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Anomalous topological pumps and fractional Josephson effects

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We discover novel topological pumps in the Josephson effects for superconductors with time-reversal symmetry. The phase difference, which is odd under the chiral symmetry defined by the product of time-reversal and particle-hole symmetries, acts as an anomalous adiabatic parameter. In contrast to topological pumps with conventional parameters, these pumps are characterized by $\mathbb{Z} \times \mathbb{Z}$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$ strong invariants. We determine the general classifications in class AIII, and those in class DIII with a single anomalous parameter. For the $\mathbb{Z}_2 \times \mathbb{Z}_2$ topological pumps in class DIII, the first $\mathbb{Z}_2$ invariant describes the coincidence of fermion parity and spin pumps whereas the second $\mathbb{Z}_2$ invariant reflects the non-Abelian statistics of Majorana Kramers pairs, leading to three distinct fractional Josephson effects.

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Introduction. Topological insulators (TIs) and superconductors (SCs) have attracted tremendous interest [1–3] in condensed matter physics. Electronic systems with energy gaps subject to time-reversal symmetry (TRS) and/or particle-hole symmetry (PHS) can be classified topologically. Nontrivial topological classes are associated with protected gapless modes on the boundary. The topological phases for gapped free fermion systems with different symmetries and dimensions fit together into an elegant "periodic table" [4–6] that unifies and generalizes the integer quantum Hall states [7,8], the chiral p wave SCs [9–11], and the $\mathbb{Z}_2$ TIs [12–15]. This framework has been extended to classify topological defects and pumping cycles [6], which are characterized by a Hamiltonian $\mathcal{H}(\boldsymbol{k},\boldsymbol{r})$. Here $\boldsymbol{k}$ is a d_k dimensional momentum variable defined in the Brillouin zone (BZ), whereas $\boldsymbol{r}$ is a set of d_r adiabatic parameters describing spatial and/or temporal variation of the Hamiltonian. $\boldsymbol{k}$ and $\boldsymbol{r}$ are distinguished by their behaviors under TRS and PHS: $\boldsymbol{k} \to -\boldsymbol{k}$ and $\boldsymbol{r} \to \boldsymbol{r}$. It was found that the topological classes for $\mathcal{H}(\boldsymbol{k},\boldsymbol{r})$ only depend on the combination $d_k - d_r$ [6]. Thus, the invariants characterizing defects and pumps for $d_r \neq 0$ are related to the invariants (given by $\mathbb{Z}$, $\mathbb{Z}_2$, or 0) in the original table [4,5] in which $d_r = 0$.

In this Rapid Communication we introduce a class of adiabatic pumping cycles with anomalous parameters that have a mixed behavior under TRS and PHS. Such a pump naturally arises in the theory of a Josephson junction coupling TRS invariant topological SCs [16,17], as well as a junction mediated by TI edge states [18]. Consider a Josephson junction in which the phase difference ϕ is an adiabatic parameter. Since ϕ is odd under TRS, a 2π cycle crosses two TRS invariant points at $\phi = 0$ and π, similar to the $\mathbb{Z}_2$ spin pump [19]. However, unlike the spin pump, the Bogoliubov–de Gennes Hamiltonian [20] has PHS for any ϕ, so ϕ is even under PHS. Unlike both k and r, ϕ is odd under the combination of TRS and PHS, which defines the unitary chiral symmetry. We will refer to parameters with this property as anomalous. One may also consider another type of anomalous parameter θ which is even (odd) under TRS (PHS), and anticipate an extended new table which should depend on $d_k - d_r$ and $d_\phi - d_\theta$.

We find that anomalous parameters lead to topological classes that substantially differ from those in the original table [4–6]. We work out the general classification in class AIII [21] (which only has the chiral symmetry) and show that the classification is $\mathbb{Z} \times \mathbb{Z}$ when the numbers of normal and anomalous parameters are both odd. We further determine the case in class DIII [21] (which has both TRS and PHS) with $d_\phi = 1$. In particular, for class DIII with $d_k = d_\phi = 1$ we show there is a $\mathbb{Z}_2 \times \mathbb{Z}_2$ strong topological invariant. One $\mathbb{Z}_2$ invariant describes a TRS (PHS) invariant version of the fermion parity (spin) pump, whereas the other Z_2 reflects the non-Abelian statistics of Majorana Kramers pairs, leading to three distinct fractional Josephson effects, as TRS invariant topological pumping cycles in SCs. Our main results are summarized in Fig. 1 and Tables I and II.

$\mathbb{Z} \times \mathbb{Z}$ invariant. We first analyze the simplest case, class AIII, in which antiunitary symmetries are absent and show the chiral symmetry (Π) leads to a $\mathbb{Z} \times \mathbb{Z}$ invariant. We will use this result later to derive a $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant in class DIII. Moreover, on its own it can be used to classify the pumps in Josephson effects with a mirror symmetry, in which each mirror eigenspace by itself has chiral symmetry [17], but no TRS or PHS.

Consider a gapped Hamiltonian satisfying

$$\Pi^{-1}\mathcal{H}(k,\phi)\Pi = -\mathcal{H}(k,s\phi), \tag{1}$$

where $s = \pm$. Focusing on the strong invariant, we may think of k (ϕ) as the azimuth (polar) angle of a sphere, where $-\pi \leqslant k,2\phi \leqslant \pi$. The chiral symmetry (1) requires the valence (v) and conduction (c) band Berry curvature to satisfy $\mathcal{F}^v(k,\phi) = s\mathcal{F}^c(k,s\phi)$, whereas the completeness relation of wave functions restricts $\sum_{i=c,v}\mathcal{F}^i(k,\phi) = 0$, leading to $\mathcal{F}^v(k,\phi) = -s\mathcal{F}^v(k,s\phi)$. Consequently, for normal cases [4–6] in which $s = +$, the Chern number must vanish, whereas for anomalous parameters with $s = -$, the Chern number survives. Moreover, along the equator, $\mathcal{H}(k,0)$ describes a normal one-dimensional insulator in class AIII, which has an integer winding number [4,17,22].

Now we demonstrate that the Chern number N_c and the winding number N_w are distinct but related. Winding numbers may be evaluated [17] by introducing a continuous deformation that trivializes $\mathcal{H}(k,0)$ to a constant Π, formulated by $\mathcal{H}_0(k,\phi') = \mathcal{H}(k,0)\cos\phi' + \Pi\sin\phi'$. In this trivial gauge, Stokes' theorem relates the loop integral of Berry connection along the equator to the integral of Berry curvature over the

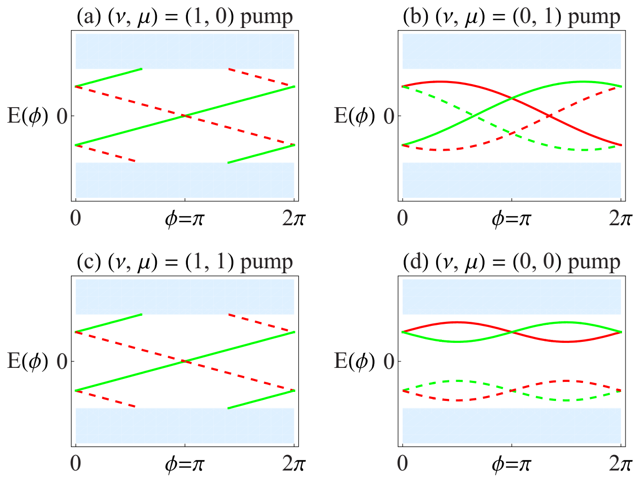

FIG. 1. (Color online) The three $\mathbb{Z}_2$ topological and one trivial pumps in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ Josephson effects for class DIII SCs. Green and red (solid and dashed) states are related by TRS (PHS). The shaded areas are the bulk continuum.

upper hemisphere (u.h.),

$$\pi N_w = \oint \mathcal{A}^v(k)dk = \int_{\text{u.h.}} \mathcal{F}_0^v(k,\phi')dkd\phi'. \tag{2}$$

Moreover, a Chern number is produced if one glues together the two u.h. integrals of $\mathcal{F}_0^v(k,\phi')$ and $\mathcal{F}^v(k,\phi)$ along the common equator, i.e.,

$$2\pi N_d = \int_{\text{u.h.}} \mathcal{F}_0^v(k,\phi')dkd\phi' - \int_{\text{u.h.}} \mathcal{F}^v(k,\phi)dkd\phi. \tag{3}$$

As a result of $\mathcal{F}^v(k,\phi) = \mathcal{F}^v(k,-\phi)$ derived above, the last integral is πN_c. We therefore conclude

$$N_w = N_c + 2N_d. \tag{4}$$

Thus, the winding number along the equator and the Chern number over the sphere are distinct but only differ by an even integer. To further illustrate this unprecedented relation, consider the following smooth and nonsingular Hamiltonian flattened on a unit sphere:

$$\mathcal{H} = \cos(3\phi)[\cos(Nk)\sigma_x + \sin(Nk)\sigma_y] + \sin(3\phi)\sigma_z. \tag{5}$$

Here $\Pi = \sigma_z$; the integer $N = N_w$ when $|\phi| \leqslant \pi/6$ and $N = N_d$ otherwise. Along the equator $\mathcal{H}(k,0)$ has a winding number N_w in the trivial gauge, whereas over the sphere $\mathcal{H}(k,\phi)$ has a Chern number $N_w - 2N_d$.

This result can be generalized to higher "dimensions." When $d_k + d_\phi = 2n$ is even, Eq. (1) with $s = -$ leads to

$$\text{Tr}[\mathcal{F}(\boldsymbol{k},\boldsymbol{\phi})^n] = (-1)^{d_\phi+1}\,\text{Tr}[\mathcal{F}(\boldsymbol{k},-\boldsymbol{\phi})^n], \tag{6}$$

where the Berry curvature $\mathcal{F} = d\mathcal{A} + \mathcal{A}\wedge\mathcal{A}$ and $\mathcal{A}$ is the non-Abelian Berry connection. The nth Chern number is an integral of Eq. (6) over the extended BZ spanned by $\boldsymbol{k}$ and $\boldsymbol{\phi}$. Thus, the Chern number vanishes when d_ϕ is even, whereas it survives when d_ϕ is odd. When d_k is odd, there is a winding number along the equator spanned by $\boldsymbol{k}$. These results are summarized in Table I.

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Josephson effects. Before advancing the $\mathbb{Z}_2 \times \mathbb{Z}_2$ strong invariant for class DIII with $d_\phi = d_k = 1$, we first analyze the boundary consequence, i.e., the spectra of Andreev bound states (ABSs) as a function of the phase difference ϕ, which naturally arises from the Josephson effects for topological SCs with TRS [16,23–26].

Figure 1(a) describes the spectrum of two ABSs with a single crossing at $\phi = \pi$ and $E = 0$. This twist reminds us of the fractional Josephson effect [10,18,27,28], yet the switching must occur at $\phi = \pi$ or 0 as required by TRS. This crossing is also reminiscent of the TI edge state [1–3,12], with an extra feature being the PHS between the conduction and valence bands. This pump is even robust against TRS (PHS) breaking as long as one symmetry is intact, and thus is the coincidence of spin and fermion parity pumps. A symmetry-allowed perturbation can only gap an even number of such spectra, indicating that this pumping cycle is characterized by a $\mathbb{Z}_2$ index $\nu = 1$. Such a topological pump can be realized by proximity coupling TI edge states [18] or hybridized double Rashba wires [25] to an s wave Josephson junction.

Figure 1(b) depicts the spectrum of four ABSs exhibiting a pair of zero-energy crossings. The degeneracies at $\phi = 0$ and π are required by TRS, whereas the crossings at $E = 0$ are protected by local conservation of fermion parity. By examining a model below, we find that the zero-energy crossings cannot be annihilated without breaking TRS or PHS, even if they are brought together. However, an even number of such double crossings can be gapped out by a symmetry-allowed disturbance. These features, in sharp contrast to Fig. 1(a), imply a distinct $\mathbb{Z}_2$ index $\mu = 1$. Importantly, this topological pump explicitly shows the non-Abelian statistics of Majorana Kramers pairs protected by TRS. In the fermion parity (0 or 1) basis of each Kramers partner (↑ or ↓), the adiabatic pumping of fermion parity and spin follows

$$|0_\uparrow 0_\downarrow\rangle \to |1_\uparrow 0_\downarrow\rangle \to |1_\uparrow 1_\downarrow\rangle \to |1_\uparrow 0_\downarrow\rangle \to |0_\uparrow 0_\downarrow\rangle, \tag{7}$$

in which ϕ advances by π in each step. This topological pump can be achieved [16] through proximity coupling a Rashba wire to an $s_\pm$ wave Josephson junction.

Since μ and ν are independent indices, it is possible to have a third $\mathbb{Z}_2$ topological pump with $\nu = \mu = 1$, which is shown in Fig. 1(c) [29] and will be discussed later. For comparison, the trivial pump is plotted in Fig. 1(d).

TABLE I. Topological classifications of gapped Hamiltonians in class AIII with and without anomalous parameters.

$d_k - d_r$	Even	Odd
$d_\phi - d_\theta$ = even	0	$\mathbb{Z}$
$d_\phi - d_\theta$ = odd	0	$\mathbb{Z}\times\mathbb{Z}$

TABLE II. Topological classifications of adiabatic pumps in class DIII, with zero or one anomalous parameter effectively.

$(d_k - d_r)$ mod 8	0,4,5,6	1	2	3	7
$d_\phi - d_\theta = 0$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$2\mathbb{Z}$
$d_\phi - d_\theta = 1$	0	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}\times\mathbb{Z}$	$2\mathbb{Z}\times 2\mathbb{Z}$

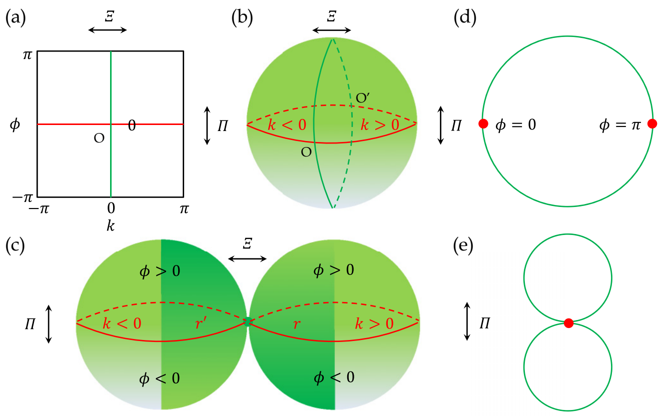

FIG. 2. (Color online) (a) The extended BZ of $\mathcal{H}(k,\phi)$ in class DIII. (b)–(e) Schematics of the contraction and deformation of the special lines and points on the torus (a) in our homotopy theory.

Homotopy argument. Now we derive the above $\mathbb{Z}_2 \times \mathbb{Z}_2$ strong invariant using a homotopy argument in the spirit of the Moore-Balents [13] argument on the $\mathbb{Z}_2$ TIs. This approach has an advantage of being model independent. When folded into each other, the two SCs coupled at a Josephson junction may be described by a Hamiltonian $\mathcal{H}(k,\phi)$ in class DIII. The Josephson effects sketched in Fig. 1 can thus be interpreted as the boundary consequences of the bulk invariant of $\mathcal{H}(k,\phi)$, which inherits PHS and TRS constraints:

$$\Xi^{-1}\mathcal{H}(k,\phi)\Xi = -\mathcal{H}(-k,\phi), \tag{8}$$

$$\Theta^{-1}\mathcal{H}(k,\phi)\Theta = \mathcal{H}(-k,-\phi), \tag{9}$$

the combination of which determines the chiral symmetry specified by Eq. (1) with $s = -$. We focus on deriving the strong invariants instead of the weak ones, so we assume the topologically nontrivial physics only occurs near $k,\phi = 0$. The $k,\phi = \pi$ lines may be trivialized and contract to a point O'. Hence the torus in Fig. 2(a), the extended BZ of $\mathcal{H}(k,\phi)$, is topologically equivalent to a sphere in Fig. 2(b). Because of the topological triviality of $\mathcal{H}(0,\phi)$ which will be demonstrated below, the $k = 0$ circle may further contract to a trivial point, as described in Fig. 2(c). The resulting two spheres are related by PHS (TRS), whereas each one only respects the chiral symmetry and thus has a $\mathbb{Z} \times \mathbb{Z}$ strong invariant, as we have demonstrated in Eq. (4).

However, there are multiple topologically inequivalent contractions from the $k = 0$ circle to a point. These ambiguities reduce the $\mathbb{Z} \times \mathbb{Z}$ invariant to a $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant. For a homotopic deformation, it is required that at each stage the contracted circle has the same symmetry constraints as the original one. Thus the contraction is naturally parametrized by r, which is even under both TRS and PHS, as shown in Fig. 2(c). The two hemispheres forming in the contraction can be glued into a sphere, which can be described by a Hamiltonian $\mathcal{H}(r,\phi)$ in class DIII. From Table I, $\mathcal{H}(r,\phi)$ has a $\mathbb{Z} \times \mathbb{Z}$ invariant. Yet the antiunitary symmetries require both integers to be even. Indeed, the original table [6] has revealed that the winding number of $\mathcal{H}(r,0)$ is $2\mathbb{Z}$ in class DIII, and that the Chern number of $\mathcal{H}(r,\phi)$ is also $2\mathbb{Z}$ in both class D and AII. Similarly to Eq. (4), here the winding number and the Chern number are distinct but only differ by $4\mathbb{Z}$. This not only explains that there are $2\mathbb{Z} \times 2\mathbb{Z}$ topologically distinct contractions of the $k = 0$ circle, but also determines the invariant of $\mathcal{H}(r,\phi)$ in class DIII.

The remaining task is to show $\mathcal{H}(\phi)$ is topologically trivial in class DIII. In the original table [4,5], zero-dimensional DIII SCs are trivial and thus we can glue together the two points $\phi = 0$ and π, as done in Fig. 2(e). The two resulting circles are related by TRS (chiral symmetry), whereas each one has PHS, i.e., in class D. As $\mathcal{H}(\phi)$ in class D is trivial as shown in the table [6], we conclude that $\mathcal{H}(\phi)$ is indeed trivial in class DIII.

So far, we have established the strong invariants for $\mathcal{H}(k,\phi)$, $\mathcal{H}(r,\phi)$, and $\mathcal{H}(\phi)$ in class DIII, which are summarized in Table II. Now we demonstrate the remaining nontrivial entries in this table. Consider the case for $d_\phi = 1$ and $d_k = 3$ in Table I; the second Chern number is compatible with having TRS and PHS, as it exists in class AII with $d_k + d_\phi = 4$ [5,14] and in class D with $d_k - d_\phi = 2$ [6]. The winding number along the equator is exactly the integer invariant in class DIII with $d_k = 3$ [4,22]. Hence, $\mathcal{H}(\boldsymbol{k},\phi)$ with $d_k = 3$ in class DIII has a $\mathbb{Z} \times \mathbb{Z}$ invariant. As for $\mathcal{H}(\boldsymbol{k},\phi)$ with $d_k = 2$ in class DIII, there can be a weak $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant in each of the two "planes" with $d_k = d_\phi = 1$, as we have demonstrated above. If one plane is trivial, then the invariant in the other plane becomes a strong invariant, which is analogous to the relation between two- and three-dimensional $\mathbb{Z}_2$ TIs [13]. In light of these analyses, we complete Table II, which has Bott periodicity 8 in $d_k - d_r$ [5,6].

Effective theory. Table II suggests a dimension reduction rule, generalizing the case [14] with no anomalous parameter. This becomes more clear in a minimal effective theory near $\boldsymbol{k} = 0$ and $\phi = \pi$. We choose a gauge in which PHS and TRS operators are $\Xi = \mathcal{K}$ and $\Theta = \sigma_y\mathcal{K}$. Consider a flattened four-band (eight-band) model,

$$\mathcal{H} = k_x\sigma_x s_x + k_z\sigma_z + \delta\phi\sigma_y(\tau_z) + k_y\sigma_x s_z + M\sigma_x s_y, \tag{10}$$

where $M = m - \boldsymbol{k}^2 - \delta\phi^2$ and $\hat{k}_y$ is normal to the Josephson junction. The boundary problem is specified by the last two terms in Eq. (10), with m switching signs [30].

The four-band model has $N_c = N_w = 1$, and we can derive the ABS spectrum,

$$\bar{\mathcal{H}} = k_x\sigma_x + k_z\sigma_z + \delta\phi\sigma_y, \tag{11}$$

which resembles a "Weyl fermion." Any perturbation in $\bar{\mathcal{H}}$ is prohibited by *both* TRS and PHS. Even an even number of such spectra cannot be gapped, consistent with the invariants being integers. When one or two k terms are taken off, $\bar{\mathcal{H}}$ describes the lower dimensional cases. Although any disturbance is still prohibited, a pair of such spectra can be gapped without breaking any symmetry, indicating the $\mathbb{Z}_2$ character.

The eight-band model has $N_c = 0$ and $N_w = 2$, and the corresponding ABSs can be described by

$$\tilde{\mathcal{H}} = k_x\sigma_x + k_z\sigma_z + \delta\phi\sigma_y\tau_z, \tag{12}$$

resembling a "Dirac fermion." In the presence of *both* TRS and PHS, a perturbation in $\tilde{\mathcal{H}}$ is allowed but cannot gap the spectrum for the $d_k = 1,2,3$ cases. Without breaking a symmetry, two copies of $\widetilde{\mathcal{H}}$ can be gapped for the $d_k = 1,2$ cases but not for the $d_k = 3$ case, reflecting their $\mathbb{Z}_2$ and integer invariants, respectively.

In light of the above model analysis, the three $\mathbb{Z}_2$ topological pumps in Figs. 1(a)–1(c) can be described respectively by $\bar{H}_a = \delta\phi\sigma_y$, $\widetilde{H}_b = \delta\phi\sigma_y\tau_z + \Delta\sigma_{x,z}\tau_y$, and $\bar{H}_c = -\delta\phi\sigma_y$ [29]. Two copies of each $\mathbb{Z}_2$ pump can be gapped out without breaking any symmetry, whereas combining any two different $\mathbb{Z}_2$ pumps leads to the third $\mathbb{Z}_2$ pump. Evidently, for the $d_k = 3$ cases, the protection of the gapless nature of $\bar{\mathcal{H}}$ requires no symmetry, whereas that of $\widetilde{\mathcal{H}}$ requires both symmetries. This difference, together with the difference in N_w and N_c, distinguishes the two $\mathbb{Z}_2$ invariants deduced from the two models.

Discussion. The $\mathbb{Z}_2$ topological pump described in Fig. 1(b) is a general feature of the Josephson effect for two TRS invariant topological SCs [16,25,31]. This may be most easily achieved [16] by proximity coupling a Rashba wire to an $s_\pm$ wave Josephson junction. The $\mathbb{Z}_2$ topological pump depicted in Figs. 1(a) or 1(c) may be realized by proximity coupling TI edge states [18] or hybridized double Rashba wires [25] to an s wave Josephson junction. Note that when $\nu = 1$, the second $\mathbb{Z}_2$ index μ which distinguishes Figs. 1(a) and 1(c) is not gauge invariant. In this case, $\mu = 0$ and $\mu = 1$ topological pumps can be switched by advancing the phases of both SCs by π. Yet, their distinction is meaningful, as coupling two copies of Fig. 1(a) or Fig. 1(c) yields to Fig. 1(d), whereas coupling Fig. 1(a) and Fig. 1(c) produces Fig. 1(b).

The 4π Josephson effect in Fig. 1(b) was first proposed in Ref. [16] and further studied in Refs. [25,31]. However, each one requires an *extra* mirrorlike symmetry to decompose the class DIII pump into two decoupled pumps in class AIII or in class D. Importantly, as demonstrated in this Rapid Communication, the fractional Josephson effect in Fig. 1(b) and the non-Abelian statistics of Majorana Kramers pairs can be protected by the TRS even in the absence of any extra symmetry.

With similar homotopy arguments on "bulk" invariants, stability analysis of "boundary" consequences, and the Clifford algebra of representative models, our results can be generalized to another three symmetry classes with the chiral symmetry and to the DIII cases with more than one anomalous parameters. Thus, we anticipate an extended new table with $8 \times 8 \times 10$ entries, specified by $d_k - d_r$, $d_\phi - d_\theta$, and symmetry classes, to completely classify topological phases for gapped free fermion systems. Besides the strong invariants that we have established, we note there exist multiple weak invariants in lower dimensional subspaces of the extended BZ.

Note added in proof. Recently, it has been discovered [32] that the dissipative $\mathbb{Z}_2$ Josephson effect, shown in Fig. 1(a) or Fig. 1(c), becomes a dissipationless $\mathbb{Z}_4$ Josephson effect in the presence of electron-electron interactions.

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