Coulomb blockade for tunneling through a long island

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We consider Coulomb blockade effects for tunneling through a piece of wire with large resistance $R \gg 1$. This system can not be treated as a zero-dimensional one, as the dynamics of internal inhomogeneous degrees of freedom is crucial. At moderately high temperatures the linear conductance G of the system is suppressed due to the one-dimensional Coulomb zero bias anomaly effect. At low T, besides the standard activational factor, there is an additional T-independent (though also exponentially strong) suppression of G. It arises due to the tunneling evolution of the charge in the wire to the equivipotential distribution. In the intermediate range of T the $G(T)$ dependence is a power law, as in the phenomenological environmental theory. The effective "environmental resistance" entering the power exponent is found explicitly. It depends on the length of the wire and on the positions of the contacts.

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I. INTRODUCTION

A phenomenon of Coulomb blockade of the electronic transport through a small grain, connected to two massive electrodes by high-resistance tunnel contacts, has been well known for many decades (see, e.g., Refs. [1] and [2]). It was extensively studied both experimentally and theoretically. In most theoretical studies the grain was assumed to be an effectively zero-dimensional object without any internal dynamics. Under this assumption the effective linear conductance of the system in the so-called "orthodox Coulomb blockade regime" is

$$G^{(0)}_{\rm ort} \sim \frac{g_A g_B}{2(g_A+g_B)}\left[\exp\left\{-\frac{E^{(+)}_C}{T}\right\}+\exp\left\{-\frac{E^{(-)}_C}{T}\right\}\right], \quad (1)$$

where $g_A \ll 1$ and $g_B \ll 1$ are dimensionless conductances of the left and the right contacts correspondingly:

$$E^{(\pm)}_C = E_C(1 \pm 2q), \quad (2)$$

where the $(+)$ sign corresponds to the state with an extra electron on the dot and $(-)$ to an extra hole. The equilibrium charge of the dot $q = q(U)$ $(|q| < 1/2)$ can be tuned by changing a gate voltage U. For $T \ll E_C$ the conductance is dominated by a process that corresponds to minimal activation energy, so that

$$G^{(0)}_{\rm ort} \sim \frac{g_A g_B}{2(g_A+g_B)}\exp\left\{-\frac{E_{\rm act}}{T}\right\},$$
$$E_{\rm act} = \min\{E^{(+)}_C, E^{(-)}_C\} = E_C \delta q, \quad (3)$$
$$\delta q \equiv 1 - 2|q|.$$

In the orthodox Coulomb blockade regime a particle (an electron or a hole) first borrows an energy $E_{\rm act}$ from the thermostat and tunnels to the dot from one of the electrodes (say, the electrode A). It stays at the dot *in a real state* for a long "waiting time," proportional to $1/g$. Then it falls down to the electrode B, and the energy $E_{\rm act}$ is again dissipated—it goes back to the thermostat.

The formulas (1) and (3) are written with an exponential accuracy: there may be an additional preexponential factor that can contain a power-law T dependence. This factor is not universal, it depends on the details of the model.

This process dominates the transport through the dot at relatively high temperatures, while at low enough temperatures the orthodox two-stage incoherent process is replaced by the cotunneling [3]. In this regime the dot remains in a (virtual) charged state only for a very short time, the tunneling of a particle to the dot from the electrode A is immediately followed by the tunneling of a particle from the dot to the electrode B. If the second tunneling leaves the dot in the very same state, as before the first tunneling, then the entire process is called *elastic cotunneling*. If the dot after the process changes its state (namely, it excites one extra electron-hole pair) then we deal with *inelastic cotunneling*. The corresponding effective conductances are

$$G^{(0)}_{\rm cot} \sim g_A g_B \begin{cases} (T/E_{\rm act})^2, & \text{(inelastic cotunneling)}, \\ \delta/E_{\rm act}, & \text{(elastic cotunneling)}, \end{cases} \quad (4)$$

where δ is the characteristic level spacing in the dot. The crossover from the elastic to the inelastic cotunneling takes place at $T \sim T_{el} \sim (E_{\rm act}\delta)^{1/2}$. The crossover from the inelastic cotunneling to the orthodox regime takes place at $T \sim T_{\rm cot}$, where

$$T^{(0)}_{\rm cot} \sim E_{\rm act}/\ln[1/(g_A+g_B)]. \quad (5)$$

The above consideration is only applicable for an effectively zero-dimensional dot. In our paper [4] we have shown that the expression for $G_{\rm cot}$ is strongly modified in the case of the extended dot. We have considered a "long island" (see Fig. 1) that has the form of a wire of total length L and diameter a. The charging energy for such a system is

$$E_C = (e^2/\epsilon L)\ln(L/a), \quad (6)$$

where ϵ is the effective dielectric constant. The classic dimensionless (in the units of h/e^2) resistance of the wire

$$R = L/\xi \gg 1, \quad (7)$$

where length ξ is related with one-dimensional conductivity of the wire $\tilde{\sigma} = e^2\xi/2\pi\hbar$; the same length can be estimated as

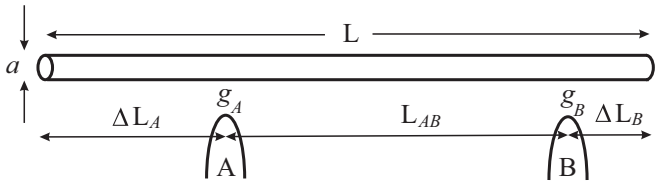

FIG. 1. Electrons tunnel between a wire of length L and diameter a and two leads A and B, placed at distance L_{AB} from each other.

$\xi \sim N_{\rm ch} l$, where l is the elastic mean free path, and $N_{\rm ch} \gg 1$ is the number of channels in the wire.

We have shown that the inelastic cotunneling in the long island is described by the formula

$$G_{\rm cot} \sim g_A g_B (T/T_{c2})^{\alpha_{\rm cot}}, \quad T_{c2} = E_C/R, \quad \alpha_{\rm cot} = 2R_{AB} = 2RL_{AB}/L, \tag{8}$$

provided the resistance $R_{AB} = L_{AB}/\xi$ of a piece of the wire between the contacts is large: $R_{AB} \gg 1$.

In the present paper we study the modification of the standard formula (3) for the incoherent sequential tunneling, which arise in the case of an extended dot (in particular, in the case of the long island). The question that we address is especially relevant in the "resonant" case, when the gate voltage is tuned so that $E_{\rm act} \ll E_C$, and/or for very low conductances g_A, g_B, so that the cotunneling regime is realized at very low temperature.

The paper is organized as follows: In Sec. II we list the main results, obtained in this paper for the linear case (i.e., for the case of small bias voltage), and discuss different regimes, occurring on the phase diagram on the $T - q$ plane. In Sec. III we describe a general technique for semiclassical treatment of multimode thermoactivated processes with special attention for the proper treating of the time zero mode at finite temperature. We also discuss the space-time structure of the solutions of the arising instanton equations. The technique is applicable for arbitrary (not necessarily small) bias voltages. In Sec. IV we apply the developed method to the case of relatively large temperatures and/or bias voltages, when the discreetness of the charge on the wire is not relevant. Here the regime of the zero bias anomaly is settled, the only effect of the finite length of the wire being the dependence of the current on the distance between the contact and the end of the wire. In Sec. V we consider the case of relatively low temperature, when the discreetness of the charge (and, therefore, the zero mode) becomes crucial and the Arrhenius temperature dependence of the current is established. The resonant case, in which the activation energy is especially small (e.g., due to the tuning of the gate voltage) is discussed in Sec. VI. In Sec. VII we show how the results obtained in the previous sections are manifested in the linear conductance of the system. Finally, Sec. VIII presents our conclusions.

II. PRINCIPAL RESULTS

In this section we formulate the results of our study as applied to the linear conductance of the system; their derivation will be given in the subsequent sections of this paper. Note that the general formalism, developed in Sec. III, applies to the case of high bias voltages as well. However, we have chosen not to discuss the results of the nonlinear theory in this paper, and leave it to a separate publication. Such a discussion would require a detailed consideration of many different cases and subcases.

The conductance $G_{\rm ort}$ due to sequential incoherent tunneling (the orthodox Coulomb blockade regime) can be expressed in terms of the local single-particle density of states

$$G_{\rm ort} = G_{\rm ort}^{(+)} + G_{\rm ort}^{(-)}, \tag{9}$$

$$G_{\rm ort}^{(\pm)} = \frac{g_A Z^{(\pm)}(x_A) g_B Z^{(\pm)}(x_B)}{2[g_A Z^{(\pm)}(x_A) + g_B Z^{(\pm)}(x_B)]}, \tag{10}$$

where $Z^{(+)}(x)$ and $Z^{(-)}(x)$ are the T-dependent factors of suppression of the local densities of states of, correspondingly, electrons and holes at the Fermi level due to Coulomb effects. The origin of the formulas (9) and (10) will be clarified in Sec. VII. Here we only want to stress that they are in a sense precise: They would produce a correct preexponential factor for the conductance if the correct preexponential factors for $Z^{(\pm)}(x)$ were used. We are not going to derive the latter prefactors in what follows, however. The fact of splitting of the current in the independent contributions of electrons and holes is also precise at low temperatures. Although at high temperatures such a splitting does not take place, the formulas remain effectively valid, since the difference between contributions of electrons and holes disappears in this limit.

There are three characteristic crossover temperatures

$$T_{c3} = E_C R, \quad T_{c2} = \frac{E_C}{R}, \quad T_{c1} = \frac{E_{\rm act}}{R} = T_{c2}\delta q. \tag{11}$$

There is always a strong inequality $T_{c2} \ll T_{c3}$ between two of these scales. The third one, T_{c1}, is relevant only in the resonant case ($\delta q \ll 1$) when $T_{c1} \ll T_{c2}$, while in the nonresonant case ($\delta q \sim 1$) it does not constitute any distinct energy scale since $T_{c1} \sim T_{c2}$.

A. High temperatures $T \gg T_{c2}$

At highest temperatures $T \gtrsim T_{c3}$ there is no T dependence of the conductance: $Z = 1$ and $G = g_A g_B/(g_A + g_B)$. In the range $T_{c2} \ll T \ll T_{c3}$ the renormalization of the local density of states is manifested in the well-known one-dimensional Coulomb zero-bias anomaly (see Refs. [5–9]) :

$$Z^{(\pm)}(x) \sim \exp\left\{-0.76\, RB(\Delta L/L_c)\left(\frac{T_{c2}}{T}\right)^{1/2} - \frac{E_C^{(\pm)}}{2T}\right\}. \tag{12}$$

The only modification of the formula (12) compared to the standard Coulomb ZBA factor for an infinitely long wire is the factor $B(\Delta L/L_c)$ [the function $B(z)$ is shown in Fig. 2].

This factor is relevant in the case when any of the contacts is close to one of the ends of the wire ($\Delta L \equiv L/2 - |x|$), and the charge spreading is therefore impeded in the corresponding direction. In particular, if the contact is placed at the very end of the wire ($\Delta L \ll L_c$), then the tunneling action is increased by a factor of two [$B(0) = 2$] compared to the case of effectively infinite wire ($\Delta L \gg L_c$).

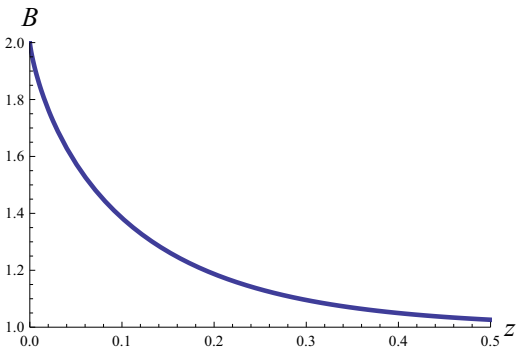

FIG. 2. (Color online) Plot of the function $B(z)$, defined by Eq. (52). This function describes the dependence of the local density of states on the position of the contact in the case of relatively high temperature.

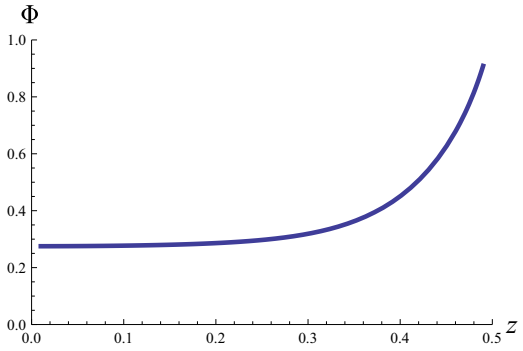

FIG. 3. (Color online) The plot of the function $\Phi(z,1)$, defined in Eq. (70). This function describes the dependence of the local density of states on the position of the contact for relatively low temperature in the nonresonant case.

The "spreading length"

$$L_c(T) = (L/2\pi)(2T_{c2}/T)^{1/2} \tag{13}$$

defines the spatial scale to which the cloud of screening charge proliferates on the tunneling stage of the process. For $L_c \ll L$ (i.e., at $T \gg T_{c2}$) the wire is effectively infinite: The tunneling stage of the charge-spreading process is terminated before the charge has a chance to reach both boundaries of the system and establish the equipotential distribution. Under this condition the length L does not enter the result for $Z(x)$.

B. Low temperatures $T \ll T_{c1}$

At low temperatures $T \ll T_{c1}$ one finds

$$Z^{(\pm)}(x) \sim \exp\{-E_C^{(\pm)}/T - R\Phi(|x|/L, 1 \pm 2q)\}, \tag{14}$$

where x is the position of the contact, accounted for with respect to the center of the wire. The second, temperature independent, term in the exponent of (14) is responsible for the tunneling spreading of the inhomogeneous charge distribution in the wire. It is small compared to the first, activational, term but still large compared to unity, and, therefore, important. The function $\Phi(z,\delta q)$ is of the order of unity in the nonresonant case, when δq is not especially small. For $\delta q = 1$, $\Phi(z,1)$ is plotted in Fig. 3. For general δq the function Φ obeys the rule

$$\Phi(1/2,\delta q) = 4\Phi(0,\delta q), \tag{15}$$

which means that the suppression of the tunneling for the charge injected through a junction, placed at the end of the wire, is much stronger than that for the junction in the center.

C. The resonant case and the environmental theory

In the resonant case (for $\delta q \to 0$) the function $\Phi(z,\delta q)$ can be evaluated analytically:

$$\Phi(z,\delta q) \approx 2\lambda(z)\ln(1/\delta q), \quad \lambda(z) = \tfrac{1}{12} + z^2, \tag{16}$$

so that

$$\begin{aligned} Z^{(\mathrm{res})}(x) &\sim (\delta q)^{2R_{\mathrm{eff}}(x)} \exp\{-E_{\mathrm{act}}/T\}, \\ R_{\mathrm{eff}}(x) &= R\lambda(x/L). \end{aligned} \tag{17}$$

The spreading resistance $R_{\mathrm{eff}}(x)$ entering (17) gives an explicit expression for the effective environmental resistance which was introduced in Refs. [6] and [10] phenomenologically, and therefore could not be explicitly related to the geometry of the experiment. In Sec. VI C of this paper we also give an explicit prescription for finding the spreading resistance for an arbitrary geometry of the quantum dot, not necessarily quasi-one-dimensional.

Since there is a strong inequality $T_{c1} \ll T_{c2}$ in the resonant case, an intermediate temperature range $T_{c1} \ll T \ll T_{c2}$ arises, where the principal (power-law) T dependence of Z arises due to shakeup of soft environmental modes (c.f. Refs. [6] and [10–12]). In this range

$$Z^{(\mathrm{res})}(x) \sim (T/T_{c2})^{2R_{\mathrm{eff}}(x)} \exp\{-E_{\mathrm{act}}/2T\}. \tag{18}$$

It is important to note that the activational term in the exponent of Z dominates the temperature dependence only at lowest temperatures $T \ll T_{c1}$ [i.e., in the formulas (14) and (17), but not in formula (18)]. For higher T the principal T dependence comes from the power-law renormalization of conductances, while the activational factor gives only a subleading contribution. We, however, keep this activational factor also in Eq. (18), since it contains the q dependence, while the leading terms in (18) are q independent. We note that power-law behavior of the type of Eq. (18) is also known in the theory of Luttinger liquids, see for example Ref. [13].

D. The summary of regimes

In the range $T_{c1} \ll T \ll T_{c2}$ the T dependence of both orthodox sequential tunneling rate (G_{ort}) and of the cotunneling rate (G_{cot}) is a power law; the only difference is in the corresponding exponents:

$$\begin{aligned} G_{\mathrm{cot}} &\propto g^2(T/T_{c2})^{\alpha_{\mathrm{cot}}}, \\ \alpha_{\mathrm{cot}} &= 2R_{AB} = \frac{2L}{\xi}\left(1 - \frac{\Delta L_A}{L} - \frac{\Delta L_B}{L}\right), \end{aligned} \tag{19}$$

$$\begin{aligned} G_{\mathrm{ort}} &\propto g(T/T_{c2})^{\alpha_{\mathrm{ort}}}, \\ \alpha_{\mathrm{ort}} &= 2R_{\mathrm{eff}} = \frac{2L}{\xi}\left(\frac{1}{3} - \frac{\Delta L}{L} + \left(\frac{\Delta L}{L}\right)^2\right), \\ \Delta L &= \max\{\Delta L_A, \Delta L_B\}. \end{aligned} \tag{20}$$

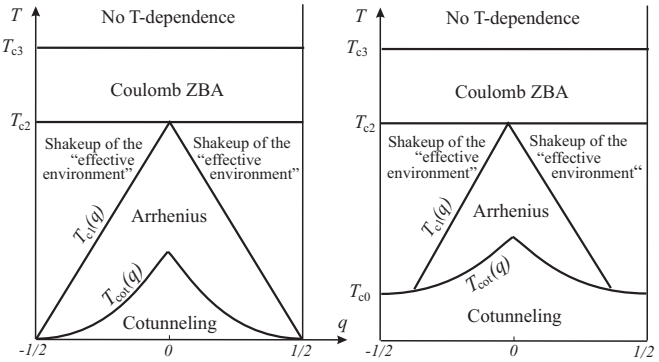

FIG. 4. The phase diagram for different regimes on a plane $T-q$. Left (right) panel: the case $\alpha_{\rm cot} > \alpha_{\rm ort}$ ($\alpha_{\rm cot} < \alpha_{\rm ort}$). The conductance $G(T)$ is described by Eq. (12) in the Coulonb ZBA regime, by Eq. (18) in the "shakeup of the environment" regime, by Eq. (14) or (17) in the Arrhenius regime, and by Eq. (8) in the regime of cotunneling.

The relation between $\alpha_{\rm cot}$ and $\alpha_{\rm ort}$ depends on the setup geometry. If the contacts are relatively far from each other (say, near the opposite ends of the wire), then $\alpha_{\rm cot} > \alpha_{\rm ort}$; if the contacts are very close to each other, then $\alpha_{\rm cot} < \alpha_{\rm ort}$. The scenario of the crossover to the cotunneling regime depends on this relation. If $\alpha_{\rm cot} > \alpha_{\rm ort}$, then for all values of $E_{\rm act}$ there is an intermediate temperature range $T_{\rm cot} < T < T_{c1}$ where the Arrhenius law (75) is valid and

$$T_{\rm cot} \sim \frac{E_{\rm act}}{\ln(1/g) + (\alpha_{\rm cot} - \alpha_{\rm ort})\ln(1/\delta q)} \ll T_{c1}. \qquad (21)$$

If $\alpha_{\rm cot} < \alpha_{\rm ort}$, then the crossover temperature $T_{\rm cot}$ is still described by Eq. (21) for not too small $\delta q > g^{1/(\alpha_{\rm ort}-\alpha_{\rm cot})}$, so that the denominator in Eq. (21) remains positive. For $T_{c1} < T_{c0}$, however, the Arrhenius domain vanishes and there is an immediate crossover between the two power laws that takes place at $T_{\rm cot} \approx T_{c0}$, where

$$T_{c0} \sim T_{c2} g^{1/(\alpha_{\rm ort}-\alpha_{\rm cot})}. \qquad (22)$$

The phase diagram for different regimes on a plane $T-q$ is shown in Fig. 4. Note that $E_{\rm act}$, T_{c1}, $T_{\rm cot}$ depend on q and are, therefore, tuneable by the gate voltage U, while T_{c2}, T_{c0}, and T_{c3} are q independent and not tuneable.

III. TUNNELLING DENSITY OF STATES: GENERAL FORMALISM

Let us now turn to the regular derivation of the results, presented above. In the "orthodox" regime the transport of electrons through the island from contact A to contact B comes about as a sequence of two incoherent one-particle processes: On the first stage an extra particle (an electron at contact A, or a hole at contact B) comes to the dot; on the second stage it leaves the dot at the opposite contact.

In contrast to the two-particle cotunneling process, the total charge of the system is changed at each of the two stages of the one-particle tunneling. To work out an adequate language for the description of this process, we have to modify the finite-system approach, introduced in Ref. [4], in order to take into account the charge conservation (irrelevant for the two-particle cotunneling, discussed in Ref. [4]: for such "neutral" processes the charge conservation was automatically ensured). The approach is based on the Levitov and Shytov semiclassical description [8] of the charge-spreading process in terms of the hydrodynamic equations of the under-barrier motion of the charge ρ and current j distributions in the wire. In the imaginary time τ these equations take the form

$$\frac{\partial\rho}{\partial t} + \frac{\partial j}{\partial x} = \mathcal{J}(x,\tau), \qquad (23)$$

$$j = -D\frac{\partial\rho}{\partial x} - \tilde{\sigma}\frac{\partial}{\partial x}\int_{-L/2}^{L/2} dx'\rho(x',t)\frac{1}{\epsilon|x-x'|}, \qquad (24)$$

where $\tilde{\sigma} = e^2\xi/2\pi\hbar$ is effective one-dimensional conductivity, and D is a diffusion constant. The source

$$\mathcal{J}(x,t) = \pm[\delta(t-t_{\rm in}) - \delta(t-t_{\rm out})]\delta(x-x_0), \qquad (25)$$

and the instanton solution is chosen in a form of a symmetric bounce: The electron (sign $+$) or a hole (sign $-$) is injected into the system at a point $x = x_0$ at the moment $t = t_{\rm in} = t_0 - i\tau_0$ and is evacuated from the same point at the moment $t = t_{\rm out} = t_0 + i\tau_0$. In the quasi 1D case, and at not very low temperatures, the diffusional flow [the first term on the right hand side of Eq. (24)] can be neglected. The proper solution of Eqs. (23) and (24) should obey the initial condition, fixing the total charge $Q(t) = \int dx\rho(x,t)$ of the wire at the initial stage of the process (before the injection):

$$Q(t < t_{\rm in}) = q_{\rm in}(N) \equiv q + N, \qquad (26)$$

where $N = 0, \pm1, \pm2, \ldots$ is the number of extra electrons in the initial state.

Another important condition is the fact that the injected electron/hole brings to the system additional energy $\pm\varepsilon$, so that the total energy of the system undergoes a jump at the injection point:

$$E_{\rm tot}(\tau = -\tau_0 + 0) - E_{\rm tot}(\tau = -\tau_0 - 0) = \pm\varepsilon. \qquad (27)$$

We note that this condition is the only place where the initial energy ε of the external electron comes into play.

The under-barrier action $S^{(\pm)}(\tau_0, x_0, \varepsilon, q_{\rm in})$, corresponding to the solution of Eqs. (23) and (24) with the initial conditions (26) and (27), is a function of unknown duration of the instanton core (the "injection time") τ_0. The actual value $\tau_0^* = \tau_0^*(T,\varepsilon,q_{\rm in})$ can be found from the condition

$$\partial S^{(\pm)}(\tau_0, x_0, \varepsilon, q_{\rm in})/\partial\tau_0|_{\tau_0=\tau_0^*} = 0. \qquad (28)$$

In fact, $S^{(\pm)}(\tau_0^*, \varepsilon, q_{\rm in})$ is the action which enters the final expression for the tunneling probability:

$$Z^{(\pm)}(x_0,\varepsilon,q_{\rm in}) \propto \exp\{-S^{(\pm)}(\tau_0^*, x_0, \varepsilon, q_{\rm in})\}. \qquad (29)$$

In what follows we will omit the arguments ε and $q_{\rm in}$ and simply write $Z^{(\pm)}(x)$, but in due time we will recall its dependence on ε and on N (through $q_{\rm in}$).

A. Contribution of the zero mode and charge conservation

The most important difference of the charged one-particle processes in a finite system, discussed in the present paper, from the neutral two-particle ones, studied in Ref. [4], is the necessity to accurately take into account the contribution of

the "zero mode" of the action S. This is a very tricky point, where our results differ from those of Ref. [10], so we discuss it here in detail.

It is very important that the solution of the linear equations of motion (23) and (24) is not unique: To any particular solution one can always add the *free solution* (corresponding to the absence of any external source) multiplied by an arbitrary coefficient. The uniqueness is only restored with the help of the initial condition (26) that fixes the coefficient. The free solution is τ independent and proportional to the zero mode $\rho_0(x)$, which carries no currents [$j_0(x) \equiv 0$]. The corresponding distribution of the potential $\varphi_0 = const$ and charge $\rho_0(x)$ obeys the condition of the equipotentiality

$$[\hat{U}\rho_0](x) = \text{const}. \tag{30}$$

The rules for regularization of the singular operator

$$[\hat{U}\rho](x) = \text{reg}\int_{-L/2}^{L/2} \frac{\rho(x')dx'}{\epsilon|x-x'|} \tag{31}$$

were discussed in Ref. [4]: The short-distance logarithmic divergency is cut off at the width of the wire a. Strictly speaking, the solution of Eq. (30) is inhomogeneous: $\rho_0(x) \propto 1 + \frac{\ln(1-(2x/L)^2)}{2\ln(L/a)}$. However, the x-dependent term plays a role only very close to the ends of the wire, at the distances of the order of the wire's diameter. Therefore this effect is weak, and we will neglect it in the leading logarithmic approximation, assuming $\rho_0 = \text{const}$.

It is convenient to split the source $\mathcal{J}$ in Eq. (25) into two parts: $\mathcal{J} = \mathcal{J}^{(0)} + \tilde{\mathcal{J}}$

$$\begin{aligned} \mathcal{J}^{(0)} &= \pm(1/L)[\delta(t-t_{\text{in}}) - \delta(t-t_{\text{out}})], \\ \tilde{\mathcal{J}} &= \pm\tilde{J}[\delta(t-t_{\text{in}}) - \delta(t-t_{\text{out}})], \\ \tilde{J} &= \delta(x-x_0) - 1/L. \end{aligned} \tag{32}$$

We will see that $\mathcal{J}^{(0)}$ affects only the zero mode, which is responsible for the dynamics of the systems total charge, while the source $\tilde{\mathcal{J}}$ excites all the modes but the zero one.

Consequently, the action $S^{(\pm)}$ is also split,

$$S = S_0^{(\pm)} + \tilde{S}^{(\pm)}, \tag{33}$$

into the contributions of zero ($S_0^{(\pm)}$) and nonzero ($\tilde{S}^{(\pm)}$) modes.

The total charge of the wire Q is conserved, except for the injection moments $\tau = \mp\tau_0$, where an extra electron (hole) comes to the wire, or leaves the wire, so that

$$Q^{(\pm)}(\tau) = q_{\text{in}} \pm \theta(\tau_0 - |\tau|) \tag{34}$$

is the unique solution, satisfying the initial condition (26). There are no currents in the zero-mode solution, therefore its contribution to the action is due solely to the potential term, accounted for with respect to the equilibrium value, and from the additional energy ε, which comes with the injected electron [see Eq. (27)]:

$$\begin{aligned} S_0^{(\pm)} &= \int d\tau E_C\left\{\left[Q_{\text{tot}}^{(\pm)}(\tau)\right]^2 - q_{\text{in}}^2\right\} - 2\tau_0\varepsilon \\ &= 2\tau_0\tilde{E}_C^{(\pm)}(\varepsilon), \end{aligned} \tag{35}$$

$$\tilde{E}_C^{(\pm)}(q_{\text{in}},\varepsilon) = E_C \mp (\varepsilon - 2q_{\text{in}}E_C). \tag{36}$$

Some comments are due at this point, since the result (35) essentially differs from the corresponding action

$$S_0^{\text{KG}} = 2\tau_0 E_C(1 - 2T\tau_0), \tag{37}$$

derived in Ref. [10] for the case $q_{\text{in}} = 0$ and $\varepsilon = 0$ (so that $\tilde{E}_C^{(\pm)} = E_C$). The charge can be formally obtained by means of integration of the equation of motion (23). The result is

$$Q^{(\pm)}(\tau) = \pm 2\sum_{\omega} \sin(\omega\tau_0)\cos(\omega\tau)/\omega, \tag{38}$$

where the Matsubara frequency summation runs over even frequencies $\omega = 2\pi Tn$, with $n = 0,1,2,\ldots$. The ambiguity of this solution comes from the ill-defined term with $\omega = 0$. The result (37) arises, if one simply omits the $\omega = 0$ term in the sum (38). Then

$$Q_{\text{scr}}^{(\pm)}(\tau) = \pm[\theta(\tau_0 - |\tau|) - 2T\tau_0], \tag{39}$$

and, substituting $Q_{\text{scr}}^{(\pm)}(\tau)$ for $Q^{(\pm)}(\tau)$ in Eq. (35), one obtains the result (37) for the action. However, the charge $Q_{\text{scr}}^{(\pm)}(\tau)$ given by Eq. (39) does not coincide with the true total charge of the system $Q^{(\pm)}(\tau)$. Their difference is given by the "zero-mode" contribution $Q_{\text{free}}^{(\pm)} \equiv Q^{(\pm)}(\tau) - Q_{\text{scr}}^{(\pm)}(\tau)$. Qualitative difference between screening and zero-mode contributions is clearly demonstrated by the spatial dependencies of the corresponding space distributions of charge: $\rho_{\text{scr}}^{(\pm)}(\tau,x)$ and $\rho_{\text{free}}^{(\pm)}(\tau,x)$. Indeed, one can show that $\rho_{\text{scr}}^{(\pm)}(\tau,x)$ is concentrated close to the location of the tunneling electron and constitutes the screening cloud, whereas $\rho_{\text{free}}^{(\pm)}(\tau,x) \propto \rho_0(x)$ is nearly x independent [up to the corrections, mentioned after Eq. (31)]. We emphasize that both contributions should be taken into account in order to obtain correct exponential dependence of the tunneling rate with activation energy E_C.

At $T > T_{c1}$ the contribution $S_0^{(\pm)}$ to the action plays only a secondary role, and in the leading approximation one obtains

$$\tau_0^* = 1/4T, \quad Q_{\text{scr}}^{(\pm)}(\tau) = \pm(1/2)\text{sign}(1/4T - |\tau|). \tag{40}$$

If one naively neglects the contribution of the free solution, then the formula (40) seems to remain valid also for $T < T_{c1}$. It is the erroneous substitution of the screening charge $|Q_{\text{scr}}^{(\pm)}| = 1/2$ in place of the true total charge (34), that results in the mysterious low-temperature activation energy $E_C/4$, following from the action (37).

B. Contribution of nonzero modes

The action $\tilde{S}^{(\pm)}$ is analogous to the corresponding action for the case of the two-particle neutral source (see Ref. [4])

$$\begin{aligned} \tilde{S}^{(\pm)}(\tau_0,x_0) &= \frac{e^2}{2}\sum_{\omega}\left\{-\frac{j\cdot j}{\tilde{\sigma}\omega} + \rho\hat{U}\rho\right\} \\ &= \frac{e^2}{2}\sum_{\omega}\frac{1}{\omega}\tilde{\mathcal{J}}\hat{\mathcal{G}}^{+}\hat{U}\tilde{\mathcal{J}} \\ &= \frac{e^2}{2}\sum_{\omega}\frac{4\sin^2(\omega\tau_0)}{\omega}[(\omega - \tilde{\sigma}\hat{U}\hat{\Delta})^{-1}\hat{U}](x_0,x_0), \end{aligned}$$

$$\rho(x,\omega) = \hat{\mathcal{G}}\tilde{\mathcal{J}}, \quad j(x,\omega) = \tilde{\sigma}\hat{\nabla}\hat{U}\rho = \tilde{\sigma}\hat{\nabla}\hat{U}\hat{\mathcal{G}}\tilde{\mathcal{J}},$$
$$\hat{\mathcal{G}} = [\omega - \tilde{\sigma}\hat{\Delta}\hat{U}]^{-1}, \quad \hat{\mathcal{G}}^{+} = [\omega - \tilde{\sigma}\hat{U}\hat{\Delta}]^{-1},$$
$$\hat{\Delta} \equiv \frac{d^2}{dx^2}. \tag{41}$$

To evaluate $\tilde{S}^{(\pm)}$ it is convenient to use an orthonormal basis of eigenfunctions $\varphi_m(x)$ and eigenvalues Λ_m of the Hermitian operator $\hat{\mathcal{G}}^{+} \cdot \hat{U}$ entering the expression (41):

$$\tilde{S}^{(\pm)} = \frac{e^2}{2}\sum_{\omega,m\neq 0}(4\Lambda_m/\omega)\sin^2(\omega\tau_0)|\varphi_m(x_0)|^2,$$

$$\varphi_m(x_0) = \sqrt{2U_m/L}\cos(\pi m w), \quad \varphi_0(x_0) = \sqrt{U_m/L}, \tag{42}$$

$$\Lambda_m = \left\{\omega + \frac{\pi^2 m^2 U_m \tilde{\sigma}}{L^2}\right\}^{-1}, \quad U_m = \frac{2}{\epsilon}\ln\left[\frac{2L}{a(m+1)}\right]$$

(see Ref. [4] for details). Here

$$w = 1/2 - |x_0|/L \equiv \Delta L/L, \tag{43}$$

ΔL being the distance from the contact to the closest end of the wire. The eigenfunctions $\varphi_m(x)$ are nothing else but the "potential modes;" the corresponding "charge modes" are $\rho_m(x) = [\hat{U}^{-1}\varphi_m](x)$. To simplify the expression (42) we note that U_m only logarithmically depends on m, so that one can take it from under the summation sign, putting $U_m \to \overline{U} \equiv U_{\overline{m}}$, where $\overline{m}$ is the characteristic value of m, corresponding to terms, that give the principal contribution to the sum over m in Eq. (42). The value of $\overline{m}$ will be found *a posteriori*. As a result,

$$\tilde{S}(\tau_0,x_0) = \frac{e^2}{L}\overline{U}\sum_{\omega}\frac{4\sin^2\omega\tau_0}{\omega}\sum_{m=1}^{\infty}\frac{\cos^2(\pi m w)}{\omega + \pi^2 m^2\tilde{\sigma}\overline{U}/L^2}. \tag{44}$$

Finally, for the total action we get

$$S^{(\pm)}(\tau_0,x_0) = 2\tau_0\tilde{E}_C^{(\pm)}(\varepsilon) + \frac{e^2}{L}\overline{U}\sum_{\omega}\frac{4\sin^2\omega\tau_0}{\omega}\sum_{m=1}^{\infty}\frac{\cos^2(\pi m w)}{\omega + \pi^2 m^2\tilde{\sigma}\overline{U}/L^2}. \tag{45}$$

We note that the contribution of nonzero modes depends neither on ε nor on q: These variables only enter $S_0^{(\pm)}$ through $\tilde{E}_C^{(\pm)}(\varepsilon)$.

Obviously,

$$\partial\tilde{S}/\partial\tau_0|_{\tau_0=1/4T} = 0, \tag{46}$$

so that in all those cases, when the total action is dominated by the nonzero modes (i.e., when $\tilde{S} \gg S_0$)

$$\tau_0^* = 1/4T. \tag{47}$$

In the case of $\tilde{S} \lesssim S_0$ the optimal duration τ_0^* can not be found from Eq. (46), and, therefore, $\tau_0^* \neq 1/4T$. In particular, it is so for very low T and/or high bias voltage V.

C. Space-time structure of the instanton solution

It should be noted that the duration of the instanton core τ_0^* does not coincide with the conventional value $\tau_{\rm tun} = 1/2T$ of the under-barrier time for general thermoactivated tunneling.

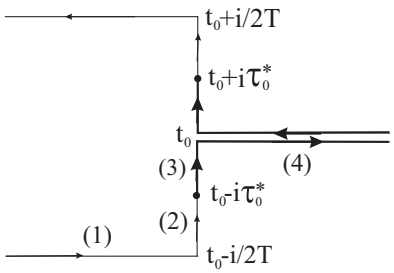

FIG. 5. The Keldysh time contour in the complex t plane for the one-particle tunneling. Parts of the contour, where an extra electron is in the wire, are shown by thick lines. About physical meaning of different segments of the contour, see the text.

Though it seems to be a paradox, this is only due to a misinterpretation of τ_0^* as a tunneling time. Indeed the full tunneling time is still $\tau_{\rm tun} = 1/2T$ also in our case, as it is illustrated in Fig. 5. Let us consider a sequence of processes on the lower part of the Keldysh contour (see Fig. 5), describing the amplitude of the transition (while the upper part corresponds to its complex conjugate). It consists of four segments.

(i) On the initial segment (1) the cloud of screening charge, that will later accommodate an extra electron, is partly created due to purely classic thermodynamic fluctuation (with a proper Gibbs probability). The corresponding countercharge is moved to spatial infinity and distributed over the system homogeneously. An extra electron, which will later jump from the contact to the wire, is still in the lead. The entire system is above the barrier (in a classically accessible region) on the stage (1), the fact of which is reflected by the horizontal direction of the (real) time flow. Note that the probability of that part of the fluctuation, which is provided classically, is temperature dependent.

(ii) The moment $t = t_0 - i/2T$ is just a classical turning point: Up to this point [on the stage (1)] the system evolved in the classically accessible domain, while on the stage (2) it enters the classically forbidden one, and the tunneling begins. Since the charge is transferred to the system only at the moment $t_{\rm in} = t_0 - i\tau_0^*$, the current and charge distribution is changed both before and after this moment of injection. On the stage (2), between $t = t_0 - i/2T$ and $t_{\rm in}$, an extra electron is still absent; "native" electrons of the wire do the job on their own, just "out of hospitality." However, further contraction of the screening cloud costs additional energy, so that the system of electrons in the wire goes under the barrier: The time here flows in the imaginary direction.

(iii) Then, finally, at the moment $t_{\rm in} = t_0 - i\tau_0^*$ an extra electron hops from the contact into the center of the partly prepared screening cloud in the wire. However, since this cloud is only "half-ready," the entire system still finds itself under the barrier. On the stage (3) the system makes its way from under the barrier: The composite cloud, now consisting of the native electrons plus the guest one, proliferates and becomes smoother, so that the potential energy of the system decreases.

(iv) At the moment $t = t_0$ the system gets from under the barrier, the time flow again turns to the real direction. On the stage (4) the composite cloud continues to proliferate,

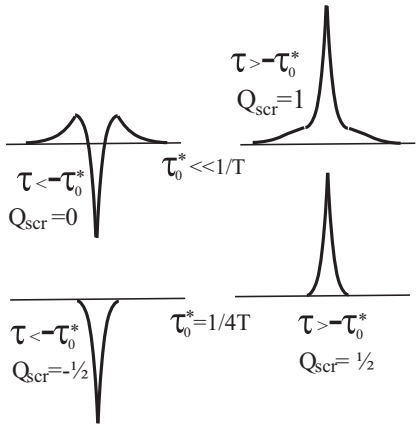

FIG. 6. Imaginary time evolution of the spatial shape of the instanton. Upper panel: nonlinear case ($\tau_0^* \ll 1/T$); the charge separation proceeds in the tunneling mode, the screening charge $Q_{\rm scr} = 0$ before the injection of an electron at $\tau = -\tau_0$, and $Q_{\rm scr} = 1$ after the injection. Lower panel: linear regime ($\tau_0^* = 1/4T$); the countercharge has escaped to spatial infinity already on the classical stage of the process, so that the tunneling inhomogeneous density has nonzero charge $Q_{\rm scr} = -1/2$ at $\tau < -\tau_0$.

and the released potential energy (its total amount being determined by the initial classic fluctuation) is gradually dissipated.

The evolution of the spatial shape of the density of the screening charge is shown in Fig. 6. Note that the above multistage scenario is quite common. In this exact way the hopping of small polarons [14] and the self-trapping processes [15] are organized: The hop of an electron is always preceded by the formation of a preliminary fluctuation (partly classical and partly quantum). The idea that an electron just hops unexpectedly into a completely unprepared system, and only then the accommodation process starts, is incorrect. Though such a misconception normally leads to qualitatively reasonable conclusions, the corresponding under-barrier action is always considerably overestimated, which indicates that the underlying scenario is not the optimal one.

IV. ONE-DIMENSIONAL ZERO BIAS ANOMALY IN A FINITE SYSTEM

In this section we discuss the case of relatively high temperature $T \gg T_{c1}$, when the discreetness of charge is not crucial and the contribution of zero-mode $S_0 \ll \tilde{S}$ and can be treated perturbatively.

A. Low energies: temperature dominated regime

If the bias voltage is relatively low, then in the zero approximation the formula (47) is valid. In the first approximation one can write

$$S^{(\pm)} = \tilde{E}_C^{(\pm)}/2T + \frac{4e^2\overline{U}}{\pi L}\sum_{k=0}^{\infty}\frac{1}{2k+1} \times \sum_{m=1}^{\infty}\frac{\cos^2(\pi m w)}{2\pi T(2k+1)+\pi^2 m^2\tilde{\sigma}\overline{U}/L^2}. \quad (48)$$

If, moreover, $T \gg T_{c2} \gg T_{c1}$, then the summation over m in Eq. (48) can be replaced by integration. Introducing new variable $u = \frac{\pi m L_c}{L\sqrt{2k+1}}$ we get

$$S^{(\pm)} = \tilde{E}_C^{(\pm)}/2T + (2/\pi)\sqrt{2T_{c3}/T}\sum_{k=0}^{\infty}(2k+1)^{-3/2} \times \int_{-\infty}^{\infty}\frac{du}{2\pi}\frac{1+\cos[2u\sqrt{2k+1}(\Delta L/L_c)]}{1+u^2}, \quad (49)$$

where the spreading length

$$L_c(T) = (\xi/2\pi)(2T_{c3}/T)^{1/2} \approx (L/2\pi)(2T_{c2}/T)^{1/2} \quad (50)$$

defines the spatial scale on which the cloud of screening charge proliferates on the tunneling stage of the process. Finally, we arrive at the result

$$Z^{(\pm)}(x_0) \sim \exp\left\{-0.76\, B\left(\frac{\Delta L}{L_c}\right)\left(\frac{T_{c3}}{T}\right)^{1/2} - \frac{\tilde{E}_C^{(\pm)}(\varepsilon)}{2T}\right\}, \quad (51)$$

where the universal function shown in Fig. 2 is given by

$$B(z) = [\zeta(3/2)(1-2^{-3/2})]^{-1}\sum_{k=0}^{\infty}(2k+1)^{-3/2} \times \{1+\exp[-2z\sqrt{2k+1}]\} \approx \begin{cases}1, & \text{for } z \gg 1,\\ 2, & \text{for } z \ll 1.\end{cases} \quad (52)$$

The result (51) is valid in the temperature range

$$T_{c2} \ll T \ll T_{c3}, \quad (53)$$

and for low energies

$$\varepsilon \ll (T T_{c3})^{1/2}. \quad (54)$$

Under these conditions the exponent in Eq. (51) is dominated by the first term; the second one is a relatively small correction (though it still can be large, compared to unity). We keep this correction mostly because it contains the dependence on parameters ε and q, while the leading term does not depend on them.

It is important to note that the exponent in Eq. (51) (the accommodation action) is increased by a factor of 2 if the contact is moved to the end of the wire: The density of states at the ends of the wire is suppressed much stronger than in the middle.

B. High energies: voltage dominated regime

For large ε, as we will see, $\tau_0^* \ll 1/T$ and the summation over ω can be replaced by integration. For small τ_0

$$\tilde{S}(\tau_0) = 2e^2\overline{U}\int_0^{\infty}\frac{\sin^2(\omega\tau_0)d\omega}{\pi\omega}\int\frac{dq}{2\pi}\frac{1+\cos(2q\Delta L)}{\omega+q^2\tilde{\sigma}\overline{U}} = e^2\left(\frac{\overline{U}}{\tilde{\sigma}}\right)^{1/2}\int_0^{\infty}\frac{\sin^2(\omega\tau_0)d\omega}{\pi\omega^{3/2}} \times \left\{1+\exp\left(-2\Delta L\sqrt{\frac{\omega}{\tilde{\sigma}\overline{U}}}\right)\right\} = 2(T_{c3}\tau_0)^{1/2}\tilde{B}\left(\frac{\Delta L}{L_c(\tau_0)}\right), \quad (55)$$

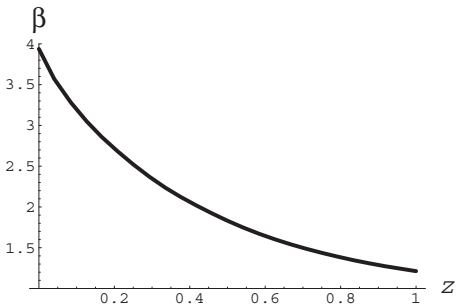

FIG. 7. Plot of the universal function $\beta(z)$, defined by Eq. (59).

where the spreading length $L_c(\tau_0) = \xi\sqrt{T_{c3}\tau_0/\pi}$, and the universal function

$$\tilde{B}(z) = \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{\sin^2 t^2 dt}{t^2}\,(1+\exp\{-2zt\}) \approx \begin{cases} 1, & \text{for } z \gg 1, \\ 2, & \text{for } z \ll 1, \end{cases} \tag{56}$$

only slightly differs from $B(z)$, defined by Eq. (52). Substituting Eq. (55) into Eq. (28) we obtain

$$\tau_0^* \approx \frac{T_{c3}}{(2\varepsilon)^2}\mu\left(\frac{\Delta L}{\overline{L}_c}\right)^2, \quad L_c(\tau_0^*) = \overline{L}_c\mu\left(\frac{\Delta L}{\overline{L}_c}\right), \tag{57}$$

$$\overline{L}_c = \frac{\xi}{2\sqrt{\pi}}\frac{T_{c3}}{\varepsilon}, \quad S \approx \beta\left(\frac{\Delta L}{\overline{L}_c}\right)\frac{T_{c3}}{2\varepsilon}, \tag{58}$$

where functions $\mu(z)$ and $\beta(z)$ are defined as follows: We define a function $\mathcal{B}(\mu,z) \equiv \mu^2(2\tilde{B}(z/\mu)/\mu - 1)$, find the position of its maximum as a function of μ, and denote the corresponding value of μ as $\mu(z)$; finally the value of $\mathcal{B}[\mu(z),z]$ is denoted as $\beta(z)$:

$$\beta(z) = \max_\mu\{\mu^2(2\tilde{B}(z/\mu)/\mu - 1)\} \approx \begin{cases} 1, & \text{for } z \gg 1, \\ 4, & \text{for } z \ll 1, \end{cases}. \tag{59}$$

The function $\beta(z)$ is shown in Fig. 7. As a result

$$Z^{(\pm)}(x_0) \sim \exp\left\{-\beta\left(\frac{\Delta L}{\overline{L}_c}\right)\frac{T_{c3}}{2\varepsilon}\right\}, \tag{60}$$

which is valid under conditions

$$T \ll T_{c3}, \qquad (TT_{c3})^{1/2}, \quad E_C \ll \varepsilon \ll T_{c3}. \tag{61}$$

In this range the density of states Z does not depend on temperature. Again, the suppression of Z is much stronger at the ends of the wire: Here the accommodation action is four times larger than in the middle.

V. ARRHENIUS REGIME: LOW TEMPERATURES, LOW ENERGIES

At lowest temperatures $T \ll T_{c1}$ the contribution of the zero mode to the action plays a crucial role. In this case the relation (47) is not valid anymore, we will see that $\tau_0^* \approx 1/2T$. More precisely,

$$\tau_0^* = \frac{1}{2T}\theta(\tilde{E}_C^{(\pm)}(\varepsilon)) - \eta\cdot\mathrm{sign}(\tilde{E}_C^{(\pm)}(\varepsilon)), \tag{62}$$

with $\eta \ll 1/T$, so that

$$S^{(\pm)} = E_{\rm act}^{(\pm)}(\varepsilon)/T + \Delta S^{(\pm)}, \tag{63}$$

where

$$E_{\rm act}^{(\pm)}(\varepsilon) = \tilde{E}_C^{(\pm)}(\varepsilon)\theta(\tilde{E}_C^{(\pm)}(\varepsilon)) = E_C(1 \pm 2q_{\rm in}) \mp \varepsilon, \tag{64}$$

$$\Delta S^{(\pm)}(\eta) = \frac{e^2}{L}\overline{U}\sum_\omega \frac{4\sin^2\omega\eta}{\omega}\sum_{m=1}^\infty \frac{\cos^2(\pi m w)}{\omega + \pi^2 m^2\tilde{\sigma}\overline{U}/L^2} - 2\eta|\tilde{E}_C^{(\pm)}(\varepsilon)|. \tag{65}$$

Because of the inequality $\eta \ll 1/T$ one can replace summation over ω in Eq. (64) by integration:

$$\Delta S^{(\pm)}(\eta) = -2\eta|\tilde{E}_C^{(\pm)}(\varepsilon)| + \frac{e^2}{L}\overline{U}\int_0^\infty \frac{4\sin^2\omega\eta}{\pi\omega}d\omega \times \sum_{m=1}^\infty \frac{\cos^2(\pi m w)}{\omega + \pi^2 m^2\tilde{\sigma}\overline{U}/L^2}, \tag{66}$$

so that the T independence of ΔS becomes obvious in this limit. Introducing the dimensionless energy $y = \omega/\pi T_{c2}$, we get

$$\Delta S^{(\pm)} = -2|\tilde{E}_C^{(\pm)}(\varepsilon)|\eta + \frac{2R}{\pi^2}\int_0^\infty 4\sin^2[y(\pi T_{c2}\eta)]\frac{dy}{y} \times \sum_{m=1}^\infty \frac{\cos^2(\pi m w)}{y + m^2} = R\,\mathcal{U}(\pi T_{c2}\eta, |x_0|/L, |\tilde{E}_C^{(\pm)}(\varepsilon)|/E_C)), \tag{67}$$

where the universal function

$$\mathcal{U}(\gamma,z,\theta) = -(2\theta\gamma/\pi) + 8\int_0^\infty \sin^2\left(\frac{\gamma u^2}{\pi^2}\right)\frac{du}{u^2} \times \left\{\frac{\cosh(2zu) + \cosh u}{2\sinh u} - \frac{1}{u}\right\}. \tag{68}$$

The condition $\partial U/\partial\gamma = 0$ follows from Eq. (28), and we arrive at

$$\Delta S^{(\pm)} = R\,\Phi(z, 1 \pm 2q), \tag{69}$$

$$\Phi(z,\theta) = \max_\gamma \mathcal{U}[\gamma,z,\theta]. \tag{70}$$

Using the definition (68) of $\mathcal{U}$ it is easy to derive the property

$$\Phi(1/2,\delta q) = 4\Phi(0,\delta q). \tag{71}$$

In the nonresonant case (when $|\tilde{E}_C^{(\pm)}(\varepsilon)|$ is not very small) we need to know $\Phi(z,\theta)$ for $\theta \sim 1$, which can only be found numerically. The function $\Phi(z,1)$ is plotted in Fig. 3.

As a result, at $T \ll |\tilde{E}_C^{(\pm)}(\varepsilon)|$ we obtain

$$Z^{(\pm)}(x_0) \sim \exp\left\{-\frac{E_{\rm act}^{(\pm)}(\varepsilon)}{T} - R\Phi\left(\frac{|x_0|}{L}, \frac{|\tilde{E}_C^{(\pm)}(\varepsilon)|}{E_C}\right)\right\}. \quad (72)$$

Below the threshold, for $\tilde{E}_C^{(\pm)}(\varepsilon) > 0$ the tunnel density of states for electrons with energy ε (holes with energy $-\varepsilon$) obeys the Arrhenius law with the activation energy $E_{\rm act}^{(\pm)}(\varepsilon) > 0$. The exponent in Eq. (72) is dominated by the first (activational) term. The second (temperature-independent) term, being relatively small, is, however, large, compared to unity. Moreover, it depends on both x_0 and ε. Because of the property (15) the corresponding T-independent suppression of Z is much stronger, if the contact is placed close to the end of the wire.

Above the threshold, for $\tilde{E}_C^{(\pm)}(\varepsilon) < 0$ the activational term is absent, and the suppression of Z is due solely to the second, T-independent term in the exponent of Eq. (72), which is symmetric with respect to the threshold. This suppression is also enhanced, if the system is driven to the resonance (i.e., $\tilde{E}_C^{(\pm)}(\varepsilon) \to 0$) from either side. The resonant case is considered in the next section.

VI. RESONANT CASE

A. Low temperatures

In the resonant case we can use the small-θ and large-γ asymptotics

$$\mathcal{U} \approx \lambda(z)\ln\gamma - 2\theta\gamma/\pi, \quad \lambda(z) = (1/12) + z^2. \quad (73)$$

The maximum of $\mathcal{U}(\gamma,z)$ is then reached at $\gamma = \gamma^* \approx \lambda(z)/\theta$, so that one easily arrives at

$$\Phi(z,\theta) \approx 2\lambda(z)\ln(1/\theta), \quad (74)$$

and, finally,

$$Z^{(\pm)}(x_0) \sim \left(\frac{|\tilde{E}_C^{(\pm)}(\varepsilon)|}{E_C}\right)^{2R_{\rm eff}(x_0)} \exp\left\{-\frac{E_{\rm act}^{(\pm)}(\varepsilon)}{T}\right\}, \quad (75)$$

$$R_{\rm eff}(x_0) = R\lambda(x_0/L). \quad (76)$$

The result (75) is only valid under condition $T \ll |\tilde{E}_C^{(\pm)}(\varepsilon)|/R$. Indeed, the above consideration was based on the assumption $\eta \ll 1/T$, while the actual value of $\eta = \eta^*$ in the resonant case is given by

$$\eta^* = \gamma^*/\pi T_{c2} \approx R_{\rm eff}(x_0)/\pi|\tilde{E}_C^{(\pm)}(\varepsilon)|. \quad (77)$$

Using the explicit expression (77) for η^*, it is easy to show that the condition $\eta^* \ll 1/T$ is equivalent to $T \ll |\tilde{E}_C^{(\pm)}(\varepsilon)|/R$.

B. Intermediate temperatures

What happens in the intermediate temperature range $|\tilde{E}_C^{(\pm)}(\varepsilon)|/R \ll T \ll T_{c2}$? Here we come back to the situation where the contribution of zero mode is relatively small and, therefore, the relation (47) is approximately valid. It means that formula (48) can be used. Moreover, since the temperature in our present case is relatively low ($T \ll T_{c2}$), the sum over k in Eq. (48) is logarithmic and can be replaced by integration over ω:

$$\begin{aligned}\tilde{S} &\approx \frac{4e^2\overline{U}}{\pi L}\sum_{k=0}^{T_{c2}/T}\frac{1}{2k+1}\sum_{m=1}^{\infty}\frac{\cos^2(\pi m w)}{\pi^2 m^2\tilde{\sigma}\overline{U}/L^2}\\ &= (4R/\pi^2)\ln(T_{c2}/T)\sum_{m=1}^{\infty}\cos^2(\pi m w)/m^2\\ &= 2\lambda(x_0/L)\,R\ln(T_{c2}/T)\\ &= 2R_{\rm eff}(x_0)\ln(T_{c2}/T). \end{aligned} \quad (78)$$

Combining this result with Eq. (75), we can write

$$Z^{(\pm)}(x_0) \sim \exp\{-E_{\rm act}^{(\pm)}(\varepsilon)/T\} \times (\max\{|\tilde{E}_C^{(\pm)}(\varepsilon)|, RT\}/E_C)^{2R_{\rm eff}(x_0)}, \quad (79)$$

which is valid in the entire range $T \ll T_{c2}$.

C. Environmental theory: how to find the spreading resistance

The expression (79) is very similar to the well-known result of the "environmental theory" [6], where, instead of $R_{\rm eff}(x_0)$, the ohmic resistance of the environment $R_{\rm env}$ appears. It means that, in the resonant case, the zero mode plays the role of the charge degree of freedom, corresponding to an effective zero-dimensional island with the charging energy $E_{\rm act}^{(\pm)}$, while the rest of the modes form an effective resistive environment. The spreading resistance $R_{\rm eff}(x_0)$ entering Eq. (75) gives an explicit expression for the phenomenological environmental resistance, which can therefore be explicitly related to the geometry of the experiment. Below we give a general recipe for finding $R_{\rm eff}$ in arbitrary geometry.

Exploring our calculation, we note that, in the leading logarithmic approximation, we have actually neglected ω-term in the denominator of $\hat{\mathcal{G}}^+$. It means that

$$\begin{aligned}\tilde{S} &= \frac{e^2}{2}(\tilde{J}\cdot\hat{\mathcal{G}}^+|_{\omega=0}\cdot\hat{U}\cdot\tilde{J})\sum_{\omega\sim T}^{T_{c2}}\frac{4\sin^2(\omega\tau_0)}{\omega}\\ &= -(2/\xi)\ln(T_{c2}/T)\,(\tilde{J}\cdot\hat{\Delta}^{-1}\cdot\tilde{J}). \end{aligned} \quad (80)$$

The matrix element $(\tilde{J}\cdot\hat{\Delta}^{-1}\cdot\tilde{J})$ is easy to evaluate in the coordinate representation. Having in mind the definition (32) of $\tilde{J}$, we find the potential distribution

$$\varphi(x) \equiv \hat{\Delta}^{-1}\tilde{J} = (x-x_0)\theta(x-x_0) - (L/2+x)^2/2L$$

and rederive

$$(\tilde{J}\cdot\hat{\Delta}^{-1}\cdot\tilde{J}) = \int\varphi(x)\tilde{J}(x)dx = -L\lambda(x_0/L). \quad (81)$$

This result suggests the following general recipe for finding the effective environmental resistance $R_{\rm eff}(\mathbf{r}_0)$, valid for an island of an arbitrary shape (not necessarily quasi-one-dimensional): Suppose that the current I_0 is injected into the island at the point $\mathbf{r}_0$ and collected homogeneously from the entire volume V of the sample (see Fig. 8):

$$\tilde{J}(\mathbf{r}) = I_0\,(\delta(\mathbf{r}-\mathbf{r}_0) - 1/V). \quad (82)$$

For such a setup one should find the distribution of the potential $V(\mathbf{r})$ from the continuity equation $\sigma\Delta V = \tilde{J}$ and then calculate the power W emitted in the external circuit.

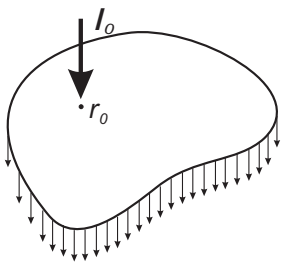

FIG. 8. The distribution of the injected and collected currents, corresponding to the effective spreading resistance $R_{\rm eff}(\mathbf{r}_0)$.

Finally, the effective resistance is

$$R_{\rm eff}(\mathbf{r}_0) = W/I_0^2 = -\left(1/I_0^2\right)\int \tilde{J}(\mathbf{r})V(\mathbf{r})d\mathbf{r}. \qquad (83)$$

If the contact has its own ohmic resistance $R_{\rm con}$, then it is connected in series with $R_{\rm eff}(\mathbf{r}_0)$, so that the total effective resistance in Eq. (75) takes a form $R_{\rm eff}^{\rm (tot)} = R_{\rm eff}(\mathbf{r}_0) + R_{\rm con}$.

D. Illustration: a toy model

A physical interpretation of the strong suppression of Z factors by the power-law factors depending on temperature [see Eqs. (75) and (79)] can be proposed in terms of a single-mode toy model. Suppose that the system can be qualitatively described by a single configurational coordinate X which somehow reflects the proliferation of the screening cloud in the wire. The corresponding "adiabatic potential" $U_{\rm eff}(X)$, shown in Fig. 9, diverges at a certain point X_0 at which an extra particle is injected into the system. The divergency is, however, weak enough that the singularity does not dominate the tunneling action; that remains finite. The system tunnels under the barrier with the optimal energy $\varepsilon_{\rm tun}$, which is determined by the standard condition $\tau_{\rm tun}(\varepsilon_{\rm tun}) = 1/2T$ (see Ref. [15]). The Coulomb ZBA regime ($T \gg T_{c2}$) is characterized by $\varepsilon_{\rm tun} \gg E_C$. For $T \ll T_{c2}$ the under-barrier action is dominated by the large-X tail of the adiabatic potential, which decreases, as X^{-2}, so that the action logarithmically diverges. This divergency is cut at the turning point $X \sim X_c(\varepsilon_{\rm tun})$, which is controlled either by finite temperature (if $E_{\rm act} \ll \varepsilon_{\rm tun} \ll E_C$) or by finite $E_{\rm act}$ (if $\varepsilon_{\rm tun} - E_{\rm act} \ll E_{\rm act}$). Note that, although this simplistic model is useful for qualitative understanding of the nature of different regimes of tunneling, it is too crude to reproduce any quantitative results.

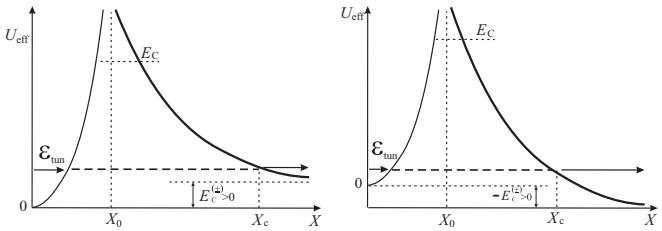

FIG. 9. The adiabatic potential for the single-mode toy model. Left panel: $\tilde{E}_C^{(\pm)} > 0$, right panel: $\tilde{E}_C^{(\pm)} < 0$. The effective configurational coordinate X tunnels under the barrier at optimal T-dependent energy $\varepsilon_{\rm tun}$. At point $X = X_0$ an extra particle is injected into the wire.

VII. THE LINEAR CONDUCTANCE

The results of the preceding Secs. III–VI depend on the values of initial number of extra electrons in the wire N and the initial energy of the tunneling particle ε through the combination $\tilde{E}_C^{(\pm)}(q_{\rm in},\varepsilon)$ [see Eqs. (36) and (26)]. For high bias voltages the probability distribution $P^{(\pm)}(N,\varepsilon)$ for N and ε is driven out of equilibrium, and should be found from the kinetic equations. This issue we will discuss in detail in a separate publication. In the present paper we concentrate on the linear case, when the equilibrium is not destroyed, and

$$P^{(\pm)}(N,\varepsilon) \propto \exp\left\{-\frac{E_N}{T}\right\} f_F(\pm\varepsilon), \qquad (84)$$

$$E_N = E_C(N^2 + 2qN), \quad f_F(\varepsilon) = \frac{1}{1+e^{\varepsilon/T}}. \qquad (85)$$

In the linear case the characteristic value of ε is of the order of T.

In the low temperature domain (for $T \ll E_{\rm act}$) the probability to have $N \neq 0$ is exponentially small: $P_N \approx \delta_{N,0}$. However, one should be accurate at this point, because the subsequent tunneling events are not independent; it would be wrong just to calculate the current, say, through the contact A, using the equilibrium distribution function P_N. The correlations should be properly taken into account.

Indeed, when an electron (a hole) tunnels into the wire, the number N changes: $N \to N+1$ ($N \to N-1$). This change leads to an exponentially strong enhancement of a subsequent tunneling of a hole (an electron). As a result, the process of the charge transfer through the system consists of correlated pairs of events. Such a pair may be tunneling of an electron almost immediately followed by tunneling of a hole, or vice versa. In other words, at low temperatures $T \ll E_{\rm act}$ the current consists of two independent contributions: The contribution of electors is due to processes in which an electron first tunnels to the system and then leaves it, while no other tunneling events have a chance to take place between these two. Similarly, the contribution of holes involves tunneling of a hole to the system with subsequent tunneling away, again, without any interference with possible other events. The corresponding currents are

$$I^{(+)} = \frac{1}{2}g_A Z^{(+)}(x_A, N=0)V \times \frac{g_B Z^{(+)}(x_B, N=1)}{g_A Z^{(+)}(x_A, N=1) + g_B Z^{(+)}(x_B, N=1)}, \qquad (86)$$

$$I^{(-)} = \frac{1}{2}g_B Z^{(-)}(x_B, N=0)V \times \frac{g_B Z^{(-)}(x_B, N=-1)}{g_A Z^{(-)}(x_A, N=-1) + g_B Z^{(-)}(x_B, N=-1)}, \qquad (87)$$

where

$$Z^{(\pm)}(x,N) = \int_0^\infty f_F(\pm\varepsilon)d\varepsilon Z^{(\pm)}(x,N,\pm\varepsilon) \qquad (88)$$

within the exponential accuracy $Z^{(\pm)}(x,N) \sim Z^{(\pm)}(x,N,0)$. The factors in the lower lines of Eqs. (86) and (87) are nothing but the branching ratios: the probabilities for an electron (hole)

that was injected through the contact $A(B)$ to escape the wire through the contact $B(A)$. Note that only these sequences of events end up in a net current through the system. The factors $1/2$ in the first lines of Eqs. (86) and (87) take into account the fact that only electrons (holes) participate in each contribution $I^{(+)}$ or $I^{(-)}$ and, effectively, only one half of the total density of states is involved in each case.

In the equilibrium state the detailed balance should be established, therefore

$$P_N Z^{(\pm)}(x,N) = P_{N\pm1} Z^{(\mp)}(x,N \pm 1). \tag{89}$$

Substituting the relation (89) into (86) and (87), we finally get the results (9) and (10), where, for brevity, we have written $Z^{(\pm)}(x)$ instead of $Z^{(\pm)}(x,\varepsilon = 0,N = 0)$.

It should be stressed that the results (9) and (10) are rigorous at $T \ll E_{\rm act}$: In principle, it would allow for finding the correct preexponential factor for the current, provided that valid preexponential factors were known for $Z^{\pm}$ functions. In this paper, however, we have calculated $Z^{\pm}$ only with an exponential accuracy, so that the potential abilities of the general results (9) and (10) were not fully utilized, and the preexponential factor of the current was not explicitly obtained.

It is also interesting to note that the results (9) and (10) remain valid in the high temperature case as well, when there is no correlation between subsequent tunneling events, and the formulas (86) and (87) do not seem to work. However, effectively, they still do work, because at $T \gg E_{\rm act}$ the factor $Z^{(\pm)}(x,N)$ ceases to depend on N and on the sort of particle (electron or hole). Thus, the formulas (9) and (10) are exact in both limiting cases $T \ll E_{\rm act}$ and $T \gg E_{\rm act}$. In the intermediate case $T \sim E_{\rm act}$ we were not able to prove them rigorously, but we expect them to be qualitatively correct.

VIII. CONCLUSION

We have developed in this paper a detailed theory of charge transport in the presence of Coulomb blockade for a quasi-one-dimensional extended conductor poorly connected to Ohmic contacts. Such a theory is expected to be applicable, for example, for the description of transport measurements on multiwall carbon nanotubes, see experimental paper [16] and review [17].

Our theory provides results both for the lowest temperatures where Arrhenius dependence of the Coulomb blockaded conductance is predicted and for the intermediate temperature range where major $T-$ dependence is of power-law type. We have shown that an accurate treatment of the zero mode, with proper accounting for the initial condition, leads to the low-temperature Arrhenius dependence of the Coulomb blockaded conductance, with the correct activation energy $E_{\rm act} = E_C(1 - 2|q|)$, as required by the thermodynamics. Besides this activational exponent, there is an additional T-independent exponent, due to the tunneling penetration of the barrier in the configurational space.

An intermediate temperature range, where the T dependence is a power law, is especially broad in the nearly-resonant case when effective charging energy is strongly suppressed. Here, in addition to the previously studied [4] cotunneling scenario, we provided a detailed calculation of power-law exponents which describe the effect of "shakeup" of the low-energy environmental modes, as in the phenomenological theory of environmental Coulomb anomaly. The role of effective environment here is played by the inhomogeneous modes of the charge distribution in the wire. We derive explicit expressions for the corresponding effective environmental resistance $R_{\rm eff}$ and predict its strong dependence on the position of the contacts: $R_{\rm eff} = R/3$ for the contact at the end of the wire, and $R_{\rm eff} = R/12$ for the contact at the center. The result of competition between the activation and cotunneling scenarios also strongly depends on the position of the contacts. If the contacts are moved from the ends of the wire towards its center, the crossover to the cotunneling mechanism is shifted to higher temperatures.

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