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Temperature dependence of the electron effective mass in InSb†

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The electron effective mass in indium antimonide was measured at temperatures between 4.2 and 138 K using far-infrared cyclotron resonance at magnetic fields of 14.08, 16.09, and 18.05 kOe. The effective mass exhibits a maximum at 55 K which is 0.4% above the mass at 4.2 K and decreases with increasing temperature above 55 K. This behavior is explained reasonably well by the temperature dependence of the dilational component of the change in energy gap between the conduction and valence bands using Kane's equation for the band-edge effective mass. There is a small discrepancy between the dilational and measured mass change. Possible causes of this discrepancy are discussed.

I. INTRODUCTION

Indium antimonide is a III-V compound semiconductor which has a nonparabolic conduction band and an unusually small electron effective mass with a value of about 0.0138 m_e at zero temperature and momentum. Both of these properties stem from a small energy gap, about 0.24 eV, which leads to a strong interaction between the conduction and valence bands. Kane[1] derived a band structure for InSb by treating the $\vec{k}\cdot\vec{p}$ interaction and the k-independent spin-orbit interaction of the conduction and valence bands. Second-order perturbation theory was used to estimate the effects of higher and lower bands. For small values of k^2, he obtained an equation relating the electron effective mass at the bottom of the conduction band to the energy gap.

Changes in the energy gap as a function of temperature are due to effects of thermal expansion, called the dilational contribution, and electron-phonon interactions.[2] The temperature dependence of the energy gap of InSb measured in early optical experiments[3] is -2.9×10^{-4} eV/K. However, the effective mass is predicted to be independent of the optical energy gap because the electron-phonon component is not expected to change the band curvature.[4] It is predicted that only the dilational contribution should effect band curvatures and thus be the dominant term in the temperature dependence of the effective mass. The thermal expansion of InSb, measured by a number of workers,[5-7] has a negative thermal-expansion coefficient below 55 K. This interesting behavior should be reflected in the dilational contribution to the temperature dependence of the mass.

The temperature dependence of the effective mass in InSb was deduced by Stradling and Wood[8] from magnetoresistance measurements showing the magnetophonon effect. The use of this effect limited the experiment to the temperature range 40–260 K and to an accuracy of 1–2%. The change in mass below 100 K was only slightly larger than experimental error. However, they concluded that the dilational change in effective mass accounts for the temperature dependence of the mass over the temperature interval that was studied.

Cyclotron resonance is a direct and accurate method of determining effective masses. In the present experiment, cyclotron masses were determined with an error of less than 0.2% between 4.2 and 138 K using a far-infrared Michelson interferometer. The work was carried out to obtain accurate effective masses for InSb to determine whether dilation fully accounts for its temperature dependence.

II. EXPERIMENTAL METHOD

The InSb studied in these experiments was obtained from the Consolidated Mining and Smelting Co. of Canada Limited. The n-type single crystal had a nominal electron mobility of 6×10^5 cm^2 V^{-1} sec^{-1} and a net carrier concentration of about 10^{14} cm^{-1} at liquid-nitrogen temperatures. Sample slices, cut in a (110) plane, were lapped with fine sandpaper to make a thin sample in the form of a wedge varying in thickness between 0 and 0.1 mm.

The Michelson interferometer was a model FS-720 built by Research and Industrial Instruments of England. The light output of the interferometer was filtered with black polyethylene and a slice of sapphire. Radiation was detected using a gallium-doped germanium detector situated in a light-collecting spherical cavity and kept at a temperature of 2 K. The responsivity and noise-equivalent power of the detector were 5×10^5 V/W and 6×10^{-13} W/Hz$^{1/2}$, respectively.

Broad-band far-infrared radiation was provided by a medium-pressure mercury lamp. The light was chopped in a double-beam system in order to reduce lamp noise. The signal at the light-chopping frequency was amplified by a modified version of a preamplifier designed by Zwerdling *et al.*[9] and a low-noise lock-in amplifier. The output of the amplifier was digitized and recorded on magnetic tape. With the data-acquisition system, it was possible to read and record eight interferogram points per second. Fourier transformation of the digitized interferogram was performed on a CDC 6400 computer to produce a spectrum with a resolution of 0.2 cm^{-1}.

The sample was situated in a small chamber that was connected to the detector cavity and to the interferometer by thin-wall-brass light pipes and was thermally isolated from the detector and the liquid-helium bath. The temperature of the sample was stabilized to ± 1 K by controlling the power in a small carbon-resistor heater with the output of a copper-constantan thermocouple situated on the copper mount holding the sample. This thermocouple was calibrated with a second thermocouple which was attached to the sample and entered the sample chamber via the light pipe. Error in measurement of sample temperature was ± 2 K.

The magnetic field was provided by a 12-in. electromagnet capable of providing 19 kOe in a 2-in. gap. The magnetic field was measured with nuclear magnetic resonance.

III. EXPERIMENTAL RESULTS

Cyclotron-resonance absorption at 16.09 kOe is shown in Fig. 1 at three temperatures. At 18 K, the dominant absorption (at 100.6 cm^{-1}) is caused by transitions between $n = 0$ and $n = 1$ spin-up Landau states, and weaker absorption (at 96.4 cm^{-1}) results from transitions between $n = 0$ and $n = 1$ spin-down states. Transitions between the impurity states associated with the $n = 0$ and $n = 1$ Landau levels account for the absorption at 107.5 cm^{-1}. At 68 and 108 K, ionization of the impurity levels has occurred and the thermal distribution of electrons results in almost the same amount of absorption from spin-up and spin-down transitions.

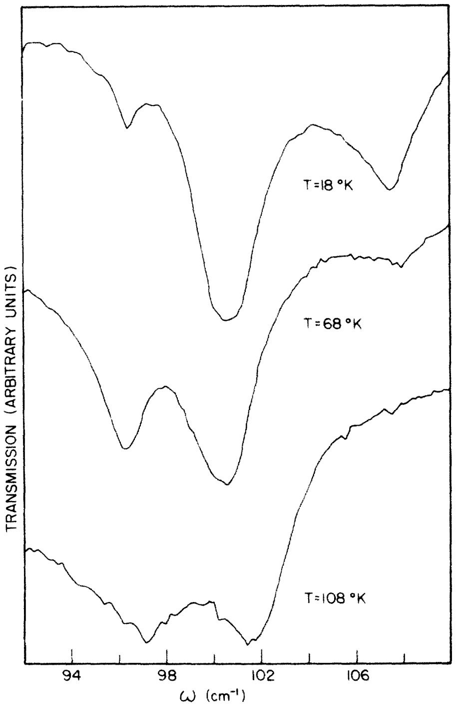

FIG. 1. Cyclotron-resonance absorption in InSb at three different temperatures with a magnetic field of 16.09 kOe.

The resonance frequencies of the spin-split $n = 0$ to $n = 1$ transitions at 14.08, 16.09, and 18.05 kOe were measured at temperatures up to 138 K and no difference in the temperature-dependent behavior of the effective mass attributable to the magnitude of the magnetic field was found. The change in frequency with temperature was quite small and the error of measurement increased as the signal-to-noise ratio decreased at higher temperature. Thus, the frequencies of the spin-up and spin-down transitions at each magnetic field value were normalized and averaged to get a statistical mean of the frequency change.

Corrections were applied to the observed resonant-cyclotron-frequency values to offset the frequency shifts caused by finite instrument aperture, polaron-mass enhancement, and plasma coupling. All frequency corrections were small. The magnitude of the first two effects was calculated in a straightforward manner. The plasma frequency ω_p was calculated using the nominal carrier concentration of 10^{14} cm^{-3} and also from the intercept of a plot of ω_c^2 vs $(H/m^*)^2$ which is equal to ω_p^2 where ω_c is the cyclotron frequency. Both methods gave a value for ω_p of 6 cm^{-1}.

The experimental points relating to the temperature dependence of the cyclotron mass are shown in Fig. 2. The cyclotron mass increases from 4.2 K by 0.4% up to a maximum at 55 ± 5 K after which the cyclotron mass decreases with increasing temperature.

IV. DISCUSSION

The Kane[1] expression for the band-edge effective mass of conduction electrons, m^*, in a narrow-gap semiconductor contains three parameters:

$$\frac{m_e}{m^*} = 1 + \left(\frac{2m_e P^2}{3h^2}\right)\left(\frac{2}{\epsilon_g} + \frac{1}{\epsilon_g + \Delta}\right) , \qquad (1)$$

where m_e is the free-electron mass. The spin-orbit-splitting parameter Δ is relatively independent of temperature since it represents an interaction that takes place deep within an atom. Further, according to Eq. (1), m^* is only weakly dependent on Δ so that any small change in its value has a negligible effect on the effective mass. P^2, the momentum matrix element of the interaction

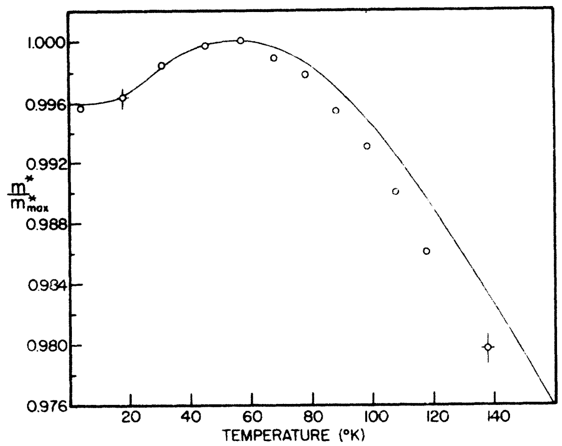

FIG. 2. The electron effective mass in InSb normalized to the maximum mass as a function of temperature. The circles denote experimental values with an error 50% larger than the circle diameter unless otherwise indicated. The solid line is the calculated theoretical curve.

between the conduction and valence bands, is insensitive to temperature.[10] Also, Ehrenreich[11] has shown that P^2 is constant, to within 20%, for III-V compounds with energy gaps ranging from 0.25 to 2.5 eV. The change in effective mass is therefore ascribed to the temperature dependence of the energy gap, ϵ_g.

The change in energy gap as a function of temperature at constant pressure is the sum of two components.[2]

$$\left(\frac{\partial\epsilon_g}{\partial T}\right)_P = \left(\frac{\partial\epsilon_g}{\partial T}\right)_V + \left[V\left(\frac{\partial P}{\partial V}\right)_T\right]\left[\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P\right]\left(\frac{\partial\epsilon_g}{\partial P}\right)_T . \quad (2)$$

The first term describes the energy shift due to the electron-phonon interaction. It is predicted to have a negligible effect on the conduction-electron effective mass because the coupling of electrons to the total phonon field has little or no effect on the curvature of the conduction band as a function of temperature.[4,12]

The second term in Eq. (2) expresses the energy shift in ϵ_g as a result of lattice dilation. It can be written in the form

$$\left[\left(\frac{\partial\epsilon_g}{\partial T}\right)_P\right]_{\text{dilation}} = -3\alpha_T B\left(\frac{\partial\epsilon_g}{\partial P}\right)_T , \quad (3)$$

where α_T is the linear coefficient of thermal expansion and B is the bulk modulus. The latter parameter is found from measured elastic constants to be only slightly temperature dependent.[13] The pressure dependence of ϵ_g at constant temperature was also observed to be only slightly temperature dependent in optical[14] and electrical[15,16] measurements. On the other hand, α_T undergoes a large change with temperature[5-7]; InSb first contracts and then expands with increasing temperature from liquid-helium temperatures. This lattice dilation changes ϵ_g according to Eq. (3).

Ideally, Eq. (1) should be tested by measuring the band-edge effective mass since it is rigorously correct only at $k = 0$. However, determination of the band-edge mass from the extrapolation of the mass to zero field is lengthy and can cause an uncertainty which is larger than the measured mass unless many measurements are taken. We found that it was possible to use the cyclotron mass in nonzero field for our study of the temperature dependence of the mass because the temperature dependences for the spin-up and spin-down transitions measured at three magnetic field values were found to be identical within experimental error.

There is also some question about the effects of thermal broadening on the position of the cyclotron-resonance peaks. For k_H not equal to zero, the energy difference between adjacent Landau levels decreases and so the cyclotron-resonance effective mass increases. However, since there is a maximum in the density of electron states at $k_H = 0$, little shift in peak position is expected from thermal effects.

Values of α_T measured by Gibbons[5] and B determined by Slutsky and Garland[13] were used to calculate the temperature dependence of the cyclotron mass. The change in mass, normalized to the maximum mass, is shown in Fig. 2 as the solid line. There is good agreement with the measured mass change at low temperatures and the position of the maximum in the measured and calculated curves agree. The general agreement shows that the change in energy gap caused by lattice dilation is the dominant mechanism for the change in mass with temperature.

However, there is a small discrepancy between the observed and calculated temperature dependence of the mass. This is most evident at high temperatures where the rate of change with temperature of the observed mass is greater than that of the calculated mass. At 138 K, the observed mass relative to the maximum mass is 0.4% less than the same calculated mass ratio. The discrepancy cannot be ascribed to electrons with $k_H \neq 0$ since such effects would tend to increase the calculated mass at higher temperatures. Neither is the absence of higher-band terms in Eq. (1) the cause. The theoretical curve was recalculated using an expression for the effective mass of the electron which includes higher-band terms.[17] No significant difference was found.

The discrepancy appears to be real so that the question of its cause arises. The pressure dependence of the electron effective mass of InSb at constant temperature has been measured by several workers. In effect, they were varying ϵ_g with pressure rather than with temperature. Itskevich and Sukhoparov[18] using magnetophonon resonance concluded that Kane's equation underestimated the change in mass from the change in ϵ_g by an amount that is similar to the discrepancy at high temperatures found in this experiment. Assuming the discrepancies to have similar causes, the problem may be ascribed to inadequacies in the Kane formulation. However, in a later study[19] of the pressure dependence of the effective mass in InSb using similar techniques, no discrepancy was found between experiment and theory. If this is the case and Kane's effective-mass expression for a rigid lattice can be accepted as correct, the discrepancy in the present experiment must be ascribed to temperature-dependent effects.

Thus far, we have neglected the temperature dependence of the energy gap due to the electron-phonon interaction since the conduction-band curvature in semiconductors is predicted to be independent of this process. However, it is possible that higher-order terms contribute to the temperature dependence of the effective mass. It is therefore worthwhile to extend Fröhlich's theory to look for this effect. Experimentally, resolution of the disagreement between the two experiments on the pressure dependence of the effective mass would be of help in determining why dilation does not fully account for the temperature dependence of the electron effective mass in InSb.

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