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Tunable Floquet Majorana fermions in driven coupled quantum dots

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We propose a system of coupled quantum dots in proximity to a superconductor and driven by separate ac potentials to realize and detect Floquet Majorana fermions. We show that the appearance of Floquet Majorana fermions can be finely controlled in the expanded parameter space of the drive frequency, amplitude, and phase difference across the two dots. While these Majorana fermions are not topologically protected, the highly tunable setup provides a realistic system for observing the exotic physics associated with Majorana fermions as well as their dynamical generation and manipulation.

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I. INTRODUCTION

Majorana fermions are spin one-half particles that are their own antiparticles. Their existence as elementary particles in nature is speculated in several theories, though conclusive experimental evidence is still lacking [1,2]. They can also exist as collective quasiparticles in condensed matter systems, in particular, as bound states at the edges or vortices of a topological superconductor [3,4]. In addition to their interesting physical properties, Majorana bound states in a superconductor encode non-Abelian exchange statistics [5]. More recently, Majorana fermions have also been theoretically found to exist as steady states in a periodically driven system even under conditions for which the instantaneous static system would not host them anywhere in the range of the drive [6–11]. The exchange statistics of these "Floquet Majorana fermions" are predicted to be similar to their static counterparts, thus providing a new avenue for applications in fault-tolerant quantum information processing [12]. Unlike static Majorana fermions that exist only as zero-energy bound states, Floquet Majorana fermions can exist as nonequilibrium steady states of nonzero energy—a manifestation of the richer topological classification of driven systems [13]. The recent observation of Floquet states in a topological insulator [14] heralds a new frontier in realizing Floquet topological states [15–17] and in our understanding of nonequilibrium quantum systems.

Though preliminary signatures of the existence of static Majorana fermions have been reported in several experiments [18–22], the results have been subject to interpretation [23–25], in part due to the ambiguity of the mechanism leading to the observed signatures and the complexity of the physical setup. Another setup using quantum dots has been proposed and studied by several authors [26–30]. In the simplest case of just two quantum dots, Majorana fermions lose their topological protection and the system must be finely tuned by a rather intricate use of variable magnetic fields at the nanoscale. However, the advantages of unambiguous detection and simpler design favor these proposals in some laboratory settings. Observing Floquet Majorana fermions in the solid state is further complicated by the screening effects of the superconductor which restrict the use of a laser source to drive the system.

In this Rapid Communication, we propose to overcome these shortcomings at once in a highly tunable system of coupled quantum dots driven periodically via external gates. Even though there is no topological protection in the simplest case of two quantum dots, the exquisite, all-electric tunability of our proposed setup can be used to counteract the detuning effects of disorder and impurities. We also propose a clear detection scheme, thus providing a realistic system for observing the rich nonequilibrium physics of Floquet Majorana fermions in the laboratory.

The schematic setup of our proposal is shown in Fig. 1. The quantum dots are proximity coupled to a superconductor which induces Cooper pair correlations across the dots. For simplicity of our analysis, we assume that the lowest state in each dot is well separated from the rest and is nondegenerate. (The experimental conditions for realizing such a system are discussed later.) Steady-state Floquet Majorana fermions are generated by the external drive instead of a variable magnetic field, which also adds several highly tunable experimental knobs to address the fine-tuning requirement. Floquet Majorana fermions are detected unambiguously by measuring the charge of a probe dot as its energy level is varied adiabatically via a gate potential.

II. MODEL

The effective Hamiltonian of the system is $H(t) = H_0 + V(t)$. The static part

$$H_0 = \sum_i \mu_i \left(d_i^\dagger d_i - \frac{1}{2} \right) + (\lambda d_1^\dagger d_2 + \Delta d_1^\dagger d_2^\dagger + \text{H.c.}), \quad (1)$$

where $d_i^\dagger$ and d_i are the creation and annihilation operators of the state of dot $i = 1,2$ with energy μ_i measured with respect to the chemical potential of the superconductor, λ is the effective hopping amplitude, and Δ is the effective pairing amplitude between the dots. Due to charge screening by proximity to the superconductor we neglect any interdot Coulomb interaction, whereas intradot Coulomb interaction is irrelevant as only one effective energy level is available at each dot. Additionally, the dots are also subject to ac potentials,

$$V(t) = A_0 \cos(\Omega t) d_1^\dagger d_1 + A_0 \cos(\Omega t + \theta) d_2^\dagger d_2, \quad (2)$$

where A_0 is the driving amplitude, Ω the frequency, and θ the phase difference between the ac potentials. The amplitudes

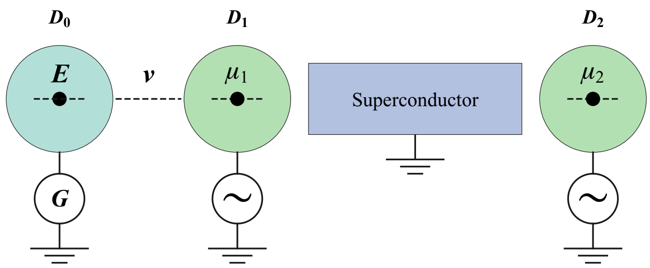

FIG. 1. (Color online) The schematic of the proposed setup. Two dots, D_1 and D_2, each with a single level at μ_1 and μ_2, respectively, are coupled by proximity to a superconductor. An external ac voltage is applied to D_1 and D_2 to realize steady-state Floquet Majorana fermions. A probe dot D_0 is coupled to the other dots with tunneling amplitude v and used to detect the Floquet Majorana fermions hosted by D_1 and D_2 by varying the energy E of the resonant level of D_0 through gate G.

of the two driving potentials are assumed to be the same for simplicity. We have checked numerically that different amplitudes do not change our conclusions.

III. FLOQUET MAJORANA FERMIONS

When the system is driven with period $T \equiv 2\pi/\Omega$, owing to the discrete time translation symmetry, the solutions of the Schrödinger equation can be written as $|\psi_\alpha(t)\rangle = e^{-i\varepsilon_\alpha t}\,|\phi_\alpha(t)\rangle$ with the periodic Floquet wave function $|\phi_\alpha(t+T)\rangle = |\phi_\alpha(t)\rangle$ and quasienergy $\varepsilon_\alpha \in (-\Omega/2,\Omega/2]$, respectively, the eigenvector and eigenvalue of the Schödinger-Floquet equation $[H(t) - i\partial_t]\,|\phi_\alpha(t)\rangle = \varepsilon_\alpha\,|\phi_\alpha(t)\rangle$ ($\hbar = 1$ throughout). Accordingly, an extended Hilbert space may be defined [31] with the inner product $\langle\langle\phi|\phi'\rangle\rangle = (1/T)\int_0^T \langle\phi(t)|\phi'(t)\rangle\,dt$. The Floquet wave functions and the quasienergies can be computed by solving the eigensystem

$$U(T,0)\,|\phi_\alpha(0)\rangle = \exp(-i\varepsilon_\alpha T)\,|\phi_\alpha(0)\rangle, \qquad (3)$$

where $U(t,t')$ is the time evolution operator $U(t,t')\,|\psi(t')\rangle = |\psi(t)\rangle$. The particle-hole symmetry of the Hamiltonian, $H \mapsto -H$ when $d_i \mapsto d_i^\dagger$, requires the quasienergies to come in pairs $(\varepsilon_\alpha, -\varepsilon_\alpha)$. Floquet Majorana fermions are states with quasienergy $\varepsilon_0 = 0$ or $\varepsilon_\pi = \pi/T = \Omega/2$ [6].

IV. LARGE FREQUENCY APPROXIMATION

We may expand the periodic Hamiltonian and Floquet wave function in a Fourier series $H(t) = \sum_m e^{im\Omega t} H^{(m)}$ and $|\phi_\alpha(t)\rangle = \sum_m e^{im\Omega t}|\phi_\alpha^{(m)}\rangle$ and approximate the Schrödinger-Floquet equation as $H_{\rm eff}|\phi_\alpha^{(0)}\rangle = \varepsilon_\alpha|\phi_\alpha^{(0)}\rangle$, where the effective Hamiltonian [32]

$$H_{\rm eff} = H^{(0)} + \frac{[H^{(-1)},H^{(1)}]}{\Omega} + O(A_0/\Omega)^2. \qquad (4)$$

It is useful to use the *rotating wave* basis

$$|\{n_i\},m\rangle = \exp\left[im\Omega t - i\int_0^t V(s)ds\right]|\{n_i\}\rangle\,, \qquad (5)$$

where $|\{n_i\}\rangle$ are the energy eigenstates of the static system H_0 with n_i occupation in the dot i and $m \in \mathbb{Z}$ labels the Fourier component. In this basis $H_{\rm eff}$ has the same form as H_0 with λ and Δ renormalized to

$$\tilde\lambda = \lambda f\left(\frac{2A_0}{\Omega}\sin\frac{\theta}{2}\right), \quad \tilde\Delta = \Delta f\left(\frac{2A_0}{\Omega}\cos\frac{\theta}{2}\right), \qquad (6)$$

where $f(x) = J_0(x) - x\,J_1(x)$. The details of this calculation are given in the Supplemental Material [33].

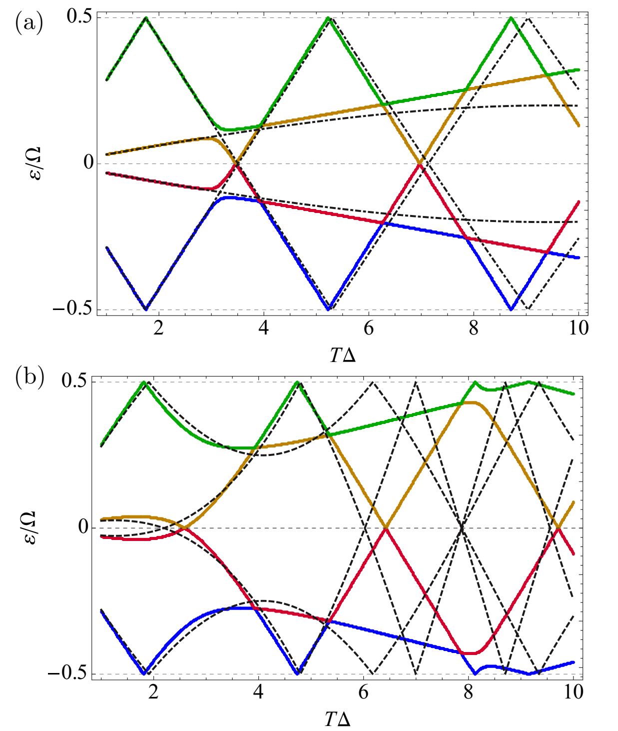

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FIG. 2. (Color online) The appearance of Floquet Majorana fermions. Quasienergies ε of the coupled-dot system are shown as a function of period $T = 2\pi/\Omega$ of the ac voltage $A_0\cos(\Omega t)$ on both dots. Here $\mu_1 = \mu_2 = 0$, the ratio of electron hopping to superconducting amplitude $\lambda/\Delta = 0.8$, and (a) $A_0/\Delta = 0.1$ and (b) $A_0/\Delta = 0.75$. The solid lines are exact numerical solutions. The dashed lines are the approximate solutions of the effective Hamiltonian in the large frequency regime. The Floquet Majorana fermions appear at quasienergies 0 and $\pm\Omega/2$. The effective Hamiltonian approximation works well for small $A_0 \lesssim \Omega$.

V. NUMERICS

In Fig. 2, we show typical quasienergy spectra of the system. Floquet Majorana fermions with quasienergies ε_0 and ε_π appear frequently at nonuniversal, parameter-dependent values of frequency. We also compare the quasienergies obtained in a large frequency approximation with the exact numerical result in Fig. 2. For $A_0/\Omega \lesssim 1$ we obtain a reasonably good agreement with the exact numerical diagonalization of the Schrödinger-Floquet equation. We note that the second term in (4) results in a significant improvement of the approximation, essentially producing the first few Floquet Majorana fermions for both ε_0 and ε_π.

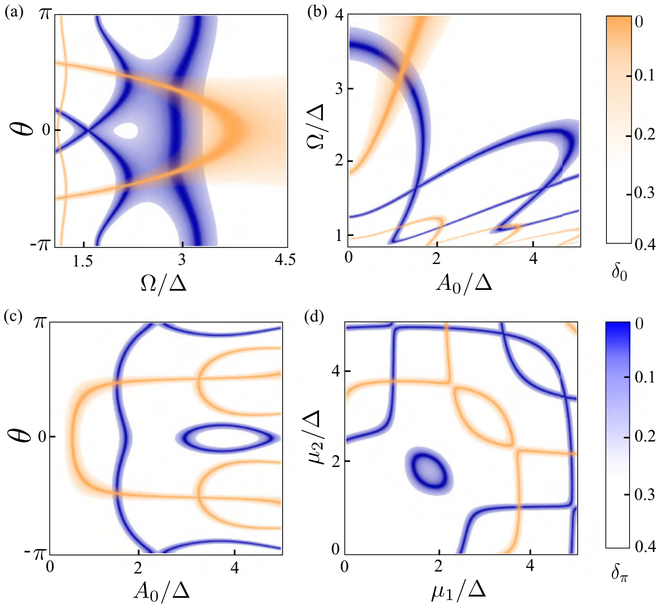

FIG. 3. (Color online) Tuning Floquet Majorana fermions in the parameter space. The regions of existence of Floquet Majorana fermions are shown for ac voltages $A_0\cos(\Omega t)$ and $A_0\cos(\Omega t+\theta)$ on the two dots. The light (orange) and dark (blue) shades show, respectively, the quasienergy gaps $\delta_0=\varepsilon_1 T$ and $\delta_\pi=\pi-\varepsilon_2 T$, where $0<\varepsilon_1<\varepsilon_2$ are the quasienergies. Whenever fixed, the parameters are $\lambda/\Delta=0.8$, $\mu_1=\mu_2=0$, $\theta=0$, $A_0/\Delta=1.5$, and $\Omega/\Delta=2.1$. The system can be tuned easily to host Floquet Majorana fermions with quasienergy 0 (orange), $\Omega/2$ (blue), or both. The thicker the shaded area, the more robust are the Floquet Majorana fermions.

VI. FINE TUNING AND PROTECTION

Due to mixing between the two dots, the Floquet Majorana fermions in this system lack topological protection against perturbations and require fine tuning to exist. In the case where either $\mu_1=0$ or $\mu_2=0$, the condition to host ε_0 Floquet Majorana fermions can be found [27] from the zero-energy states of H_0, i.e., $|\tilde{\lambda}|=|\tilde{\Delta}|$. It can therefore be seen from the shape of f that for $A_0/\Omega\sim 1$ varying θ provides a very effective way of fine tuning the system to host Floquet Majorana fermions. More generally, the fine-tuning problem is addressed by a large, all-electric, highly tunable manifold of control parameters that allow one to counteract the detuning effects of impurities and retune the system to stabilize the Floquet Majorana fermions.

In Fig. 3, we show the phase diagram depicting the occurrence of Floquet Majorana fermions at ε_0 and ε_π from the exact numerical solution in different planes of the parameter space. This phase diagram is our first main result. Several significant features of the phase diagram are as follows: (1) Floquet Majorana fermions appear in an extended region of the parameter space. (2) Varying the system parameters produces many Floquet Majorana fermion states. In an experiment, both A_0 and θ can be varied with a high degree of control and precision [34], thus allowing the system to be finely tuned. (3) For sizable ranges of parameters, Floquet Majorana fermions are insensitive to changes in one or more parameters. This is seen as near vertical or horizontal shaded lines in Fig. 3. In particular, near the crossing points in Fig. 3(d), a fluctuation in just one of μ_1 and μ_2 cannot remove the Floquet Majorana fermions. While not topological, this provides some robustness for the Floquet Majorana fermions in this system. In equilibrium a "quadratic" robustness exists [27] near $\mu_1=\mu_2=0$ with the Majorana fermion energy splitting proportional to $\mu_1\mu_2/2\Delta$. In the driven system, our numerical results [33] show the splitting is quadratic or linear, reflecting the structures shown in Fig. 3.

VII. DETECTION

The detection of Majorana fermions presents several challenges. They may be detected by spectroscopic measurements, such as tunneling or conductance through leads that overlap with their wave functions. However, the topological character of Majorana fermions is difficult to ascertain in such experiments. In the case of Floquet Majorana fermions the dynamical nature of the states can be exploited for a clear identification. Two of us [11] have shown before that the (zero-temperature) conductance σ satisfies the Floquet sum rule with bias V, $\sum_{n\in\mathbb{Z}}\sigma(V+n\Omega)=2e^2/h$, for $V=\varepsilon_0$ and/or ε_π if and only if Floquet Majorana fermions are present at $\varepsilon=\varepsilon_0$ and/or ε_π. Even though there is no topological protection, the Floquet sum rule may still be used to detect the Floquet Majorana fermions once the system is fine tuned to host them.

A more suitable detection setup for the quantum-dot system is obtained by coupling a probe quantum dot D_0 with a single resonant level at energy E to one or both of the dots, as depicted in Fig. 1. An effective Hamiltonian can be written as $Ed_0^\dagger d_0+[v(t)d_0^\dagger+v(t)^*d_0]\gamma_{12}(t)$, where d_0 is the annihilation operator of the electron in D_0, $\gamma_{12}(t)$ is the Majorana operator for steady state in the double-dot system with quasienergies $\pm\varepsilon$, and $v(t)\propto\langle\psi(t)|\eta\rangle$ is the overlap between the wave functions η of the probe dot and $\psi(t)$ of the coupled quantum dots. Since the fermion parity of the whole system is preserved, we can restrict ourselves to either the odd or even parity subspace spanned by the $|n_0 n_{12}\rangle$ occupation basis, where $n_0,n_{12}=0,1$ are, respectively, the occupations of D_0 and the pair of states in D_1 and D_2. Then, the reduced tunneling Hamiltonian takes the form

$$H_{\rm tun}(t)=\begin{pmatrix}0 & v(t)\\ v(t)^* & E\end{pmatrix}. \qquad (7)$$

In the static system v is constant and the adiabatic variation of E results in a Landau-Zener transition that switches the occupation of D_0 [26]. For the Floquet Majorana fermions, the state of the coupled quantum dots is a superposition of the steady states with quasienergies $\pm\varepsilon$, $\psi(t)=c_-e^{i\varepsilon t}\phi_{-\varepsilon}(t)+c_+e^{-i\varepsilon t}\phi_\varepsilon(t)$. Precisely at quasienergy $\varepsilon=0$ ($\Omega/2$), $v(t)$ becomes a periodic function with period T ($2T$). Applying the Floquet formalism to $H_{\rm tun}(t)$, we can see that this results in avoided crossings of the quasienergies at $E=k\Omega$ [$E=(k+\frac{1}{2})\Omega$] with $k\in\mathbb{Z}$. As E is adiabatically varied, each avoided crossing results in a switching transition in the occupation (charge) of the probe dot, as the system moves within the same degenerate parity subspace. These characteristic charge switchings at multiples of integral or half-integral drive frequencies can happen only in the presence

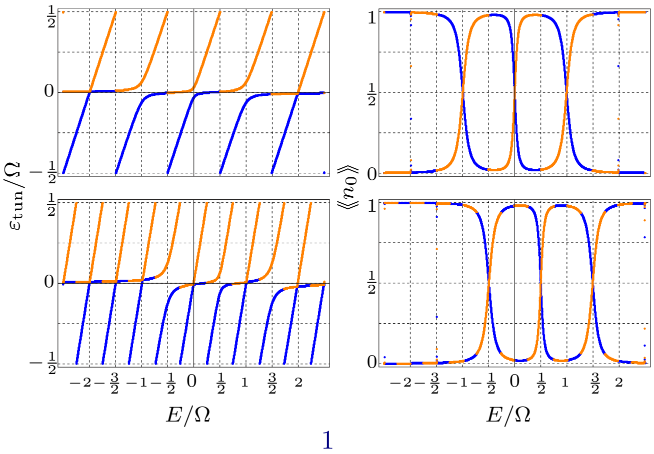

FIG. 4. (Color online) Adiabatic detection of Floquet Majorana fermions. For the reduced system consisting of the probe dot D_0 and two degenerate Floquet Majorana fermions in the coupled dots D_1 and D_2, the tunneling quasienergy $\varepsilon_{\rm tun}$ (left) and the time-averaged probability $\langle\langle n_0 \rangle\rangle$ of finding an electron in D_0 (right) are shown as the energy E of resonant level in D_0 varies. A simple profile of the tunneling amplitude $v(t) = e^{i\varepsilon t}(v_0 + v\cos\Omega t)$ is assumed with $v_0 = 0.2\Omega/2\pi$, $v = 0.8\Omega/2\pi$, which is periodic for a Floquet Majorana fermion at quasienergy $\varepsilon = 0$ (top panels) and antiperiodic for $\varepsilon = \Omega/2$ (bottom panels). The dark (blue) and light (orange) curves correspond to, respectively, steady states with lower and higher tunneling quasienergies. The probability follows a smooth curve that matches the crossing and anticrossing points of the quasienergy spectrum.

of ϵ_0 or ϵ_π quasienergies. As the only driven parts of our system are the quantum dots, these can be used to detect Floquet Majorana fermions unambiguously. This is our second main result.

In Fig. 4, we show a typical quasienergy spectrum of the tunneling Hamiltonian $H_{\rm tun}$ (left panel) and the corresponding time-averaged value of the probe charge $\langle\langle n_0 \rangle\rangle$ (right panel) for a simplified profile of $v(t)$. More details and results for the full three-dot system are given in the Supplemental Material [33].

VIII. DISCUSSION

The setup proposed here can be realized in the laboratory using well-established tools developed for fabricating and measuring quantum dots in order to create, detect, and manipulate Floquet Majorana fermions in a highly tunable fashion using an all-electric circuit with great precision. A potential challenge in creating this system is in obtaining an appreciable pairing amplitude between spin-filtered resonant levels of the two quantum dots through the superconductor bridge. For a singlet superconductor, this can be addressed in at least three ways: (i) by exploiting the spin-orbit interaction with a characteristic length smaller than the size of the quantum dots [28,35]; (ii) by applying opposite Zeeman fields on the two dots; or (iii) by applying a uniform external Zeeman field and field tuning the quantum-dot g factors [36] to achieve opposite signs on the two dots. We shall now provide a range of parameters for a realistic setup. Showing the level spacing at the active quantum-dot state by δE, we must have $\Omega \gg \delta E$ so that the quasienergy levels in each dot can be taken to be the same as the static ones. Also we would like to minimize Rabi oscillations in each period of the external drive, that is, $\sqrt{A_0^2 + (\delta E - \Omega)^2} \ll \Omega$, which, together with the previous condition, yields $A_0 \ll \sqrt{\delta E \Omega}$.

A major obstacle in the clear detection of Majorana fermions is ruling out other mechanisms leading to bound states with similar spectroscopic signatures, e.g. disorder-induced Andreev bound states or Kondo resonances [23,24,37]. We have not considered the Kondo effect in response to the ac drive in our setting [38]. However, we note that in static equilibrium the Kondo fixed point is unstable and the phase of the system is instead controlled by another fixed point induced by the Majorana fermions [39]. It would be interesting to study the interplay between the Kondo interaction and Floquet Majorana fermions. In this regard, an advantage of Floquet Majorana fermions over the static ones is the existence of ε_π states that are distinct from ε_0 states and other nondynamical bound states and whose presence can be clearly identified in the experiments we have described.

To summarize, we propose realizing Floquet Majorana fermions in a double-dot system coupled through a superconductor and subject to ac gate potentials. The driving amplitude, frequency, and the ac phase difference between the dots provide a large, highly tunable set of control knobs that make it possible to observe the signatures of Floquet Majorana fermions in a solid-state system and to test their exotic properties. Extensions of the proposed architecture to many coupled dots as a platform to exploit the non-Abelian statistics of Floquet Majorana fermions [8,40] for quantum information processing and nonlocal quantum entanglement is an interesting problem for the future.

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