

Hall field-induced resistance oscillations in a *p*-type Ge/SiGe quantum well

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We report on a magnetotransport study in a high-mobility 2D hole gas hosted in a pure Ge/SiGe quantum well subject to dc electric fields and high-frequency microwave radiation. We find that under applied dc bias the differential resistivity exhibits a pronounced maximum at a magnetic field which increases linearly with the applied current. We associate this maximum with the fundamental peak of Hall field-induced resistance oscillations (HIRO) which are known to occur in 2D electron gases in GaAs/AlGaAs systems. After taking into account the Dingle factor correction, we find that the position of the HIRO peak is well described by the hole effective mass $m^* \approx 0.09 m_0$, obtained from microwave photoresistance in the same sample.

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Magnetotransport in very high Landau levels of high-mobility two-dimensional electron gases (2DEG) hosted in GaAs/AlGaAs quantum wells is known to exhibit a variety of fascinating phenomena [1]. For example, when a 2DEG is exposed to microwave radiation and weak perpendicular magnetic field B , the magnetoresistance acquires prominent oscillations [2,3], which are controlled by $\epsilon_\omega = \omega/\omega_c$, where $\omega = 2\pi f$ is the microwave frequency and $\omega_c = eB/m^*$ is the cyclotron frequency of the charge carrier with the effective mass m^* . These oscillations, known as microwave-induced resistance oscillations (MIRO), are usually explained in terms of the displacement mechanism [4–7], which originates from the radiation-induced modification of scattering off impurities, and the inelastic mechanism [8,9], which stems from the radiation-induced changes in the distribution function. Both mechanisms predict that the photoresistivity $\delta\rho_\omega$ oscillates as

$$\delta\rho_\omega/\rho_D = -2\pi\eta\mathcal{P}\lambda^2\epsilon_\omega \sin 2\pi\epsilon_\omega, \quad (1)$$

where ρ_D is the Drude resistivity, η is the dimensionless scattering rate, which contains both displacement and inelastic contributions [10], \mathcal{P} is the dimensionless microwave power [9,11], $\lambda = \exp(-\pi/\omega_c\tau_q)$ is the Dingle factor, and τ_q is the quantum lifetime. In extremely clean 2DEG [12], the MIRO minima can develop into zero-resistance [13,14] or zero-conductance [15] states. Microwave-induced magnetoconductance oscillations and associated zero-conductance states [15] have been also realized in a nondegenerate 2D system, electrons on liquid helium surface [16,17]. Most recently, MIRO have been observed in a two-dimensional hole gas (2DHG) in Ge/SiGe quantum wells [18].

Another class of magneto-oscillations appears in the differential resistivity r of a Hall bar-shaped structures under applied direct current [19–21]. These oscillations, known as Hall field-induced resistance oscillations (HIRO), originate exclusively from the displacement mechanism [22,23] as a result of the commensurability between the cyclotron diameter $2R_c$ and the spatial separation between Landau levels, tilted in space by Hall electric field $E = \rho_H j$, where j is the current density and ρ_H is the Hall resistivity. More specifically, whenever $\epsilon_j = eE(2R_c)/\hbar\omega_c$ is close to an integer value, the probability for an electron to make an elastic transition to a higher Landau level as a result of backscattering off short-range disorder is maximized, giving rise to a maximum in r . At $2\pi\epsilon_j \gg 1$, the resultant correction to the differential

resistivity δr is given by

$$\frac{\delta r}{\rho_D} = \frac{16}{\pi} \frac{\tau}{\tau_\pi} \lambda^2 \cos 2\pi\epsilon_j, \quad (2)$$

where τ is the transport scattering time and τ_π is the backscattering time. As suggested by Eq. (2), HIRO can be used to obtain the effective mass of the charge carrier. This can be achieved using, e.g., the dependence of the magnetic field of the fundamental HIRO peak ($\epsilon_j = 1$) on j ,

$$B_1 \approx \frac{4\pi m^*}{e^2 k_F} j, \quad (3)$$

where k_F is the Fermi wave number. To date, studies of HIRO have been limited almost exclusively [24] to high-mobility 2DEG in GaAs/AlGaAs heterostructures [19–21,25–29]. While Eq. (3) was successfully employed in the above studies, it disregards the B dependence of λ^2 which, as we show below, becomes important at sufficiently low τ_q values.

In this Rapid Communication we report on a nonlinear magnetotransport study in a new material system, a high-mobility two-dimensional hole gas hosted in a pure Ge/SiGe quantum well [30]. Under applied dc bias, the differential resistivity exhibits a pronounced maximum which shifts to higher B with increasing j . In agreement with Eq. (3), we observe a roughly linear relationship between the peak position B_1 and j . However, direct employment of Eq. (3) yields an estimate for the hole effective mass of $m^* \approx 0.11 m_0$, noticeably larger than $m^* \approx 0.09 m_0$ obtained in recent MIRO experiments [18]. To investigate this discrepancy, we have measured microwave photoresistance in the same sample, from which we have obtained $m^* \approx 0.087 m_0$, in good agreement with Ref. [18]. The high quality of our MIRO data also allowed us to perform a Dingle analysis from which we obtained $\tau_q \approx 2.8$ ps. We demonstrate that in our 2DHG, Eq. (3) underestimates the position of the fundamental HIRO maximum due to (i) the strong B dependence of the Dingle factor, which is neglected in Eq. (3), and (ii) the approximate nature of Eq. (2) near $\epsilon_j \approx 1$. Once the above factors are taken into account, we find that HIRO in our 2DHG are well described by $m^* \approx 0.09 m_0$.

Our lithographically defined 50- μm -wide Hall bar sample was fabricated from a fully strained, 20-nm-wide Ge/Si_{0.2}Ge_{0.8} quantum well grown by reduced pressure chemical vapor

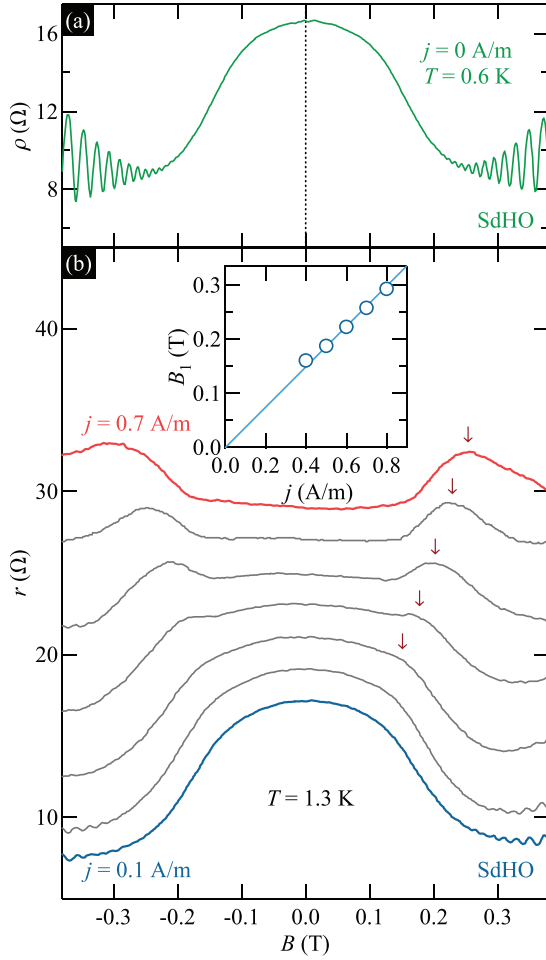


FIG. 1. (Color online) (a) Magnetoresistivity $\rho(B)$ measured at $T \approx 0.6$ K. (b) Differential resistivity $r(B)$ at current densities from $j = 0.1$ A/m to 0.7 A/m, in steps of 0.1 A/m, measured at $T \approx 1.3$ K. The traces are vertically offset for clarity by 2Ω . The inset shows the position of the fundamental HIRO peak B_1 as a function of the current density j . Fit to $B_1 = 4\pi m^* j / e^2 k_F$ yields $m^* \approx 0.11 m_0$.

deposition [30]. Holes were supplied by a 10-nm-wide Boron-doped layer separated from the Ge channel by a 26-nm-wide undoped $\text{Si}_{0.2}\text{Ge}_{0.8}$ spacer. At $T \approx 1$ K, our 2DHG has the hole density $p \approx 2.8 \times 10^{11} \text{ cm}^{-2}$ and the mobility $\mu \approx 1.3 \times 10^6 \text{ cm}^2/\text{Vs}$. The resistivity ρ and the differential resistivity r were measured in sweeping B using a standard four-terminal lock-in technique.

In Fig. 1(a) we present magnetoresistivity $\rho(B)$, measured at $T \approx 0.6$ K, which exhibits significant negative magnetoresistance effect, similar to what has been recently observed in 2DEG in GaAs/AlGaAs quantum wells [31–37], and Shubnikov–de Haas oscillations. Figure 1(b) shows the differential resistivity r as a function of B under applied direct currents with densities from $j = 0.1$ A/m to 0.7 A/m, in steps of 0.1 A/m, measured at $T \approx 1.3$ K. Distinct peaks (cf. \downarrow) start showing up at $j = 0.3$ A/m, symmetrically at both magnetic field directions. As expected for HIRO, the peaks move to higher B and grow in magnitude with increasing current.

According to Eq. (2) the fundamental HIRO maximum should occur close to $\epsilon_j = 1$ and, as a result, its position should

scale linearly with j ; see Eq. (3). In the inset of Fig. 1(b) we plot B_1 as a function of j and observe the expected linear dependence. The fit to the data using Eq. (3) yields an effective mass of $m^* \approx 0.11 m_0$. This value is about 20% higher than the effective mass, $m^* \approx 0.09$, obtained in a recent MIRO experiment [18] on a lower mobility Ge/SiGe quantum well.

There are at least two factors which might lead to an overestimated value of m^* obtained from our HIRO data using Eq. (3). First, Eq. (2) is valid only in the limit of $2\pi\epsilon_j \gg 1$, a condition which is only marginally met at $\epsilon_j \approx 1$. For a more accurate description of HIRO, $\cos 2\pi\epsilon_j$ in Eq. (2) should be replaced by [22]

$$\mathcal{F}_j(\epsilon_j) \approx \frac{\pi}{2} [J_1^2(\pi\epsilon_j) - 2\pi\epsilon_j J_0(\pi\epsilon_j) J_1(\pi\epsilon_j)], \quad (4)$$

where J_0 and J_1 are the Bessel functions of the first kind. The second factor which contributes to an overestimated value of m^* stems from the dependence of the Dingle factor on ϵ_j , $\lambda^2(\epsilon_j) = \exp(-\epsilon_j/f_j\tau_q)$, where $f_j = 2j/ek_F$.

In order to more accurately access the value of ϵ_j which corresponds to the position of the fundamental HIRO peak B_1 , it is necessary to know the quantum lifetime τ_q . In contrast to the HIRO experiments on 2DEG in high-mobility GaAs/AlGaAs quantum wells, which routinely exhibit multiple oscillations [21,27,29,38], the data in our 2DHG reveal only one HIRO maximum and are not of sufficient quality to perform systematic Dingle analysis. We can, however, obtain τ_q from microwave photoresistance, as discussed below.

In Fig. 2(a) we present microwave-induced resistance oscillations [39] measured at $f = 179$ GHz and $T \approx 0.6$ K in the same sample. We note that in our previous MIRO study [18], which used a lower mobility ($\mu \approx 0.4 \times 10^6 \text{ cm}^2/\text{Vs}$) device and lower microwave frequencies ($f \leq 110$ GHz), only two MIRO maxima and one minimum were resolved. In this experiment, however, we detect four pairs of maxima and minima, occurring on the opposite sides of the corresponding cyclotron resonance harmonics. According to Eq. (1), the photoresistance is expected to vanish at integer values of ϵ_ω , providing an accurate way to obtain the effective mass value. As shown in Fig. 2(a), vertical lines which are drawn at $\epsilon_\omega = 1, 2, 3$, and 4 , calculated using $m^* = 0.087 m_0$, cross all observed zero-response nodes, $\delta\rho_\omega = 0$. The obtained value is in good agreement with an earlier estimate of $m^* = 0.09 m_0$ obtained in Ref. [18].

The high quality of the photoresistance data shown in Fig. 2(a) allows us to perform a proper Dingle analysis of the MIRO amplitude in 2DHG hosted in a Ge/SiGe quantum well. In Fig. 2(b) we present a reduced MIRO amplitude $\delta\rho_\omega/\epsilon_\omega$ as a function of ϵ_ω using a log-linear scale and observe well-behaved exponential dependence over more than two orders of magnitude. The fit to the data using $\exp(-\epsilon_\omega/f\tau_q) \equiv \exp(-2\pi/\omega_c\tau_q)$ yields $\tau_q \approx 2.8$ ps. This value is considerably lower than τ_q in high-mobility 2DEG in GaAs/AlGaAs, where it ranges between 10 and 20 ps [21,33,40–42]. Using $m^* = 0.09 m_0$ and $\tau_q = 2.8$ ps we can estimate λ^2 at $B = 0.13$ T, the field where one can expect to observe the second HIRO maximum at current density $j = 0.7$ A/m [43]. The obtained value of $\lambda^2 \sim 10^{-4}$ is about two orders of magnitude smaller than λ^2 at $B = 0.25$ T, explaining why no second HIRO maximum is detected. This observation is consistent with the

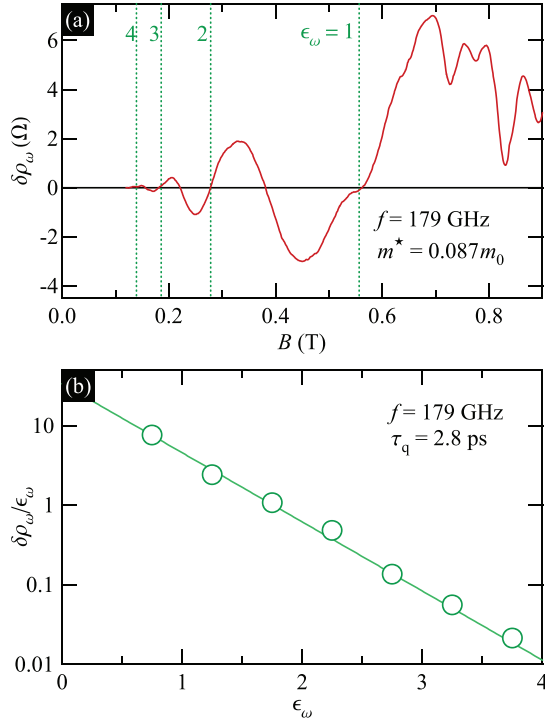


FIG. 2. (Color online) (a) Microwave photoresistance $\delta\rho_\omega$ as a function of B measured at $f = 179$ GHz and $T = 0.6$ K. Vertical lines are drawn at the cyclotron resonance harmonics, $\epsilon_\omega = 1, 2, 3, 4$ calculated using $m^* = 0.087 m_0$. (b) Reduced MIRO amplitude $\delta\rho_\omega/\epsilon_\omega$ as a function of ϵ_ω on a log-linear scale. The fit to $\exp(-\epsilon_\omega/f\tau_q)$ generates $\tau_q \approx 2.8$ ps.

fact that MIRO also cease to exist at $B \lesssim 0.13$ kG (see also Ref. [18]).

Having obtained the quantum lifetime, we now demonstrate that incorporating the B dependence of the Dingle factor and using Eq. (4), instead of approximate expression given by Eq. (2), can indeed explain the larger value of the effective mass obtained using Eq. (3). In Fig. 3(a) we present $\lambda^2 \mathcal{F}_j$ calculated using Eq. (4) with $j = 0.7$ A/m, $\tau_q = 2.8$ ps, $m^* = 0.087 m_0$ as a function of B . The vertical lines are drawn at $\epsilon_j = 1, 2$. As one can see, only a single HIRO peak is observed, in agreement with our experimental data. Furthermore, this peak occurs at $B \approx 0.25$ T, which is very close to what we observe in experiment [cf. the top trace in Fig. 1(b)] and is almost exactly 20% higher than B_1 obtained from Eq. (3). Based on this observation, we conclude that both MIRO and HIRO are well described by $m^* \approx 0.09 m_0$. We emphasize that to obtain an accurate position of the fundamental HIRO peak it is important to both include the Dingle factor correction and to use Eq. (4), instead of approximate Eq. (2). We also notice that the Dingle factor correction does not affect the accuracy of m^* obtained from MIRO since we are using zero-response nodes, according to Eq. (1), exactly correspond to integer values of ϵ_ω .

Finally, to qualitatively confirm the value of τ_q , obtained from the Dingle plot in Fig. 2(b), we present in Fig. 3(b) $\lambda^2 \mathcal{F}_\omega = -2\pi\epsilon_\omega\lambda^2 \sin 2\pi\epsilon_\omega$ calculated using $f = 179$ GHz, $\tau_q = 2.8$ ps, and $m^* = 0.087 m_0$ as a function of B . We observe that the calculated curve very well reproduces

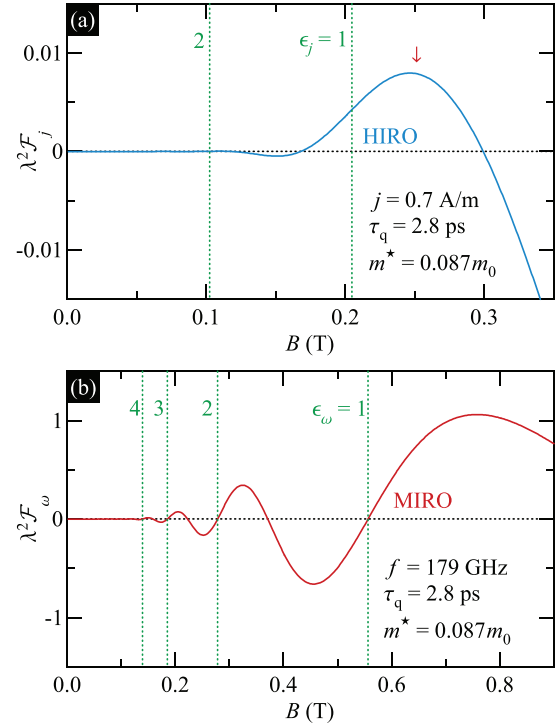


FIG. 3. (Color online) (a) $\lambda^2 \mathcal{F}_j$ calculated using Eq. (4) with $j = 0.7$ A/m, $\tau_q = 2.8$ ps, $m^* = 0.087 m_0$ as a function of B . Vertical lines are drawn at $\epsilon_j = 1, 2$. Notice that the fundamental HIRO peak (cf. \downarrow) occurs at B which is 20% higher than B_1 given by Eq. (3) (cf. vertical line marked by $\epsilon_j = 1$). (b) $\lambda^2 \mathcal{F}_\omega = -2\pi\epsilon_\omega\lambda^2 \sin 2\pi\epsilon_\omega$ calculated with $f = 179$ GHz, $\tau_q = 2.8$ ps, $m^* = 0.087 m_0$ as a function of B . Vertical lines are drawn at $\epsilon_\omega = 1, 2, 3, 4$.

our experimental findings—the same number of oscillations, same magnetic field onset, and same locations of zero-response nodes (cf. vertical lines).

In summary, we have studied nonlinear magnetotransport in a high-mobility two-dimensional hole gas hosted in a pure Ge/SiGe quantum well. Under applied dc bias, the differential resistivity shows a pronounced maximum which moves to higher magnetic fields with increasing direct current. We associate this maximum with the fundamental peak of Hall field-induced resistance oscillations (HIRO), which are frequently observed in 2DEG in GaAs/AlGaAs heterostructures. Our analysis shows that the position of the HIRO peak and microwave-induced resistance oscillations observed in the same sample are both well described by the hole effective mass $m^* \approx 0.09 m_0$. Since previous studies of MIRO and HIRO were limited exclusively to GaAs/AlGaAs systems, our findings establish that a strained Ge/SiGe quantum well is a promising 2D hole system, with a light carrier mass and high mobility, for further exploration of nonequilibrium transport phenomena.

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