Universal short-time response and formation of correlations after quantum quenches

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(Received 24 April 2014; revised manuscript received 21 July 2014; published 11 August 2014)

The short-time evolutions of two distinct systems, the pump and probe experiments with a semiconductor and the sudden quench of cold atoms in an optical lattice, are found to be described by the same universal response function. This analytic formula at short time scales is derived from the quantum kinetic-theory approach observing that correlations need time to form. The demand of density conservation leads to a reduction of the relaxation time by a factor of 4 in quench setups. The influence of the finite-trapping potential is derived and discussed along with Singwi-Sjølander local-field corrections including the proof of sum rules.

DOI: 10.1103/PhysRevB.90.075303 PACS number(s): 78.47.J−, 42.65.Re, 82.53.Mj, 71.45.Gm

I. INTRODUCTION

The experimental progress with cold atoms triggered by Bose-Einstein condensation has led to an enormous undertaking to understand the time-dependent formation of correlations. Besides pump and probe experiments in semiconductors like GaAs [1] or InP [2], where the formation of collective modes and quasiparticles has been observed with the help of femtosecond spectroscopy, it is now possible to measure the time-dependent occupation of Hubbard-like setups and to observe the formation of correlations [3]. The ultrafast excitations in semiconductors, clusters, or plasmas by ultrashort laser pulses are characterized by long-range Coulomb interactions reflected in the time dependence of the dielectric function [1,2,4] in the terahertz regime [5]; for an overview of theoretical and experimental work see [6]. Calculating nonequilibrium Green's functions [7,8] allows one to describe the formation of collective modes [2,9], screening [7], and even exciton population inversions [10].

The thermalization of cold atoms is characterized by short-range interactions mostly described within Hubbard-like models [11]. Different kinds of quenches are applied and studied [12,13], and the importance of local conservation laws has been pointed out [14]. In regard to these conservation laws, several existing many-body approximations in the literature have been analyzed by numerical solution of Kadanoff and Baym equations [15,16]. Special preparation of cold atoms in optical lattices allows us to study the local relaxation [3,17] and to explore dissipation mechanisms [18]. At intermediate time scales a quasistationary state has been found during the thermalization process [19].

Different physical systems, the long-range Coulomb [20] and short-range Hubbard systems, can be described by a common theoretical approach leading to a unique formula to explain the formation of correlations at short time scales, as we will demonstrate. This could be of interest since, normally, the formation is treated by numerically demanding calculations solving Green's functions [7,8] or renormalization group equations [3].

What is the basic reason why such different systems show similar features? It is just the fact that correlations need time to form. In other words higher-order correlations need more time to be built up than low-order ones. Although this statement is strictly valid for only weakly correlated systems, we adopt it here and see that it leads to good results at short time scales even in the strongly correlated regime. Therefore it is suggested a conjecture that the lowest level, the mean-field approximation, is sufficient to describe the basic features of the short-time formation of correlations. We will follow a relaxation-time approximation while explicitly imposing local conservation laws where the mean fields are time dependent, which is an extension of the idea of Mermin [21,22]. In order to estimate the influence of higher-order correlations on the response, we will derive the time-dependent local-field corrections on the level of the Singwi-Sjølander approximation.

The outline of this paper is as follows. In the next section we briefly sketch the kinetic equations in the mean-field and conserving relaxation-time approximation and linearize the solution. The influence of a harmonic trap is derived exactly. In Sec. III we solve the kinetic equation for the situation of sudden quench of atomic lattices like Hubbard models, leading to an analytical formula describing the time dependence of the occupation, and compare them with the experimental and renormalization-group (RG) data. This formula brings the first important result of the paper. The influence of the trapping potential is discussed in the regime of the experiments which allows a perturbative treatment. Section IV is devoted to the response function when the system is initially uncorrelated. Here it is found that the short-time response has the same form for long-range Coulomb interactions and for short-range Hubbard interactions. This universal time dependence is the second important result of the paper, and the sum rules are discussed and the local-field corrections as an expression of higher-order correlations are derived. The time dependence of the response function for different approximations is illustrated by the movies in the Supplemental Material [23]. A summary tries to encourage the usage and further comparison with other approaches.

II. KINETIC THEORY APPROACH

The advantage of the equation of motions is that the linearization leads to a higher-order response function than used as an approximation in the kinetic equation. This means, e.g., that a mean-field equation leads to random-phase-approximation (RPA) response, which has been pointed out

in [24,25]. Therefore we will briefly present the ingredients of such a kinetic approach and restrict ourselves to the mean field as lowest order since higher-order correlations need time to form if one begins in an uncorrelated state [26,27].

We consider models with the Hamiltonian

$$H = \sum_k \epsilon_k a_k^+ a_k + \frac{1}{2}\sum_{kpq} V_q a_k^+ a_p^+ a_{p+q} a_{k-q}, \qquad (1)$$

where the energy dispersion of quasiparticles in Coulomb systems like semiconductors is given by an effective mass $\epsilon_p = p^2/2m$. As a model for the short-range lattices we consider the Hubbard model of constant hopping J and constant Coulomb repulsion $V_q = U$. For short-range lattices of length a and with hopping element J the dispersion is given by $\epsilon_p = 2J(1-\cos ap/\hbar)$ such that $1/m \approx 2Ja^2/\hbar^2$ near the band minima.

We consider the time evolution of the reduced density matrix $\langle p+\frac{1}{2}q|\delta\rho|p-\frac{1}{2}q\rangle = \delta f(p,q,t)$, which is given by linearization, $\delta[H,\rho] = [\delta H,\rho_0] + [H_0,\delta\rho]$, of the kinetic equation

$$\dot\rho + i[H,\rho] = \frac{\rho^{\rm l.e.}-\rho}{\tau} \qquad (2)$$

with respect to an external perturbation $\delta V^{\rm ext}$. The effective Hamiltonian consists of the quasiparticle energy, the external and induced mean field $\langle p+\frac{1}{2}q|\delta H|p-\frac{1}{2}q\rangle = \delta V^{\rm ext} + V_q\delta n_q$, given by the interaction potential V_q, and the density variation δn_q. As a possible confining potential we assume a harmonic trap $V^{\rm trap} = \frac{1}{2}Kx^2$, which leads to $\langle p+\frac{1}{2}q|\delta[V^{\rm trap},\rho]|p-\frac{1}{2}q\rangle = -K\partial_p\partial_q\delta f(p,q,t)$.

The kinetic equation (2) relaxes towards a local equilibrium (Fermi/Bose) distribution but with an allowed variation of the chemical potential $\langle p+\frac{q}{2}|\rho^{\rm l.e.}-\rho|p-\frac{q}{2}\rangle = \langle|\rho^{\rm l.e.}-\rho^0|\rangle - \delta f(p,q,t) = -\frac{\Delta f}{\Delta\epsilon}\delta\mu(q,t) - \delta f(p,q,t)$. Here we use the short-hand notation $\Delta f = f_0(p+\frac{q}{2}) - f_0(p-\frac{q}{2})$ and $\Delta\epsilon = \epsilon_{p+\frac{q}{2}} - \epsilon_{p-\frac{q}{2}}$. The variation of the chemical potential is determined by the density conservation $n = \sum_p f = \sum_p f^{\rm l.e.}$, leading to a relation between density variation $\delta n(q,t) = \tilde\Pi(t,\omega=0)\delta\mu(q,t)$ and the polarization in the RPA

$$\Pi(t,t') = i\sum_p \left[f_{p+\frac{q}{2}}(t') - f_{p-\frac{q}{2}}(t')\right] e^{(i\varepsilon_{p+\frac{q}{2}} - i\varepsilon_{p-\frac{q}{2}} + \frac{1}{\tau})(t'-t)},$$

$$\tilde\Pi(t,\omega) = \int d(t-t')e^{i\omega(t-t')}\Pi(t,t'). \qquad (3)$$

The linearized kinetic equation (2) therefore reads, for $\delta f_t = \delta f(p,q,t)$,

$$\dot{\delta f}_t + \frac{\delta f_t}{\tau} + i\Delta\epsilon\delta f_t = i\Delta f\delta V_t^{\rm ext} + i\Delta f V_q\delta n_t + \frac{\Delta f}{\Delta\epsilon}\frac{\delta n_t}{\tau\Pi(q,0,t)} + iK\partial_p\partial_q\delta f_t. \qquad (4)$$

The last term in the second line describes the confining harmonic trap, and the first term comes from Mermin's correction due to the density-conserving relaxation-time approximation.

Neglecting the time derivative of the homogeneous part $f_0(p,t)$ compared to $\delta f(p,q,t)$, we can solve the kinetic equation (4) considering the momentum derivatives of the last term as a perturbation to obtain

$$\delta f(p,q,t) - \delta f(p,q,0) = i\int_{t_0}^t dt' \exp\left[\left(i\Delta\varepsilon + \frac{1}{\tau}\right)(t'-t)\right] \times\left\{\Delta f(t')\left[V_q\delta n(q,t') + V_q^{\rm ext}(t')\right] + \frac{1}{i\tau\tilde\Pi(t',0)}\frac{\Delta f(t')}{\Delta\varepsilon}\delta n(q,t') + K\partial_p\partial_q\delta f(p,q,t')\right\}. \qquad (5)$$

Further evaluation is very much dependent on the physical setup and leads to different solutions as we will demonstrate now.

III. ATOMS IN A LATTICE AFTER SUDDEN QUENCH

For cold atoms occupying every other place on a lattice $f_k = [1+(-1)^k]n/2$ we have a Fourier transform to the momentum distribution,

$$f_0(p) = a\sum_{k=-N}^{N} e^{\frac{i}{\hbar}kap} f_k = na\frac{\sin(2N+1)\frac{qp}{\hbar}}{\sin\left(\frac{ap}{\hbar}\right)} \to \pi\hbar n\delta(p) = \frac{n}{2}\delta_p, \qquad (6)$$

for a large total number of atoms N and lattice spacing a. We can now Laplace transform the time $t\to s$ in the kinetic equation (4) or (5) to get

$$\delta f_s = \frac{\delta f_0}{s+i\Delta\epsilon+\frac{1}{\tau}} + \frac{in}{2}\left(\frac{\delta_{p+q/2}}{s-ib+\frac{1}{\tau}} - \frac{\delta_{p-q/2}}{s+ib+\frac{1}{\tau}}\right) \times(\delta V^{\rm ext} + V_q\delta n_s) + \frac{1}{2\tau}\left(\frac{\delta_{p+q/2}}{s-ib+\frac{1}{\tau}} + \frac{\delta_{p-q/2}}{s+ib+\frac{1}{\tau}}\right)\delta n_s + \frac{iK}{s+i\Delta\epsilon+\frac{1}{\tau}}\partial_p\partial_q\delta f_s, \qquad (7)$$

where we introduced $\Delta\epsilon|_{p=\pm q/2} = \pm 4J\sin^2\frac{aq}{2\hbar} = \pm b$. The initial time disturbance of the distribution δf_0 is determined according to the different physical preparations.

In the case of a sudden quench the interaction is switched on suddenly, and no external perturbation will be assumed, $\delta V^{\rm ext} = 0$. Let's consider the time evolution of an empty place in a lattice where every other place was populated initially. The density $n_t = \frac{n}{2} + \delta n_t$ starts with $n_0 = 0$, which means $\delta n_0 = -n/2$ is the initial condition. If we first look at the quench without interaction ($V = 0$), we can solve (4):

$$\delta f_t = \delta f(0)e^{-i\Delta\epsilon t - \frac{t}{\tau}}, \qquad (8)$$

with a choice of $\delta f(0) = -n/2$ so that the density

$$\delta n_t = \sum_p \delta f_t = -\frac{n}{2}J_0(\sqrt{4Jbt})e^{-\frac{t}{\tau}} \qquad (9)$$

starts with $\delta n_0 = -n/2$ as desired since the Bessel function $J_0(0) = 1$. Note that $\sum_p = a/2\pi\int_0^{2\pi/a}dp$, according to the finite band. If we use the parabolic approximation near the band minimum instead, we will obtain $\mathrm{Re}[\delta n_t] = -n\sin(x)/2x$,

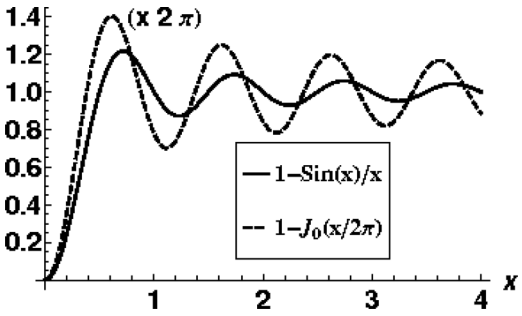

FIG. 1. Comparison of the free-particle-density short-time evolution (dashed line) of (9) with the result using a parabolic approximation (solid line), where the x axes of the latter curve are elongated by a factor of 2π.

with x being 2π times the argument of the Bessel function in (9), as illustrate in Fig. 1. Despite the $1/x$ decay instead of the correct $1/\sqrt{x}$, there are more artifacts of the parabolic approximations, e.g. the 2π faster oscillation and the appearance of an imaginary part in the density. Therefore we will use the correct $\epsilon_p = 2J(1-\cos ap/\hbar)$ dispersion and no parabolic approximation.

Now that the initial value is specified, we can integrate (7) over the momentum including the interaction to get an equation for the density. Using the Laplace transform $J_0(\sqrt{4Jbt}) \bullet\!\!-\!\!\circ 1/\sqrt{s^2+4Jb}$, let's first inspect the solution without a confining trap ($K=0$):

$$\delta n_s = -\frac{n}{2}\frac{\left(s+\frac{1}{\tau}\right)^2+b^2}{\sqrt{\left(s+\frac{1}{\tau}\right)^2+4Jb}\left(s^2+\frac{s}{\tau}+nbV_q+b^2\right)} \circ\!\!-\!\!\bullet$$

$$\delta n_t = -\frac{n}{2}J_0(\sqrt{4Jbt})e^{-\frac{t}{\tau}} - \frac{n}{4\gamma\tau^2}\int_0^t dx\, J_0(\sqrt{4Jbx})e^{-\frac{t+x}{2\tau}} \times [2\gamma\tau\cos\gamma(t-x)+(1-2bnV_q\tau^2)\sin\gamma(t-x)], \tag{10}$$

where $\gamma^2 = nbV + b^2 - 1/4\tau^2$. Without interaction, $V=0$, and damping, $1/\tau \to 0$, we obtain the exact result of [17].

In Fig. 2 we compare the experimental data [3] with (10), and we plot the interaction-free evolution together with that including interaction. The main effect of interaction is the damping, which brings the curves nearer to the experimental data. Here we use the parameter for the lattice constant given by half of the short laser wavelength, $a = \lambda/2 = 765$ nm, which provides a wave vector of $q = \pi\hbar/a$, and an initial density $n = 1/2a$ with every other place filled. The relaxation time characterizes dissipative processes, which we assume arise due to polaron scattering. These lattice deformation processes are dominated by hopping transport at high temperatures and band regime transport at low temperatures, with the transition given by $\hbar/\tau = 2J\exp(-S)$, where S describes the ratio of polaron binding to optical phonon energy. This quantity is generally difficult to calculate [28] but is on the order of 1. We will use it as a fit parameter and find a common value, $\tau = 0.6\hbar/J$, for the results in the figures presented here. A more refined theory should also take the contribution of the electron-electron correlation into account for this relaxation time.

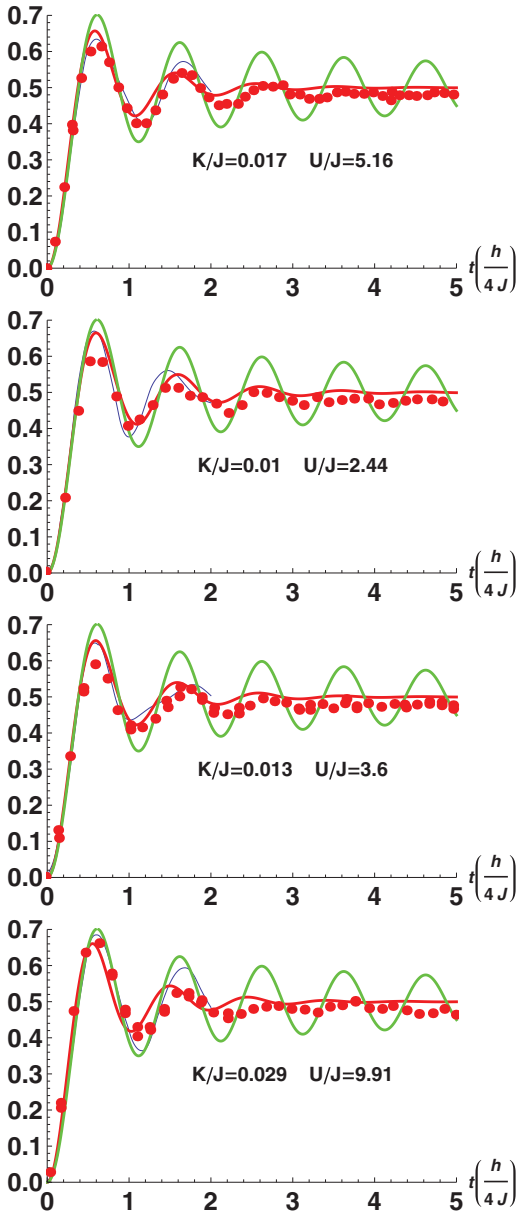

FIG. 2. (Color online) Comparison of the experimental data of [3] (dots) with the RG calculation (thin line) [17] and Mermin's conserving relaxation time $\tau = 0.6\hbar/J$ approximation (10) without (green) and with interaction (red).

The influence of Mermin's correction is visible in Fig. 3. One sees that without these corrections of conserving relaxation time approximation (brown curve) we obtain too much damping. In order to compare the relative forms of the curve we artificially increase the relaxation time by a factor of 4 (green line), which brings both curves closer together. One sees that without Mermin's correction (and the 4 times larger relaxation time) the oscillation is too fast compared to the data. Mermin's correction as an expression for the density conservation decreases the frequency, which is in better agreement with the data.

However, it is still not sufficient since the oscillations are still too fast. We need the corrections to the finite

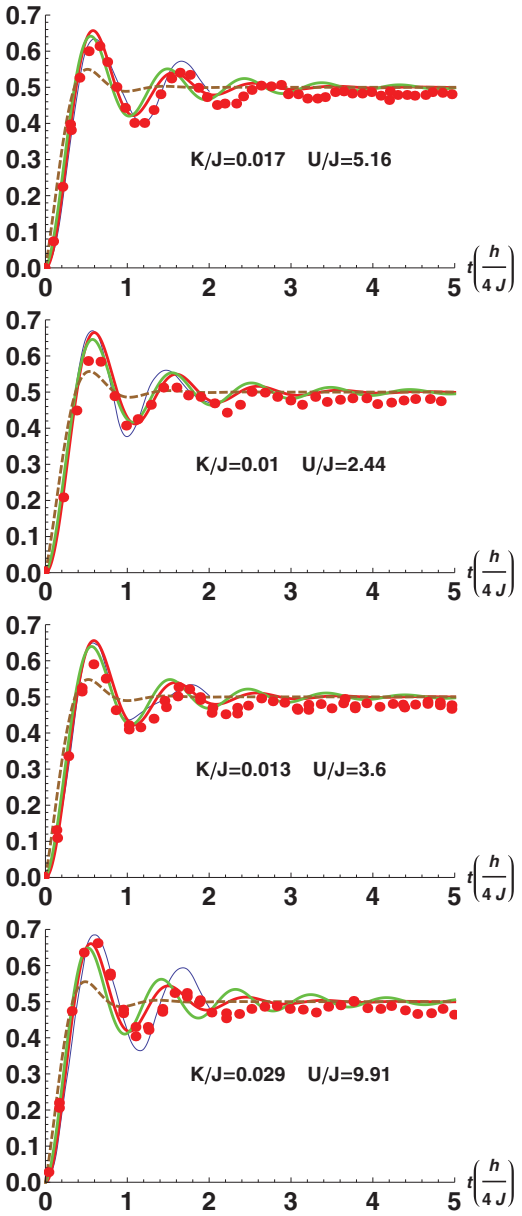

FIG. 3. (Color online) Same data as in Fig. 2 and (10), with (red) and without (brown) Mermin's conserving relaxation time approximation. The curve without Mermin's approximation but with a 4 times larger relaxation time is plotted as a green curve for comparison.

trap condensed in the parameter K/J, which we have not considered yet.

The finite-size term $\sim K/J$ is a small number such that one iteration of (7) is sufficient. With one partial integration with respect to p we obtain three terms. The first one,

$$\frac{iKn}{2}\sum_p \partial_p\left(\frac{1}{s+i\Delta\epsilon+\frac{1}{\tau}}\right)\partial_q\left(\frac{1}{s+i\Delta\epsilon+\frac{1}{\tau}}\right), \quad (11)$$

is easily seen to vanish by back transforming in time and performing the momentum integration. The second term is the q derivative of δn, which is readily integrated due to the δ_p functions, yielding

$$\frac{K}{2}\partial_q\delta n_s 4Ja\cos\frac{qa}{2}\sin\frac{qa}{2} \times\left[nbV\partial^2_{\frac{1}{\tau}}-\frac{1}{\tau}\partial^2_{\frac{1}{\tau}}\left(s+\frac{1}{\tau}\right)\right]\frac{1}{(s+\frac{1}{\tau})^2+b^2}. \quad (12)$$

Since the experimental data are performed with commensurate wavelengths $q=\hbar/a$, this term is negligibly small.

The last remaining term is the one proportional to δn_s, which results in

$$\frac{K}{2}\delta n_s 2Ja^2\sin^2\frac{qa}{2} \times\left[-nbV\partial^2_{\frac{1}{\tau}}+\frac{1}{\tau}\partial^2_{\frac{1}{\tau}}\left(s+\frac{1}{\tau}\right)\right]\frac{1}{\left(s+\frac{1}{\tau}\right)^2+b^2}, \quad (13)$$

where we again have neglected terms $\sim\cos\frac{qa}{2}$. We see that the trap potential introduces an additional s and therefore time dependence. However, these corrections range from zero at zero time ($s\to\infty$) to the maximal value at large times ($s\to 0$). Using the latter limit, we obtain a constant shift to the term nbV in (10) of

$$nbV \to nbV+k,$$
$$k = \frac{Ka^2b}{2(1+\tau^2b^2)^2}[nbV\tau^2(\tau^2b^2-3)-1+3b^2\tau^2]. \quad (14)$$

In Fig. 4 we see that these trap-potential corrections decrease the frequency further, and (10) agrees better with the data, although the corrections are small. For comparison we plotted the RG simulation [17] as thin lines. For larger interaction U/J we see that the analytic result (15) describes the data slightly better, and we can give the time evolution up to more oscillations than possible using numerical RG.

IV. SHORT-TIME RESPONSE FUNCTION

A. RPA modes

Now we are interested in the short-time response of the system to an external perturbation $V^{\rm ext}$. This is different from a sudden quench since here we initially have $\delta f(p,q,0)=0$ and the system is driven out of equilibrium by $V^{\rm ext}$. As a result, we will obtain the dielectric response, which gives microscopic access to optical properties.

Integrating (5) over momentum, one obtains the time-dependent density response

$$\delta n(q,t)=\int_{t_0}^{t}dt'\chi(t,t')V_q^{\rm ext}(t') \quad (15)$$

describing the response of the system with respect to the external field in contrast to the polarization function (3), which is the response to the induced field.

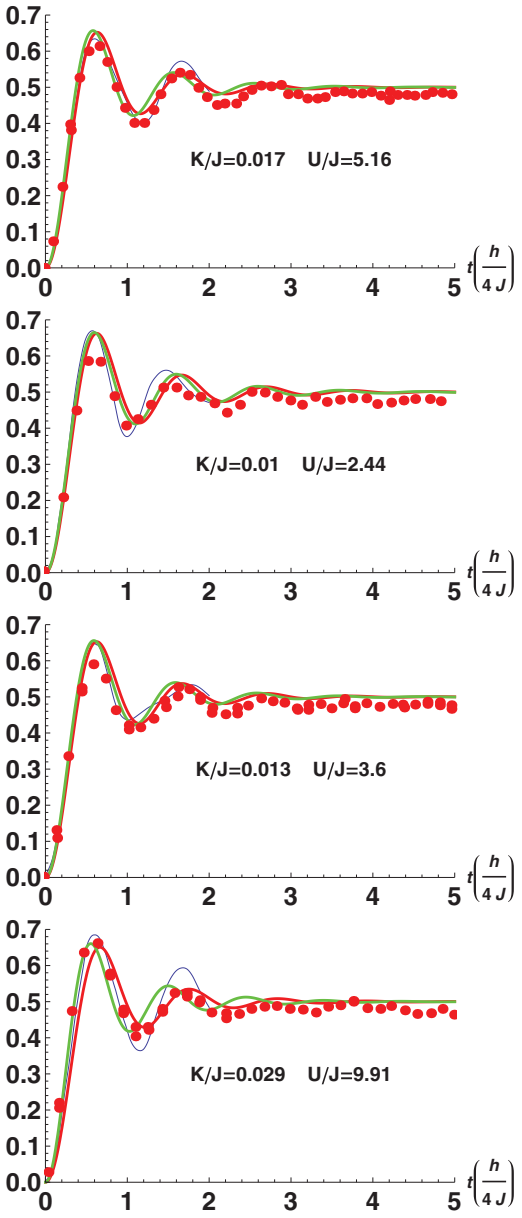

FIG. 4. (Color online) Same data as in Fig. 2, with (red) and without (green) the influence of the trapping potential K/J of (14) together with the RG calculation (thin line) of [17].

One obtains the equation for $\chi(t,t')$ from (15) by interchanging integrations in (5),

$$\chi(t,t') = \Pi(t,t') + \int_{t'}^{t} d\bar{t} \times \{[\Pi(t,\bar{t})V_q + I(t,\bar{t})]\chi(\bar{t},t') + R(t,\bar{t})\}, \quad (16)$$

with the polarization (3) and Mermin's correction,

$$I(t,t') = \sum_p \frac{f_{p+\frac{q}{2}}(t') - f_{p-\frac{q}{2}}(t')}{\varepsilon_{p+\frac{q}{2}} - \varepsilon_{p-\frac{q}{2}}} \frac{e^{(i\varepsilon_{p+\frac{q}{2}} - i\varepsilon_{p-\frac{q}{2}} + \frac{1}{\tau})(t'-t)}}{\tau\tilde{\Pi}^{\mathrm{RPA}}(t',0)}. \quad (17)$$

The confining-trap potential leads to the term

$$R(tt') = K\sum_p e^{(i\varepsilon_{p+\frac{q}{2}} - i\varepsilon_{p-\frac{q}{2}} + \frac{1}{\tau})(t'-t)}\partial_p\partial_q\delta f(p,q,t'). \quad (18)$$

For cold atoms on the lattice we have already obtained the solution (7), which we can use here with $\delta f(0) = 0$, and we have

$$\Pi(t,t') = ne^{\frac{t'-t}{\tau}}\sin[b(t'-t)],$$
$$I(t,t') = \frac{1}{\tau}e^{\frac{t'-t}{\tau}}\cos[b(t'-t)], \quad (19)$$
$$R(t,t') = \frac{JKa}{\hbar}\int_{t'}^{t} dt'(t'-t)e^{\frac{(t'-t)}{\tau}}\sin\left(\frac{qa}{2\hbar}\right)\partial_q\chi(\bar{t},t').$$

This will lead to the same response formula as that for a gas of particles with the thermal Fermi/Bose distribution for f_p. For the latter we work in the limit of long wavelengths $q \to 0$, and the leading terms are $\Pi(t,t') \approx \frac{q^2 n(t')}{m}(t'-t)e^{\frac{t'-t}{\tau}}$, $I(t,t') \approx \frac{1}{\tau}e^{\frac{t'-t}{\tau}}$, and $R(t,t') = K\int_{t'}^{t} dt'(t'-t)e^{\frac{(t'-t)}{\tau}}\frac{q\partial_q}{m}\chi(\bar{t},t')$ with the time-dependent density $n(t)$.

We introduce the collective mode of plasma/sound-velocity oscillations for a Coulomb gas and for the Hubbard models, respectively,

$$\omega_p^2 = \begin{cases} \frac{ne^2}{m\varepsilon_0} & \text{for} \quad V_q = \frac{e^2\hbar^2}{\varepsilon_0 q^2}, \epsilon_p = \frac{p^2}{2m}, \\ bnaU & \text{for} \quad V_q = Ua, \epsilon_p = 2J(1-\cos pa/\hbar), \end{cases} \quad (20)$$

where we have already used $b = 4J\sin^2\frac{aq}{2\hbar}$. For Coulomb interactions one has an optical mode, while for atoms on the lattice the mode is acoustic.

For the gas of particles it is convenient to transform (16) into a differential equation:

$$\ddot{\chi}(tt') + \frac{1}{\tau}\dot{\chi}(tt') + \omega_p^2\chi(tt') = -k\omega_p^2\partial_{\omega_p^2}\chi(tt') + o\left(k\omega_p^2\right),$$
$$\chi(t,t) = 0, \quad \dot{\chi}(t,t')|_{t=t'} = -\omega_p^2/V_q + o\left(k\omega_p^2\right), \quad (21)$$

where the influence of the trap is condensed in $k = 2K/m$ for the gas system and (14) for the Hubbard models.

Interestingly, both solutions, the one for the Hubbard lattice (7) and the one for the gas of particles (21), lead to the same result for the integral equation (16) for the two-time response function,

$$V\chi(t,t') = -\frac{\omega_p^2}{\gamma}e^{-\frac{t-t'}{2\tau}}\sin\gamma(t-t'), \quad (22)$$

but with different collective modes, $\gamma = \sqrt{\omega_p^2 + k - \frac{1}{4\tau^2}}$ for the Coulomb gas and $\gamma = \sqrt{\omega_p^2 + b^2 + k - \frac{1}{4\tau^2}}$ for cold atoms. In this sense we consider (22) to be universal short-time behavior.

In the further analysis we will closely follow the experimental way of analyzing the two-time response function [2]. The pump pulse creates charge carriers in the conduction band, and the probe pulse tests the time evolution of this occupation. The time delay after this probe pulse, $T = t - t_0$, is Fourier transformed into frequency. Similarly, we set the half-empty lattice of cold atoms to relax at t_0. The frequency-dependent

inverse dielectric function associated with the actual time t is then given by

$$\frac{1}{\varepsilon(\omega,t)} = 1 + \int_0^{t-t_0} dT e^{i\omega T} V \chi(t, t-T), \tag{23}$$

which is exactly the one-sided Fourier transform introduced in Ref. [29]. Integral (23) with (22) can be expressed in terms of elementary functions. Without the last term due to the confining potential it is the solution derived for Coulomb systems in [20].

The virtue of (23) is that the long-time limit correctly yields the Drude formula ($K \to 0$),

$$\lim_{t\to\infty} \frac{1}{\varepsilon} = 1 - \frac{\omega_p^2}{\gamma^2 - \omega\left(\omega + \frac{i}{\tau}\right)}, \tag{24}$$

which is not easy to achieve within short-time expansions [29] and which had provided the wrong long-time limit of $1 - \omega_p^2/[\omega_p^2 - (\omega + i/\tau)^2]$ before.

B. Sum rules

Checking on the sum rules we first see that the f-sum rule is completed independent of time. Indeed, using $\int_0^\infty \omega \sin \omega T = -\delta'(T)\pi$, we obtain from (23)

$$\int_0^\infty \omega \mathrm{Im}\varepsilon^{-1} = -\pi\omega_p^2. \tag{25}$$

The compressibility sum rule becomes time dependent and reads

$$\int_0^\infty \frac{d\omega}{\omega} \mathrm{Im}\varepsilon^{-1} = -\frac{\pi\omega_p^2}{2\gamma} \int_0^{t-t_0} dT e^{-\frac{T}{2\tau}} \sin \gamma T$$

$$\to -\frac{\pi}{2}\frac{\omega_p^2}{\gamma^2 + k} = -\frac{\pi}{2} \begin{cases} \frac{\omega_p^2}{\omega_p^2 + k} \\ \frac{\omega_p^2}{\omega_p^2 + b^2 + k} \end{cases} \quad \text{for} \quad t \to \infty \tag{26}$$

for the gas of particles and lattice atoms, respectively.

This result can be confirmed from (24) by using the Kramers-Kronig relation,

$$\frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega} \mathrm{Im}\chi = \mathrm{Re}\chi(\omega = 0) = -\frac{1}{V_q} \begin{cases} \frac{\omega_p^2}{\omega_p^2 + k} \\ \frac{\omega_p^2}{\omega_p^2 + b^2 + k} \end{cases}, \tag{27}$$

in which the long wavelength limit $q \to 0$ defines the compressibility for the lattice gas,

$$\kappa = \lim_{q\to 0} \frac{1}{n^2 V_q\left(\omega_p^2 + b^2 + k\right)} = \frac{1}{n(naU - Ka^2/2)}, \tag{28}$$

where we have used the long-wavelength limit of (14).

For the long-range Coulomb gas the compressibility is defined as the response to the screened field (E instead of D),

$$\kappa = -\frac{1}{n^2} \lim_{q\to 0} \frac{\chi(q,0)}{1 + V_q \chi(q,0)} = \lim_{q\to 0} \frac{\omega_p^2}{n^2 V_q (s^2 q^2 + k)}, \tag{29}$$

where one needs to expand one step further in powers of q in the polarization function to get the sound velocity s. The compressibility vanishes if we have a trap $k \neq 0$. Without a trap we have $\kappa = 1/nms^2$ for the gas.

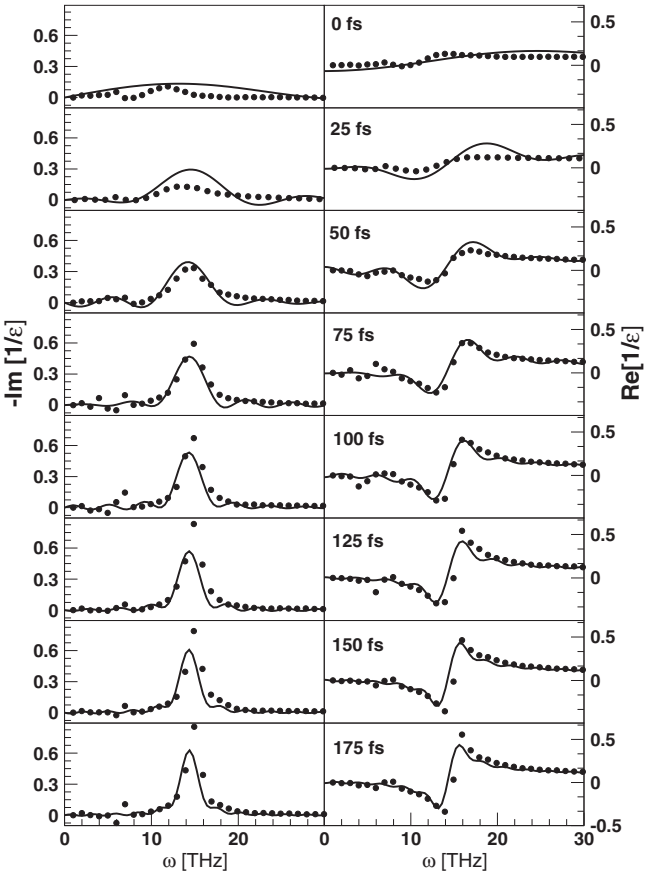

FIG. 5. The time evolution of the inverse dielectric function in GaAs. The labels from top to bottom denote the time t. The pump pulse was at $t_0 = -40$ fs, and the probe pulse has a FWHM of 27 fs. Circles are data from Refs. [1,4], and solid lines show the electronic part (22) published in [20]. The plasma frequency is given by ω_p =14.4 THz, and the relaxation time is $\tau = 85$ fs.

In spite of the numerous approximations, Eq. (23) with (22) fits well the experimental data, as shown in Fig. 5 for the polar semiconductor GaAs. Formula (22) results in a slightly too fast buildup of the collective mode at time $t = 25$ fs. This is, however, just the time duration of the experimental pulse and, consequently, the time of populating the conduction band which we have approximated by an instant jump [20].

For the atoms on a lattice we obtain the time evolution of the dielectric function as plotted by snapshots in Fig. 6. We see that the influence of finite-trap correction diminishes the frequency of the collective mode, as we had already seen with the sudden quench. Further the damping is lowered, which is visible by a sharpening of the mode. Note the instability around times of $1h/4J$ at frequencies of $4J/h$ where Imε becomes negative.

C. Local-field correction

For strong interactions like the cold atoms feel in the lattice one might expect that the mean-field response is not sufficient. Although it works well at short times since the interactions need time to form, for larger times we expect to see the influence of the so-called local-field corrections. They describe the interaction cloud around the atom which changes

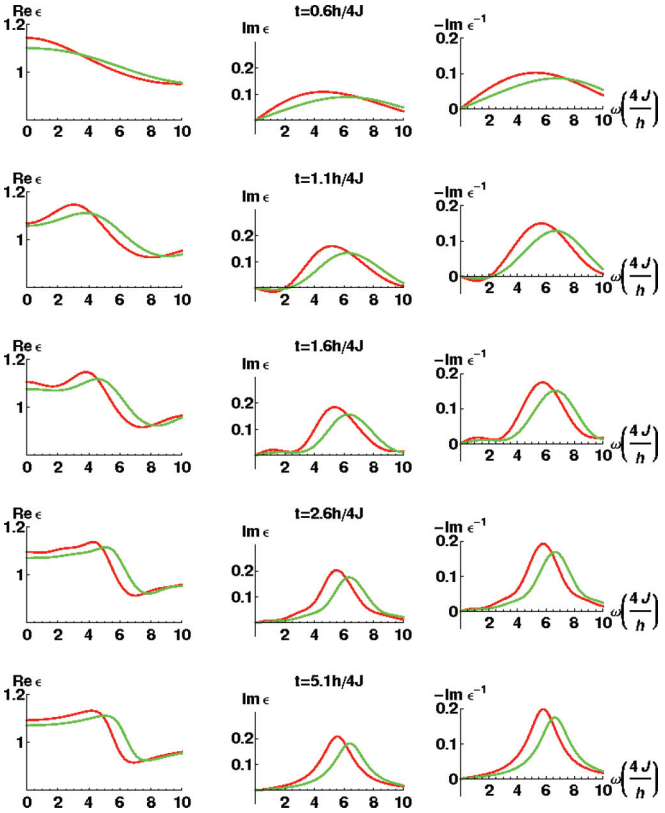

FIG. 6. (Color online) The time evolution of the dielectric function for the atomic lattice for $U/J = 9.91$ of Fig. 3 with (red) and without (green) trap correction according to (14) (see the movie in the Supplemental Material [23]).

the potential locally. We will derive it here for the lattice atoms in the sense of Singwi and Sjølander [30]. For this purpose we calculate the second time derivative of the reduced density matrix since this describes the dynamics. With the help of $i\dot{\rho}_k = [\rho_k, H]$ the Hamiltonian (1) leads to

$$\begin{aligned}\ddot{\rho}_k = &-\sum_{k_1}\left(\epsilon_{k_1+k}-\epsilon_{k_1}\right)^2 a^+_{k_1}a_{k_1+k}\\ &-\frac{1}{2}\sum_{k_1 q} V_q\left(\epsilon_{k_1+k}-\epsilon_{k_1}-\epsilon_{k_1+k-q}+\epsilon_{k_1-q}\right)\\ &\times\left(\rho_q a^+_{k_1}a_{k_1+k-q}+a^+_{k_1}a_{k_1+k-q}\rho_q\right).\end{aligned} \tag{30}$$

The trick now is to write the mean-field term $k = q$ in the second sum in front of the sum. For lattice atoms with the dispersion $\epsilon_p = 2J(1-\cos pa)$ one obtains

$$\begin{aligned}\ddot{\rho}_k = &-4b^2\sum_{k_1}\left(\frac{\sin\frac{k_1 a}{2}}{\sin\frac{ka}{2}}\right)^2\cos^2\frac{k_1 a}{2}f_{k_1,k}\\ &-nbV_k\sum_{k_1}\cos(k_1 a)(\rho_k f_{k_1,0}+f_{k_1,0}\rho_q)\\ &\times\left[1+\frac{\sum_q \frac{V_q\sin\frac{qa}{2}}{V_k\sin\frac{ka}{2}}\sum_{k_1}\cos(k_1 a)(\rho_q f_{k_1,k-q}+f_{k_1,k-q}\rho_q)}{\sum_{k_1}\cos(k_1 a)(\rho_k f_{k_1,0}+f_{k_1,0}\rho_k)}\right],\end{aligned} \tag{31}$$

and one sees that the effect of correlations beyond the mean field can be recast into a local field $V_k \to V_k[1+G_k(t)]$. Introducing the cosine-weighted density $\rho^c_q = \sum_k \cos(ka) f_{k,q}$, this local field is read off (31),

$$G_k = \sum_q \frac{V_q\sin\frac{qa}{2}}{V_k\sin\frac{ka}{2}}\frac{\rho_q\rho^c_{k-q}+\rho^c_{k-q}\rho_q}{\rho_k\rho^c_0+\rho^c_0\rho_k}. \tag{32}$$

We now linearize the Wigner function $f_{p,q} = a^+_{p-\frac{q}{2}}a_{p+\frac{q}{2}} = \frac{n}{2}\delta_p + \delta f_{p,q}$ according to the special equilibrium distribution (6) and the density operator as an integral over p, where $\rho_q = \rho_0 + \delta\rho_q$. Then $\rho_0 = \rho^c_0 = n/2$, and we assume $\delta\rho_q = \delta\rho_{-q}$ to arrive for Hubbard models $V_k = Ua$ at a wave-vector-independent local field,

$$\delta G(t) = \sum_q \cos\left(\frac{qa}{2}\right)\frac{\delta\rho^c_q(t)}{\rho^c_0}. \tag{33}$$

This local-field correction means that we have to replace the term $V_q\delta n \to V_q\delta n + nV_q\delta G$ in the kinetic equation (4). It is not hard to see from the solution (5) that $\delta n^c = \delta n\cos\frac{qa}{2}$ holds. We obtain for the Laplace transform of the response (15) and (16), which was $\chi_s(k) = -nb(k)/[s^2 + \frac{s}{\tau} + nb(k)V_k + b(k)^2]$, the modification

$$\begin{aligned}\delta n_s(k) &= \chi_s(k)\left[V^{\rm ext}_s(k) + aU\sum_q\cos^2\frac{qa}{2}\delta n_s(q)\right]\\ &= \frac{\chi_s(k)}{1-V\delta\bar{G}_s}V^{\rm ext}_s(k) = \chi^{\rm eff}_s(k)V^{\rm ext}_s(k),\end{aligned} \tag{34}$$

where $b(q) = 4J\sin^2(qa/2)$. To solve (34) we integrate the first line with $\cos^2$, leading to

$$\begin{aligned}\delta\bar{G}_s &= -\sum_q\frac{nb(q)\cos^2\left(\frac{qa}{2}\right)}{s^2+\frac{s}{\tau}+nb(q)V+b(q)^2}\\ &= -\frac{1}{\pi V}\int_0^1 dy\frac{\sqrt{y-y^2}}{z-y+\frac{4J}{nV}y(y-2)},\end{aligned} \tag{35}$$

where $z = [(s+\frac{1}{2\tau})^2+\gamma^2]/4nVJ$ and γ is from (22).Since the response function without local-field correction is just $\chi = -nb/z$, it is advantageous to expand with respect to $1/z$ to obtain

$$V\chi^{\rm eff}_s = -\sin^2\frac{qa}{2}\left(\frac{1}{z}-\frac{1}{8z^2}-\frac{3+\frac{22J}{nV}}{64z^3}+o(z^{-4})\right). \tag{36}$$

This is easily back transformed into time,

$$\begin{aligned}V\chi^{\rm eff}_t = &-\omega_p^2 e^{-\frac{t}{2\tau}}\left[1+\frac{\omega_p^2}{16\gamma}\frac{\partial}{\partial_\gamma}\left(1-\frac{3+\frac{22J}{nV}}{256\gamma}\omega_p^2\frac{\partial}{\partial_\gamma}\right)\right]\\ &\times\frac{\sin\gamma t}{\gamma},\end{aligned} \tag{37}$$

and the correction is discussed in Fig. 7. We see that the local-field correction shifts the collective mode towards higher energies and sharpens the mode. Further it leads to more structure at smaller frequencies which heal out at larger times.

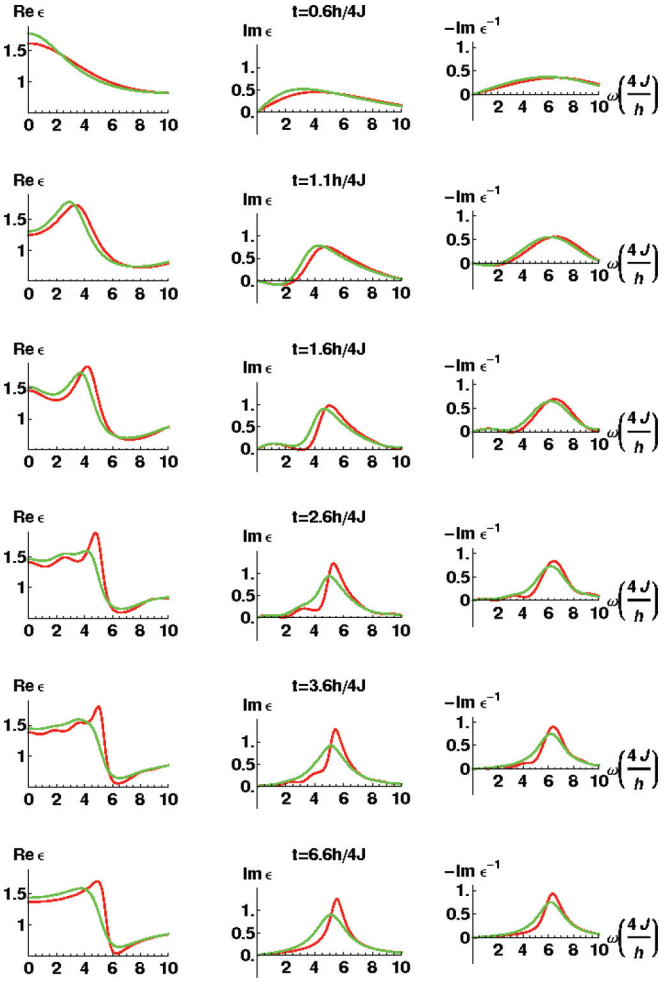

FIG. 7. (Color online) The time evolution of the dielectric function for the atomic lattice of Fig. 6 with $U/J = 9.91$ with (red) and without (green) local-field correction of (37) (see the movie in the Supplemental Material [23]).

V. SUMMARY

The aim of the present paper was to separate the gross features of the formation of collective modes at transient times which are due to mean-field fluctuations. Assuming higher-order correlations described by conserving relaxation-time approximations, this has resulted in an analytic formula for the time dependence of the dielectric function and for sudden quench dynamics. Subtracting this gross feature from the data allows one to extract the effects which are from higher-order correlations and which have to be simulated by quantum kinetic theory [7–10] and response functions with approximations beyond the mean field [24]. These treatments are numerically demanding such that analytic expressions for the time dependence of some variables [31] are useful for controlling the numerics.

We have also discussed here the Singwi-Sjølander local-field corrections in order to estimate the influence of higher-order correlations [32] on short-time behavior. We find the same time scale for the formation of correlation as for the mean-field response but with more details in the response function due to the local-field corrections. This deviation between the two levels of approximations is considered an estimate of the validity of the simple mean-field formula.

The final answer, however, regarding to what time the presented short-time expansion works can only be given by solving the time evolution of the complete correlation functions [15]. Here a selective comparison with experiments and RG simulations in the strong-coupling regime has been chosen as a first test. Further comparisons are needed and encouraged.

As a further result of this paper we derive the influence of a finite trapping potential on the kinetic equation and the time evolution of the population. We find that the main effect of correlations consists of lowering the collective frequency and a damping.

To conclude, we have derived an analytic formula for the formation of correlations after a sudden quench and for the density response to an external perturbing field. By considering two distinct physical systems, the pump and probe dynamics in semiconductors and the dynamics of atoms in an optical lattice, we find the same short-time feature of the formation of quasiparticles, which we suggest is universal. The simplicity of the presented result is extremely practical and offers a wide range of applications. It could spare a lot of computational power to simulate ultrashort-time behavior of new nanodevices, and it could help us understand and describe the formation of collective modes during processes which are not experimentally accessible in the early phase of reactions like in nuclear collisions.

Note added in proof. There is a remarkable activity to investigate the time which correlations need to spread known as effective light cone [33] based on the Lieb-Robinson bound [34] which supports the here used conjecture that higher-order correlations need more time to develop than lower ones.

ACKNOWLEDGMENT

The financial support by the Brazilian Ministry of Science and Technology is acknowledged.

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