epsilon_R, \pm(\epsilon_R - \epsilon_L)$ and integration of $\Gamma_{RL}(\epsilon_L,\epsilon_R)$ and $\Gamma_{LR}(\epsilon_L,\epsilon_R)$ over $\epsilon_L + \epsilon_R$ then yield $\Gamma_{RL}(\Delta E)$ and $\Gamma_{LR}(\Delta E)$, respectively. If the setup is symmetric, i.e., $\gamma^L = \gamma^R$, extraction of the two other rates, $\Gamma_{LL}(\Delta E)$ and $\Gamma_{RR}(\Delta E)$, is possible. At $eV = 0$ there is no net current, and the electric current noise is proportional to the sum of two equal rates, $\Gamma_{RL}(\epsilon_L,\epsilon_R)$ and $\Gamma_{LR}(\epsilon_L,\epsilon_R)$. By taking $1/2$ of the measured noise we obtain each of those equal rates, as well as $\Gamma_{LL}(\epsilon_L,\epsilon_L') = \Gamma_{RR}(\epsilon_R',\epsilon_R)$ with $\epsilon_L' = \epsilon_R, \epsilon_R' = \epsilon_L$. At finite eV, the rates $\Gamma_{LL}(\Delta E)$ and $\Gamma_{RR}(\Delta E)$ remain unchanged. Note that restricting ourselves to zero temperature, the PDF is dominated now by a single rate $[\Gamma_{RL}(\Delta E)]$, and our analysis does not require the knowledge of $\Gamma_{LL}(\Delta E)$ and $\Gamma_{RR}(\Delta E)$.

Two extra QDs tuned to resonances at energies ϵ_L and ϵ_R can be used to implement the energy filters. The resulting energy resolution will be of the order of the level width of the filters. We require that this width is determined by the coupling of the filter to the respective lead rather than to the central QD. We note that under these conditions, the addition of filters indeed modifies the transport properties of the setup. What is significant to our analysis is the fact that within the allowed filter windows the cotunneling process is hardly modified (the modification is small in the ratio of QD-filter coupling and the filter-lead coupling).

To further improve the energy resolution of the filters, one may introduce a junction with three entry/exit directions in between the QD and each of the filters. Each junction should be connected to the QD, to the nearby filter, and to an additional drain. By breaking the time reversal symmetry the junction can be tuned such that most backscattered electrons are drained out of the circuit through the additional drain and hence do not affect the measurement.

The results reported here constitute a step towards understanding the energy characteristics of nanoscopic setups. Quantum dots, being a pillar of such systems, play an important role in such investigations. By studying a QD operating in the cotunneling regime, the energy characteristics of the QD in the "deep" quantum limit have been addressed directly.

To conclude, we have analyzed energy dissipation in a QD operating in the cotunneling regime, where energy is transferred to the QD in the form of particle-hole excitations. The QD is in contact with an environment, which supplies an equilibration mechanism to the excess energy deposited on the QD by the cotunneling electrons (this energy may also be negative). The time scale associated with the equilibration of the QD is assumed to be the shortest one in the system. We have analytically obtained the cumulant generating function, which supplies complete information on the statistics of energy dissipation in the QD. Specifically, the average, variance, and Fano factor have been evaluated. We have further obtained numerically the corresponding PDF. The analysis of the results in the context of the recently discovered fluctuation relations underlines that fluctuation relations should be applied with caution. Our results are amenable to experimental verification with thermometry, or, with the more common transport measurement of charge.

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[16] See, e.g., H. Bruus and K. Flensberg, *Many-Body Quantum Theory in Condensed Matter Physics: An Introduction* (Oxford University Press, Oxford, U.K., 2004), Sec. 10.3.

[17] This initial condition introduces an additional contribution to the cumulant generating function discussed below, namely, $-\sigma^2\tau^2/2$. This adds to the variance of the PDF a contribution equal to σ^2. We ignore this contribution in the analytical treatment below, since, in principle, σ can be as small as one wishes.

[18] The contribution of elastic cotunneling processes is negligible since the QD is metallic.