Odd-parity superconductivity from phonon-mediated pairing: Application to $Cu_xBi_2Se_3$

P. M. R. Brydon,* S. Das Sarma, Hoi-Yin Hui, and Jay D. Sau

Condensed Matter Theory Center and Joint Quantum Institute, University of Maryland, College Park, Maryland 20742-4111, USA

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Motivated by the proposed topological state in $Cu_xBi_2Se_3$, we study the possibility of phonon-mediated odd-parity superconductivity in spin-orbit coupled systems with time-reversal and inversion symmetry. For such systems, we show that, in general, pure electron-phonon coupling can never lead to a triplet state with a higher critical temperature than the leading singlet state. The Coulomb pseudopotential, which is the repulsive part of the electron-electron interaction and is typically small in weakly correlated systems, is therefore critical to stabilizing the triplet state. We introduce a chirality quantum number, which identifies the electron-phonon vertex interactions that are most favorable to the triplet channel as those that conserve chirality. Applying these results to $Cu_xBi_2Se_3$, we find that a phonon-mediated odd-parity state may be realized in the presence of weak electronic correlations if the chirality-preserving electron-phonon vertices are much stronger than the chirality-flipping vertices.

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I. INTRODUCTION

The discovery that gapped electronic systems can be topologically nontrivial has sparked enormous interest [1,2]. While there now exists several clear examples of topological insulators, such as Bi_2Se_3 [3] and SnTe [4], the unconventional gap structure of topological superconductors make these systems much rarer [5]. Intriguingly, a superconducting state appears upon doping some topological insulators, most notably $Cu_xBi_2Se_3$ [6]. Fu and Berg have proposed that this system realizes a topological superconductor, with a novel odd-parity (triplet) pairing state [7].

$Cu_xBi_2Se_3$ has subsequently been the subject of intense study [8–16]. Experiments show a full gap [9,10], and anomalies in the dc magnetization [11] and an upper critical field that exceeds the Pauli limit indicate triplet pairing [10]. This interpretation is supported by point-contact spectroscopy measurements of the expected topologically protected surface subgap states [12–14], but other experiments find no subgap structure [15], consistent with nontopological s-wave pairing. Although the experimental situation in $Cu_xBi_2Se_3$ has not yet been settled, similar signatures of unconventional superconductivity have been observed in $Sn_{1-x}In_xTe$ [17] and Bi_2Se_3 under pressure [18]. This raises the tantalizing possibility of an entire class of topological superconductors obtained by doping topological insulators.

The origin of a triplet pairing state in any of these doped semiconductors is mysterious, as they are likely free of the strong correlations thought to be an essential aspect [19] of the triplet superconductors UPt_3 [20] and Sr_2RuO_4 [21]. Rather, the electron-phonon interaction is expected to play the dominant role in the pairing [7,16,17]. This is quite surprising, however, as it is widely believed that phonon-mediated pairing generically yields a singlet state [22], although a definitive proof has been lacking. Furthermore, previous analyses did not include the strong spin-orbit coupling characteristic of topological insulators which may favor triplet pairing [7,17]. As such, they cannot exclude the possibility that the electron-phonon interaction indeed stabilizes a triplet state in these materials.

In this paper we study the fundamental question of when electron-phonon interactions stabilize a triplet state and thus evaluate the conditions required for the proposed topological superconductivity in $Cu_xBi_2Se_3$. We first prove a theorem, showing that for the BCS theory the symmetries of the electron-phonon vertex functions ensure that, purely with electron-phonon coupling, the critical temperature of the leading triplet state *never* exceeds that of the leading singlet. Therefore, the stabilization of the triplet state must depend on the so-called Coulomb pseudopotential, which may not be small [23]. We then define a generalized chirality operator, which allows us to identify electron-phonon coupling vertices that would stabilize a triplet gap. Materials where chirality preserving vertices dominate could be candidates for electron-phonon mediated triplet superconductivity. Finally, we apply these insights to a model of $Cu_xBi_2Se_3$ [7] and identify the electron-phonon vertices that cause an attractive interaction in the triplet channel. If these terms dominate the electron-phonon interaction, the topological state could be realized in the presence of weak correlations.

II. ELECTRON-PHONON INTERACTION AND PAIRING

We start by considering a strongly spin-orbit coupled system with inversion $\mathcal{I}$ and time-reversal $\mathcal{T}$ symmetries, so that every eigenstate is at least doubly degenerate [24]. Assuming for simplicity that a single band crosses the Fermi energy, we can index the degenerate momentum eigenstates by a pseudospin variable $s = \pm$, such that $\mathcal{I}|\mathbf{k},s\rangle = |-\mathbf{k},s\rangle$ and $\mathcal{T}|\mathbf{k},s\rangle = s|-\mathbf{k},-s\rangle$. In the presence of strong spin-orbit coupling the electron-phonon interaction may not conserve pseudospin (in contrast to Ref. [22]), and so we have the general form

$$H_{\text{e-p}} = \sum_{\mathbf{k},\mathbf{k}'} \sum_{s,s'} \sum_{\eta} g^{\eta}_{s',s}(\mathbf{k}',\mathbf{k})(b^{\dagger}_{\mathbf{k}-\mathbf{k}',\eta} + b_{\mathbf{k}'-\mathbf{k},\eta}) c^{\dagger}_{\mathbf{k}',s'} c_{\mathbf{k},s} \,, \tag{1}$$

where $b_{\mathbf{q},\eta}$ is the annihilation operator for a phonon in mode η with momentum $\mathbf{q}$, and $c_{\mathbf{k},s}$ is the annihilation operator for an

*pbrydon@umd.edu

electron in state $|\mathbf{k},s\rangle$. The inversion and time-reversal symmetries require that the vertex functions satisfy $g^{\eta}_{s',s}(\mathbf{k}',\mathbf{k}) = \pm_{\eta} g^{\eta}_{s',s}(-\mathbf{k}',-\mathbf{k})$ and $g^{\eta}_{s',s}(\mathbf{k}',\mathbf{k}) = ss'[g^{\eta}_{-s',-s}(-\mathbf{k}',-\mathbf{k})]^*$, respectively, where the sign $\pm_{\eta}$ under inversion depends on the phonon mode.

Within the BCS approximation, the electron-phonon coupling generates the pairing interaction

$$V_{s_2,s_1;s_3,s_4}(\mathbf{k}',\mathbf{k}) = -\sum_{\eta} \frac{g^{\eta}_{s_1,s_3}(\mathbf{k}',\mathbf{k}) g^{\eta}_{s_2,s_4}(-\mathbf{k}',-\mathbf{k})}{\omega_{\mathbf{k}'-\mathbf{k},\eta}} \times \Theta(\omega_D - |\epsilon_{\mathbf{k}}|)\Theta(\omega_D - |\epsilon_{\mathbf{k}'}|)\,, \quad (2)$$

where $\omega_{\mathbf{q},\eta}$ is the dispersion of phonon mode η, $\epsilon_{\mathbf{k}}$ is the electronic dispersion, and ω_D is a cutoff on the order of the Debye energy. The pairing interaction is the kernel of the linearized BCS equation for the matrix gap function $\hat{\Delta}(\mathbf{k})$, which is formulated as an eigenvalue problem

$$\lambda \Delta_{s_1,s_2}(\mathbf{k}') = -\sum_{\mathbf{k},s_3,s_4} V_{s_2,s_1;s_3,s_4}(\mathbf{k}',\mathbf{k}) \Delta_{s_3,s_4}(\mathbf{k})\,. \quad (3)$$

Only solutions with positive eigenvalues have a finite critical temperature, and the solution with the largest eigenvalue is the leading instability. Inversion symmetry limits physical solutions to either even-parity pseudospin singlet or odd-parity pseudospin triplet states.

III. SINGLET VS TRIPLET PAIRING

In the conventional case, i.e., in the absence of spin-orbit coupling, electron-phonon coupling is expected to lead to the singlet channel being dominant. Such a singlet pairing state is described by a gap function $\hat{\Delta}^{(s)}(\mathbf{k}) = f^{(s)}(\mathbf{k})(i\hat{\sigma}^y)$, where $f^{(s)}(\mathbf{k})$ gives the momentum dependence of the pairing function, and $\hat{\sigma}^y$ is the y Pauli matrix. For the general electron-phonon interaction, the symmetries of the electron-phonon vertices yield a gap equation in the singlet channel of the form

$$\lambda^{(s)} f^{(s)}(\mathbf{k}') = {\sum_{\mathbf{k},s,\eta}}' \frac{\left|g^{\eta}_{ss}(\mathbf{k}',\mathbf{k})\right|^2 + \left|g^{\eta}_{s\bar{s}}(\mathbf{k}',\mathbf{k})\right|^2}{\omega_{\mathbf{k}-\mathbf{k}',\eta}} f^{(s)}(\mathbf{k})\,, \quad (4)$$

where the momenta are restricted to the shell of thickness ω_D about the Fermi surface, which is indicated by the prime on the summation. The singlet gap function is therefore an eigenstate of a matrix with nonnegative entries. It follows from the Perron-Frobenius theorem [25] that the gap function $f^{(s)}(\mathbf{k})$ of the dominant instability has no sign changes as a function of the wave vector $\mathbf{k}$, as is characteristic of conventional s-wave singlet pairing.

We now consider the triplet pairing function with the largest critical temperature, $\hat{\Delta}^{(t)}(\mathbf{k})$. To compare with the singlet channel, we apply a momentum dependent pseudospin-rotation transformation so that it is recast in the form $\hat{\Delta}^{(t)}(\mathbf{k}) = \chi_{\mathbf{k}} f^{(t)}(\mathbf{k})\hat{\sigma}^x$, where $f^{(t)}(\mathbf{k})$ and $\chi_{\mathbf{k}}$ are the magnitude and sign of the triplet gap, respectively, and $\hat{\sigma}^x$ is the x Pauli matrix. In other words, we have rotated the pseudospin at $\mathbf{k}$ and $-\mathbf{k}$ so that in the new pseudospin basis the triplet pair formed from these states has vanishing z component of pseudospin. Note that this rotation does not affect the singlet pairing, nor does it alter the symmetry properties of the electron-phonon vertices. The gap magnitude $f^{(t)}(\mathbf{k})$ satisfies the eigenvalue equation

$$\lambda^{(t)} f^{(t)}(\mathbf{k}') = {\sum_{\mathbf{k},s,\eta}}' \chi_{\mathbf{k}'}\chi_{\mathbf{k}} \frac{\left|g^{\eta}_{ss}(\mathbf{k}',\mathbf{k})\right|^2 - \left|g^{\eta}_{s\bar{s}}(\mathbf{k}',\mathbf{k})\right|^2}{\omega_{\mathbf{k}-\mathbf{k}',\eta}} f^{(t)}(\mathbf{k})\,. \quad (5)$$

The magnitude of the matrix elements in Eq. (5) are bounded by the corresponding elements in the singlet gap equation. By a corollary to the Perron-Frobenius theorem [25], the maximal eigenvalue of Eq. (5) therefore cannot exceed the maximal singlet eigenvalue. Since the leading triplet gap satisfies Eq. (5), we have our first major result which can be stated as the following theorem: in a system with inversion and time-reversal symmetry, the critical temperature of the leading triplet gap *never exceeds* that of the leading singlet gap for a purely phonon-mediated pairing interaction.

Our analysis implies that electronic correlations are vital to stabilizing a triplet state. In particular, the spatial separation of the electrons in a triplet Cooper pair reduces the pair-breaking effect of the Coulomb pseudopotential compared to a s-wave singlet state. A sufficiently large Coulomb pseudopotential may therefore reduce the critical temperature of the leading singlet state below that of the triplet [22]. Such a strong Coulomb pseudopotential is the necessary condition for the triplet superconductivity to emerge in the system.

IV. DEGENERATE SINGLET AND TRIPLET STATES

While the singlet pairing typically may be expected to dominate over triplet pairing, it was pointed out by Fu and Berg [7] that the singlet and triplet states would be degenerate if the Dirac-like Hamiltonian considered by them commuted with a chirality operator. Motivated by this, we generalize the notion of "chirality" to index the doubly degenerate states near the Fermi surface of an arbitrary electronic system. Specifically, the pseudospin states $|\mathbf{k},s\rangle$ become the chiral states $|\mathbf{k},\nu\rangle$ where the chirality $\nu = s\chi_{\mathbf{k}}$, and $\chi_{\mathbf{k}}$ is the sign of the leading triplet gap as defined above. We hence replace the pseudospin indices in the gap equations (4) and (5) by chirality indices using $g^{\eta}_{s',s}(\mathbf{k}',\mathbf{k}) = g^{\eta}_{\nu',\nu}(\mathbf{k}',\mathbf{k})\delta_{\nu',s'\chi_{\mathbf{k}'}}\delta_{\nu,s\chi_{\mathbf{k}}}$, obtaining

$$\lambda^{(\alpha)} f^{(\alpha)}(\mathbf{k}') = {\sum_{\mathbf{k},\nu,\eta}}' \frac{\left|g^{\eta}_{\nu\nu}(\mathbf{k}',\mathbf{k})\right|^2 \pm \left|g^{\eta}_{\nu\bar{\nu}}(\mathbf{k}',\mathbf{k})\right|^2}{\omega_{\mathbf{k}-\mathbf{k}',\eta}} f^{(\alpha)}(\mathbf{k}), \quad (6)$$

where the plus (minus) sign in the summand holds for $\alpha = s\,(t)$. Comparing the transformed equations in the singlet and triplet channels, it is clear that the singlet and triplet eigenvalues are identical if the electron-phonon vertices do not flip the chirality index, i.e., $\lambda^{(s)} = \lambda^{(t)}$. We see that electron-phonon vertices which preserve an appropriately defined chirality index generate attractive interactions in the triplet channel, while chirality-flipping vertices are always triplet pair breaking. This is the second major result of our paper. Note that in other works, unconventional pairing is achieved via a strongly forward-scattering electron-phonon interaction, which promotes attractive interactions in many pairing channels [16,26]. In contrast, our condition precisely determines the electron-phonon interactions that generate the triplet state, and there is no requirement that these vertices involve small momentum transfers.

We make our discussion more concrete by using the chirality index to define a chirality operator $\mathcal{O}_{\rm ch}(\mathbf{k}) = \sum_\nu \nu|\mathbf{k},\nu\rangle\langle\mathbf{k},\nu|$. When only electron-phonon interactions which commute with $\sum_{\mathbf{k}} \mathcal{O}_{\rm ch}(\mathbf{k})$ are present, every singlet solution $\hat{\Delta}^{(s)}(\mathbf{k})$ is degenerate with a triplet solution $\hat{\Delta}^{(t)}(\mathbf{k}) = U(\mathbf{k})\hat{\Delta}^{(s)}(\mathbf{k})U(-\mathbf{k})$, where $U(\mathbf{k}) = \exp(i\pi\mathcal{O}_{\rm ch}(\mathbf{k})/4)$. On the other hand, an electron-phonon interaction which does not commute with the chirality operator is triplet pair-breaking. Crucially, it is not necessary to solve the gap equations to define the chirality index, as this only depends upon the sign structure of the triplet gap. This is very convenient, as it is common to approximate the exact solution of the gap equations by a simple function consistent with the point group. Given such a time-reversal-invariant triplet state, we can hence define a chirality operator which relates it to a singlet state with a nonnegative gap. The effective coupling constants for these two states, obtained by taking the inner product of the gap functions with the pairing interaction Eq. (2), are then degenerate if only electron-phonon vertices which preserve the chirality are present.

V. APPLICATION TO $Cu_xBi_2Se_3$

The proposed odd-parity pairing state of $Cu_xBi_2Se_3$ provides an excellent illustration of the preceding discussion. We start by introducing an effective Hamiltonian valid near the Fermi surface, where the electronic states are primarily derived from the Se p_z orbitals at the top and bottom of the quintuple-layer unit cell. Denoting these two distinct sites by $s^z = \pm 1$, the low-energy spectrum is described by the $\mathbf{k}\cdot\mathbf{p}$ model [7]

$$H_0 = \sum_{\mathbf{k}} \psi^\dagger(\mathbf{k})[-\mu\hat{s}^0\otimes\hat{\sigma}^0 + m\hat{s}^x\otimes\hat{\sigma}^0 + v_z k_z\hat{s}^y\otimes\hat{\sigma}^0 + v(k_x\hat{s}^z\otimes\hat{\sigma}^y - k_y\hat{s}^z\otimes\hat{\sigma}^x)]\psi(\mathbf{k}). \quad (7)$$

Here $\psi(\mathbf{k}) = (c_{\mathbf{k},1,\uparrow}, c_{\mathbf{k},1,\downarrow}, c_{\mathbf{k},-1,\uparrow}, c_{\mathbf{k},-1,\downarrow})^T$, where $c_{\mathbf{k},n,\sigma}$ destroys an electron with momentum $\mathbf{k}$ and spin σ at site n. The Pauli matrices in site and spin space are denoted by $\hat{s}^\mu$ and $\hat{\sigma}^\mu$, respectively. The chemical potential is denoted by μ, m is the mass, and v_z and v are velocities along the z-axis and in the x-y plane, respectively. We consider the physical case where the chemical potential lies in the conduction band, i.e., $\mu > m$. The Hamiltonian is symmetric under inversion ($\mathcal{I} = \hat{s}^x\otimes\hat{\sigma}^0$) and time reversal ($\mathcal{T} = i\hat{s}^0\otimes\hat{\sigma}^y\mathcal{K}$), and so the eigenstates of Eq. (7) can be labeled by a pseudospin [13].

The site degree of freedom allows odd-parity superconducting states in a relative s wave, such as the A_{1u} state $\Delta_{A_{1u}} i\hat{s}^y\otimes\hat{\sigma}^x$ proposed in Ref. [7]. As it opens a full gap on the Fermi surface [7,13], and has surface bound states consistent with point-contact spectroscopy measurements [12–14], it is one of the most promising candidates for a topological state in $Cu_xBi_2Se_3$. We have seen, however, that the phonon-mediated pairing interaction generally favors an even-parity state with a full gap. The simplest example of this is the topologically trivial A_{1g} state $\Delta_{A_{1g}} i\hat{s}^0\otimes\hat{\sigma}^y + \Delta'_{A_{1g}} i\hat{s}^x\otimes\hat{\sigma}^y$ [7].

In the absence of the mass term in Eq. (7), the Bogoliubov Hamiltonian for the A_{1g} state with $\Delta'_{A_{1g}} = 0$ can be mapped into that for the A_{1u} state by the unitary transformation $U = \exp(i\pi\hat{s}^y\otimes\hat{\sigma}^z/4)$ [7]. This immediately identifies the chirality operator as $\mathcal{O}_{\rm ch}(\mathbf{k}) = \hat{s}^y\otimes\hat{\sigma}^z$. In the compact notation of Eq. (7), we have the general electron-phonon interaction Hamiltonian

$$H_{\text{e-p}} = \sum_{\mathbf{k},\mathbf{k}'}\sum_{\eta}\sum_{\mu,\nu} f^{\eta}_{\mu,\nu}(\mathbf{k}',\mathbf{k})(b^\dagger_{\mathbf{k}-\mathbf{k}',\eta} + b_{\mathbf{k}'-\mathbf{k},\eta}) \times \psi^\dagger(\mathbf{k}')\hat{s}^\mu\otimes\hat{\sigma}^\nu\psi(\mathbf{k}). \quad (8)$$

If only vertex functions $f^{\eta}_{\mu,\nu}(\mathbf{k}',\mathbf{k})$ for which $\hat{s}^\mu\otimes\hat{\sigma}^\nu$ commutes with the chirality operator $\hat{s}^y\otimes\hat{\sigma}^z$ are nonzero, it follows from the discussion above that the coupling constants for the A_{1u} and A_{1g} states are identical. Vertex functions for which $\hat{s}^\mu\otimes\hat{\sigma}^\nu$ anticommutes with $\hat{s}^y\otimes\hat{\sigma}^z$ are generally expected to be present, however, giving the A_{1g} state the higher coupling constant.

In the general case of a finite mass gap, the Fu and Berg A_{1g} and A_{1u} Hamiltonians cannot be mapped into one another by a chirality transformation. In the vicinity of the Fermi surface, however, we can define a chirality operator that relates the two gaps [27]. This is sufficiently close to the chirality operator in the massless limit that the classification of the electron-phonon vertices obtained above remains valid to good approximation. Specifically, the chirality-preserving electron-phonon vertices for the massless case are now either still chirality preserving, or contain chirality-flipping terms which are smaller by a factor of $m/\mu \approx 0.3$ than the chirality preserving [8]. A similar analysis holds for the vertices which flip the chirality in the $m = 0$ limit. Our classification of the electron-phonon vertices is the starting point for a detailed microscopic analysis of the pairing instability in $Cu_xBi_2Se_3$.

We make this concrete by considering a toy model where the electrons couple to a dispersionless optical mode with frequency ω_0. From Eq. (8) we include only the $(\mu,\nu) = (0,0)$ and $(x,0)$ terms, representing chirality-preserving and flipping vertices, respectively. We assume that the corresponding vertex functions g_0 and g_x are constant. The Fu and Berg A_{1g} and A_{1u} states are then exact eigenstates of the phonon-mediated pairing interaction, with eigenvalues $\lambda_{A_{1g}} = (g_0^2 + g_x^2 + 2|g_x g_0|m/\mu)/\omega_0$ and $\lambda_{A_{1u}} = (g_0^2 - g_x^2)[1-(m/\mu)^2]/\omega_0$, respectively. The A_{1g} state is the leading instability for nonzero g_x or m, while the A_{1u} state only has a finite critical temperature for $|g_x| < |g_0|$. We also include the on-site repulsion $H_{\text{e-e}} = (U/V)\sum_{\mathbf{q}}\sum_{s=\pm}\rho_{s,\uparrow}(\mathbf{q})\rho_{s,\downarrow}(-\mathbf{q})$, where $\rho_{s,\sigma}(\mathbf{q}) = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}+\mathbf{q},s,\sigma}c_{\mathbf{k},s,\sigma}$ and V is the volume. As the first A_{1g} gap $\Delta_{A_{1g}}$ involves on-site pairing, a finite $U > 0$ will tend to lower its critical temperature. On the other hand, the intersite A_{1u} state is unaffected by $H_{\text{e-e}}$.

We study the pairing in our model within the mean-field approximation. For simplicity, the conduction band is assumed to extend from $m-\mu$ below the Fermi surface to $W-\mu$ above, with a constant density of states ν_0 and $W \gg m$. Deriving the gap equations, we find that the critical temperature of the A_{1g} state satisfies

$$\det\begin{vmatrix} \left(\frac{g_x^2+g_0^2}{\omega_0} - \frac{U}{2}\right)\chi_0 - 1 & -\frac{U}{2}\chi & \frac{g_0 g_x}{\omega_0}\chi_{01} \\ -\frac{U}{2}\chi_0 & -\frac{U}{2}\chi - 1 & 0 \\ \frac{g_0 g_x}{\omega_0}\chi_{01} & 0 & \frac{g_x^2+g_0^2}{\omega_0}\chi_1 - 1 \end{vmatrix} = 0, \quad (9)$$

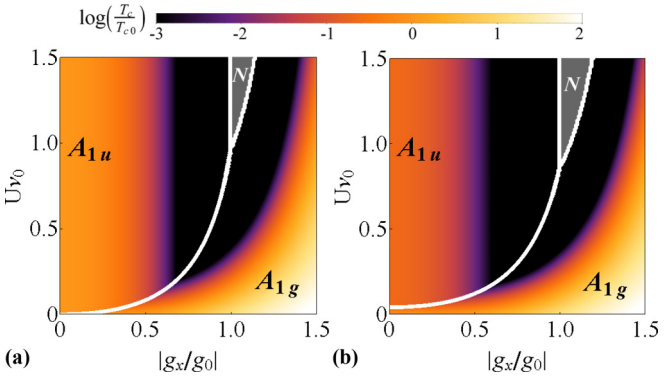

FIG. 1. (Color online) Phase diagram for our toy model of $Cu_xBi_2Se_3$ with (a) vanishing ($m = 0$) and (b) nonzero ($m = 0.4\mu$) mass gaps, showing the leading superconducting instability as a function of $|g_x/g_0|$ and $U\nu_0$. The logarithmic color scale shows the critical temperature T_c relative to the critical temperature T_{c0} at $m = U = g_x = 0$. In the gray region N, the system remains normal down to zero temperature. We set $W = 10\mu$, $\omega_D = 0.1\mu$, and $g_0^2/\omega_0 = 0.1225/\nu_0$.

while for the A_{1u} state we have to solve $\lambda_{A_{1u}}\chi_0 = 1$. Following the notation of Ref. [7], the gap equations are expressed in terms of $\chi_0 = \nu_0 \int_{-\omega_D}^{\omega_D} d\epsilon \tanh(\epsilon/2k_bT_c)/\epsilon$, $\chi_{01} = (m/\mu)\chi_0$, $\chi_1 = (m/\mu)^2\chi_0$, and $\chi = \nu_0 \int_{m-\mu}^{W-\mu} d\epsilon \tanh(\epsilon/2k_bT_c)/\epsilon - \chi_0$. The resulting phase diagram is shown in Fig. 1 for the cases of (a) vanishing and (b) nonzero mass gap. In the absence of on-site repulsion the A_{1g} state has a higher critical temperature than the A_{1u}, except for $m = g_x = 0$ where the two are degenerate. Sufficiently strong on-site repulsion suppresses the critical temperature of the A_{1g} state below that for the A_{1u}. For small ratios $|g_x/g_0| \lesssim 0.5$, this requires only a relatively weak repulsion $U \approx 0.1W$. If $|g_x/g_0|$ is close to unity, however, a repulsive potential on the order of the bandwidth is necessary, and the critical temperature will be very small. Since $Cu_xBi_2Se_3$ is likely weakly correlated, we conclude that the A_{1u} state could be realized if the chirality-preserving electron-phonon vertices are much larger than the chirality flipping, which is the final major result of our work. It is not obvious that this should be the case, however, and this problem requires detailed microscopic modeling beyond the present discussion. Interestingly, Wan and Savrasov have recently proposed that a strongly forward-scattering phononic modulation of the spin-orbit coupling is generic to layered semiconductors [16], although a nodal A_{2u} state then has highest eigenvalue in the triplet channel.

VI. SUMMARY

In this paper we have shown that within the BCS theory the leading instability of a phonon-mediated pairing interaction can be a triplet state, but this must be degenerate with a singlet solution. Our analysis relies only on the symmetries of the electron-phonon vertex functions. We have additionally formulated a condition in terms of a chirality operator for when this degeneracy holds. We have hence identified the electron-phonon vertices that produce an attractive interaction in the triplet channel and those that are pair breaking, which we apply to the topological state proposed for $Cu_xBi_2Se_3$. If the former dominate the latter, we show that weak electronic correlations could stabilize the odd-parity state. Large-scale (and quantitatively accurate) first principles calculations can in principle determine whether specific systems (e.g., $Cu_xBi_2Se_3$, $Sn_{1-x}In_xTe$, etc.) satisfy the necessary theoretical constraints derived in our work, providing a route to the realization of topological superconductivity in ordinary electronic materials.

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