Electromagnetic and gravitational responses of two-dimensional noninteracting electrons in a background magnetic field

Alexander G. Abanov[1,2] and Andrey Gromov[1]
[1]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*
[2]*Simons Center for Geometry and Physics, Stony Brook University, Stony Brook, New York 11794, USA*
(Received 1 February 2014; revised manuscript received 14 July 2014; published 29 July 2014)

We compute electromagnetic, gravitational, and mixed linear response functions of two-dimensional free fermions in an external quantizing magnetic field at an integer filling factor. The results are presented in the form of the effective action and as an expansion of currents and stresses in wave vectors and frequencies of the probing electromagnetic and metric fields. In addition to the well-studied $U(1)$ Chern-Simons and Wen-Zee terms we find a gravitational Chern-Simons term that controls the correction to the Hall viscosity due to the background curvature. We relate the coefficient in front of the term with the chiral central charge.

DOI: 10.1103/PhysRevB.90.014435 PACS number(s): 73.43.−f, 67.10.Jn, 71.45.Gm

I. INTRODUCTION

Recent interest in the Hall viscosity in the theory of the fractional quantum Hall effect (FQHE) and the interest to the interplay of defects and mechanical stresses with the electromagnetic properties of materials motivates studies of gravitational, electromagnetic, and mixed responses in condensed matter physics. The gravitational field in condensed matter systems can be understood either as a way to represent the deformational strains present in the material under consideration or as a technical tool allowing to extract the correlation functions involving stress tensor components.

It is always important to have a simple model system for which such responses can be calculated exactly. For the quantum Hall effect one can consider two-dimensional electron gas in a constant magnetic field (2DEGM) as such a model. When the density of fermions is commensurate with the magnetic field the integer number of Landau levels is filled and one expects a local and computable response to weak external fields. This model is an important starting point of analysis for quantum Hall systems as a free electron gas for the theory of metals. However, while some electromagnetic responses for 2DEGM can be found in the literature, we were not able to find the complete results for mixed and gravitational linear responses. The goal of this paper is to compute these responses providing the analog of the Lindhard [1] function, both electromagnetic (e/m) and gravitational, for 2DEGM. We compute the effective action encoding linear responses in the presence of external inhomogeneous, time-dependent, slowly changing electromagnetic and gravitational fields.

We compare and find an agreement of the obtained responses with known e/m responses [2–5] and with known results for Hall viscosity at integer fillings [6,7]. In addition, we find the stress, charge, and current densities induced by perturbations of spatial geometry. Another point of comparison is given by phenomenological hydrodynamic models for FQHE [8–13] and Ward identities following from the exact local Galilean symmetry (also known as nonrelativistic diffeomorphism) of the model [14,15].

The main results of this paper are presented in Eqs. (11) and (30). The first of these states that the low-energy effective action for the integer quantum Hall system is *not* completely captured by the Wen-Zee arguments [16] and the correct coefficient in front of the gravitational Chern-Simons term is *not* completely determined by the orbital spin and the filling fraction, but, in addition, requires the knowledge of the chiral central charge. The second of these states that the chiral central charge manifests itself in a curved space and shifts the value of the Hall viscosity. In particular, Eq. (30) implies that one could determine the chiral central charge, and therefore, thermal Hall conductivity [17,18] from the Hall viscosity computed on a curved space.

II. MODEL

Our starting point is the system of a two-dimensional free nonrelativistic fermions interacting with an external gauge A_μ and spatial metric g_{ij} fields. We assume that the spatial metric can depend on time. The action has the form

$$S = \int d^2x dt \sqrt{g}\left[\frac{i}{2}\hbar\psi^\dagger\partial_0\psi - \frac{i}{2}\hbar(\partial_0\psi^\dagger)\psi + eA_0\psi^\dagger\psi - \frac{\hbar^2}{2m}g^{ij}(D_i\psi)^\dagger D_j\psi + \frac{g_s B}{4m}\psi^\dagger\psi\right]. \quad (1)$$

We assume that the fermions are spin polarized and treat the ψ field as a complex Grassman scalar. We have also added a Zeeman term with the g factor g_s. For the case of electrons in a vacuum $g_s = 2$, but it is convenient to keep it arbitrary for potential condensed matter applications. The covariant derivative $D_i = \partial_i - i\frac{e}{\hbar c}(\bar{A}_i + A_i)$ and includes both the vector potential of the constant background magnetic field $B_0 = \partial_1\bar{A}_2 - \partial_2\bar{A}_1$ and a weak perturbation. In the curved background the magnetic field is defined as $B = \frac{1}{\sqrt{g}}(\partial_1\bar{A}_2 - \partial_2\bar{A}_1 + \partial_1 A_2 - \partial_2 A_1)$, so it transforms as a (pseudo)scalar under coordinate transformations. We separate it into constant part and perturbation as $B = B_0 + b$. In this work we use the expression linear in fields $b = B - B_0 \approx \partial_1 A_2 - \partial_2 A_1 - \frac{1}{2}\delta g_{ii}$. Here δg_{ij} is a deviation from the flat background $g_{ij} = \delta_{ij} + \delta g_{ij}$.

We omit the chemical potential term in Eq. (1) for brevity, but assume throughout the paper that the lowest N Landau levels are completely filled in the ground state. We use conventional notations for metric fields so that g_{ij} and g^{ij} are reciprocal matrices and an invariant spatial volume is given by $\sqrt{g}\, d^2x$ with $g = \det(g_{ij})$.

 ©2014 American Physical Society

To find the linear responses of the system (1) with respect to perturbations in the gauge and metric fields we compute the effective (induced) action of the theory in quadratic approximation. The effective action S_{eff} is defined as a path integral over the fermionic fields

$$e^{\frac{i}{\hbar}S_{\text{eff}}[A_\mu,g_{ij}]} \equiv \int D\big(g^{\frac{1}{4}}\psi\big)D\big(g^{\frac{1}{4}}\psi^\dagger\big)e^{\frac{i}{\hbar}S}\,. \tag{2}$$

The notation [19,20] $D(g^{\frac{1}{4}}\psi)$ serves as a reminder that the path integral is taken over the space of functions $\psi(x),\psi^\dagger(x)$ equipped with the invariant scalar product given by

$$(\psi,\phi) \equiv \int d^2x\,\sqrt{g}\psi^\dagger\phi\,. \tag{3}$$

We stress that only a *finite* number of the Landau levels is filled, therefore, only a finite number of eigenmodes contributes to the fluctuation determinant. There are no divergencies and no renormalization is required.

III. SPIN CONNECTION

In the external magnetic field the electron is moving along a circular orbit. There is an orbital spin $\bar{s}$ associated to this motion. The orbital spin is not a part of the original action (1), but an *emergent* phenomenon [6] that appears after the averaging over the "fast" cyclotron motion of the electron. The orbital spin couples to the *emergent* $SO(2)$ spin connection, just like electric charge couples to the vector potential.

The Levi-Civita $SO(2)$ spin connection can be expressed in terms of the vielbeins as [14]

$$\omega_0 = -\frac{\epsilon^{ab}}{2}e^{aj}\partial_0 e^b_j, \quad \omega_i = -\frac{\epsilon^{ab}}{2}e^{aj}\partial_i e^b_j + \frac{\epsilon^{jk}}{2\sqrt{g}}\partial_j g_{ik}\,. \tag{4}$$

This connection transforms like an abelian gauge field under the local $SO(2)$ rotations [14].

There are several general arguments that explain why the spin connection has to be a part of the low-energy description of the FQH states [16,21,22]. Nevertheless, there is a confusion in the literature about the Chern-Simons term that can be constructed from ω_μ. Methods based on the local Galilean invariance cannot say anything about the term or the coefficient in front of it because it is too far in the gradient expansion. The methods of the authors of [16] give the wrong prediction for the coefficient in front of the gravitational Chern-Simons (gCS) term. The major result of this paper is the direct computation of the coefficient. We notice that the mismatch between our computation and the result of [16] is captured *precisely* by the gravitational anomaly.

IV. EFFECTIVE ACTION

The effective action defined in Eq. (2) can be computed as a regular expansion in background fields $A_\mu(x,t)$ and $g_{ij}(x,t)$ and their gradients. In the following we expand the effective action to quadratic order in the external fields. It is convenient to separate it as

$$S_{\text{eff}} = S^{(1)}_{\text{eff}} + S^{(\text{geom})}_{\text{eff}} + S^{(2)}_{\text{eff}}\,. \tag{5}$$

The first contribution is given by

$$S^{(1)}_{\text{eff}} = \int d^2x dt\sqrt{g}[-\epsilon_0 + \rho_0 A_0 + s_0\omega_0]\,, \tag{6}$$

where ω_0 is the time component of the spin connection and ϵ_0, ρ_0, and s_0 are the energy density, density, and the orbital spin density in the ground state. They are given, respectively, by

$$\rho_0 = \frac{N}{2\pi l^2}\,, \quad \epsilon_0 = \rho_0\,\hbar\omega_c\frac{2N-g_s}{4}\,, \quad s_0 = \rho_0\hbar\frac{N}{2}\,. \tag{7}$$

Here and throughout the paper we use conventional notations for magnetic length and cyclotron frequency given in terms of the constant part of the background magnetic field B_0 as

$$l^2 = \frac{\hbar c}{eB_0}\,, \quad \omega_c = \frac{eB_0}{mc}\,. \tag{8}$$

We notice here that although Eq. (6) includes all terms linear in A_μ, g_{ij} they also contain several quadratic terms. Indeed, the expansion of the $\sqrt{g}$ in terms of the deviations from the flat background is

$$\sqrt{g} = 1 + \tfrac{1}{2}\delta g_{ii} - \tfrac{1}{8}\big[(\delta g_{11}-\delta g_{22})^2 + 4\delta g_{12}\delta g_{21}\big] + \cdots \tag{9}$$

and Eq. (6) should be re-expanded and truncated to the terms up to the second order in fields.

The second term in Eq. (5) contains the topological and *geometrical* contributions to the effective action (with $\hbar = c = e = 1$)

$$S^{(\text{geom})}_{\text{eff}} = \frac{N}{4\pi}\int\left(AdA + NAd\omega + \frac{2N^2-1}{6}\omega d\omega\right), \tag{10}$$

where we used the "form notation" $\int AdA \equiv \int d^2x dt\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$. The coefficients of the three terms in Eq. (10) give, respectively, the Hall conductivity $\sigma_H = \frac{N}{2\pi}$, the average orbital spin per particle $\bar{s} = \frac{N}{2}$ (corresponding to the Wen-Zee shift $\mathcal{S} = N$), and the gCS coefficient $\frac{N(2N^2-1)}{24\pi}$.

The following comment is in order. The action (1) is written in terms of the gauge potential A_μ and metric g_{ij}. It does not require spin connection ω_μ as it is already covariant due to the fact that ψ is a scalar field. Thus, the $S^{(\text{geom})}_{\text{eff}}$ should also depend solely on the vector potential and metric. It is, however, instructive to write $S^{(\text{geom})}_{\text{eff}}$ in terms of A_μ and ω_μ as we did in Eq. (10). With the accuracy used in this paper the dependence on ω_μ can be restored with the help of linearized version of (4), i.e., $\omega_i \leftrightarrow -\frac{1}{2}\epsilon^{jk}\partial_j\delta g_{ik}$ and $\omega_0 \leftrightarrow \frac{1}{2}\epsilon^{jk}\delta g_{ij}\delta\dot{g}_{ik}$.

It is illuminating to present Eq. (10) as an explicit sum over Landau levels

$$S^{(\text{geom})}_{\text{eff}} = \sum_{n=1}^{N}\int\left[\frac{1}{4\pi}(A+\bar{s}_n\omega)d(A+\bar{s}_n\omega) - \frac{c}{48\pi}\omega d\omega\right], \tag{11}$$
$$c = 1,$$

where $\bar{s}_n = \frac{2n-1}{2}$ is the orbital spin per particle on the nth Landau level and the last term is an anomalous gCS contribution the same for all Landau levels. It is equal to the nonrelativistic limit of the well-known relativistic gCS term [23]. This last is related to the gravitational anomaly via the usual anomaly inflow arguments. Its presence shows that the spin connection *does not* simply combine with the vector potential in the effective action as suggested in [16,21]. We speculate that the offset is due to the gravitational anomaly experienced by the chiral edge modes in the curved space.

The physical meaning of Chern-Simons and Wen-Zee terms have been extensively discussed in the literature. The relativistic gCS term is usually related to the transverse heat transport via Luttinger's argument [23–27]. The last term in Eq. (5) gives the remaining second-order terms

$$S^{(2)}_{\rm eff} = \int d^2x dt\, \mathcal{L}^{(2)}\,,$$

$$\mathcal{L}^{(2)} = \frac{1}{2}\left(A_\mu \Pi^{\mu\nu} A_\nu + A_\mu \Theta^{\mu}_{ij}\delta g^{ij} + \delta g^{ij}\Lambda_{ijkl}\delta g^{kl}\right), \quad (12)$$

where differential operators Π,Θ,Λ encode the electromagnetic, mixed, and gravitational responses, respectively. These operators can be computed exactly as infinite series in time and spatial derivatives or as a series in frequency and wave vectors in Fourier representation. We will present the details of the computation elsewhere and give here only the results obtained in the lowest orders in gradients

$$\frac{4\pi}{N}\mathcal{L}^{(2)} = ml^2E_i^2 - \frac{N}{m}b^2 - \frac{3N}{2}l^2 b(\partial_i E_i) - \frac{2N^2-1}{4m}bR\,, + \frac{2N^2-1}{6}l^2R(\partial_iE_i) + \frac{N(N^2-1)}{8m}R^2 + \cdots\,, \quad (13)$$

where R is the scalar curvature given by $R = \partial_i\partial_j\delta g_{ij} - \Delta\delta g_{ii}$ after linearization. While the first three terms of the expansion (13) can be found in the literature [2] the other terms are new.

The effective action presented above is, probably, the most compact way to summarize linear responses. However, we find it convenient to have direct formulas for observables such as charges, currents, and stresses in a dynamic and inhomogeneous background. We present the explicit expressions and their physical meaning for linear responses in the next sections. For illustration purposes and to lighten up the equations in the following we consider only the lowest Landau level filled, i.e., $N = 1$.

V. DENSITY

The expectation value of the electric charge density is given by the variational derivatives of the action (2) with respect to the scalar potential

$$\rho(x) \equiv \frac{1}{\sqrt{g}}\frac{\delta S_{\rm eff}}{\delta A_0(x)} = \langle\psi^\dagger\psi\rangle. \quad (14)$$

In the curved background the density has to be understood as the number of particles per invariant volume element

$$\rho-\rho_0 = \frac{1}{2\pi}\left(1+\frac{3-g_s}{4}l^2\Delta\right)b + \frac{1}{8\pi}\left(1+\frac{1}{3}l^2\Delta\right)R + \frac{ml^2}{2\pi}\left(1+\frac{3}{8}l^2\Delta\right)(\partial_iE_i)\,, \quad (15)$$

where Δ is the Laplacian.

Integrating Eq. (15) over a closed manifold we obtain the shift of the degeneracy of the lowest Landau level due to the topology of the manifold

$$Q = \int d^2x\sqrt{g}\rho = \int d^2x\sqrt{g}\left(\frac{B}{2\pi}+\frac{R}{8\pi}\right) = N_\phi + \frac{1}{2}\chi\,, \quad (16)$$

where N_ϕ is the total magnetic flux and χ is the Euler characteristics of the manifold [16]. The correction to the density due to curvature gradients in Eq. (15) is in agreement with [13,28]. Extending Eq. (16) to the case of an isolated conic singularity with the deficit angle θ we find

$$\delta Q = \int d^2x\sqrt{g}(\rho-\rho_0) = \frac{1}{8\pi}\int d^2x\sqrt{g}R = \frac{1}{4\pi}\theta\,. \quad (17)$$

The points of higher positive curvature suck particles in and increase local density. Although the derivation presented here cannot be rigorously applied to the case of conic singularity where the curvature $R = 2\theta\delta(x)$ is highly singular, the integral formula (17) is exact and can be checked by direct computation of the density on a surface of the cone.

A detailed discussion of the dynamic response functions requires an analysis of the local Galilean invariance and is beyond the scope of this paper. In the following we illustrate some structures arising as the time dependence is introduced.

In the flat background and for $N = 1$, $g_s = 0$ we have

$$\frac{\rho(\omega)}{\rho_0} = \frac{1}{1-\omega^2}\left(1 + l^2b + ml^4\partial_iE_i - \frac{3}{2}l^2\Delta\frac{2l^2b+ml^4\partial_iE_i}{4-\omega^2} + \cdots\right),$$

where ω is measured in units of ω_c. The overall pole at $\omega = 1$ is expected even in the presence of interactions as a consequence of the Kohn's theorem. The poles at $\omega = n$, $n = 2,3,\ldots$, corresponding to transitions between different Landau levels occur in the next terms of the gradient expansion.

Expanding in frequency and including the gravitational perturbations we have the leading term (first order in the time derivative)

$$\rho(x,t) = \rho(x,0) + \frac{3}{16\pi ml^2}\epsilon_{ij}\partial_i\partial_k\dot{g}_{ik} \quad (18)$$

with $\rho(x,0)$ given by Eq. (15).

VI. ELECTRIC CURRENT

The expectation value of the electric current density is given by the variational derivative of the action (2) with respect to the vector potential

$$J^i(x) \equiv \frac{1}{\sqrt{g}}\frac{\delta S_{\rm eff}}{\delta A_i(x)} = \left\langle\frac{g^{ij}}{2mi}[\psi^\dagger D_j\psi - (D_j\psi)^\dagger\psi]\right\rangle. \quad (19)$$

We find

$$\langle J_i\rangle = \epsilon^{ij}\left[\sigma_H E_j + \frac{2-g_s}{4\pi m}\partial_j\left(b+\frac{R}{8}\right)\right], \quad (20)$$

where the wave-vector-dependent Hall conductivity is given by

$$\sigma_H(k) = \frac{1}{2\pi}\left(1 - \frac{3-g_s}{4}|kl|^2 + \frac{22-9g_s}{96}|kl|^4\right). \quad (21)$$

The correction of the order of k^2 is in full agreement with the general results for Galilean-invariant systems [7,14]. The k^4 term calculated here is new.

The second term in Eq. (20) is another new result of this work. It shows that in low orders of the gradient expansion the gradient of the magnetic field and curvature affect current similarly to the electric field. We also point out that in agreement with [22] the $m \to 0$ limit is regular for $g_s = 2$.

VII. STRESS TENSOR

The expectation value of the stress tensor is given by

$$T_{ij} \equiv -\frac{2}{\sqrt{g}}\frac{\delta S_{\text{eff}}}{\delta g^{ij}(x)} = \frac{1}{2m}\langle (D_i\psi)^\dagger D_j\psi + (D_j\psi)^\dagger D_i\psi \rangle - \frac{1}{4m} g_{ij}(\Delta_g + g_s B)\langle \psi^\dagger\psi\rangle. \tag{22}$$

Here Δ_g is the Laplace-Beltrami operator defined as $\Delta_g\rho = \frac{1}{\sqrt{g}}\partial_i(g^{ij}\sqrt{g}\partial_j\rho)$ [29].

Using Eq. (22) we find the stress tensor in the leading order in gradients

$$T_{ij} = \frac{1}{8\pi}(\partial_i E_j + \partial_j E_i) + \delta_{ij}\left[\epsilon_0 - \frac{4-g_s}{8\pi}\partial_k E_k + \frac{2-g_s}{8\pi m l^2}\left(b + \frac{R}{8}\right)\right]. \tag{23}$$

The stress tensor has a regular limit $m \to 0$ limit for $g_s = 2$.

The action (1) is Weyl-invariant. The Weyl symmetry implies a relation between one-point correlation functions of the energy density and pressure

$$\epsilon = \frac{1}{2}T_i^{\ i}, \tag{24}$$

so Eq. (23) can be used to extract the energy density in the ground state in the presence of external fields. Keeping only the lower gradients we obtain the correction to the energy density

$$\epsilon - \epsilon_0 = -\frac{4-g_s}{8\pi}\partial_i E_i + \frac{2-g_s}{8\pi m l^2}\left(b + \frac{R}{8}\right). \tag{25}$$

In the case of an isolated conic singularity we get a contribution to the total energy $\frac{\delta E}{E_0} = \frac{\theta}{8\pi}$ per singularity [30].

VIII. HALL VISCOSITY

The time-dependent part of the stress tensor is related to another quantity of great interest: the Hall viscosity. We are looking for the parity odd terms in the stress tensor containing no more than two spatial derivatives.

$$T^{\text{odd}}_{ij} = \tfrac{1}{2}\eta_H(\epsilon_{ik}\dot{g}_{kj} + \epsilon_{jk}\dot{g}_{ki}) + \tfrac{1}{2}\eta_H^{(2)} l^2[\epsilon_{il}\partial_l\partial_j + \epsilon_{jl}\partial_l\partial_i]\dot{g}_{kk}, \tag{26}$$

where $\eta_H = \eta_H(\omega,k)$ is a generalization of the Hall viscosity to finite wave number and frequency (here $N = 1$ and we measure ω in units of ω_c)

$$\frac{\eta_H(\omega,k)}{\eta_H^{(0)}} = \frac{4}{4-\omega^2} + |kl|^2\left(\frac{1}{1-\omega^2} - \frac{6}{4-\omega^2} + \frac{6}{9-\omega^2}\right).$$

Here the conventional Hall viscosity

$$\eta_H(\omega = 0, k = 0) \equiv \eta_H^{(0)} = \tfrac{1}{2}\rho_0\bar{s}\,. \tag{27}$$

At zero wave vector $\eta_H(\omega,k=0)/\eta_H^{(0)} = 4/(4-\omega^2)$ is in full agreement with [7]. For the coefficient in front of the second tensor [second line of Eq. (26)] we have

$$\eta_H^{(2)} = \frac{1}{8}\rho_0\left(\frac{2}{1-\omega^2} - \frac{4}{4-\omega^2}\right). \tag{28}$$

In the static limit and for general N we rewrite the expression for the Hall viscosity as a sum over Landau levels

$$\eta_H(k,\omega = 0) = \frac{1}{2\pi l^2}\sum_{n=1}^{N}\left(\frac{\bar{s}_n}{2} + \frac{1}{4}\left[\bar{s}_n^2 - \frac{c}{12}\right]|kl|^2\right). \tag{29}$$

One has to recall that $c = 1$ and that the orbital spin per particle at the nth Landau level is $\bar{s}_n = \frac{2n-1}{2}$. We remark here that the gCS term gives a long wave k^2 correction to the Hall viscosity in a fashion similar to how the Wen-Zee term produces the long wave correction to the Hall conductivity [14]. In fact, the k^2 correction to the Hall viscosity (29) comes solely from the gCS term.

We note that the gCS term also corrects the value of the Hall viscosity in the presence of constant background curvature R_0. Indeed, the gCS term gives a contribution $\sqrt{g}\frac{1}{2}R_0\omega_0$ to the effective action, which results in $\delta\eta_H = \frac{N(2N^2-1)}{96\pi}R_0$. Then the total value of the Hall viscosity is given by

$$\eta_H = \frac{1}{2\pi l^2}\sum_{n=1}^{N}\left(\frac{\bar{s}_n}{2} + \frac{1}{8}\left[\bar{s}_n^2 - \frac{c}{12}\right]R_0 l^2\right). \tag{30}$$

The second term gives the correction due to the curvature of the background and should be compared to Eq. (29). If the coefficient c is indeed the chiral central charge then Eq. (30) suggests a very nontrivial relation. One could determine the chiral central charge (and, therefore, thermal Hall conductivity) simply measuring the Hall viscosity on a sphere.

Equation (30) is also in (somewhat surprising) correspondence with [31], where the same (for $N = 1$) curvature-induced shift of the relativistic version of the Hall viscosity was observed.

IX. CONCLUSION

We have considered noninteracting two-dimensional fermions in the background electromagnetic and metric fields (1). We have computed the effective action in the second order in deviations from the background of flat metric and constant magnetic field for an integer filling factor. The results are presented both in terms of the effective action and as linear response formulas for density, current, and stress. The effective action features topological Chern-Simons and geometric Wen-Zee and gravitational Chern-Simons terms. The last of these controls the correction to the Hall viscosity due to the presence of the background curvature. The coefficient in front of the term is related to the chiral central charge. The higher-gradient corrections to Hall conductivity and Hall viscosity have been computed.

The model considered is known to be of fundamental importance for the understanding of the quantum Hall effect and topological phases of matter. Our results provide a good starting point for refined variational and hydrodynamic approaches to QHE and elucidate phenomenological and symmetry-based relations between linear responses in Galilean-invariant systems found recently. A possible generalization of this work is to include torsion into the background to analyze responses to dislocations [32]. When the paper was already completed we learned about the work [33] where some of the results presented here have been extended to FQHE.

ACKNOWLEDGMENTS

We thank B. Bradlyn, M. Goldstein, T. Hughes, K. Jensen, G. Monteiro, P. van Nieuwenhuizen, and P. Wiegmann for helpful discussions. The work of A.G.A. was supported by the NSF under Grant No. DMR-1206790.

[1] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (Courier Dover, New York, 2003).
[2] Y.-H. Chen, F. Wilczek, E. Witten, and B. I. Halperin, Int. J. Mod. Phys. B **3**, 1001 (1989).
[3] S. Randjbar-Daemi, A. Salam, and J. Strathdee, Nucl. Phys. B **340**, 403 (1990).
[4] J. D. Lykken, J. Sonnenschein, and N. Weiss, Phys. Rev. D **42**, 2161(R) (1990).
[5] P. B. Wiegmann, Phys. Rev. Lett. **65**, 2070 (1990).
[6] N. Read, Phys. Rev. B **79**, 045308 (2009).
[7] B. Bradlyn, M. Goldstein, and N. Read, Phys. Rev. B **86**, 245309 (2012).
[8] S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B **33**, 2481 (1986).
[9] N. Read, Phys. Rev. Lett. **62**, 86 (1989).
[10] S. C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989).
[11] M. Stone, Phys. Rev. B **42**, 212 (1990).
[12] P. Wiegmann, Phys. Rev. B **88**, 241305(R) (2013).
[13] A. G. Abanov, J. Phys, A: Math. Theor. **46**, 292001 (2013).
[14] C. Hoyos and D. T. Son, Phys. Rev. Lett. **108**, 066805 (2012).
[15] D. Son and M. Wingate, Ann. Phys. (NY) **321**, 197 (2006).
[16] X. G. Wen and A. Zee, Phys. Rev. Lett. **69**, 953 (1992).
[17] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **55**, 15832 (1997).
[18] A. Cappelli, M. Huerta, and G. R. Zemba, Nucl. Phys. B **636**, 568 (2002).
[19] K. Fujikawa, Phys. Rev. Lett. **44**, 1733 (1980).
[20] S. W. Hawking, Commun. Math. Phys. **55**, 133 (1977).
[21] J. Fröhlich and U. M. Studer, Rev. Mod. Phys. **65**, 733 (1993).
[22] D. T. Son, arXiv:1306.0638.
[23] M. Stone, Phys. Rev. B **85**, 184503 (2012).
[24] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[25] T. Kimura and T. Nishioka, Prog. Theor. Phys. **127**, 1009 (2012).
[26] S. Ryu, J. E. Moore, and A. W. W. Ludwig, Phys. Rev. B **85**, 045104 (2012).
[27] Z. Wang, X.-L. Qi, and S.-C. Zhang, Phys. Rev. B **84**, 014527 (2011).
[28] P. Wiegmann and A. G. Abanov, arXiv:1311.4479.
[29] We remark here that the term $-\frac{1}{4m} g_{ij} \Delta_g \rho$ in (22) comes from the path integral measure (2) while the rest of (22) can be obtained in a conventional way from the variation of (1) over the metric.
[30] $E_0 = \epsilon_0/\rho_0$ is the energy per particle in the unperturbed state.
[31] T. L. Hughes, R. G. Leigh, and O. Parrikar, Phys. Rev. D **88**, 025040 (2013).
[32] The role of torsion was discussed before [34,31] where it was shown that the Hall viscosity appears as a response to torsion in relativistic theory.
[33] T. Can, M. Laskin, and P. Wiegmann (unpublished).
[34] T. L. Hughes, R. G. Leigh, and E. Fradkin, Phys. Rev. Lett. **107**, 075502 (2011).