Quantum theory of acoustoelectric interaction

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Within the self-consistent-field approximation, a quantum-mechanical derivation is given for the dielectric response function of an arbitrarily degenerate free-electron gas which is subjected to a drift field. Neglecting in the equation of motion for the one-electron density operator a convection term, significant in the classical-collision-dominated regime only, the dielectric response function and the acoustic gain factor for a piezoelectrically active sound wave are obtained for the quantum and semiclassical-microscopic regimes. The manner in which the theory can be extended to the collision-dominated regime is discussed. For a collision-free electron gas, the requirements of energy and momentum conservation in individual electron-phonon interactions lead to a cutoff in the acoustoelectric coupling when the acoustic wave number exceeds the characteristic electron wave number. The broadening of this cutoff due to collisions is investigated and compared with thermal broadening. A number of useful approximations for the acoustic gain factor are derived.

I. INTRODUCTION

The interaction of acoustic lattice waves with the free carriers of a piezoelectric semiconductor has been of considerable interest for some time.[1] When the carrier drift velocity exceeds the velocity of sound, this interaction may lead to a buildup of an intense acoustic flux by amplification of thermal-background lattice vibrations. The amplified sound flux causes the drift current to saturate or to oscillate, the latter case being associated with the formation and propagation of acoustoelectric high-field domains.

The acoustoelectric interaction can be classified into three regimes according to the manner in which the electron gas responds to the acoustic perturbation. In the classical-collision-dominated regime,[2] the electron mean free path is much smaller than the acoustic wavelength. Electron drift and diffusion are then determined by the local field and the local-carrier-density gradient, respectively. In this regime, the interaction can be treated as that between a sound wave and a space-charge wave, with the application of a drift field leading to a collective drift of acoustically bunched electrons. In the semiclassical-microscopic regime,[3] the electron mean free path is comparable with or longer than the acoustic wavelength, and nonlocal effects occur. In this regime, the interaction can be described in terms of the Landau damping mechanism[4] as a resonant interaction between the sound wave and individual electrons traveling with a velocity component in the direction of acoustic propagation equal to the velocity of sound. Application of a drift field, by shifting the dc part of the carrier distribution in reciprocal space, changes the relative number of electrons with velocity components just below and just above the velocity of sound. Finally, in the quantum regime, the acoustic wavelength is comparable with the characteristic electron de Broglie wavelength. The electron recoil in a single-phonon emission or absorption process then becomes significant,[5] and a cutoff in the electron-phonon coupling occurs when the phonon momentum becomes too large to be given up or absorbed by an electron in an energy-conserving process. In this regime, the interaction can be considered as a phonon-maser process.[6]

So far, most experimental work has been concerned with the classical-collision-dominated regime,[1] although the semiclassical-microscopic regime has also been investigated to some extent.[7,8] The introduction of x-ray-scattering techniques[9,10] to obtain the spectral distribution of piezoelectrically amplified sound flux in the 100-GHz region has permitted an extension of experimental work to the quantum regime, and has thereby stimulated renewed theoretical interest in this area. The high-frequency cutoff in the acoustoelectric coupling has recently been observed by Carlson and Segmüller[10,11] in x-ray-diffraction experiments performed on highly doped GaAs single crystals. For nondegenerate electron statistics, the acoustoelectric coupling has been discussed by Nakamura[12] in terms of individual electron-phonon interactions which conserve energy and momentum. To analyze the results of the above-mentioned x-ray-diffraction experiments, these calculations were extended by the present author[13] to treat the case of an arbitrarily degenerate electron gas, and to consider effects of impurity banding. The phonon-maser model adopted in these papers treats the screening of the electron-phonon coupling in an approximative manner. Furthermore, it is difficult to include in that model effects arising from finite electron collision times. For these reasons, an alternative approach has been adopted in which the

acoustic attenuation constant is obtained from the quantum-mechanically calculated longitudinal dielectric response function. This method seems first to have been applied by Takimoto[14] to calculate acoustic absorption in metals. A similar calculation was done by Spector[15] for electron–optical-phonon interaction in semiconductors. Gurevich and Kagan[16] used the dielectric-response-function method to discuss acoustoelectric coupling for a collision-free electron gas at zero drift field. Electron collisions were taken into account by Carlson and Segmüller[10] to evaluate the collision broadening of the high-frequency cutoff of the zero-field attenuation constant. A dielectric-response-function approach has also been adopted by Spector[17] and by Sharma and Thornton[18] to study acoustoelectric effects in magnetic fields and in crossed electric and magnetic fields, respectively. To the best of the author's knowledge, a quantum theory of dielectric response and acoustoelectric coupling in the presence of a static electric field and a scattering mechanism has not previously been published. This latter case differs from the aforementioned by the fact that stationary electron wave functions in the presence of a drift field do not exist. Using time-dependent electron eigenfunctions, Spector[19] has given a quantum theory of dielectric response for a collision-free electron gas subjected to arbitrary electric and magnetic fields. In the present paper the simultaneous effects of collisions and a drift field are discussed using the representation provided by the time-independent free-electron wave functions.

In Sec. II we solve the equation of motion for the one-electron density operator in the presence of a drift field, a scattering mechanism, and a piezoelectrically active sound wave. It is assumed that the carrier collisions can be treated in the relaxation-time approximation. The dc part of the density operator will then be diagonal in the representation provided by the plane-wave eigenfunctions of the kinetic-energy operator, and to first order in the electric field the diagonal elements will correspond to a drifted thermal-equilibrium distribution function for the electrons in reciprocal space.

The equation of motion for the density operator contains a term which describes the convection or collective drift of acoustically bunched electrons in the applied drift field. This term gives rise to the characteristic resonant field dependence of the classical acoustoelectric gain factor,[2] but the term is negligible outside the collision-dominated regime.[3] Since we shall concern ourselves with acoustoelectric effects in the quantum regime, a simplified solution of the equation of motion is obtained by neglecting the convection term. From this solution, the longitudinal dielectric response function is calculated. In Secs. III and IV, we derive a number of limiting expressions for the acoustoelectric gain factor.

II. ELECTRON RESPONSE WITH A DRIFT FIELD

We consider a piezoelectrically active acoustic wave $e^{i\vec{q}\cdot\vec{r}-i\omega t}$ interacting with an arbitrarily degenerate gas of conduction electrons in the presence of a drift field $\vec{F}_0$ and a scattering mechanism. $\vec{q}$ is the acoustic wave vector and ω the angular acoustic frequency. Within the self-consistent-field approximation,[20] the response of the electron gas to the external disturbance can be obtained from the one-electron density operator ρ. The equation of motion to be satisfied by this operator is

$$i\hbar\frac{\partial\rho}{\partial t}=[H_0+H_F+H_{1s},\rho]-i\hbar(\rho-\rho_s)/\tau\ , \tag{1}$$

where $H_0=p^2/2m$ is the kinetic-energy operator, m being the effective electron mass, and $\vec{p}$ the momentum operator. $H_F=e\vec{F}_0\cdot\vec{r}$ is the potential energy in the applied drift field, and $H_{1s}=V_{1s}\times e^{i\vec{q}\cdot\vec{r}-i\omega t}$ is the potential energy in the self-consistent electric field accompanying the sound wave. e is the elementary charge, τ is the electron collision time, and ρ_s is the equilibrium density operator corresponding to the local electron density. The requirement of carrier conservation in the scattering processes may be expressed as[21]

$$\mathrm{Tr}\{\delta(\vec{r}_0-\vec{r})[\rho-\rho_s]\}=0\ , \tag{2}$$

which implies that ρ and ρ_s correspond to the same local electron density. In the calculations to follow, this condition serves to determine ρ_s. Although not always justified in practice,[22] the assumption of an energy-independent relaxation time will suffice for the present purpose. The quantum-mechanical cutoff of the acoustoelectric interaction is affected by electrons in a relative narrow range of energy, and will be controlled by the collision time corresponding to these energies.

Let us first consider the equation of motion in the absence of an acoustic disturbance. In this case, a time independent density operator exists which satisfies the relation

$$[H_0+H_F,\ \rho]=i\hbar(\rho-\rho_0)/\tau\ , \tag{3}$$

with ρ_0 being the thermal-equilibrium density operator. Inserting into this equation $\rho=\rho_0+\rho_d$, and expanding to first order in the electric field, we obtain

$$i\hbar\rho_d/\tau=[H_0,\rho_d]+[H_F,\rho_0]\ . \tag{4}$$

The thermal-equilibrium density operator ρ_0 has the property that $\rho_0|\vec{k}\rangle=f_0(\vec{k})|\vec{k}\rangle$, where $f_0(\vec{k})$ is the thermal-equilibrium electron distribution function and $|\vec{k}\rangle$ denotes a plane-wave eigenfunction for the kinetic-energy operator, i.e., $H_0|\vec{k}\rangle=E(\vec{k})|\vec{k}\rangle$, with $E(\vec{k})=\hbar^2k^2/2m$. Evaluating matrix

elements on both sides of (4) between the eigenfunctions $|\vec{k}\rangle$ and $|\vec{k}'\rangle$, and using the commutator relation[23]

$$[H_F, \rho_0] = ie\vec{F}_0 \cdot \nabla_{\vec{k}} \rho_0 \,, \tag{5}$$

we obtain

$$\langle \vec{k}|\rho_d|\vec{k}'\rangle = (e\tau/\hbar)\vec{F}_0 \cdot \nabla_{\vec{k}} f_0(\vec{k})\delta(\vec{k}-\vec{k}') \,, \tag{6}$$

and consequently

$$\langle \vec{k}|\rho|\vec{k}'\rangle = [f_0(\vec{k}) + (e\tau/\hbar)\vec{F}_0 \cdot \nabla_{\vec{k}} f_0(\vec{k})]\delta(\vec{k}-\vec{k}') \,. \tag{7}$$

To first order in the drift field, the density operator is thus diagonal in the representation provided by the free-electron eigenstates $|\vec{k}\rangle$, and the diagonal elements correspond to a drifted electron distribution function

$$f_0(\vec{k}-\vec{k}_d) \cong f_0(\vec{k}) - \vec{k}_d \cdot \nabla_{\vec{k}} f_0(\vec{k}) \,, \tag{8}$$

with the displacement in reciprocal space being $\vec{k}_d = -e\tau\vec{F}_0/\hbar$.

Since the argument can be generalized to higher order in the field, we shall from now on define the solution of (3) to be the density operator ρ_F and assume that it has the property

$$\rho_F|\vec{k}\rangle = f(\vec{k})|\vec{k}\rangle \,, \tag{9}$$

with $f(\vec{k})$ being the dc part of the electron distribution function in the presence of the drift field. This procedure appears to be rigorous in the relaxation-time approximation, although a more detailed description of the electron collisions under certain conditions may lead to off-diagonal elements in ρ_F, e.g., for ionized-impurity scattering with the scattering centers arranged in some sort of a regular lattice.[23] Such problems are outside the scope of the present paper.

Let us now return to our original problem as stated in (1). In order to linearize the solution, we expand the density operators ρ and ρ_s to first order in the wave amplitudes

$$\rho = \rho_F + \rho_1 \quad \text{and} \quad \rho_s = \rho_0 + \rho_2 \,, \tag{10}$$

where ρ_1 and ρ_2 vary in time and space as $e^{i\vec{q}\cdot\vec{r}-i\omega t}$. Since the collision processes tend to restore an equilibrium distribution of electrons relative to the lattice frame, ρ_s should be expanded in terms of an undrifted density operator. Retaining only first-order terms in the acoustic amplitude, we then find

$$i\hbar\frac{\partial\rho_1}{\partial t} = [H_0, \rho_1] + [H_F, \rho_1] + [H_{1s}, \rho_F] - i\hbar(\rho_1 - \rho_2)/\tau \,, \tag{11}$$

which evaluated between the eigenfunctions $|\vec{k}\rangle$ and $|\vec{k}-\vec{q}\rangle$ becomes

$$[i\hbar/\tau + \hbar\omega - E(\vec{k}) + E(\vec{k}-\vec{q}) - ie\vec{F}_0 \cdot \nabla_{\vec{k}}]\langle \vec{k}|\rho_1|\vec{k}-\vec{q}\rangle = [f(\vec{k}-\vec{q}) - f(\vec{k})]V_{1s} + (i\hbar/\tau)\langle \vec{k}|\rho_2|\vec{k}-\vec{q}\rangle \,. \tag{12}$$

In the limit of small acoustic wave number relative to the characteristic electron wave number, this equation reduces to

$$[i\hbar/\tau + \hbar\omega - \hbar^2\vec{k}\cdot\vec{q}/m - ie\vec{F}_0 \cdot \nabla_{\vec{k}}]f_1(\vec{k}) = -V_{1s}\vec{q}\cdot\nabla_{\vec{k}} f(\vec{k}) + (i\hbar/\tau)f_2(\vec{k}) \,, \tag{13}$$

which is identical to Eq. (2.4) in Spector's paper.[3] The quantum theoretical expression (12) therefore applies in the semiclassical and classical regimes covered by Spector's theory, as well as in the quantum regime.

Associated with the acoustically generated electron-density modulation we have a wave-periodic variation of the local Fermi energy ϵ_F, i.e.,

$$\epsilon_F = E_F + H_2 \,, \tag{14}$$

where E_F is the thermal-equilibrium Fermi energy and H_2 varies as $e^{i\vec{q}\cdot\vec{r}-i\omega t}$. Since ρ_2 is the equilibrium density operator corresponding to the simultaneous action of the self-consistent potential energy H_{1s} and of the variation of the Fermi level H_2, we have

$$\langle \vec{k}|\rho_2|\vec{k}-\vec{q}\rangle = \frac{f_0(\vec{k}) - f_0(\vec{k}-\vec{q})}{E(\vec{k}) - E(\vec{k}-\vec{q})}(V_{1s} - V_2) \,, \tag{15}$$

with $V_2 = \langle \vec{k}|H_2|\vec{k}-\vec{q}\rangle$. The minus sign in the factor $V_{1s} - V_2$ arises because the electron density is a function of the difference between the local potential energy and the local Fermi energy.

The electron-density modulation is determined by

$$n_1 = V^{-1}\,\mathrm{Tr}(e^{-i\vec{q}\cdot\vec{r}}\rho_1) = V^{-1}\sum_{\vec{k}} \langle \vec{k}|\rho_1|\vec{k}-\vec{q}\rangle \,, \tag{16}$$

or, using the condition of carrier conservation in in the scattering processes,

$$n_1 = V^{-1}\,\mathrm{Tr}(e^{-i\vec{q}\cdot\vec{r}}\rho_2) = V^{-1}\sum_{\vec{k}} \langle \vec{k}|\rho_2|\vec{k}-\vec{q}\rangle \,. \tag{17}$$

The summation extends over all allowed wave vectors. V is the sample volume.

The first-order differential equation (12) has the solution

$$\langle \vec{k}|\rho_1|\vec{k}-\vec{q}\rangle = (i/eF_0)\int_{-\infty}^{k_x} v(k_x', k_y, k_z)\,u(k_x, k_x')\,dk_x' \,, \tag{18}$$

with

$$v(\vec{k}') = [f(\vec{k}'-\vec{q}) - f(\vec{k}')]V_{1s} + [f_0(\vec{k}'-\vec{q}) - f_0(\vec{k}')]\,2ik_1(V_{1s} - V_2)/(q - 2k_x') \tag{19}$$

and

$$u(k_x, k_x') = \exp\left(-\frac{i\hbar^2 q}{2meF_0}[(q + 2k_0 + 2ik_1)(k_x - k_x') - (k_x^2 - k_x'^2)]\right) \,. \tag{20}$$

Here we have taken the applied drift field and the direction of acoustic propagation to be along the x axis of our coordinate system. The wave numbers k_0 and k_1 are given by $k_0 = m\omega/\hbar q$ and $k_1 = m/\hbar q\tau$. Using (15)–(18), V_2, $\langle \vec{k}|\rho_1|\vec{k}-\vec{q}\rangle$, and $\langle \vec{k}|\rho_2|\vec{k}-\vec{q}\rangle$ may now be eliminated to obtain the electron-density modulation n_1 from which the dielectric response function may be evaluated. A detailed discussion of this problem will be the subject of a future publication. We only note that the dielectric response function obtained in this way will apply for all three regimes of electron response.

For the present discussion of acoustoelectric effects in the quantum regime, the convection term $\vec{F}_0 \cdot \nabla_{\vec{k}} \langle \vec{k}|\rho_1|\vec{k}-\vec{q}\rangle$ in the equation of motion for the density operator may be neglected.[3] The results to be obtained in the rest of this paper will then apply for $ql \gg 1$ only.[24] l is the electron mean free path. Neglecting the convection term in (12), elimination of V_2, $\langle \vec{k}|\rho_1|\vec{k}-\vec{q}\rangle$, and $\langle \vec{k}|\rho_2|\vec{k}-\vec{q}\rangle$ yields for the electron-density modulation

$$n_1 = \frac{-V_{1s} L(q,\omega)}{1 + R_0(q,\omega)/L_0(q,0)} , \tag{21}$$

with

$$L(q,\omega) = V^{-1} \sum_{\vec{k}} \frac{f(\vec{k}-\vec{q}) - f(\vec{k})}{E(\vec{k}) - E(\vec{k}-\vec{q}) - \hbar\omega - i\hbar/\tau} , \tag{22}$$

$$L_0(q,0) = V^{-1} \sum_{\vec{k}} \frac{f_0(\vec{k}-\vec{q}) - f_0(\vec{k})}{E(\vec{k}) - E(\vec{k}-\vec{q})} , \tag{23}$$

and

$$R_0(q,\omega) = V^{-1} \times \sum_{\vec{k}} \frac{(i\hbar/\tau)[f_0(\vec{k}-\vec{q}) - f_0(\vec{k})]}{[E(\vec{k}) - E(\vec{k}-\vec{q})][E(\vec{k}) - E(\vec{k}-\vec{q}) - \hbar\omega - i\hbar/\tau]} . \tag{24}$$

As before, $f_0(\vec{k})$ is the thermal-equilibrium electron distribution function and $f(\vec{k})$ is the dc part of the distribution function in the presence of the drift field. For the dielectric response function we then find

$$\epsilon(q,\omega) = 1 - e^2 n_1/\epsilon_0 q^2 V_{s1} = 1 + \frac{(e^2/\epsilon_0 q^2) L(q,\omega)}{1 + R_0(q,\omega)/L_0(q,0)} , \tag{25}$$

with ϵ_0 being the static dielectric constant of the material.

From the dielectric response function, we may finally obtain the acoustoelectric gain factor β using the expression[5]

$$\beta = K^2 q \, \mathrm{Im}[\epsilon^{-1}(q,\omega)] , \tag{26}$$

where K is the electromechanical coupling constant. In Secs. III and IV, we shall discuss a number of limiting expressions for the acoustic gain factor applicable, respectively, for the arbitrarily degenerate electron gas with no collisions, and for the fully degenerate electron gas with few collisions.

III. COLLISION-FREE ELECTRON GAS

For a collision-free electron gas, our expression (25) for the dielectric response function reduces to the well-known Lindhard formula[25]

$$\epsilon(q,\omega) = 1 + (e^2/\epsilon_0 q^2 V) \times \sum_{\vec{k}} \frac{f(\vec{k}-\vec{q}) - f(\vec{k})}{E(\vec{k}) - E(\vec{k}-\vec{q}) - \hbar\omega - i\alpha} , \tag{27}$$

where α is a small positive energy introduced to define the convergence properties of the summation. For zero drift field, a discussion of the acoustoelectric interaction based on this formula has previously been given by Gurevich and Kagan.[16] Their expressions are fairly complicated to apply, however, and it is worth while deriving a few simple approximations.

We shall require that the acoustic wavelength be much larger than the lattice spacing so that the material can be considered an elastic continuum. This condition in fact is necessary if the phenomenological constitutive equations of the piezoelectric, and therefore also Eq. (26), are to be used. The acoustic dispersion relation will then be linear, i.e., $\omega = qv_s$, with v_s being the velocity of sound. In most cases of physical interest, v_s will be much smaller than the thermal-electron velocity $v_T = (2k_B T/m)^{1/2}$, so that an expansion to first order in v_s/v_T will be appropriate. $k_B T$ is the thermal energy.

The drift field is assumed to be applied along the direction of acoustic propagation which we choose to be the x axis of our coordinate system. The drift field is taken into account by inserting for the electron distribution function a displaced thermal-equilibrium distribution, i.e.,

$$f(\vec{k}) = \left[\exp\left(\frac{\hbar^2}{2mk_B T}[(k_x - k_d)^2 + k_y^2 + k_z^2] - \frac{E_F}{k_B T} \right) + 1 \right]^{-1} , \tag{28}$$

where E_F is the Fermi energy as measured from the conduction-band edge. The displacement of the electron distribution in reciprocal space is given by $k_d = e\tau F_0/\hbar = mv_d/\hbar$, v_d being the electron drift velocity. It will be assumed that k_d is much smaller than characteristic electron wave numbers and that v_d is much smaller than the thermal-electron velocity v_T. This may be considered part of a justification for the assumption of a rigidly shifted electron distribution in reciprocal space. For a further discussion of this point see Sec. V. It is noted that this assumption is commonly used to calculate acoustoelectric amplification factors and is known to give results in good agreement with experiments.

It proves convenient to introduce the function

$$Q(q,\omega) = [\epsilon(q,\omega) - 1] q^2 . \tag{29}$$

The physical significance of this function may be realized by noting that for $\omega = 0$ and for q much smaller than characteristic electron wave numbers, $Q(q, \omega)$ reduces to [25]

$$Q(q, 0) \cong q_0^2 = \frac{e^2}{\epsilon_0} \frac{\partial n_c}{\partial E_F} \ , \tag{30}$$

where q_0 is the screening wave number for the arbitrarily degenerate electron gas as obtained from the Thomas-Fermi approach. n_c is the thermal-equilibrium density of conduction electrons. In terms of $Q(q, \omega)$, the acoustoelectric gain factor may be expressed as

$$\beta = \frac{-K^2 q^3 \, \mathrm{Im}Q}{(q^2 + \mathrm{Re}Q)^2 + (\mathrm{Im}Q)^2} \ . \tag{31}$$

Expanding now to first order in the parameter $(v_s - v_d)/v_T$ we find

$$\mathrm{Im}Q \cong -B \frac{\hbar\omega}{k_B T} \, y f_0(a) \tag{32}$$

and

$$\mathrm{Re}Q \cong \frac{2B}{\pi} \int_0^\infty \frac{dx}{x} \left[G(\tfrac{1}{2}\hbar q - x) - G(\tfrac{1}{2}\hbar q + x) \right] \equiv g_0^2(q), \tag{33}$$

with

$$B = e^2 m^2 k_B T / 2\pi\epsilon_0 \hbar^4 q \ , \tag{34}$$

$$y = (v_d - v_s)/v_s \ , \tag{35}$$

$$a = \hbar^2 q^2 / 8 m k_B T \ , \tag{36}$$

$$f_0(a) = (e^{a-\eta} + 1)^{-1} \ , \tag{37}$$

and

$$G(p) = \ln\{1 + \exp[(E_F - p^2/2m)/k_B T]\} \ . \tag{38}$$

$\eta = E_F / k_B T$ is the normalized Fermi energy. $f_0(a)$ is the usual Fermi factor giving the probability that an electron state of energy $a k_B T = \hbar^2 q^2 / 8m$ be occupied in thermal equilibrium. This factor arises because an electron must have a minimum wave number of (approximately) $\frac{1}{2}q$ to emit or absorb a phonon of wave number q in an energy- and momentum-conserving process. [13] At high phonon wave numbers, only a small fraction of the free electrons possesses this momentum, and the acoustoelectric interaction is reduced as result of the decrease of $|\mathrm{Im}Q|$. $g_0(q)$ denotes the screening wave number which, quantum-mechanically calculated, is a function of the acoustic wave number. As q becomes comparable with the characteristic electron wave number, $g_0(q)$ starts to decrease and approaches zero in the limit of very-high wave numbers.

With these approximations, and noting that $\mathrm{Re}Q \gg |\mathrm{Im}Q|$, our expression for the acoustoelectric gain factor reduces to

$$\beta = (\pi)^{1/2} \frac{K^2 \omega}{v_T} \frac{q^2 q_c^2}{(q^2 + g_0^2)^2} \, y f_0(a) \ , \tag{39}$$

where

$$q_c^2 = e^2 N_c / \epsilon_0 k_B T \tag{40}$$

and

$$N_c = 2(m k_B T / 2\pi\hbar^2)^{3/2} \ . \tag{41}$$

N_c is the effective density of states for the conduction band and q_c is the Debye screening wave number for a nondegenerate electron gas of density N_c.

For an arbitrarily degenerate electron gas, the integral in (33) can not be analytically evaluated. (For fully degenerate statistics the integration is straightforward, [25] and for nondegenerate statistics the integral can be expressed in terms of Dawson's integral. [16,26]) The variation of the screening wave number with acoustic wave number is a second-order effect of the inability of the electron gas to respond to the acoustic perturbation. In many cases of practical interest, one may therefore neglect the dependence of the screening wave number on q by replacing g_0 by q_0 in the expression (39) for the acoustoelectric gain factor. One then retains the expression for the gain factor found on the basis of the phonon-maser model[13] in which the screening of the electron-phonon coupling was treated by the Thomas-Fermi approach. The neglect of the q dependence of g_0 was found[13] to give rise to an underestimate of the acoustic gain factor at the frequency of maximum gain by 12% for a GaAs sample with a free-carrier density of $n_c = 1.35 \times 10^{17}$ cm^{-3} at 20 K. (These values of doping and temperature relate to the x-ray-diffraction experiments of Ref. 10.)

An improved approximation can be obtained by expanding $g_0(q)$ as given by (33) to second order in q. Performing such an expansion for nondegenerate and fully degenerate statistics, one finds

$$g_D^2 \cong q_D^2 (1 - 2a/3) \tag{42}$$

and

$$g_T^2 \cong q_T^2 (1 - q^2 / 12 k_F^2) \ , \tag{43}$$

respectively, with

$$q_D^2 = e^2 n_c / \epsilon_0 k_B T \tag{44}$$

and

$$q_T^2 = 3 e^2 n_c / 2 \epsilon_0 E_F \ . \tag{45}$$

q_D is the Debye screening wave number for the nondegenerate electron gas, and q_T is the Thomas-Fermi screening wave number for the fully degenerate gas. $k_F = (2mE_F/\hbar^2)^{1/2}$ is the radius of the Fermi sphere. As before $a = \hbar^2 q^2 / 8 m k_B T$. We have found that interpolation between the two for-

mulas (42) and (43) can be accomplished using the relation

$$g_0^2 \cong q_0^2[1-(2a/3)F_{-3/2}(\eta)/F_{-1/2}(\eta)] , \tag{46}$$

with

$$q_0^2 = \frac{e^2}{\epsilon_0}\frac{\partial n_c}{\partial E_F} = q_D^2 F_{-1/2}(\eta)/F_{1/2}(\eta) . \tag{47}$$

The functions $F_{1/2}(\eta)$, $F_{-1/2}(\eta)$, and $F_{-3/2}(\eta)$ belong to a class of tabulated Fermi integrals defined by[27]

$$F_j(\eta) = \frac{1}{\Gamma(j+1)}\int_0^\infty \frac{x^j\,dx}{\exp(x-\eta)+1} , \tag{48}$$

where $\Gamma(j+1)$ is the usual Γ function. For the numerical example considered above, using the approximation (46) for the screening wave number reduces the underestimate of the gain factor from 12% to 3%.

To illustrate the quantum-mechanical cutoff of the acoustoelectric coupling, we have plotted in Fig. 1 the acoustic gain factor β as a function of angular acoustic frequency ω for a GaAs sample with a free-electron density of 1.35×10^{17} cm^{-3} at 20 K.[10] For the effective electron mass and for the squared electromechanical coupling constant we have assumed $m = 0.067m_0$ and $K^2 = 3.7\times10^{-3}$, respectively. m_0 is the free electron mass. The value adopted for K^2 applies for fast transverse acoustic waves propagating along the [110] direction of GaAs. The drift parameter is $y = (v_d - v_s)/v_s = 5$. For angular acoustic frequencies above 5×10^{11} sec^{-1}, the probability that an electron possesses the momentum required to emit or absorb a phonon decreases quickly, and the gain factor is reduced.

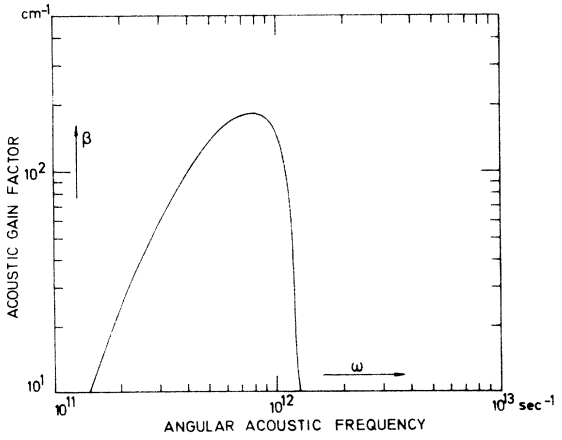

FIG. 1. Acoustic gain factor as a function of angular frequency for fast transverse waves propagating along the [110] direction of GaAs with a free-carrier density of 1.35×10^{17} cm^{-3} at 20 K. The electron gas is almost completely degenerate, and the rapid high-frequency cutoff of the gain factor reflects the sharpness of the Fermi surface. Electron collisions were neglected in these calculations.

IV. COLLISION BROADENING OF THE HIGH-FREQUENCY CUTOFF

In Sec. III, we discussed the acoustoelectric coupling for a collision-free electron gas. A cutoff of the electron-phonon coupling was then found to occur when the phonon momentum becomes too large to be given up or absorbed by an electron in an energy-conserving process. Mathematically, this cutoff is described by the factor $f_0(a)$ appearing in the formula (32) for ImQ. The cutoff function

$$f_0(a) = [\exp\{\hbar^2(q^2 - 4k_F^2)/8mk_BT\} + 1]^{-1} \tag{49}$$

gives the thermal-equilibrium probability that an electron state of energy $ak_BT = \hbar^2q^2/8m$ be occupied. In the case of complete degeneracy, the cutoff function reduces to the step function

$$\Theta(2k_F - q) = \begin{cases} 1 \text{ for } q < 2k_F \\ 0 \text{ for } q > 2k_F \end{cases} , \tag{50}$$

corresponding to an infinitely sharp cutoff of the electron-phonon coupling when the acoustic wave number exceeds twice the radius of the electron Fermi sphere.

In the presence of electron collisions, an electron wave can propagate in phase with the sound wave over finite periods of time only. Consequently, energy need no longer be strictly conserved in individual electron-phonon interactions, and a broadening of the high-frequency cutoff of the acoustoelectric coupling can be expected when the collision time becomes comparable with the acoustic period. In this section we discuss the collision broadening effect for a fully degenerate electron gas.

With complete degeneracy, our expressions for $L(q,\omega)$, $L_0(q,0)$, and $R_0(q,\omega)$ can be integrated to give

$$L(q,\omega) = \pi C[A(\tfrac{1}{2}q + k_d - k_0 - ik_1) + A(\tfrac{1}{2}q - k_d + k_0 + ik_1)] , \tag{51}$$

$$L_0(q,0) = 2\pi C\,A(\tfrac{1}{2}q) , \tag{52}$$

and

$$R_0(q,\omega) = \frac{ik_1\pi C}{k_0+ik_1}[A(\tfrac{1}{2}q - k_0 - ik_1) + A(\tfrac{1}{2}q + k_0 + ik_1) - 2A(\tfrac{1}{2}q)] , \tag{53}$$

with

$$k_0 = mv_s/\hbar , \quad k_1 = m/\hbar q\tau , \tag{54}$$

$$C = 2m/(2\pi)^3\hbar^2 q , \tag{55}$$

and

$$A(x) = 2xk_F + (k_F^2 - x^2)\ \ln\{(x+k_F)/(x-k_F)\}. \quad (56)$$

In the limit of zero drift field, these results reduce to those stated by Carlson and Segmüller.[10] Again, it is worthwhile deriving a simple approximate expression for the acoustic gain factor by expanding to first order in the parameter $(k_d - k_0)/k_F$. In this way we obtain

$$\mathrm{Im}Q \cong -B\,\frac{\hbar\omega}{k_B T}\,y\,\varphi(q, k_1) \quad (57)$$

with the cutoff function

$$\varphi(q, k_1) = \pi^{-1}\left[\tfrac{1}{2}\pi + \tan^{-1}\left(\frac{2k_F - q}{2k_1}\right) - \frac{k_1}{q}\ln\frac{(2k_F+q)^2}{(2k_F-q)^2 + 4k_1^2}\right]. \quad (58)$$

Here, the dependence of the collision broadening upon the applied drift field has been suppressed. Furthermore, we have treated k_1 as a small quantity to be neglected in comparison with the sum $k_F + \frac{1}{2}q$, though k_1 may be significant relative to the difference $k_F - \frac{1}{2}q$. As before $B = e^2m^2k_BT/2\pi\epsilon_0\hbar^4q$.

In Fig. 2, we have plotted $\varphi(q, k_1)$ as a function of the reduced phonon wave number $q/2k_F$ with $s = m/2\hbar k_F^2\tau$ as a parameter. The three curves correspond to $s = 2.5\times10^{-2}$ (1), 5×10^{-2} (2), and 0.1 (3), respectively. For a GaAs sample with a free-electron density of 1.35×10^{17} cm^{-3}, these values for s correspond to $\tau = 4.4\times10^{-13}$, 2.2×10^{-13}, and 1.1×10^{-13} sec, respectively. These values for τ again correspond to $\omega\tau = 0.48$, 0.24, and 0.12, respectively, at the cutoff frequency $\omega = 2k_Fv_s$. For comparison with the collision-broadening effect, we have plotted in Fig. 3, $f_0(a)$ as a function of reduced phonon wave number. $f_0(a)$ describes the thermal broadening of the high-frequency cutoff. In this figure, the parameter is $k_T/k_F = (2mk_BT)^{1/2}/\hbar k_F$, and the values $k_T/k_F = 0.2$, 0.3, and 0.4 were used to calculate curves 1, 2, and 3, respectively. For the above-mentioned GaAs sample, these values for k_T/k_F approximately correspond to $T = 6$, 15, and 25 K. If the slopes of the cutoff functions at $q/2k_F = 1$ are taken as measure of the broadening effect, for our representative GaAs sample it is found that a collision time of $\tau = 2\times10^{-13}$ sec[10] is approximately equivalent to a temperature of 15 K.

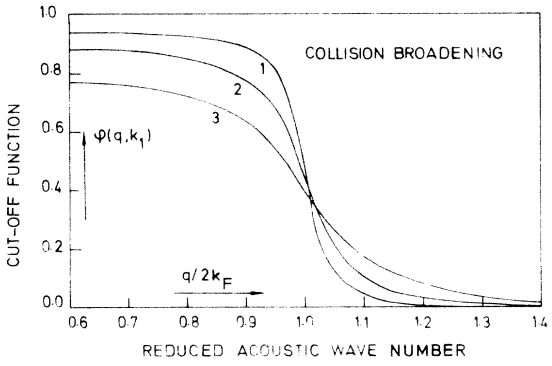

FIG. 2. Collision broadening of the high-frequency cutoff of the electron-phonon coupling. The cutoff function $\varphi(q, k_1)$ is plotted as a function of reduced phonon wave number $q/2k_F$ for a fully degenerate electron gas. For a GaAs sample with a free-electron density of 1.35×10^{17} cm^{-3} at 0 K, the curves correspond to $\omega\tau = 0.48$ (1), 0.24 (2), and 0.12 (3), respectively, at the cutoff frequency $\omega = 2k_Fv_s$. q is the acoustic wave number, k_F is the radius of the electron Fermi sphere, ω is the angular acoustic frequency, and τ is the electron collision time. v_s is the velocity of sound.

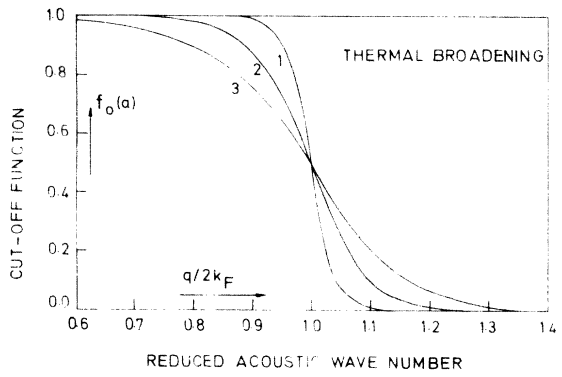

FIG. 3. Thermal broadening of the high-frequency cutoff of the electron-phonon coupling. The cutoff function $f_0(a)$ is plotted as a function of reduced phonon wave number $q/2k_F$. For a GaAs sample with a free-electron density of 1.35×10^{17} cm^{-3}, these curves correspond to $T = 6$, 15, and 25 K, respectively. a is the energy (measured relative to thermal energy) that an electron must have to emit a phonon of wave number q. k_F is the radius of the electron Fermi sphere.

For a partly degenerate electron gas, the significance of electron collisions in broadening the cutoff of the electron-phonon coupling is somewhat reduced. We shall not attempt a detailed discussion of this case, however.

V. DISCUSSION

To analyze acoustoelectric effects at very-high frequencies where the phonon momentum become comparable with characteristic electron momenta, we have developed a quantum theory of electron response in the presence of a drift field and a scattering mechanism. The response theory may also find applications in the study of such phenomena as plasma-wave propagation and acoustic attenuation in nonpiezoelectric semiconductors.

From the dielectric response function we have evaluated the acoustic gain factor for a piezoelec-

trically active sound wave, and a number of useful approximations have been obtained by inserting for the electron distribution a displaced thermal-equilibrium distribution and expanding to first order in the drift field. The assumption of a displaced thermal-equilibrium distribution is consistent with the assumption of an energy-independent collision time. Considering the expression (27) for the dielectric response function of a collision-free electron gas to be generally applicable independent of the form that the electron distribution may take under the influence of the drift field, it is possible to discuss effects arising from the deviation of the distribution function from a displaced thermal-equilibrium distribution. We expect a uniform heating of the electron gas in the field to give rise to a reduction of the acoustic frequency of maximum gain, a reduction of the gain maximum itself, and a broadening of the high-frequency cutoff in the electron-phonon coupling. These effects are analogous to those obtained by increasing the sample temperature. Streaming of the electron distribution along the field direction will cause a reduction of the electron-phonon coupling at lower frequencies and an upshift of the quantum cutoff.

For applications to highly doped semiconductors, the assumption of a simple parabolic band for the free electrons may be inappropriate. It is well known that the overlap of bound-electron wave functions associated with shallow impurity states for heavily doped semiconductors leads to the formation of impurity bands[28] which may distort the principal bands of the material. The effect of impurity banding on acoustoelectric coupling was studied in Ref. 13 in terms of an extremely simple two-band model. Impurity banding was found to give rise to an enhancement of the electron-phonon coupling (associated with the reduced degeneracy of the free-electron gas) and to an upshift of the high-frequency cutoff (associated with the presence of impurity-band electrons with relatively high effective mass).

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