Evolution of a neutron-initiated micro big bang in superfluid ^{3}He-*B*

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A nuclear capture reaction of a single neutron by ultracold superfluid ^{3}He results in a rapid overheating followed by the expansion and subsequent cooling of the hot subregion, in a certain analogy with the big bang of the early universe. It was shown in a Grenoble experiment that a significant part of the energy released during the nuclear reaction was not converted into heat even after several seconds. It was thought that the missing energy was stored in a tangle of quantized vortex lines. This explanation, however, contradicts the expected lifetime of a bulk vortex tangle, 10^{-5}–10^{-4} s, which is much shorter than the observed time delay of seconds. In this paper we propose a scenario that resolves the contradiction: the vortex tangle, created by the hot spot, emits isolated vortex loops that take with them a significant part of the tangle's energy. These loops quickly reach the container walls. The dilute ensemble of vortex loops attached to the walls can survive for a long time, while the remaining bulk vortex tangle decays quickly.

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I. INTRODUCTION

It is generally presumed that the universe started with a big bang, subsequently expanding very rapidly while cooling and going through a cascade of gauge phase transitions, in which the four fundamental forces of nature separated out. Today, the universe exhibits inhomogeneous large-scale structures: galaxies that form clusters, arranged in turn in superclusters such as the "great wall." The clustering takes the form of long chains or filaments, which are separated by large voids, regions empty of visible mass. These inhomogeneities might have been nucleated during a rapid nonequilibrium transition known as a "*quench*." Many types of topological defects can be created at such a transition: domain walls, cosmic strings, and monopoles [1,2]. Cosmic strings have received particular attention among cosmologists since they provide possible seeds for galaxy formation.

The development of cosmology is greatly limited by the fact that one cannot apply methods of experimental physics. The alleged "big bang" occurred a long time ago, and now, after many stages of its nontrivial evolution, we can only observe its consequences. The early stages of the formation of the universe can only be reconstructed with an unclear degree of certainty. Therefore, there are many different theories of early evolution of the universe, including even the theory of the multiverse [3]. In order to formulate a quantum multiparametric theory that adequately describes the universe, one can use a different physical system that models the fundamental properties of the universe but allows experimental studies. One of the fundamental problems of the early universe is the formation of topological defects at phase transitions. In order to model this process one has to find an adequate physical system with the gauge or orientation phase transitions. Unfortunately, attempts to use superfluid ^{4}He [4] failed to observe any vortices due to various reasons [5,6]. In superconductors [7] and in ferromagnets [8], the formation of defects was observed; however, their symmetries are quite different from that in our early universe.

Liquid ^{3}He below the temperature $T_c \simeq 10^{-3}$ K exhibits a phase transition to superfluidity characterized by a simultaneous breaking of the orbital, spin, and gauge symmetries that are thought to be the best analogy to those broken after the big bang [9]. Moreover, ^{3}He can be locally heated by a nuclear fusion reaction of a single low-energy neutron with a ^{3}He nucleus: $n + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + p$. Each capture deposits an energy of 764 keV, which thermalizes within a distance on the order of 30 μm [10], creating a "hot spot" of normal ^{3}He which quickly expands and cools down, giving birth to quantized vortices. This was observed in the Helsinki [11,12] and Grenoble [13] experiments, in agreement with the predictions of the Zurek modification [14] of the Kibble scenario [2] of the formation of cosmic strings after the big bang. In addition to these experiments, active collaborations between cosmologists and low-temperature physicists began in Europe (e. g., European Science Foundation Networks "Non-equilibrium Field Theory in Particle Physics, Condensed Matter and Cosmology" in 1998–2000, COSLAB in 2002–2007, and the European Microkelvin Collaboration in 2009–2013). Results of these studies were presented in the most detailed form in the book by Volovik [9], in which many aspects of the analogy between the universe and superfluid ^{3}He are presented. Some of them already led to new results, such as the prediction of the second Higg's boson [15] and the Majorana fermion in ^{3}He [16,17], which was just recently observed experimentally [18]. Studies of the *A*-*B* interface in superfluid ^{3}He as a model of a cosmological brane [19] became a new direction in research of the universe-^{3}He analogy. Finally, recent studies, directly related to the results of experiments discussed in this paper, are a manifestation of the multiverse model with a first-order phase transition between the *A* and *B* phases of superfluid ^{3}He after the neutron-initiated overheating [20]. These results may explain the nature of dark energy as a result of the interaction of our universe with other universes, similar to the interaction between the *A* and *B* phases of superfluid ^{3}He [9,21–25]. Furthermore, after the recent publication of the new analysis of the results of Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck satellite missions of cosmic microwave background radiation [26] this idea has become popular [3].

Unfortunately, earlier analysis of the Grenoble experiment [13] faced a serious contradiction between the very short expected lifetime of a bulk vortex tangle (about 10^{-4} s) and the very long lifetime (above 1 s) of the hidden, presumably in these vortices, energy. This makes the attractive idea to consider quantitatively the neutron-^{3}He quench experiment as an analog of the universe evolution questionable. Therefore, a careful analysis of the Grenoble experiment [13] in light of current understanding of the quantized vortex dynamics in superfluid ^{3}He seems to be timely and highly desirable. This is the subject of this paper, which includes a detailed discussion of the relevant importance of numerous relaxation mechanisms of the tangle of quantized vortex lines in the ^{3}He hot spot after the quench. In particular we suggest a scenario of the evolution of such tangles that resolves the long-standing contradiction between the lifetimes of the bulk vortex tangle and hidden energy.

We therefore conclude that the neutron-initiated quench experiment can really serve as currently the best available experimental model of the early evolution of the universe (or multiverse?) [27]. This calls for further experimental and theoretical investigation of this phenomenon. In particular, a new set of experiments at different temperatures, pressures, and external magnetic fields may clarify important details of the vortex creation and relaxation and the role of the A and B phases. The open questions for the theoretical analysis include, among others, estimating the critical temperature of the time-dependent, space-inhomogeneous superfluid phase transition and generalization of the Kibble-Zurek scenario for more complicated symmetries [other than the simplest U(1)] of quantum vacuum in 3He and of the early universe. We hope that additional experimental, analytical, and numerical studies of the "micro-big-bang" phenomenon may shed further light on the initial evolution of the universe.

II. THE KIBBLE-ZUREK QUENCH SCENARIO IN HOMOGENEOUS SUPERFLUID ^{3}He

In a homogeneous quench, the new phase begins to form simultaneously in many independent regions of the system. Kibble [2] suggested that inhomogeneity may originate from fluctuations. With the growth of coherent regions of the low-temperature phase, they begin to come in contact with each other. At the boundaries, where different regions meet, the order parameters do not necessarily match each other, and consequently, a domain structure forms. If the broken symmetry is the U(1) gauge symmetry, these are domains with different phases of the order parameter. Such a random domain structure reduces to a network of linear defects, which are quantized vortex lines in superfluids and superconductors or cosmic strings in the early universe. If the broken symmetry is more complicated, as is the case for ^{3}He superfluids, then defects of different dimensionality and structure can form. Later, Zurek proposed [14,28] a phenomenological approach allowing us to estimate the mean intervortex distance,

$$\ell \simeq f\xi_0(\tau_Q/\tau_T)^{1/4}, \quad \tau_Q = T_c/[dT(t)/dt]_{T=T_c}, \tag{1}$$

and the resulting vortex-line density $\mathcal{L} = \ell^{-2}$ (the vortex length per volume). Here ξ_0 is the zero-temperature coherent length, τ_T is the thermal relaxation time of the ordered phase, and τ_Q is the quench time. The prefactor $f > 1$ accounts for the fact that the naïve random-walk arguments, leading to Eq. (1) without f, underestimate the density $\mathcal{L}$. Early estimates of $f \sim 10$, reported in the past [29–31], are probably too large. The estimate $f \simeq 2.5$ was suggested recently [32]. Notice that the expected value of f depends on the details of the interatomic potential [32], and these are not well known. Thus, this estimate cannot be considered as final either.

III. THE HOT-SPOT EVOLUTION

The Grenoble experiments [13] were conducted in a cubic box of size $X = 5$ mm at a background temperature $T_0 \simeq 0.1\,T_c \simeq 100$ μK, at which the superfluid ^{3}He may be considered to be a quantum vacuum with an extremely dilute gas of thermal excitations (Bogoliubov quasiparticles). The energy of $E_0 = 764$ keV deposited by reaction increases their number. After a delay of about 1 s their residual density was measured by a specially designed sensitive bolometer: a box of volume of about 0.1 cm^3 with two vibrating-wire thermometers, immersed in superfluid ^{3}He [33,34]. A detailed analysis of the energy balances shows that an essential part, $\Delta E_{st} \equiv E_0 - Q \simeq 85$ keV (at zero pressure), of the energy E_0 was not fully converted into heat Q [35]. It was assumed [13] that the energy deficit ΔE_{st} was stored in the kinetic energy of flow in the form of quantized vortex lines.

An accurate verification of this hypothesis requires a detailed analysis of the initial stage of the spreading dynamics of the temperature following the neutron capture. However, it is sufficient for our purposes to do this on a semiquantitative level by comparing E_0 with the energy of the vortex lines created by the quench. To this end we use the continuous media approximation with the temperature diffusion equation

$$\frac{\partial T(r,t)}{\partial t} = D_T\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right), \tag{2}$$

with the temperature independent thermal diffusion coefficient $D_T \simeq 5\,\mathrm{cm}^2/\mathrm{s}$. For simplicity we will ignore the possible temperature dependence since we are predominantly interested in the later evolution, when the temperature in the center of the hot sphere is below T_c. Here we ignore the fact that the mean free path of thermal excitations is not small compared to the characteristic size of the region in which the products of the neutron absorption thermalize. The self-similar solution of (2) is

$$T(r,t) = T_0 + \frac{E_0}{C_V(4\pi D_T t)^{3/2}}\exp\left(-\frac{r^2}{4D_T t}\right), \tag{3}$$

where $C_V = C_V(T_c) \simeq 5.83 \times 10^3$ erg K^{-1}cm^{-3} is the specific heat per unit volume [36]. Equation (3) allows us to estimate the quench time $\tau_Q \simeq 0.4\,\mu$s and to see that the temperature in the center of the hot sphere drops below T_c in some $t_0 = 3\tau_Q/2 \simeq 0.6$ μs. It then continues to cool down quickly and reaches $0.5T_c$ in a further $\sim$0.4 μs. The radius of the sphere $R(t,T_*)$, for which $T(r,t)$ is equal to T_c, depends on t as

$$R(t,T_*) = \sqrt{-4tD_T\ln[(T_c - T_0)(4\pi tD_T)^{3/2}C_V/E_0]}. \tag{4}$$

One can see that $R(t,T_*) = 0$ at $t = 0$ and $t = [E_0/C_V(T_* - T_0)]^{2/3}/4\pi D_T$. It reaches its maximum [37]

$$R_{max} = \sqrt{3/2\pi e}[E_0/C_V(T_c - T_0)]^{1/3} \quad (5)$$

at

$$t_{max} = [E_0/C_V(T_c - T_0)]^{2/3}/4\pi e D_T\ . \quad (6)$$

For $T_0 = 0.1T_c$ this gives $R_{max} \approx 26\,\mu$m.

IV. ESTIMATE OF THE INITIAL VORTEX-TANGLE ENERGY E_{VOR}

Taking in Eq. (1) the phase relaxation time $\tau_T \approx 1.3$ ns [38], the quench time $\tau_Q \simeq 0.4\,\mu$s, and $f \simeq 2.5$, we estimate the theoretical distance between the vortices, created by the Kibble-Zurek mechanism, as $\ell \simeq 10.8\,\xi_0$. With the coherent length of ^{3}He $\xi_0 \simeq 0.077\,\mu$m this corresponds to $\ell \simeq 0.83\,\mu$m and to the density of vortex lines $\mathcal{L} \simeq 1.5 \times 10^8$ cm^{-2}. Hence, the total vortex length inside the hot sphere (with $T > T_c$) of radius $R_{max} \simeq 26\,\mu$m is about $L = \mathcal{L}\,(4\pi R^3_{max}/3) \simeq 11$ cm.

The energy of this tangle may be estimated by assuming that the vortex orientations are uncorrelated at separations above ℓ (i.e., there is no large-scale flow). Then we can use the energy of a quantized vortex line per unit length $\gamma = \gamma_0 \ln(\ell/\xi_0)$. Here $\gamma_0 = \rho_s\kappa^2/4\pi \simeq 1.76$ keV/cm (for the superfluid component density ρ_s equal to the total ^{3}He density, $\rho = 0.0814$ g/cm^3) and the quantum of circulation $\kappa = h/(2m_3) = 6.6 \times 10^{-4}$ cm^2/s (here m_3 is the atomic mass of ^{3}He). With $\ell \simeq 10.8\,\xi_0$, we arrive at $\gamma \simeq 4.2$ keV/cm, which results in the vortex energy $E_{vor} \simeq 46$ keV, which is comparable to the experimentally observed residual energy deficit $\Delta E_{st} \simeq 85$ keV. Bearing in mind that Eq. (1) gives only an order-of-magnitude estimate of ℓ, we have to consider this quantitative agreement between E_{vor} and ΔE_{st} as a success of the Kibble-Zurek scenario. Now we need to analyze the characteristic times of different channels of dissipation of E_{vor}.

V. DECAY AND DIFFUSION OF VORTEX TANGLE

The free evolution of the vortex tangle in the continuous-media approximation can be described by the phenomenological Vinen's equation [39], supplemented by the diffusion term [40–43],

$$\frac{\partial \mathcal{L}(\boldsymbol{r},t)}{\partial t} = -\nu'\mathcal{L}^2 + D_L\nabla^2\mathcal{L}. \quad (7)$$

Here $\nu' \simeq 0.1\kappa$ [44] is the effective kinematic viscosity, and the estimates of the diffusion coefficient D_L vary between 0.1κ [42] and 2.2κ [43]. This equation has two characteristic time scales: the decay time of a homogeneous tangle $\tau_{dec} \simeq 1/[\nu'\mathcal{L}(0)] = \ell_0^2/\nu'$ and the diffusion time of a sphere of initial radius $R(0)$, $\tau_{dif} \simeq R(0)^2/D_L$. Having in mind that $\nu' \sim D_L$ but in the initial tangle $\ell_0 \simeq 0.8\,\mu$m $\ll R(0) \simeq 26\,\mu$m, we conclude that $\tau_{dec} \ll \tau_{dif}$. This means that the diffusive spreading is irrelevant for the problem at hand, and the tangle decays in the time $\tau_{dec} \simeq \ell_0^2/\nu' \simeq 10^{-4}$s. In other words, it is impossible to preserve the initial energy of the tangle, $\simeq$85 keV, for longer than $\sim$$10^{-4}$ if the tangle is confined to a sphere of radius $R(0)$ as small as $\sim$26 μm.

VI. EMISSION (EVAPORATION) OF BALLISTIC VORTEX LOOPS

Numerical simulations [45,46] show that the radial profile of vortex density $\mathcal{L}(r)$ has a steep drop in an external shell of width ℓ near its boundary where the continuous-media model, Eq. (7), fails. This shell may emit small vortex loops of size ℓ. Barenghi and Samuels [45] come up with the lifetime of tangles in this process,

$$\tau_{em} \simeq \ell_0^2/\kappa\ , \quad (8)$$

close to the time scale of the bulk decay $\tau_{dec} \simeq \ell_0^2/\nu'$. This means that evaporated loops can take a substantial fraction of total energy of the tangle.

The estimate $\tau_{em} \simeq \ell_0^2/\kappa$ is independent of the initial radius of the tangle R_0, which is probably only valid for sparse tangles with $\ell_0 \sim R_0$. Below we employ a simple model for the dynamics of evaporation of dense tangles, which gives an R_0-dependent lifetime. Namely, we approximate any instantaneous configuration of the outer layer as an ensemble of vortex loops of mean radius $\sim\ell$, half of which are moving outwards. For these, the probability of reconnecting with another loop becomes small, and they escape into open space. For simplicity we assume a uniform density $\mathcal{L}$ (no bulk diffusion), no bulk decay, and no counterflow. We approximate vortex loops by rings of radius $\sim\ell$ that travel in all directions with velocity $v \sim \kappa/\ell$ and have bulk mean free path $\sim\ell$. All prefactors of order unity are dropped.

The tangle's outer shell of thickness $\sim\ell$ disappears in time $\sim\ell^2/\kappa$. About half of the loops escape into open space (their fraction may be enhanced if the vortex loops are outwardly polarized due to their interaction with the thermal counterflow). We thus have $dR/dt = -\kappa\big/\ell$, giving $R(t) \simeq R_0 - \kappa\, t/\ell$. The radius collapses [to $R(\tau_{em}) = 0$] in time

$$\tau_{em} \simeq R_0\ell_0/\kappa \sim 10^{-3}\ \text{s}\ . \quad (9)$$

This approach gives a result which is longer than Eq. (8) (because $R_0 > \ell_0$) yet still much shorter than the experimentally observed time of $\sim$1 s.

The bottom line is that the evaporation should spare a substantial part of the initial energy in the form of vortex loops with a size of order $\ell \sim \mathcal{L}_0^{-1/2}$, which are expected to reach the container walls in some $\tau \sim X\ell/\kappa \sim 2 \times 10^{-2}$ s. This conclusion is supported by recent numerical simulations of vortex reconnections by Kursa *et al.* [47] and of the decay of a vortex tangle by Kondaurova and Nemirovskii [46], which show that indeed the leading mechanism of the vortex-line (and, correspondingly, energy) loss in a compact tangle at zero temperature might be the evaporation of vortex loops from its surface.

VII. PINNED OR FRUSTRATED REMNANT VORTICES

Upon the loop's arrival at the flat container wall at an arbitrary angle, some part of its energy (corresponding to the normal-incidence component of its impulse) might be lost into sound, while the part of the energy corresponding to the sideways sliding of the remaining semiloop should be preserved for a long time. Until the surface density of these loops becomes sufficient to create a developed tangle with frequent reconnections (estimates show that this will never happen), this energy will not be dissipated quickly.

In a container with corners and generally rough walls (which facilitate pinning), the vortices terminated at the wall can become metastable [48]. As the experimentally observed length of metastable vortex lines is independent of pressure [49], we might speculate that this is typical for the particular geometry (size), not pressure. For instance, it was shown by Awschalom and Schwarz [50] that in superfluid ^{4}He the number of pinned remnant vortex lines, upon the decay of a larger number, is quite universal, $\mathcal{L}_0 \leqslant 2\ln(X/\xi_0)/X^2 \approx 19/X^2$. Even though vortex pinning is much weaker in ^{3}He-B, one might still expect these scaling arguments to work as well. For the container size $X \sim 5$ mm, the total length of remnant vortices can thus be as high as $L \sim \ln(X/\xi_0)X \sim 10$ cm [that corresponds to stored energy $\gamma_0 \ln(X/\xi_0)L \sim 160$ keV], independent of pressure.

VIII. CONCLUSION

We discussed the evolution of a tangle of quantized vortex lines of the initial radius $R_0 \sim 26\ \mu$m and energy ∼85 keV created after a strongly nonequilibrium rapid quench from the normal into the superfluid phase of liquid ^{3}He. The tangle disappears within about 10^{-3} s due to two processes of comparable efficiency: bulk decay of quantum turbulence and evaporation of isolated vortex loops away from the tangle. The evaporated vortex loops arrive at the container walls in $\sim 10^{-2}$ s, where they remain in a metastable state for a very long time while keeping a substantial fraction of the initial energy of the vortex tangle. The "hidden energy," detected in the Grenoble calorimetric experiments with a time response of ∼1 s, should thus be comparable to the initial energy of the vortex tangle nucleated upon the quench (micro big bang).

The presented scenario of the micro big bang, although appearing to be feasible, stresses the significance of many problems that require further quantitative analysis. Among them are accounting for the temperature dependence of the thermal conductivity the heat capacity and the counterflow during the cooling process, estimating the critical temperature of the time-dependent, space-inhomogeneous superfluid phase transition, generalization of the Kibble-Zurek scenario for more complicated symmetries [other than the simplest U(1)] of quantum vacuum in ^{3}He and of the early universe, calculation of the relative fractions of the initial vortex length decaying inside the tangle and evaporating from it, and modeling the collective dynamics of short vortex loops, pinned to the surface.

Resolving these and related problems may help shed more light on the intriguing problem of fundamental importance: the evolution of the early universe.

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APPENDIX: INITIAL ENERGY BALANCE

The energy balance in the Grenoble experiment has been discussed in Refs. [34,35]. It was concluded that alternative sources of possible losses cannot explain the value of the hidden energy. Here we mention several such processes. There are defects, other than one-dimensional vortex lines, that could be generated in the superfluid transition: zero-dimensional monopoles (boojums) and two-dimensional solitons (domain walls). However, the energy of the former is very small, and the formation of the latter cannot be foreseen in the geometry of the experiment.

The thermal boundary resistance between the bolometer wall and the superfluid is enormous at these temperatures due to the Kapitza resistance, so any losses via thermal contact are out of the question. The next process could be the ionization of atoms and their scintillation. We should note that ionized atoms form dimers. Spin-singlet dimers radiate UV radiation, while triplets do not. Experiments performed in liquid ^{4}He with electron radiation have shown that helium is quite a good scintillator. It radiates about 6%–8% of the total energy deposited by high-energy electrons in ^{4}He (see Ref. [51]). One may suggest that for ^{3}He this value should be comparable. However, that is not the case. It is well known, particularly from recent experiments on the search for dark matter, that the ionization after the nuclear recoil is about 3 times smaller than for an electron for the same deposited energy. This principle is used for the separation between the dark matter and light-particle events (see, for example, [52]). Taking these circumstances into account, we may conclude that the ionization losses are below 3%. However, we should also take the energy of triplet dimers. The singlet dimers (25% of states) decay very quickly with the UV radiation. The rest, the triplet dimers (75%), live much longer and are quenched on the walls of the cell. The energy of triplet states returns to quasiparticles with a delay of about a few seconds [49]. In conclusion the ionization energy for nuclear recoil events is limited by about 10%

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