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Relationship of time-reversal symmetry breaking to optical Kerr rotation

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We prove an instance of the reciprocity theorem that demonstrates that Kerr rotation, also known as the magneto-optical Kerr effect, may only arise in materials that break microscopic time-reversal symmetry. This argument applies in the linear-response regime and only fails for nonlinear effects. Recent measurements with a modified Sagnac interferometer have found finite Kerr rotation in a variety of superconductors. The Sagnac interferometer is a probe for nonreciprocity, so it must be that time-reversal symmetry is broken in these materials.

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I. INTRODUCTION

Recently, there has been controversy [1–8] surrounding the apparent measurement of finite polarization rotation, also known as the magneto-optical polar Kerr effect, in optical reflection measurements off of a variety of high-T_c superconductors [9,10] The polar Kerr effect is characterized by the Kerr angle θ_K, which is the difference in phase angle delays yielded by oppositely circularly polarized plane-wave beams of light upon normal incidence reflection from a sample. Although the Kerr effect is often associated with magnetic materials [8,11,12], it has been suggested that the observations of Karapetyan and co-workers [9,10] are more consistent with cholesteric order [4]—an order characterized by mirror asymmetry about any plane. Although these measurements convincingly demonstrate a real and novel effect, we argue that the interpretation of cholesteric order is flawed. This discussion has broached a much more long-standing controversy [13–43] regarding the correct form of gyrotropic electromagnetic constitutive relations and whether Kerr rotation is allowed by general optically active media—media that break only mirror symmetry about a plane containing the surface normal [44]. In this Rapid Communication, we prove a general theorem that guarantees that the observation of Kerr rotation must always imply that it is microscopic time-reversal symmetry that is broken.

The idea that the Kerr effect implies microscopic time-reversal symmetry breaking has been argued by a number of authors [23–32], but those conclusions made by Halperin [33] provide a useful introduction. He considers a plane-wave source and adjacent detector, both at a fixed distance along the z axis to the sample at $z = 0$. The distance is great enough such that the source and detector may as well be considered on top of each other. Let R_{++} and R_{--} be the reflection amplitudes for circular polarization states reflecting into circular polarization states with the same sense of rotation; for incident and reflected rays propagating along the $\hat{\mathbf{z}}$ axis, $\pm$ refers to the polarization state of the electric field given by $\mathbf{E}_\pm = \mathrm{Re}\{\frac{1}{\sqrt{2}}(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})e^{ik_z z - i\omega t}\}$ as $z \to \infty$. $\pm$ also may be understood as the sign of the spin angular momentum of the light with respect to the $\hat{\mathbf{z}}$ axis and independent of the direction of propagation $k_z\hat{\mathbf{z}}$; $\pm$ is *not* the helicity. By application of Onsager's relations, Halperin demonstrates that if the material is time-reversal symmetric, then $R_{++} = R_{--}$. Since the Kerr angle is $\theta_K = \frac{1}{2}(\arg\ R_{++} - \arg\ R_{--})$, it will be zero when the material preserves time-reversal symmetry. Although his argument is satisfactory, it is deserving of a more rigorous discussion.

We begin as Halperin does. Consider a general measurement of the reflection amplitudes where the pair of sources, each collocated with a detector, are positioned arbitrarily with respect to a sample and each other. Let the sources be of arbitrary shape, but emit light, which, in the absence of all other sources or scatterers, appears as a circularly polarized plane wave at infinite distance. In the presence of scatterers, the emitted field may still be described as having a circular polarization state $\pm$ near the source, if not as a plane wave. We consider the experiment where light of the $+$ polarization state is emitted at a source located at $\mathbf{r}_1$ and the $+$ component of the reflected wave is measured at a detector at $\mathbf{r}_2$. Let, also, light of the $-$ polarization state be sourced at $\mathbf{r}_2$ and the $-$ component of the polarization be measured at $\mathbf{r}_1$. This is accomplished if the collocated detectors are such that they signal the arrival of a photon in the time reverse of the quantum optical state initially formed at the respective source. Again, the Kerr angle is the measured difference in complex arguments of the two propagation amplitudes. In the limit of $\mathbf{r}_1 \to \mathbf{r}_2 \to \infty$, the measured reflection amplitudes are the same as R_{++} and R_{--} described by Halperin.

We will demonstrate that, for the measurement described above, when the instrumentation and the sample consist of materials that are all time-reverse symmetric, the electromagnetic propagation amplitude from $\mathbf{r}_1$ to $\mathbf{r}_2$ will always be identical to that for propagation from $\mathbf{r}_2$ to $\mathbf{r}_1$. It then follows that the Kerr angle will also be zero when there is time-reversal symmetry and that broken-mirror symmetry, alone, cannot give rise to Kerr rotation.

II. PROPAGATORS FOR OPTICAL MEASUREMENTS

Photon Green's functions describe optical measurements. In the macroscopic limit, the light emitted from a source and measured by a detector is modeled by the retarded Green's function for the macroscopic Maxwell's equations,

$$\nabla \times \mathbf{E} = -\frac{1}{c}\partial_t \mathbf{B}, \qquad \nabla \cdot \mathbf{D} = \rho_f,$$
$$\nabla \times \mathbf{H} = \frac{1}{c}\partial_t \mathbf{D} + \frac{4\pi}{c}\mathbf{J}, \qquad \nabla \cdot \mathbf{B} = 0,$$

where $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -\frac{1}{c}\partial_t \mathbf{A}$ in the radiation gauge. At optical frequencies, it is sufficient to describe the material's response with just a dielectric susceptibility tensor $\tilde{\chi}(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1)$ [44,45]. The retarded Green's function $\tilde{G}^{\mathrm{ret}}$ relates the source

current $\mathbf{J} = (J_x, J_y, J_z)$ to the macroscopic vector potential $\mathbf{A}$,

$$\mathbf{A}(t_2,\mathbf{r}_2) = \frac{4\pi}{c}\int \tilde{G}^{\rm ret}(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1)\mathbf{J}(t_1,\mathbf{r}_1)dt_1 d\mathbf{r}_1. \quad (1)$$

Precise statements of the symmetries of the electromagnetic field and its measurement entail that the reflection amplitudes be considered quantum mechanically [46,47]. The quantum electrodynamic field measured at $(t_2,\mathbf{r}_2)$ by a point dipole detector, aligned to the μ linear polarization state, will be $\hat{A}_\mu(t_2,\mathbf{r}_2)|0\rangle$, where $\hat{A}_\mu(0,\mathbf{r}) = \hat{A}^\dagger_\mu(0,\mathbf{r})$ is the position-space field operator and $|0\rangle$ is the vacuum state. Likewise, supposing a pointlike dipole source creates a ν linearly polarized photon at $(t_1,\mathbf{r}_1)$, then the quantum field it initially forms will be $\hat{A}_\nu(t_1,\mathbf{r}_1)|0\rangle$. For $t_2 > t_1$, the amplitude for free-space propagation between the source and the receiver is given by

$$\langle 0|\hat{A}_\mu(t_2,\mathbf{r}_2)\hat{A}_\nu(t_1,\mathbf{r}_1)|0\rangle = \delta_{\mu\nu}\frac{\delta\left(t_2 - t_1 - \frac{1}{c}|\mathbf{r}_2 - \mathbf{r}_1|\right)}{4\pi|\mathbf{r}_1 - \mathbf{r}_2|}.$$

Squared, this is the transition probability density for the detection of a photon at time t_2 given its creation at t_1 [46]. When the sources are of a single frequency ω, the phase delay, as used to define the Kerr angle, is the complex argument of the propagator in the frequency-position domain: $\tilde{G}^{\rm ret}(\omega;\mathbf{r}_2,\mathbf{r}_1) = \int \tilde{G}^{\rm ret}(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1)e^{i\omega(t_2-t_1)}dt_2 dt_1$.

When the light is interacting with matter, then to lowest order, the linear response of the macroscopic field at the detector $\mathbf{A}(t,\mathbf{r}_2)$ for $\mathbf{r}_2$ outside of the material, to an optical source at $\mathbf{r}_1$, also outside of the material, is given by Eq. (1) [47]. The retarded Green's function is obtained by complex conjugating the negative frequency part of the following time-ordered propagator:

$$G^{\rm F}_{\mu\nu}(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1) = \langle g|T[\hat{A}_\mu(t_2,\mathbf{r}_2)\hat{A}_\nu(t_1,\mathbf{r}_1)]|g\rangle, \quad (2)$$

where T is the time-ordering operator for photons:

$T[\hat{A}_\mu(t_2,\mathbf{r}_2)\hat{A}_\nu(t_1,\mathbf{r}_1)] = \theta(t_2 - t_1)\hat{A}_\mu(t_2,\mathbf{r}_2)\hat{A}_\nu(t_1,\mathbf{r}_1) + \theta(t_1 - t_2)\hat{A}_\nu(t_1,\mathbf{r}_1)\hat{A}_\mu(t_2,\mathbf{r}_2)$. We choose to focus on the time-ordered propagator just to emphasize how propagators are calculated from quantum perturbative methods.

The expectation value is taken with respect to the many-body ground state $|g\rangle = \lim_{t\to-\infty}\hat{g}^\dagger(t)|0\rangle$ of the whole system. This ground state includes the material, the environment, and any instrumentation. If the system is at finite temperature, then a Boltzmann-weighted sum of propagators, evaluated with respect to the stationary states of the system is used in lieu of the above. In this way, even incoherent optical sources [23] may be described.

In assuming that the measurement is described exactly by Eq. (2), it is implied that the source is the perturbation to the full Hamiltonian of the world $\hat{H}$, which describes the light, the material, and the detectors. The perturbing source $\mathbf{J}(t,\mathbf{r})$ is slowly turned on from zero at $t = -\infty$ and slowly turned off at $t = \infty$. It is also assumed that the sample, the source, and the receiver do not interact in any way other than by the scattered light; this is tantamount to requiring that the operators $\hat{A}_\mu(t,\mathbf{r}_2)$ and $\hat{A}_\nu(t,\mathbf{r}_1)$ commute with each other and with $\hat{g}(t)$ and $\hat{g}^\dagger(t)$ at equal times. These are the same conditions requisite for application of the Kubo formula, and results similar to those in the next section appear in many texts in connection with it [48].

III. THE RECIPROCITY THEOREM

We will prove that, if time-reversal symmetry is respected, then no Kerr rotation is observed, by showing that this symmetry condition implies that the propagator for + polarized light traversing from $\mathbf{r}_1 \to \mathbf{r}_2$ and the propagator for − polarized light traversing from $\mathbf{r}_2 \to \mathbf{r}_1$ are identical. Of central importance is that the measurement is performed with collocated sources and detectors, which create or destroy photons in states that are the time reverse of each other. This condition is clearly true for the two pointlike dipole sources/detectors, located at $\mathbf{r}_1$ and $\mathbf{r}_2$, considered in this discussion. We later describe an example of how this is achieved in practice.

The antilinear time-reversal operator [49–52] $\mathcal{T}$ commutes with the Hamiltonian $\hat{H}$; $\mathcal{T}\hat{H}\mathcal{T}^\dagger = \hat{H}$ but still inverts the time-evolution operator $\mathcal{T}e^{-i\hat{H}t}\mathcal{T}^\dagger = e^{i\hat{H}t}$ as well as anti-commutes with all other operator generators of motion. Its action on quantum states u,v is $\mathcal{T}|u\rangle = |\bar{u}^*\rangle$ where the overbar represents the time-reversed state and $*$ refers to the fact that the map is to the "complex-conjugate Hilbert space" [51], where $\langle u^*|v^*\rangle = \langle v|u\rangle$ and $\langle u^*|e^{-i\hat{H}t}|v^*\rangle = \langle v|e^{i\hat{H}t}|u\rangle$.

The vector potential has odd time-reversal parity, so $\mathcal{T}\hat{A}_\mu(0,\mathbf{r})\mathcal{T}^\dagger = -\hat{A}_\mu(0,\mathbf{r})$. Since $\hat{A}_\mu(t,\mathbf{r}) = e^{i\hat{H}t}\hat{A}_\mu(0,\mathbf{r})e^{-i\hat{H}t}$, then $\mathcal{T}\hat{A}_\mu(t,\mathbf{r})\mathcal{T}^\dagger = -\hat{A}_\mu(-t,\mathbf{r})$. It follows that:

$$\begin{aligned}
&\langle g|T[\hat{A}_\mu(t_2,\mathbf{r}_2)\hat{A}_\nu(t_1,\mathbf{r}_1)]|g\rangle \\
&= \langle g|\mathcal{T}^\dagger\mathcal{T}T[\hat{A}_\mu(t_2,\mathbf{r}_2)\mathcal{T}^\dagger\mathcal{T}\hat{A}_\nu(t_1,\mathbf{r}_1)]\mathcal{T}^\dagger\mathcal{T}|g\rangle \\
&= \langle \bar{g}^*|T[\mathcal{T}\hat{A}_\mu(t_2,\mathbf{r}_2)\mathcal{T}^\dagger\mathcal{T}\hat{A}_\nu(t_1,\mathbf{r}_1)\mathcal{T}^\dagger]|\bar{g}^*\rangle \\
&= \langle \bar{g}^*|T[(-1)\hat{A}_\mu(-t_2,\mathbf{r}_2)(-1)\hat{A}_\nu(-t_1,\mathbf{r}_1)]|\bar{g}^*\rangle \\
&= \langle \bar{g}|T[\hat{A}_\nu(-t_1,\mathbf{r}_1)\hat{A}_\mu(-t_2,\mathbf{r}_2)]|\bar{g}\rangle \\
&= \langle \bar{g}|T[\hat{A}_\nu(t_2,\mathbf{r}_1)\hat{A}_\mu(t_1,\mathbf{r}_2)]|\bar{g}\rangle, \quad (3)
\end{aligned}$$

where the last equality follows from time-translation symmetry. It is then the case that if the ground state of the material is time-reversal symmetric, $|\bar{g}\rangle = |g\rangle$ that

$$G^{\rm F}_{\mu\nu}(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1) = G^{\rm F}_{\nu\mu}(t_2,\mathbf{r}_1,t_1,\mathbf{r}_2). \quad (4)$$

There is a similar derivation of this symmetry for the retarded propagator, or else, it is obtained from analytic continuation of the above.

We refer to this result as the "reciprocity theorem," and it is only satisfied when the ground state possesses microscopic time-reversal symmetry. Again, the restriction to pointlike dipole sources is unnecessary as an extended source is described by integrating $\mathbf{r}_1$ and $\mathbf{r}_2$ over the respective volumes. Linear absorption in the sample is inconsequential; whereas the transition amplitude for absorption of a photon from $\mathbf{r}_1$ may be different for the amplitude of absorption for a photon from $\mathbf{r}_2$, these amplitudes are not measured and do not contribute to 2. Finally, Onsager's relations [53–63] for the linear response, the Rayleigh-Carson electromagnetic reciprocity theorem [64–67], and its quantum counterpart for unitary evolution [49,52,68,69], are all known manifestations of Eq. (4).

Some scattering-matrix formulations of the theorem claim to satisfy reciprocity only in the asymptotic far-field limit [70–72]. This is because scattering-matrix elements define

transition amplitudes between free-space plane-wave electromagnetic fields in the asymptotic past or future and the perturbation expansion of the S matrix is performed in powers of the scattering material's contribution to the Hamiltonian [47]. We approach reciprocity from a different perspective and find no such restriction as we evaluate expectation values of the propagator with respect to $|g\rangle$ and perturbatively expand the measured quantity $\mathbf{A}(t,\mathbf{r})$ in powers of the optical source's semiclassical contribution to the Hamiltonian $\hat{A}_\mu(t,\mathbf{r})J_\mu(t,\mathbf{r})$ as in linear response [56,57].

To conclude the proof, circular polarization states may be represented by linear states via $\hat{A}_\pm(0,\mathbf{r}) = \frac{1}{\sqrt{2}}[\hat{A}_x(0,\mathbf{r}) \pm i\hat{A}_y(0,\mathbf{r})]$, so $\mathcal{T}\hat{A}_+(0,\mathbf{r})\mathcal{T}^\dagger = -\hat{A}_-(0,\mathbf{r})$. This is sensible since the $\pm$ photon polarization states are eigenstates of spin angular momentum and the time-reversal operator reverses its direction. We return to considering retarded propagators as they describe evolution of the system forward in time; when there is time-reversal symmetry, analytic continuation of Eq. (4) gives

$$G^{\text{ret}}_{++}(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1) = G^{\text{ret}}_{--}(t_2,\mathbf{r}_1,t_1,\mathbf{r}_2). \tag{5}$$

Then there cannot be Kerr rotation as the frequency domain propagators are also the same, so $\theta_K = \frac{1}{2}\arg G^{\text{ret}}_{++}(\omega;\mathbf{r}_2,\mathbf{r}_1) - \frac{1}{2}\arg G^{\text{ret}}_{--}(\omega;\mathbf{r}_1,\mathbf{r}_2) = 0$.

This result does not always hold for nonlinear response because the reflection amplitudes are not related by time-reversal symmetry. Consider a nonparametric process where the reflection of $+$ polarized light results in a spin excitation $\langle e_\uparrow| = \lim_{t\to\infty}\langle g|\hat{e}_\uparrow(t)$. If the equilibrium state is time-reverse symmetric, $|g\rangle = |\bar{g}\rangle$ and $\mathcal{T}\hat{e}_\downarrow(t)\mathcal{T}^\dagger = \hat{e}_\uparrow(-t)$, where $|e_\downarrow\rangle = \lim_{t\to-\infty}\hat{e}^\dagger_\downarrow(t)|g\rangle$, then applying $\mathcal{T}$ to a higher-order propagator [58,59,73] yields

$$\begin{aligned}&\langle e_\uparrow|T[\hat{A}_+(t_3,\mathbf{r}_2)\hat{A}_+(t_2,\mathbf{r}_1)\hat{A}_+(t_1,\mathbf{r}_1)]|g\rangle\\ &\quad = -\langle g|T[\hat{A}_-(t_3,\mathbf{r}_1)\hat{A}_-(t_3+t_1-t_2,\mathbf{r}_1)\hat{A}_-(t_1,\mathbf{r}_2)]|e_\downarrow\rangle.\end{aligned} \tag{6}$$

In other words, the amplitude of a process that results in the creation of an excited state of the material for light going from $\mathbf{r}_1 \to \mathbf{r}_2$ is equal to that where an initial excited state decays and emits a photon for light going from $\mathbf{r}_2 \to \mathbf{r}_1$. Because the optical field is perturbing the material from the unilluminated equilibrium state in the infinite past, although the nonlinear excitation and decay processes are both possible, the Boltzmann weight for the material beginning in the excited state will be less than that of the ground state. Although the two amplitudes above are equal up to a sign, their weightings are different, and so there may be an asymmetry with respect to time-reversal of the sum of all weighted amplitudes for light going from $\mathbf{r}_1 \to \mathbf{r}_2$ and vice versa. Kerr rotation may then be measured even if the equilibrium state of the material is time-reverse symmetric. This Kerr angle will be intensity dependent and if, as intensity is tuned to zero, the Kerr angle also approaches zero, then the equilibrium state of the material is necessarily time-reverse symmetric. Nonreciprocity is also possible if the spectral content of the incident and reflected beams differ as when there is harmonic generation or Raman shifts. This is demonstrated in a similar manner to that of the above; if light of frequency ω_1 reflects to light of frequency ω_2, there is no condition that the source for the incident light is such that the spectral weights for ω_1 and ω_2 are the same. Thus, the two weighted sums of amplitudes will differ.

Nonequilibrium systems, such as a relaxing glass or a system driven by some other external source field, are inherently changing as a function of time and so can give rise to nonreciprocity. However, there is a subtlety in that $\hat{H}$ is the Hamiltonian for the whole world, so it is inaccurate, in this argument, to speak of open systems that break time-translation symmetry and invalidate the last step in (3). In other words, Eq. (5) fails when microscopic time-reversal symmetry is broken but does not distinguish between systems in which it is broken due to a phase of matter that arises from spontaneous symmetry breaking or from an external forcing as in the spin Hall effect [74] where an applied current results in an unbalanced population of spins. Likewise, there might be a highly excited state of a material that breaks mirror symmetry and emits radiation as it relaxes asymmetrically in the two circular polarization states, again, leading to an unbalanced spin population. If these nonequilibrium systems are steady state [75], then there will still be a density matrix $\hat{\rho}$ that is not Boltzmann and is used to evaluate Eq. (2). Unless this density matrix manifestly breaks time-reversal symmetry $[\mathcal{T},\hat{\rho}] \neq 0$, then the measurement will satisfy reciprocity, and there can be no Kerr rotation.

IV. CONCLUSION

In proving Eq. (5), we have dispelled some incorrect ideas, recently promulgated [2,4,5,13–22] as well as affirmed and clarified the work of a number of studies [23–43]. To summarize: (1) Kerr rotation may only arise from microscopic time-reversal symmetry breaking as will circular dichroism in normal incidence reflection. This symmetry breaking may occur either through spontaneous symmetry breaking or by nonequilibrium processes. Optically active materials, such as those with a k-linear susceptibility or any other form of mirror-symmetry breaking, cannot give rise to Kerr rotation as they are time-reversal symmetric. (2) The proof above coincides with Onsager's relations and the electromagnetic reciprocity theorem, and all three will fail only when microscopic time-reversal symmetry is broken. The theorems do not apply for nonlinear response, however nonlinear response must exhibit intensity-dependent observables, such as Kerr rotation, or an alteration in the reflected frequency spectrum. There are nonlinear effects that are intensity independent and only alter the spectral content, such as spontaneous Raman shifts or spontaneous parametric photon down-conversion, but these effects are incoherent and yield a random-phase delay.

These results constrain the predictions of *all* constitutive relations [76] used to model time-reversal symmetric media. A common source of confusion impeding the acceptance of these arguments has been the calculations of Kerr rotation when using the mirror-symmetry-breaking k-linear constitutive relations with material constants allowed to vary with position [18,31]: $\mathbf{B} = \mathbf{H}$ and $\mathbf{D} = \epsilon^0(\mathbf{r})\mathbf{E} + \gamma(\mathbf{r})\nabla\times\mathbf{E}$ or $\mathbf{D} = \epsilon^0(\mathbf{r})\mathbf{E} + \nabla\times[\gamma(\mathbf{r})\mathbf{E}]$, where ϵ^0 is the isotropic permittivity and γ is the spatially dependent isotropic gyrotropic parameter [44]. The resolution of this paradox is that only when the material constants are homogeneous will these relations conform to the intended symmetries of the model. When

there is a surface or spatial inhomogeneity, these constitutive relations do not satisfy Onsager's relations [28,29], which means they do not explicitly satisfy time-reversal symmetry and cannot appropriately describe the system under discussion. Furthermore, in lossless media, they do not respect Poynting's theorem [39,77] or follow from a least-action principle [78].

Onsager's relations must be enforced if time-reversal symmetric media are to be modeled correctly. Consider the following permittivity tensor:

$$\epsilon_{\mu\nu}(\omega,\mathbf{r},\mathbf{r}') = \epsilon^0_{\mu\nu}(\omega,\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') - \gamma_{\mu\nu\lambda}(\omega,\mathbf{r},\mathbf{r}')\partial_\lambda\delta(\mathbf{r}-\mathbf{r}'). \quad (7)$$

This form generalizes the constitutive relations for k-linear response in homogeneous media, where $\gamma_{\mu\nu\lambda}$ and $\epsilon^0_{\mu\nu}$ will be constant, to a form where they are spatially dependent. Onsager's relations $\epsilon_{\mu\nu}(\omega,\mathbf{r},\mathbf{r}') = \epsilon_{\nu\mu}(\omega,\mathbf{r}',\mathbf{r})$ require that $\epsilon^0_{\mu\nu}(\omega,\mathbf{r}) = \epsilon^0_{\nu\mu}(\omega,\mathbf{r})$ and $\gamma_{\mu\nu\lambda}(\omega,\mathbf{r},\mathbf{r}') = -\gamma_{\nu\mu\lambda}(\omega,\mathbf{r}',\mathbf{r})$. For isotropic media, $\epsilon^0_{\mu\nu}(\omega,\mathbf{r}) = \epsilon^0(\mathbf{r})$ and $\gamma_{\mu\nu\lambda}(\omega,\mathbf{r},\mathbf{r}') \equiv \varepsilon^{\mu\nu\lambda}\eta(\mathbf{r},\mathbf{r}')$, where $\eta(\mathbf{r},\mathbf{r}')$ is a scalar symmetric function. As an example, if $\eta(\mathbf{r},\mathbf{r}') = \gamma(\frac{1}{2}\mathbf{r}+\frac{1}{2}\mathbf{r}')$, where $\gamma(\mathbf{r})$ is some other scalar function, then $\mathbf{D} = \epsilon^0(\mathbf{r})\mathbf{E} + \frac{1}{2}\gamma(\mathbf{r})\nabla\times\mathbf{E} + \frac{1}{2}\nabla\times[\gamma(\mathbf{r})\mathbf{E}]$. It can be easily checked that this form does not predict Kerr rotation [34–36,38,39], but our proof of Eq. (5) guaranteed that this would be the case for *any* choice of $\eta(\mathbf{r},\mathbf{r}')$ that is symmetric in the arguments as Onsager's relations are correctly included.

The Sagnac interferometer [24,79–83], the instrument used to measure the Kerr angle in the papers of Karapetyan and co-workers [9,10], being a unique test for reciprocity, only measures microscopic time-reversal symmetry breaking. This is so because the interferometer measures the Kerr angle by interfering two beams of light made to reflect from the sample in a fashion such that the sourcing aperture for one beam is the receiving aperture for the other and vice versa. The Sagnac interferometer conveys light of two linear polarization states to the sample by a polarization maintaining single-mode optical fiber. The end face of the fiber is an aperture for the two linear polarization states, and the two modes that couple from free space to the two fiber axes are the time reverse of those two that are emitted from it. A quarter-wave plate, with the slow axis oriented at 45° with respect to the two polarization states emerging from the fiber axes, is placed between the fiber end face and the sample. The two orthogonal linearly polarized beams of light emitted from the fiber are transformed into opposite circularly polarized states after traversing the quarter-wave plate. The circularly polarized beams of light partially reflect from the sample into the same circular polarization states and will pass through the quarter-wave plate a second time, transforming back into orthogonal linear polarization states, but now rotated 90° from before. In this way, the beams couple from one axis of the fiber to the other and interfere at a polarizer, oriented at 45° with respect to both axes of the fiber, placed at the other end of the fiber-optic cable. A lock-in amplifier technique recovers the Kerr angle from the interference intensity [82]. Because the fiber is highly birefringent and the diode light source has 8-μm coherence length, only light that couples, after reflecting from the sample, between *different* axes in the fiber will traverse optical path lengths that differ by less than a coherence length and interfere coherently at the polarizer [82].

The reciprocity theorem applies to the Sagnac interferometer exactly. The spatial filtering of the fiber ensures that the electromagnetic spatial modes that are sourced and received by the fiber are exactly the time reverse of each other. Comparing the phase delays of light exchanged between the two fiber axes uniquely tests for microscopic time-reversal symmetry breaking not only in a sample being probed, but also within the optical components that make up the instrument itself. Misalignments or imperfect optical components will not introduce spurious signals as they will have time-reversal symmetric responses.

Because of the reciprocity theorem, the suggestion [4] that the recent measurements of a Kerr effect [10] stem from an equilibrium phase of matter with mirror-symmetry breaking and without time-reversal symmetry breaking, cannot be correct. Instead, the reciprocity theorem implies that either the ground state must break time-reversal symmetry or the sample is in a highly nonequilibrium state that does as well. More tests are needed to determine if nonlinear effects are relevant.

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