Reconnection of quantized vortex filaments and the Kolmogorov spectrum

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The energy spectrum of the three-dimensional (3D) velocity field, induced by collapsing vortex filaments, is studied. One of the aims of this work is to clarify the appearance of the Kolmogorov-type energy spectrum $E(k) \propto k^{-5/3}$, observed in many numerical works on discrete vortex tubes and quantized vortex filaments in quantum fluids at zero temperature. Usually when explaining classical turbulent properties of quantum turbulence the model of vortex bundles is used. This model is necessary to mimic the vortex stretching, which is responsible for the energy transfer in classical turbulence. In my consideration I do not appeal to the possible "bundle arrangement" but instead explore the alternative idea that the turbulent spectra appear from a singular solution, which describes the collapsing line at moments of reconnection. One more aim is related to an important and intensively discussed topic—the role of hydrodynamic collapse in the formation of turbulent spectra. I demonstrated that the specific vortex filament configuration generated the spectrum $E(k)$ close to the Kolmogorov dependence and discussed the reason for this as well as the reason for deviation. I also discuss the obtained results from the points of view of both classical and quantum turbulence.

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I. SCIENTIFIC BACKGROUND AND MOTIVATIONS

I discuss the possibility of realization of the Kolmogorov-type energy spectrum $E(k) \propto k^{-5/3}$ of the three-dimensional (3D) velocity field, produced by the quantized vortex filaments, collapsing towards reconnection.

The first motivation of this work is related to the problem of modeling classical turbulence with a set of chaotic vortex tubes. This idea has been discussed for quite a long time.(For details, see, e.g., Refs. [1–3]). In classical fluids thin vortex tubes do not exist because they spread due to viscosity, so the concept of vortex filaments should just be considered as a model. Quantum fluids, where the vortex filaments are real objects, give an excellent opportunity for studying the question of whether the dynamics of a set of vortex lines is able to reproduce (at least partially) the properties of real hydrodynamic turbulence. This case is usually referred to as a quasiclassic behavior of quantum turbulence. Turbulent motion of superfluid helium had been studied for a long time (see, e.g., Ref. [4] and review articles [5,6]). The modern stage of study of turbulent motion of superfluid helium started with the work by Maurer and Tabeling [7], who observed the Kolmogorov spectrum $E(k) \propto k^{-5/3}$ in flowing helium below T_λ, and then in works of their successors (see, e.g., Refs. [8,9] and also the review [10]).

The most common view on the quasiclassical turbulence is the model of vortex bundles. The point is that the quantized vortices have a fixed core radius, so they do not possess the very important property of classical turbulence—stretching of vortex tubes with a decrease in core size. The latter is responsible for the turbulent energy cascade from large scales to the small scales. Collections of near-parallel quantized vortices (vortex bundles) do possess this property, so the idea that the quasiclassical turbulence in quantum fluids is realized via a set of vortex bundles of different sizes and intensities (number of threads) seems quite natural. Another argument in favor of the bundle structure is related to the distribution of energy in the space of scales. In the classic turbulence the most energy is concentrated on the large scales, far exceeding the viscous boundary of the inertial interval. By analogy, the energy of quasiclassic turbulence should be concentrated on scales $\gg\delta$ (here δ is the intervortex space of the order of $\mathcal{L}^{-1/2}$, where $\mathcal{L}$ is the vortex line density), implying that there must be some tendency towards bunching. Indeed, in a chaotic vortex tangle the energy is proportional to the vortex line density $\mathcal{L}$ ($E \propto \mathcal{L}$) but in the quasiclassic case it should grow faster (probably $E \propto \mathcal{L}^{4/3}$; see Ref. [3], Section 9.3).

Conception of the bundle structure is justified for high temperatures, where the coupling with normal component is strong and bundles of quantized vortices are frozen into the normal component eddies. The situation changes for very low temperatures (which I consider further), where the normal component is absent and there are no obvious reasons for the bundle formation. There exist a number of attempts (mainly numerical; analytical evidences are lacking) to demonstrate the existence of a bundle, but the corresponding efforts are preliminary (see, e.g., papers [11,12] and references therein). On the other hand, for low temperature, the regular vortex bundles, even if they are prepared artificially, they are extremely unstable (see, e.g., Refs. [3,13,14]). They can be easily destroyed by reconnections either between the neighboring threads or in collisions with the other bundles. As a result, the regular structure "melted" and vortex filaments are randomized.

It should be realized that the conception of the bundle structure is the secondary one. It originated from the idea of quasiclassic behavior of quantum turbulence. In turn the latter idea is based on several pieces of evidence, such as the decay of the vortex tangle observed in some experiments (see, e.g., papers [15–17] and also the reviews [18–20]), numerical results on the energy spectra, and also some peculiarities of flow behind of obstacles, both stationary and moving (oscillating).

Among various arguments supporting the idea of quasiclassic behavior of quantum turbulence, the strongest is the

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k dependence of the spectra of energy $E(k)$ obtained in numerical simulations. There are several works that demonstrate a dependence of $E(k)$ close to the Kolmogorov law $E(k) \propto k^{-5/3}$. These are works based on both the vortex filament method [11,21–23] and the Gross-Pitaevskii equation [24–26]. At first sight this fact demonstrates the connection with classical turbulence and tells about nonuniform distribution of energy with larger energy for small k (large scales). The vortex bundle scenario seems to follow naturally from this fact. However, the following argument must be taken into account: The modeling of the classical turbulence with a set of chaotic quantized vortices is undoubtedly an outstanding goal, which can be considered as a breakthrough in the tantalizing problem of turbulence, but it requires an enhanced level of scientific rigor: All arguments and evidences of any hypothesis should be carefully scrutinized. In particular, there should be a possibility of alternative interpretation of experimental and numerical facts. For instance, the bunching of filaments is not the only way to accumulate energy on a large scale. This can be also realized by the presence of very large vortex loops. A more advanced variant is a set of polarized vortex loops. At the stage of the putting forward of various hypotheses, this proposal is not any worse than the vortex bundle conception. Similarly, the bundle structure is not a unique variant to generate the Kolmogorov-type spectrum; moreover there are no relevant calculations, just an appeal to the analogy with the classical turbulence. Therefore, it quite tempting to find a mechanism for the appearance of the Kolmogorov-type spectrum which is directly based on the vortex lines dynamics, and I offer the collapsing vortex filaments as a candidate for this purpose.

The second motivation is related to another important and intensively discussed topic: the role of hydrodynamic collapse in the formation of turbulent spectra. The striking examples of such type of spectra are the Phillips spectrum for water-wind waves, created by white caps (wedges of water surface) or the Kadomtsev-Petviashvili spectrum for acoustic turbulence created by shocks. In works [27,28] the authors studied the collapse of the vorticity field, characterized by explosive grow of vorticity field due to stretching of vortex tubes. They also reported about velocity spectra close to the Kolmogorov law. In the case of quantum fluids, vortices do not change their inner structure and the singularity arises due to the approach of interacting vortex filaments. The result of this approach is appearance of a very acute kinks, and the energy of interaction between closely located parts can essentially exceed contributions from smooth elements of lines.

Finally, the third motivation is that the question of energy spectra induced by various configurations of vortex filaments is interesting. Indeed, moving chaotically vortex filaments explicitly determine the entire motion of the fluid and in particular the energy spectrum. The case of the reconnecting lines is of a special interest, because the reconnection events (rather frequent) are very important ingredient of dynamics of vortex lines. As a by-product, the spectra induced by reconnecting lines was discussed both in quantum and classical cases (see, e.g., [23,28,29]), but as numerical results without revelation of the principle reasons for the appearance of the Kolmogorov (power-like) spectra.

In the work I introduce the general method for calculation of the energy spectrum via the vortex line configuration, then I choose the analytic relation for the shape of kink, and conduct the mixed analytic and numerical evaluation of $E(k)$. I demonstrate that the spectrum $E(k)$ is very close to the Kolmogorov dependence $\propto k^{-5/3}$ and discuss the reason for this as well as the reason for deviation.

II. CALCULATION OF SPECTRUM

The formal relation, allowing the calculation of $E(\mathbf{k}) = \rho_s \mathbf{v_k v_{-k}}/2 = \rho_s \omega_\mathbf{k} \omega_{-\mathbf{k}}/2k^2$ (this follows, e.g., from the observation that in $\mathbf{k}$ space $\omega_\mathbf{k} = \mathbf{k} \times \mathbf{v_k}$, and $\mathbf{kv_k} = 0$ due to incompressibility) via the vortex line configuration $\{\mathbf{s}(\xi)\}$, can be written as follows (see [30,31]):

$$E(\mathbf{k}) = \frac{\rho_s \kappa^2}{16\pi^3 k^2} \oint \oint \mathbf{s}'(\xi_1)\mathbf{s}'(\xi_2) d\xi_1 d\xi_2 e^{i\mathbf{k}(\mathbf{s}(\xi_1)-\mathbf{s}(\xi_2))}. \quad (1)$$

Here ρ_s is the superfluid density, κ is the quantum of circulation equal to $h/m_{He} = 9.97 \times 10^{-4}$ ^{2}cm/s, where h is Planck's constant and m_{He} is the mass of the helium atom. The vortex line configuration $\mathbf{s}(\xi) = \bigcup \mathbf{s}_i(\xi_i)$ is the union of lines $\mathbf{s}_i(\xi_i)$, where $\mathbf{s}_i(\xi_i)$ describes the i-vortex line position parameterized by the label variable ξ_i, $\mathbf{s}_i'(\xi_i)$ denotes the derivative with respect to variable ξ_i (the tangent vector), and $\int_C = \int_C \sum_j$. In the isotropic case, the spectral density depends on the absolute value of the wave number k. Integration over the solid angle leads to the formula (see Refs. [31,32])

$$E(k) = \frac{\rho_s \kappa^2}{(2\pi)^2} \oint \oint \mathbf{s}'(\xi_1)\mathbf{s}'(\xi_2) d\xi_1 d\xi_2 \frac{\sin(k|\mathbf{s}(\xi_1) - \mathbf{s}(\xi_2)|)}{k|\mathbf{s}(\xi_1) - \mathbf{s}(\xi_2)|}. \quad (2)$$

For anisotropic situations, formula (2) is understood as the angular average. Thus, for calculation of the energy spectrum $E(k)$ of the 3D velocity field, induced by the collapsing vortex filament, I need to know the exact configuration $\{\mathbf{s}(\xi)\}$ of vortex lines.

III. SHAPE OF KINK

Despite the huge number of works devoted to the dynamics of collapsing lines both in classic and quantum fluids [33,34] (this list is far from full) the exact solution $\mathbf{s}(\xi)$ for the shape of curves has not yet been obtained. The main results are obtained by different approaches, combining analytical and numerical methods, such as the vortex filament method (in both the local induction approximation and the Biot-Savart law), as well as the full 3D studies of Navier-Stocks equations or the nonlinear Schrödinger equation for vortices in the Bose-Einstein condensate.

Qualitatively, the results of these investigations are quite similar and can be described as follows. Due to long-range interaction in the Biot-Savart integral, the initially arbitrarily oriented vortices, when they approach each other, start by reorienting their close segments so as to bring them into an antiparallel position. Further, cusps may appear on the approaching segments of two vortex lines. The curvature of these cusps may be so large that the self-induced velocity of each perturbation overcomes the repulsion from the adjoining vortex line. Further the cusps grow and approach each other closer; this increases their curvature and, correspondingly, their self-induced velocities, and this process is repeated faster

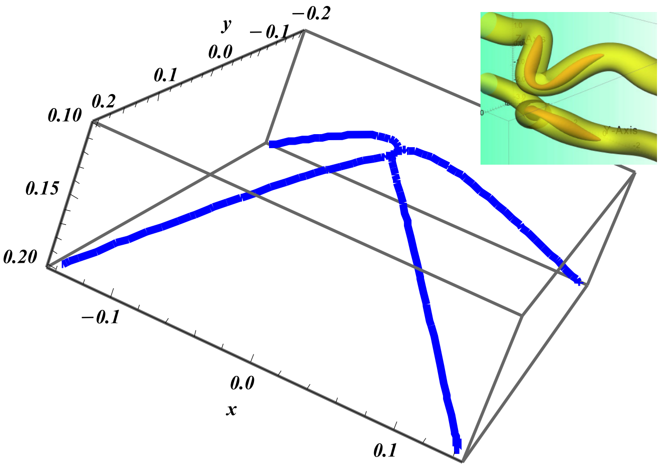

FIG. 1. (Color online) The touching quasihyperbolae describing the collapsing lines [see Eq. (3)] obtained in Ref. [34]. In the inset I set (as an example) the kinks on the antiparallel collapsing vortex tubes obtained in numerical simulation [35].

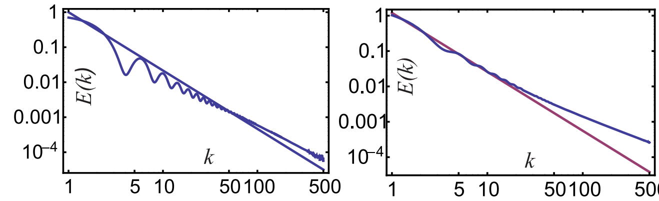

FIG. 2. (Color online) Right: The spectrum $E(k)$, obtained numerically on the base formula (2). The straight line has a slope $-5/3$. Left: The same spectrum obtained on the basis of procedure described in Sec. V [Eq. (4)].

and faster. It is important that this process grows explosively, since the distance between the two perturbed segments, Δ, decreases according to the relation $\Delta \sim (t^* - t)^{1/2}$, where t^* is some quantity depending on the relevant parameters and initial conditions. Thus, in a finite time the vortex lines collapse. Asymptotic lines are two hyperbolic curves lying on opposite sides of the pyramid (see, e.g., [36,37]). A little different scenario without the pyramid structure was observed in Ref. [38]. In a recent study [34], it was shown that the curves are not exact hyperbolas, but slightly different lines (the authors call these curves quasihyperbolae) of type $h(\xi) = \sqrt{a^2\xi^2/(a^2+\xi^2) + a^2 + \xi^2}$, and that they lie not in the planes of the pyramid sides but on the curved surfaces, bent inwards. In the moments just before the collapse, when the vortex cores nearly touch each other, the very acute kink appears. This curves may be written in parametric form (cf. formula (16) of Ref. [34]):

$$\mathbf{s}_{1,2}(\xi) = (\pm[h(\xi) - c], \pm\xi, \{h[h(\xi) - b]\}). \qquad (3)$$

The described configuration is shown in Fig. 1. The signs are chosen so that $\mathbf{s}_1'(0)\mathbf{s}_2'(0) = -1$ (the vortices are antiparallel). Quantity a is of the order of the curvature radius on the tip of the kinks; quantity b (related to a; see Ref. [34]) is responsible for bending the surfaces on which the quasihyperbolae lie. Quantity c is also of the order of a and is responsible for closeness of the filaments. All three quantities are smaller than the intervortex space $\delta = \mathcal{L}^{-1/2}$ (where $\mathcal{L}$ is the vortex line density). This vision is consistent with the results of numerous numerical works, studying the collapse of vortex lines (see, e.g., Refs. [28,35] and references therein; the decisive picture obtained in Ref. [35] is shown in the inset of Fig. 1).

IV. NUMERICAL RESULTS

In the left graphic of Fig. 2 I presented the results of numerical calculation of spectrum $E(k)$ on the base formula (2) (without prefactor before integral) using a configuration $\{\mathbf{s}(\xi)\}$ of vortex lines described by Eq. (3). I calculated only the interaction energy between the approaching parts of different lines; the self-energy in vicinity of the point of contact vanishes, since lines are antiparallel and they just annihilate. The parameters $a = 0.1, b = 0.09, c = 0.1$ (the case $a \approx c$ corresponds to nearly touching curves) had been chosen. Integration over ξ_1,ξ_2 was performed from zero to 1. The upper limit 1 corresponds to the size of the kink, which is the order of intervortex space δ (see inset of Fig. 1). Obviously, the choice of the upper limit larger than the intervortex space δ requires a knowledge of the large-scale arrangement of the tangle structure. The latter is a central question of the quantum turbulence problem and represents the tremendous and outstanding problem. Thus, my results are not applicable for the real vortex tangle in the small-k limit. But in the interval of wave numbers k between $1 \div 50$ (which corresponds to curvature of the kink and its inverse size) the slope of $E(k)$ is indeed close to $-5/3$. I discuss the origin of this in the following paragraph.

V. ANALYTIC CONSIDERATION

Because of the rapidly oscillating function, the evaluation of integral (2) is difficult, even numerically. In addition, numerical results obscure underlying physics; therefore, I intend to perform an analytical study, at least as far as possible. The integral (2) can be approximately evaluated for large k using the method of asymptotic expansion [39]. When k is large the function $\sin(k\,|\mathbf{s}(\xi_1) - \mathbf{s}(\xi_2)|)$ is a rapidly varying function; therefore, the main contribution into integral comes from points of minimal value of the separation function between points of the curves $D(\xi_1,\xi_2) = |\mathbf{s}(\xi_1) - \mathbf{s}(\xi_2)|$. This is enhanced by the fact that the distance is included in the denominator in the integrand of Eq. (2). Thus, the behavior of the phase function $D(\xi_1,\xi_2)$ near minimum is crucial for value of the integral and for its k dependence. Let us study the phase function $D(\xi_1,\xi_2)$ for the vortex configuration described by Eq. (3) just before collapse when $c \approx a$. It is convenient to introduce variables $\rho = \xi_1 - \xi_2$ and $R = (\xi_1 + \xi_2)/2$ and recast the double integral $\int_C \int_C d\xi_1 d\xi_2$ as multiple integral $\int dR \int d\rho$ in the domain bounded by lines $\rho = 2R$ and $\rho = -2R$. The upper limit for R is not essential, since the integral gains the main contribution from the vicinity of point $R = 0$. Let us consider the behavior of function $D(\rho,R)$. It is depicted in Fig. 3 (in the extended domain) at the beginning of coordinates $\rho = 0, R = 0$ function $D(\rho,R) = 0$. The important feature of function $D(\rho,R)$ is its behavior of it near points $\rho = 0$ (for different R), which is the median

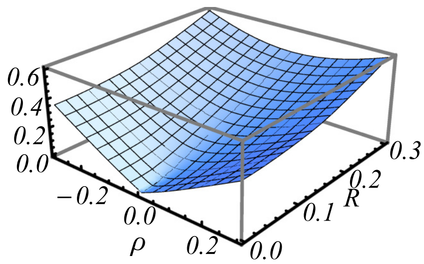

FIG. 3. (Color online) Quantity, the phase function in coordinates ρ, R.

part of domain arising from equidistant ($\xi_1 = \xi_2$) points of the touching vortex filaments. For fixed R (perpendicular to the median direction) the functions $D(\rho, R = \text{const.})$ are approximated by pieces of parabolas $\propto \rho^2$, then transferred into linear function $\propto |\rho|$ for $\rho \gtrsim a$ with the same slope for all R. Thus, all points of the median are points of the local minimum, and $\partial D/\partial\rho|_{\rho=0} = 0$ for all R. Another important feature of the phase function $D(\rho, R)$ is its dependence on R along the median $\rho = 0$.

Function $D(0,R)$ is depicted in logarithmic coordinates in Fig. 4. It is seen that it behaves as $\propto R^2$ and then passes into $\propto R^1$ for $R \sim a$ (crossover region). Thus, I have a complicated case, when point ($R = 0, \rho = 0$) is simultaneously both a corner of domain restricted by curves $\rho = 2R$ and $\rho = -2R$, and a stationary point (minimum), i.e., $\nabla D(\rho, R) = 0$. To move further I pass to polar coordinates R, θ, then by integrating over angle θ I obtain asymptotic expansion over $1/k$. The leading term has the form

$$E(k) = \int_0 dR \sqrt{\frac{\pi}{k\partial^2 D/\partial\rho^2|_{\rho=0}}} \frac{\sin\left(kD(\rho,R)\right)}{kD(\rho,R)}\Bigg|_{\rho=0}. \quad (4)$$

I used here that integration over θ is identical to integration over $d\rho$, namely $d\rho = Rd\theta$, and the median curve $\rho = 0$ is the line where function $D(\rho, R = \text{const.})$ has a local minimum; $\partial D/\partial\rho|_{\rho=0} = 0$. Therefore, the integration over θ can be carried out by the use of the stationary phase method, which gives Eq. (4). Calculating the integral in the vicinity of the stationary point I neglected the slowly changing function $\mathbf{s}'(\xi_1)\mathbf{s}'(\xi_2)$, putting it equal to -1 (recall that the lines are antiparallel). Additionally, I take $\sin(kD(\rho,R))$ as an imaginary part of $\exp(ikD(\rho,R))$.Thus, I reduced the whole problem to an evaluation of the one-dimensional (1D) integral. In the right graphic of Fig. 2 I present $E(k)$, calculated on the basis of formula (4). First, please note that spectrum calculated with

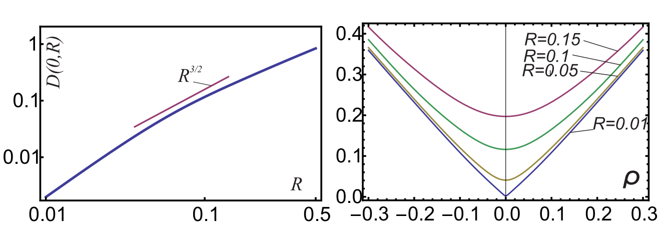

FIG. 4. (Color online) Left: Function $D(0,R)$, distance along the median in the log-log coordinates. Segment of the straight line has a slope 3/2. Right: The slices of the phase function $D(\rho,R)$ for different R.

the use of formula (4) is very close to the spectrum calculated on basis (2); this justifies the approximated procedure, as described above. Second, and more important, is that again in the interval of wave numbers k between $1 \div 50$ the slope of $E(k)$ is close to $-5/3$. To understand an appearance of the $\approx k^{-5/3}$ dependence I appeal to the so-called Erdelyi lemma [40], which says that the integral $\int_0 x^{\beta-1} f(x) e^{i\lambda x^\alpha} dx$ with a smooth enough function $f(x)$ has an expansion in asymptotic series as $\sum_m a_m \lambda^{-\frac{m+\beta}{\alpha}}$ with the leading term $\lambda^{-\frac{\beta}{\alpha}}$. In particular, this means that if I took the collapsing filaments not to be quasihyperbolas but as pure power-like functions $\mathbf{s}_{1,2}(\xi) = (\xi, \pm\xi^{3/2}, 0)$ (3/2 parabolas) and implemented the procedure described above, I would find that the spectrum had an exact $E(k) \propto k^{-5/3}$ form. Coming back to solution (3) and Fig. (4), I see that function $D(0,R)$ is not 3/2 parabola but it is more sophisticated function which behaves as $\propto R^2$ and then passes into $\propto R^1$ in the crossover region Δ, covering from 1 to 1.5 decades near quantity a. Therefore, in the crossover region where the quantity $D(0,R)$ is close to $R^{3/2}$, it should be expected that $E(k)$ is close to the Kolmogorov dependence $\propto k^{-5/3}$ for the wave numbers k of the order $2\pi/\Delta$, which indeed takes place. The crossover region lies from the scale of the bend and the scale where branches of hyperbolas tend to become straight lines. Actually it is close to the size of the bridging kink on the curves and is of the order of intervortex space $\delta = \mathcal{L}^{-1/2}$ (see the inset in Fig. 1).

VI. CONCLUSION AND DISCUSSION

Thus, I evaluated the energy spectrum of the 3D velocity field induced by collapsing vortex filaments. I obtained it both numerically and analytically. The latter allowed me to understand the reason the Kolmogorov-like spectrum appeared from the shape of collapsing vortex filaments.

Coming back to the aims of the work stated in the introduction I can suggest that the spectrum $E(k)$ close to the Kolmogorov dependence $\propto k^{-5/3}$, which was observed in numerical simulations on the dynamics of quantized vortex filaments [11,21], can appear from the reconnecting lines. The interval of wave numbers where the spectrum $E(k) \approx k^{-5/3}$ is observed is regulated by the curvature of the kink and intervortex space δ. In reality this spectrum covers a maximum $1 \div 1.5$ decades around $k \approx 2\pi/\delta$. It should be stressed, however, that in the key numerical works [11,21] the ranges for wave number are also of the order of one decade around $k \approx 2\pi/\delta$. Unfortunately, because of a lack of exact analytic solution for the configuration $\{\mathbf{s}(\xi)\}$ of the collapsing vortex filaments, the quantity $E(k)$ is approximate and relies on the asymptotic solution (3). On the other hand (as seen from the proposed analytical consideration), the spectrum depends on a few features of the collapsing lines, such as as the order of tangency and the crossover to a smooth straight line. The exact shape of kink, however, can affect mainly the region of large wave numbers k. As for the small wave numbers k (smaller than $2\pi/\delta$), the spectrum $E(k)$ should be extracted by integrating the base formula (2) over the whole vortex configuration, including the very remote elements of vortex filaments ($|\mathbf{s}(\xi_1) - \mathbf{s}(\xi_2)| \gg \delta$). Clearly, this can be done, provided that I know (at least in a statistical sense) the large-scale arrangement of the vortex tangle structure. This question is central to the quantum turbulence problem and is a tremendous and outstanding problem. Thus, my approach is not suitable to investigate the question of the form and origin of the energy spectrum of quantum turbulence in the limit of small wave numbers.

Another, more delicate question, touched on in the introduction, concerns the role of the dynamics of discrete vortices in the physics of turbulence. My results support the point of view on the role of collapse in the formation of turbulent spectra conducted in Ref. [27]. On the other hand, besides the interval of wave numbers (discussed above), other unclear questions remain. In the Kolmogorov scenario the spectrum $E(k) \propto k^{-5/3}$ was the consequence of a k-independent energy cascade P_k in the k space. In the scheme based on collapsing lines the energy cascade does not appear at all (at least in an explicit form). An assumption can be put forward that the collapse of lines, which delivers energy into a tiny region near the point of collapse (then this energy is burned in the process of full reconnection), plays the role of the vortex stretching in the transfer of energy to small scales. There are possible other scenarios related to full reconnection and formation of larger or smaller vortex loops (recombination processes), which implies redistribution of the energy between various scales, inducing the direct and inverse cascades; see Refs. [41,42]).

Of course, there are many other issues relating to higher structure functions or to the number of reconnections necessary to maintain a uniform spectrum, although the numerous number of reconnections (which is $\propto \kappa \mathcal{L}^{5/2}$; see, e.g., Refs. [43–46]) can lead to the uniform and isotropic picture. These issues, however, are outside the framework of the presented work and will be the object of future studies. As concluding remark I note that evaluation of the spectrum from reconnecting lines is performed on the basis of elegant mathematical theory, which allows understanding of the origin of the Kolmogorov spectrum grounded on the shape of collapsing curves.

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