Quantum theory of photon-drag transport phenomena in solids and its application to tellurium crystal*

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Starting with the density-matrix equation, we obtain the transport equation for the photon-drag effect arising from an optical interband transition. Using this transport equation, the current induced by the photon-drag effect is derived. The temperature dependence of the photon-drag current predicted by the theory agrees quite well with the experimental result in tellurium crystal.

INTRODUCTION

When a light beam impinges on a crystalline solid, a transfer of momentum takes place between the photons of the light beam and the electrons and holes of the solid. If the intensity of the light beam is high enough (e.g., in the megawatt range), this momentum transfer will induce an observable current or voltage in the solid. This phenomenon is called the photon-drag effect.

The first experimental observation of the photon-drag effect was by Danishevskii *et al.*[1] and Gibson *et al.*[2] in germanium crystal. The theoretical basis for this effect was considered by Grinberg,[3] based on the electron-acoustic-phonon interaction. More recently, the theory of the photon-drag effect was extended to polar and piezoelectric crystals.[4–6]

Previous theoretical work on the photon-drag effect began with the Boltzmann transport equation. The purpose of this paper is to consider the photon-drag effect starting from the density matrix equation of quantum mechanics and then to derive the Boltzmann transport equation. In addition, we will consider band structures that differ from that of the germanium crystal.

CALCULATION

The Hamiltonian for the interaction of the photon-electron-phonon system can be written as follows:

$$H_T = H_e + H_n + H_p + H_{en} + H_{ep} + H_F \quad , \tag{1}$$

where

$$H_e = \sum_{l,k} E_{kl}\, \epsilon^\dagger_{kl}\, \epsilon_{kl} \quad , \tag{2}$$

$$H_p = \sum_{q\lambda} \hbar\omega_{q\lambda}\, b^\dagger_q\, b_{q\lambda} \quad , \tag{3}$$

$$H_n = \sum_{\vec{p}} \hbar\bar{\omega}_p\, a^\dagger_p\, a_p \quad , \tag{4}$$

$$H_{en} = \sum_{lk,l'k'} N^{ll'}_{kk'}\, \epsilon^\dagger_{kl}\, \epsilon_{k'l'} \quad , \tag{5}$$

$$H_{ep} = \sum_{lk,k'l'} P^{ll'}_{kk'}\, \epsilon^\dagger_{kl}\, \epsilon_{k'l'} \quad , \tag{6}$$

$$H_F = \sum_i F(i); \quad F = -eE_{xi}\, x_{xi} \quad . \tag{7}$$

H_e, H_p, and H_n are the Hamiltonians of the electrons, the photons, and the phonons in the second quantization representation. The terms H_{ep} and H_{en} are the perturbed parts of the Hamiltonian resulting from the interaction of the electrons with the photons and the interaction of the electrons with the phonons, respectively. $\epsilon^\dagger_{kl}$ (ϵ_{kl}), $b^\dagger_{\lambda q}$ ($b_{\lambda q}$), and $a^\dagger_p$ (a_p) are the creation (annihilation) operators of the electrons, the photons, and the phonons, respectively. The quantum number $(l, \vec{k})$ designates the band index and the momentum of the electrons

$$P^{ll'}_{kk} = \frac{e}{mc} \sum_{q\lambda} (b_{q\lambda} H''_{kl,k'l'} + b^\dagger_{q\lambda} \bar{H}''_{kl,k'l'}) \tag{8}$$

and

$$N^{ll'}_{kk'} = \sum_p (a_p H'_{kl,k'l'} + a^\dagger_p \bar{H}'_{kl,k'l'}) \quad , \tag{9}$$

where $H'_{kl,k'l'}$ (or $\bar{H}_{kl,k'l'}$) and $H''_{kl,k'l'}$ (or $\bar{H}_{kl,k'l'}$) are the matrix elements between the bands l with momentum k and band l' with momentum k' arising from the interaction of the electrons with phonon and photons, respectively.

In the present work, we will only consider the photon-drag effect arising from the interband transition of the simple band structure of conduction bands and the valence bands such as shown in Figs. 1 and 2.

According to the quantum theory, the density matrix of the total system given in Eq. (1) satisfies the following operator equation:

$$i\hbar \frac{d\rho}{dt} = [H_T, \rho] \quad . \tag{10}$$

The steady-state solution of Eq. (10) is accurate to the first order in the electric field and can be obtained by studying the Laplace transform[7]

$$F(s) = s \int_0^\infty e^{-st} \rho(t)\, dt \tag{11}$$

for $s \to 0^+$.

Applying Eq. (11) to both sides of Eq. (10), we obtain

$$i\hbar s[F(s)-\rho(0)]=[(H_0+H_{ep}+H_{en}),F(s)]+[H_F,\rho(0)]\ , \tag{12}$$

where $H_0=H_e+H_n+H_p$, and $\rho(0)$ is the density matrix of the total system at $t=0^-$, which we assume can be written

$$\rho(0)=\rho_e\,\rho_p\,\rho_n\ , \tag{13}$$

where ρ_e, ρ_p, and ρ_n are the density matrices of the electrons, the photons, and the phonons, respectively, before the interaction was turned on.

In the occupation-number presentation, the wave function of the total system can be written in the following form:

$$|K\rangle=|n_e\rangle\,|n_p\rangle\,|n_n\rangle\ , \tag{14}$$

where

$$|n_e\rangle=|n_1^e,n_2^e,n_3^e,\ldots\rangle\ , \tag{15}$$

$$|n_p\rangle=|n_1^p,n_2^p,n_3^p,\ldots\rangle\ , \tag{16}$$

and

$$|n_n\rangle=|n_1^n,n_2^n,n_3^n,\ldots\rangle \tag{17}$$

are the wave functions of the electrons, the photons, and the phonons in occupation representation, respectively. In the present work, the capital letter will be used to designate the energy state of the total system.

Using the approximation Eq. (12) and the wave function given in Eq. (14), one can show, by following the same procedure of Argyres, and Kohn and Luttinger,[7,8] that the transport equation for the band l with momentum k can be obtained from the following equation:

$$\begin{aligned}0=&\sum_K n_{\bar k\bar l}\,C^0_{KK}-2\pi i\sum_{k'l'}f_{k'l'}(1-f_{\bar k\bar l})N_{\bar k-k'}\,|H'_{\bar k\bar l,k'l'}|^2\,\delta(E_{\bar k\bar l}-E_{k'l'}-\hbar\bar\omega_{\bar k-k'})-2\pi i\sum_{k'l'}f_{k'l'}\\&\times(1-f_{\bar k\bar l})\,(N_{k'-\bar k}+1)\,\delta(-E_{k'l'}+E_{\bar k\bar l}+\hbar\bar\omega_{k'-\bar k})\,|\bar H'_{\bar k\bar l,k'l'}|^2+2\pi i\sum_{kl}f_{\bar k\bar l}(1-f_{kl})\,(N_{k-\bar k})\\&\times|H'_{kl,\bar k\bar l}|^2\,\delta(-E_{\bar k\bar l}+E_{kl}-\hbar\bar\omega_{k-\bar k})+2\pi i\sum_{kl}f_{\bar k\bar l}\,(1-f_{kl})\,(N_{\bar k-k}+1)\,|H'_{kl,\bar k\bar l}|^2\,\delta(-E_{\bar k\bar l}+E_{kl}+\hbar\bar\omega_{\bar k-k})\\&-2\pi i\sum_{k'l'}f_{k'l'}(1-f_{\bar k\bar l})\,|\bar H''_{\bar k\bar l,k'l'}|^2\,P_{\bar k-k'}\,\delta(E_{\bar k\bar l}-E_{k'l'}-\hbar\omega_{\bar k-k'})-2\pi i\sum_{k'l'}f_{k'l'}\\&\times(1-f_{\bar k\bar l})\,|\bar H''_{\bar k\bar l,k'l'}|^2\,(P_{k-\bar k'}+1)\,\delta(E_{\bar k\bar l}-E_{k'l'}+\hbar\omega_{k-\bar k'})+2\pi i\sum_{kl}f_{\bar k\bar l}(1-f_{kl})\,|H''_{kl,\bar k\bar l}|^2\\&\times P_{k-\bar k}\,\delta(E_{kl}-E_{\bar k\bar l}-\hbar\omega_{k-\bar k})+2\pi i\sum_{kl}f_{\bar k\bar l}(1-f_{kl})\,|\bar H''_{kl,\bar k\bar l}|^2\,(P_{\bar k-k}+1)\,\delta(E_{kl}-E_{\bar k\bar l}+\hbar\omega_{\bar k-k})\ ,\end{aligned} \tag{18}$$

where $N_{k-\bar k}$ and $P_{k-k'}$ are the average number of the phonon and photons with wave vector $(k-\bar k)$ and $(k-k')$, respectively, where the phonon and the photon system is characterized by the distribution functions ρ_n and ρ_p. Using Wick's theorem in field theory, it can be shown that the first terms in Eq. (18) can be written[8]

$$\frac{\partial f_{F_i}}{\partial t}=-\frac{e}{\hbar}E_x\frac{\partial f_{kl_i}}{\partial k_x}\ ,\quad i=1,2\ . \tag{19}$$

Equation (18) is very general and is difficult to solve. As a result we will simplify this equation for a practical case in which only one laser beam was used in the experiment. For this case it can be shown that the transport equation for band l_1 and band l_2 takes the following form:

$$\frac{\partial f_{F_1}}{\partial t}+\frac{\partial f_{p_1}}{\partial t}+\frac{\partial f_{n_1}}{\partial t}=0\ , \tag{20}$$

$$\frac{\partial f_{F_2}}{\partial t}+\frac{\partial f_{p_2}}{\partial t}+\frac{\partial f_{n_2}}{\partial t}=0\ , \tag{21}$$

where

$$\begin{aligned}\frac{\partial f_{p_1}}{\partial t}=\frac{2\pi}{\hbar}&[f_{\vec k+\vec q,l_2}(1-f_{\vec k l_1})]\,|\bar H''_{kl_1,(\vec k+\vec q)l_2}|^2\,(P_{\vec q}+1)\,\delta(E_{\vec k+\vec q,l_2}-E_{\vec k l_1}-\hbar\omega_{\vec q})\\&-f_{\vec k l_1}[1-f_{\vec k+\vec q,l_2}]\,|H''_{(\vec k+\vec q)l_2,\vec k l_1}|^2\,P_{\vec q}\,\delta(E_{\vec k+\vec q,l_2}-E_{\vec k l_1}-\hbar\omega_{\vec q})\ ,\end{aligned} \tag{22}$$

$$\frac{\partial f_{n1}}{\partial t}=\frac{2\pi}{\hbar}\left(\sum_{\vec{p}} f_{\vec{k}-\vec{p},l_1}(1-f_{kl_1})N_{\vec{p}}\,|H'_{\vec{k}l_1,(\vec{k}-\vec{p})l_1}|^2\,\delta(E_{\vec{k}l_1}-E_{\vec{k}-\vec{p},l_1}-\hbar\bar{\omega}_{\vec{p}})+\sum_{\vec{p}} f_{\vec{k}+\vec{p},l_1}(1-f_{\vec{k}l_1})\right.$$
$$\times(N_{\vec{p}}+1)\,|\bar{H}'_{\vec{k}l_1,(\vec{k}+\vec{p})l_1}|^2\,\delta(E_{\vec{k}l_1}-E_{\vec{k}+\vec{p},l_1}+\hbar\bar{\omega}_{\vec{p}})-\sum_{\vec{p}} f_{\vec{k}l_1}(1-f_{\vec{k}+\vec{p},l_1})N_{\vec{p}}\,|H'_{(\vec{k}+\vec{p})l_1,kl_1}|^2$$
$$\left.\times\delta(E_{\vec{k}+\vec{p},l_1}-E_{\vec{k}l_1}-\hbar\bar{\omega}_{\vec{p}})-\sum_{\vec{p}} f_{\vec{k}l_1}(1-f_{\vec{k}-\vec{p},l_1})(N_{-\vec{p}}+1)\,|\bar{H}'_{(\vec{k}-\vec{p})l_1,kl_1}|^2\,\delta(E_{\vec{k}-\vec{p},l_1}-E_{\vec{k}l_1}+\hbar\bar{\omega}_{\vec{p}})\right)\,, \tag{23}$$

$$\frac{\partial f_{p2}}{\partial t}=\frac{2\pi}{\hbar}\left((1-f_{\vec{k}l_2})f_{\vec{k}-\vec{q},l_1}\,|H''_{\vec{k}l_2,(\vec{k}-\vec{q})l_1}|^2\,P_{\vec{q}}\,\delta(E_{\vec{k}l_2}-E_{\vec{k}-\vec{q},l_1}-\hbar\omega_{\vec{q}})-f_{\vec{k}l_2}\right.$$
$$\left.\times(1-f_{\vec{k}-\vec{q},l_1})(P_{\vec{q}}+1)\,|\bar{H}''_{(\vec{k}-\vec{q})l_1,\vec{k}l_2}|^2\,\delta(E_{\vec{k}l_2}-E_{\vec{k}-\vec{q},l_1}-\hbar\omega_{\vec{q}})\right)\,, \tag{24}$$

and

$$\frac{\partial f_{n2}}{\partial t}=\frac{2\pi}{\hbar}\left(\sum_{\vec{p}} f_{\vec{k}-\vec{p},l_2}(1-f_{\vec{k}l_2})N_{\vec{p}}\,|H'_{\vec{k}l_2,(\vec{k}-\vec{p})l_2}|^2\,\delta(E_{\vec{k}l_2}-E_{\vec{k}-\vec{p},l_2}-\hbar\bar{\omega}_{\vec{p}})+\sum_{\vec{p}} f_{\vec{k}+\vec{p},l_2}(1-f_{\vec{k}l_2})(N_{\vec{p}}+1)\right.$$
$$\times|\bar{H}'_{\vec{k}l_2,(\vec{k}+\vec{p})l_2}|^2\,\delta(E_{\vec{k}l_2}-E_{\vec{k}-\vec{p},l_2}+\hbar\bar{\omega}_{\vec{p}})-\sum_{\vec{p}} f_{\vec{k}l_2}(1-f_{\vec{k}+\vec{p},l_2})N_{\vec{p}}\,|H'_{(\vec{k}+\vec{p})l_2,\vec{k}l_2}|^2$$
$$\left.\times\delta(E_{\vec{k}+\vec{p},l_2}-E_{\vec{k}l_2}-\hbar\bar{\omega}_{\vec{p}})-\sum_{\vec{p}}\delta(E_{\vec{k}-\vec{p},l_2}-E_{\vec{k}l_2}+\hbar\bar{\omega}_{\vec{p}})f_{\vec{k}l_2}(1-f_{\vec{k}-\vec{p},l_2})\,|\bar{H}'_{(\vec{k}-\vec{p})l_2,\vec{k}l_2}|^2(N_{\vec{p}}+1)\right)\,. \tag{25}$$

Equations (23) and (24) are still difficult to solve. Hence, we will make the relaxation approximation

$$\frac{\partial f_{n1}}{\partial t}=-\frac{f_i-f_i^0}{\tau_i}\quad\text{and}\quad\frac{\partial f_{n2}}{\partial t}=-\frac{f_f-f_f^0}{\tau_f}\,, \tag{26}$$

where f_i^0 and f_f^0 are the equilibrium distribution function of the electrons in the first and second conduction band, respectively. In this approximation, the transport equation takes the form:

$$-\frac{e}{\hbar}E_x\frac{\partial f^0_{kl1}}{\partial k_x}+\frac{\partial f_{p1}}{\partial t}-\frac{f_i-f_i^0}{\tau_i}=0\,, \tag{27}$$

$$-\frac{e}{\hbar}E_x\frac{\partial f^0_{kl2}}{\partial k_x}+\frac{\partial f_{p2}}{\partial t}-\frac{f_f-f_f^0}{\tau_f}=0\,. \tag{28}$$

Now if we let $f_i-f_i^0=\langle f^1\rangle$ ($\alpha=1$) and $f_f-f_f^0=\langle f^2\rangle$ ($\alpha=2$), Eqs. (27) and (28) have exactly the same form as that of Eq. (2) of Valov *et al.*,[9] and the equations of Grinberg (Ref. 3, p. 532) for zero magnetic field. The difference between Eqs. (27) and (28) and that of Grinberg and of Valov *et al.* is the sign in the first terms of Eqs. (27) and (28). The reason for this is that the equations of Grinberg and of Valov *et al.* are for holes, whereas Eqs. (27) and (28) are for electrons. It should be pointed out that Eq. (1) of Grinberg and Eq. (4) used by the author[4,5] are for the photon-drag effect arising from intraband transition. On the other hand, Eqs. (27) and (28) are for the photon-drag effect due to interband transition. Now let us use Eqs. (27) and (28) to derive an equation for the current induced by the photon-drag phenomena for the band model as shown in Fig. 1.

For the case in which no external electric field is applied, $E\cong 0$ (short-circuit current), we can write the total induced current as follows:

$$J_{\hat{q}}=\frac{-2e}{(2\pi)^3}\left(\int\vec{v}_i\cdot\hat{q}\,\tau_i\frac{\partial f_{p1}}{\partial t}d\vec{k}_i+\int\vec{v}_f\cdot\hat{q}\,\tau_f\frac{\partial f_{p2}}{\partial t}d\vec{k}_f\right)\,. \tag{29}$$

Now if we use equations (22) and (24), the equation can be written as follows:

$$J_{\hat{q}}=\frac{-2e}{(2\pi)^3}\left(\frac{2\pi}{\hbar}\right)\int\vec{v}_i\cdot\hat{q}\,\tau_i\,d\vec{k}_i\,[\,|H_{(\vec{k}_i+\vec{q})l_2,k_il_1}|^2(f_{\vec{k}_i+\vec{q},l_2}-f_{\vec{k}_il_1})\,\delta(E_{\vec{k}_i+\vec{q},l_2}-E_{\vec{k}_il_1}-\hbar\omega)]-\frac{2e}{(2\pi)^3}\left(\frac{2\pi}{\hbar}\right)$$
$$\times\int\vec{v}_f\cdot\hat{q}\,\tau_f\,d\vec{k}_f\,[\,|H_{\vec{k}_fl_2,(\vec{k}_f-\vec{q})l_1}|^2(f_{\vec{k}_f-\vec{q},l_1}-f_{\vec{k}_fl_2})\,\delta(E_{\vec{k}_fl_2}-E_{\vec{k}_f-\vec{q},l_1}-\hbar\omega)], \tag{30}$$

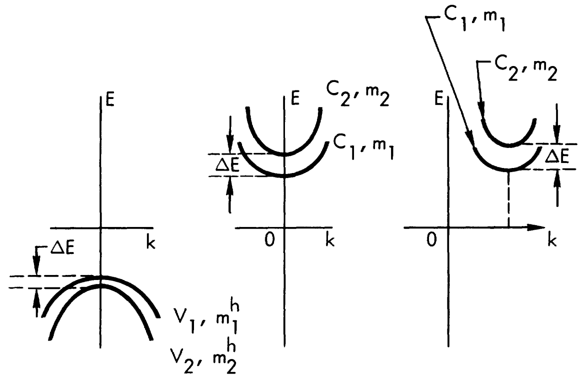

FIG. 1. Simple energy diagram of the electrons and the holes used in the present calculation.

where the subscripts i and f of $\vec{v}$, τ, and f in Eqs. (26)–(30) represent the subscripts $k_i l_1$ and $k_f l_2$, respectively, and $\hat{q}$ is the unit vector of the photon, e.g., $\vec{q}/|q|$.

In general, the matrix elements in Eq. (30) are functions of direction of polarization and of photon momentum. They are, in general, very complicated to calculate. Therefore, as a first-order approximation, we will use a procedure to average the polarization, and the momentum of the photons to obtain an average value of the matrix elements. These averages can be written as follows[10]:

$$|H_{(\vec{k}_i+\vec{q})l_2,k_i l_1}|^2_{av} = |H_{l_1 l_2}|^2 + (k_i^2)A_{l_1 l_2} , \qquad (31)$$

$$|H_{k_f l_2,(\vec{k}_f-\vec{q})l_1}|^2_{av} = |H_{l_2 l_1}|^2 + (k_f^2)A_{l_2 l_1} . \qquad (32)$$

To simplify the notation, we write $H_{l_1 l_2} = H_{if}$, $E_{k' l_1} = E_i(k')$, $E_{k' l_2} = E_f(k')$ and $f_{k' l_1} = f_i(k')$, etc. If we use the matrix elements given in Eqs. (31) and (32) and make a Taylor expansion of all the functions under the integral sign of Eq. (30) in terms of the photon wave vector $\vec{q}$, and then make an expansion around the electron wave vector $\vec{k}_0$, where $\vec{k}_0$ satisfies the equation $E_f(k_0) - E_i(k_0) = \hbar\omega$, we obtain an explicit formula for the current induced by the photon-drag effect. Because the steps involved in the expansion are straightforward, we only present the result here:

$$\delta[E_f(\vec{k}_i+q) - E_i(k_i) - \hbar\omega] = \delta(k_i - \delta k_i - k_0) \Big/ \left(\frac{\partial E_f}{\partial k_0} - \frac{\partial E_i}{\partial k_0}\right) \left\{1 - \left(\frac{\partial E_f}{\partial k_0} - \frac{\partial E_i}{\partial k_0}\right)^{-1} \left[\delta k_i \left(\frac{\partial^2 E_f}{\partial k_0^2} - \frac{\partial^2 E_i}{\partial k_0^2}\right) + \vec{q} \cdot \hat{k} \frac{\partial^2 E_f}{\partial k_0^2}\right]\right\} , \qquad (33)$$

$$\tau_j(k) = \tau_j(k_0) + \frac{\partial \tau_j}{\partial k_0} \vec{\delta} k \cdot \hat{k} \quad (j = i, \text{ or } f) , \qquad (34)$$

$$\vec{v}_j \circ \hat{q} = \frac{\hat{q} \circ \vec{k}}{\hbar} \left(\frac{\partial E_j}{\partial k_0} + \delta k \frac{\partial^2 E_j}{\partial k_0^2}\right) \ (j = i, \text{ or } f) , \qquad (35)$$

$$f_f(k_i + q) - f_i(\delta k_i) = [f_f(E_i + \hbar\omega) - f_i(E_i)] + \delta k_i \left(\frac{\partial f_f(E_i + \hbar\omega)}{\partial k_0} - \frac{\partial f_i}{\partial k_0}\right) , \qquad (36)$$

$$\delta[E_f(k_f) - E_i(k_f - \vec{q}) - \hbar\omega] = \delta(k_f - k_0 - \delta k_f) \Big/ \left(\frac{\partial E_f}{\partial k_0} - \frac{\partial E_i}{\partial k_0}\right) \left\{1 - \left(\frac{\partial E_f}{\partial k_0} - \frac{\partial E_i}{\partial k_0}\right)^{-1} \left[\vec{q} \cdot \hat{k} \frac{\partial^2 E_i}{\partial k_0^2} + \delta k_f \left(\frac{\partial^2 E_f}{\partial k_0^2} - \frac{\partial^2 E_i}{\partial k_0^2}\right)\right]\right\} , \qquad (37)$$

$$[f_i(k_f - \vec{q}) - f_f(k_f)] = [f_i(E_f - \hbar\omega) - f_f(E_f)] + \delta k_f \left(\frac{\partial f_i(E_f - \hbar\omega)}{\partial k_0} - \frac{\partial f_f(E_f)}{\partial k_0}\right) , \qquad (38)$$

where the functions f_i, f_f, $E_j(k)$ and $E_f(k)$ on the left-hand side of Eqs. (33) to (38) are evaluated at $k = k_0$ and the derivations in these equations have the following meaning:

$$\left.\frac{\partial E_i(k)}{\partial k}\right|_{k=k_0} = \frac{\partial E_i}{\partial k_0} ,$$

$$\left.\frac{\partial f_f(E_i + \hbar\omega)}{\partial k}\right|_{k=k_0} = \frac{\partial f_f(E_i + \hbar\omega)}{\partial k_0} ,$$

and

$$\left.\frac{\partial^2 E_i}{\partial k^i}\right|_{k=k_0} = \frac{\partial^2 E_i}{\partial k_0^2} \ \text{etc...} .$$

Using the fact that

$$[E_f(\vec{k}_f) - E_i(\vec{k}_f - \vec{q}) - \hbar\omega] = 0 , \qquad (39)$$

$$[E_f(\vec{k}_i + \vec{q}) - E_i(k_i) - \hbar\omega] = 0 , \qquad (40)$$

we obtain

$$\delta k_f = -(\vec{q} \cdot \hat{k}_f) \frac{\partial E_i}{\partial k_0} \left(\frac{\partial E_f}{\partial k_0} - \frac{\partial E_f}{\partial k_0}\right)^{-1} \qquad (41)$$

and

$$\delta k_i = -(\vec{q} \cdot \hat{k}_i) \frac{\partial E_f}{\partial k_0} \left(\frac{\partial E_f}{\partial k_0} - \frac{\partial E_i}{\partial k_0}\right)^{-1} , \qquad (42)$$

where $\hat{k}$ is the unit vector of the wave vector of the electron. After substituting Eqs. (33)–(42) into

Eq. (30) and simplifying, we obtain the following equation for the current:

$$J = J_1 + J_2 \ ,$$

where

$$J_1 = \frac{4}{3}\frac{e^3 qI}{m^2 c}\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\left(\frac{\partial E_i}{\partial k_0}\right)\left(\frac{\partial E_f}{\partial k_0}\right)(\tau_i)(k_0^2)[f_f(E_i+\hbar\omega)-f_i(E_i)] \times\left(\frac{\partial E_f}{\partial k_0}-\frac{\partial E_i}{\partial k_0}\right)^{-2}\left[\left(-F(k_0)\,C^{-1}+\frac{\partial}{\partial k_0}\ln G_1\right)[|H_{fi}|^2 + A_{if}\hbar^2 k_0^2] + 2k_0\hbar^2 A_{fi}\right] \ , \tag{43}$$

and

$$G_1 = [f_f(E_i+\hbar\omega)-f_i(E_i)]\,k_0^2\,\tau_i\,\frac{\partial E_i}{\partial k_0} \ , \tag{44}$$

$$-F(k_0)\,C^{-1} = \left[\frac{\partial^2 E_f}{\partial k_0^2} - \left(\frac{\partial E_f}{\partial k_0}\right)\left(\frac{\partial E_f}{\partial k_0}-\frac{\partial E_i}{\partial k_0}\right)^{-1}\left(\frac{\partial^2 E_f}{\partial k_0^2}-\frac{\partial^2 E_i}{\partial k_0^2}\right)\right]\left(\frac{\partial E_f}{\partial k_0}\right)^{-1} \ , \tag{45}$$

$$J_2 = \frac{4}{3}\left(\frac{e^3 qI}{m^2 c}\right)\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\left(\frac{\partial E_f}{\partial k_0}\right)\left(\frac{\partial E_i}{\partial k_0}\right)\left(\frac{\partial E_f}{\partial k_0}-\frac{\partial E_i}{\partial k_0}\right)^{-2}(\tau_f)k_0^2[f_i(E_f-\hbar\omega)-f_f(E_f)] \times\left[2A_{12}k_0\hbar^2 + (|H_{if}|^2 + k_0^2\hbar^2 A_{12})\left(-\bar{F}(k_0)\,(\bar{C})^{-1}+\frac{\partial}{\partial k_0}\ln(G_2)\right)\right] \ , \tag{46}$$

$$G_2 = k_0^2\,\tau_f\,\frac{\partial E_f}{\partial k_0}[f_i(E_f-\hbar\omega)-f_f(E_f)] \ , \tag{47}$$

$$-\bar{F}(k_0)\,(\bar{C})^{-1} = \left[\frac{\partial^2 E_i}{\partial k_0^2} - \frac{\partial E_i}{\partial k_i}\left(\frac{\partial E_f}{\partial k_0}-\frac{\partial E_i}{\partial k_0}\right)^{-1}\left(\frac{\partial^2 E_f}{\partial k_0^2}-\frac{\partial^2 E_i}{\partial k_0^2}\right)\right]\left(\frac{\partial E_i}{\partial k_0}\right)^{-1} \ , \tag{47a}$$

where in Eqs. (45) and (47), I is the intensity of the light, m is free electron mass, c is the velocity of the light in free space, ω is frequency of the incident light, e is the charge of the electron, q is the wave vector of the photon, and τ_f, E_i, E_f and τ_i are functions of k_0.

Now in the case in which the zero-order matrix elements $|H_{if}| \neq 0$, we can ignore the terms associated with A_{12}. In this case Eqs. (43)–(47) can be simplified as follows:

$$J_1 = \frac{4}{3}\left(\frac{e^3 qI}{m^2 c}\right)\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\left(\frac{\partial E_i}{\partial k_0}\right)\left(\frac{\partial E_f}{\partial k_0}\right)(\tau_i)(k_0^2)[f_f(E_i+\hbar\omega)-f_i(E_i)]\left(\frac{\partial E_f}{\partial k_0}-\frac{\partial E_i}{\partial k_0}\right)^{-2}\left(-F(k_0)\,C^{-1}+\frac{\partial}{\partial k_0}\ln G_1\right)|H_{fi}|^2 \ , \tag{48}$$

$$J_2 = \frac{4}{3}\left(\frac{e^3 qI}{m^2 c}\right)\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\left(\frac{\partial E_f}{\partial k_0}\right)\left(\frac{\partial E_i}{\partial k_0}\right)\left(\frac{\partial E_f}{\partial k_0}-\frac{\partial E_i}{\partial k_0}\right)^{-2}(\tau_f)(k_0^2)[f_i(E_f-\hbar\omega)-f_f(E_f)]|H_{if}|^2\left(-\bar{F}(\bar{C})^{-1}+\frac{\partial}{\partial k_0}\ln G_2\right) \ , \tag{49}$$

where τ_i and τ_f are functions of k_0.

Now let us apply the present theory to the valence band structure as shown in Fig. 1. To obtain the current for this case, we need the distribution of the holes rather than the distribution of the electrons. The hole distribution can be obtained from Eqs. (20) and (21) by making the substitutions into Eq. (18) of

$$f^h_{kl_1} = f^h_i(k) = [1 - f_i(k)] \ , \tag{50}$$

$$f^h_{kl_2} = f^h_f(k) = [1 - f_f(k)] \ . \tag{51}$$

After making the substitutions of Eqs. (50) and (51) into Eq. (18) and carrying out the same analysis as for electrons, we can obtain the hole current due to the photon-drag effect simply by changing the sign of Eqs. (43) to (47a) and replacing $f_i(E_i+\hbar\omega)$ by $f^h_i(E^h_i+\omega)$, $f_i(E_i)$ by $f^h_i(E^h_i)$, E_i by E^h_i, E_f by E^h_f, and the electron relaxation time by the hole relaxation time.

For crystals with center-of-inversion symmetry, the zero-order matrix elements vanish. Therefore, the equation for the photon-drag for hole current can be written as in the spherical-band approximation as follows:

$$J = J_1 + J_2 \ , \tag{52}$$

$$J_1 = -\frac{8}{3}\left(\frac{e^3 qI}{m^2 c}\right)\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\frac{(m_d)^2}{(m_1)(m_2)}\tau^h_i k_0^3\hbar^2 A_{if}[f^h_f(E^h_i+\hbar\omega)-f^h_i(E_i)]\left(1+\frac{k_0}{2}\frac{\partial}{\partial k_0}\ln G^h_i\right) \ , \tag{53}$$

$$J_2 = -\frac{8}{3}\left(\frac{e^3 qI}{m^2 c}\right)\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\frac{(m_d)^2}{m_1 m_2}\tau_f^h k_0^2 [f_i^h(E_f^h - \hbar\omega) - f_f(E_f^h)] A_{12} k_0 \hbar^2 \left(1 + \frac{k_0}{2}\frac{\partial}{\partial k_0}\ln G_2^h\right) . \qquad (54)$$

where $1/m_d = 1/m_2^h - 1/m_1^h$ and m_1^h and m_2^h are the effective masses of the holes in the valence bands V_1 and V_2, respectively.

For the case where the zero-order matrix elements for intervalence transition does not vanish (e.g., $|H_{if}|^2 \neq 0$), we can write the total current as

$$J_t^h = J_1^h + J_2^h \ , \qquad (55)$$

$$J_1^h = -\frac{4}{3}\left(\frac{e^3 qI}{m^2 c}\right)\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\left(\frac{\partial E_i^h}{\partial k_0}\right)\left(\frac{\partial E_f^h}{\partial k_0}\right)\tau_i^h(k_0)^2 [f_f^h(E_i + \hbar\omega) - f_i^h(E_i)]\left(\frac{\partial E_f^h}{\partial k_0} - \frac{\partial E_i^h}{\partial k_0}\right)^{-2} |H_{if}|^2\left[-F_{h1}(k_0)\,C_{h1}^{-1} + \frac{\partial}{\partial k_0}\ln G_1^h\right], \qquad (55a)$$

and

$$J_2^h = -\frac{4}{3}\left(\frac{e^3 qI}{m^2 c}\right)\frac{1}{\epsilon^{1/2}}\frac{1}{(\hbar\omega)^2}\left(\frac{\partial E_f^h}{\partial k_0}\right)\left(\frac{\partial E_i^h}{\partial k_0}\right)\left(\frac{\partial E_f^h}{\partial k_0} - \frac{\partial E_i^h}{\partial k_0}\right)^{-2}\tau_f^h k_0^2$$

$$\times [f_i^h(E_f^h - \hbar\omega) - f_f^h(E_f^h)]\,|H_{if}|^2\left[-\bar{F}_{h2}(k_0)\,(\bar{C}_{h2})^{-1} + \frac{\partial}{\partial k_0}\ln G_2^h\right] , \qquad (55b)$$

where G_1^h, G_2^h, $F_{h1}(C_{h1})^{-1}$ and $\bar{F}_{h2}(\bar{C}_{h2})^{-1}$, which occur in Eqs. (54)–(57), are given as follows:

$$G_1^h = [f_f^h(E_i^h + \hbar\omega) - f_i^h(E_i^h)]\, k_0^2\, \tau_i^h \frac{\partial E_i^h}{\partial k_0} ,$$

$$-F_{h1}(k_0)\, C_{h1}^{-1} = \left[\frac{\partial^2 E_f^h}{\partial k_0^2} - \left(\frac{\partial E_f^h}{\partial k_0}\right)\left(\frac{\partial E_f^h}{\partial k_0} - \frac{\partial E_i^h}{\partial k_0}\right)^{-1}\left(\frac{\partial^2 E_f^h}{\partial k_0^2} - \frac{\partial^2 E_i^h}{\partial k_0^2}\right)\right]\left(\frac{\partial E_f^h}{\partial k_0}\right)^{-1} ,$$

$$G_2^h = k_0^2\, \tau_f^h \frac{\partial E_f^h}{k_0}[f_i^h(E_f^h - \hbar\omega) - f_f^h(E_f^h)] ,$$

$$-\bar{F}_{h2}(k_0)\,(\bar{C}_{h2})^{-1} = \left[\frac{\partial^2 E_i^h}{\partial k_0^2} - \left(\frac{\partial E_i^h}{\partial k_i}\right)\left(\frac{\partial E_f^h}{\partial k_0} - \frac{\partial E_i^h}{\partial k_0}\right)^{-1}\left(\frac{\partial^2 E_f^h}{\partial k_0^2} - \frac{\partial^2 E_i^h}{\partial k_0^2}\right)\right]\left(\frac{\partial E_i^h}{\partial k_0}\right)^{-1} .$$

As a test of Eq. (55), we apply it to the photon-drag effect in tellurium. This effect was observed by Panyakeow *et al.*[11] The band structure of the tellurium was quite complicated and the effective mass of the holes are not scalars.[12–14] Based upon the $\vec{k}\cdot\vec{p}$ perturbation theory, together with experimental data obtained from experiments, the dispersion of the holes in valences band can be written as follows[14]:

$$E_{1,2}(k) = -ak_z^2 - bk^2 \pm (\lambda^2 + 2a\lambda\, t_0\, k_z^2)^{1/2} , \qquad (56)$$

where

$$a \cong 3.5\times10^{-15}\ \text{eV cm}^2, \quad \lambda \cong 62.5\ \text{meV} ,$$
$$b \cong 3.45\times10^{-15}\ \text{eV cm}^2, \quad t_0 \cong 1.30. \qquad (57)$$

The energy band of the holes with the energy dispersion given in Eq. (56) is shown in Fig. 2.

In the experiment of Panyakeow *et al.*,[11] the excitation source was a 10.6-μ CO_2 laser. The energy of the photon was approximately equal to that of the band gap between the two valence bands, e.g., $h\omega \cong 2\lambda = \Delta E$. This means that all the absorption will take place around $k = 0$. Hence in the first-order approximation, we can expand the third term in Eq. (56) in a Taylor series, and write the dispersion in the following way:

$$E_1 = \frac{h^2 k_z^2}{2m_{z1}} + \frac{h^2}{2m_{1\perp}} k_1^2 , \quad E_2 = \frac{h^2 k_1^2}{2m_{2\perp}} + \frac{h^2 k_z^2}{2m_{z2}} \qquad (58)$$

where

$$\frac{h^2}{2m_{z1}} = -a(1 - t_0), \quad \frac{h^2}{2m_{z2}} = -a(1 + t_0), \quad \frac{h^2}{2m_{1\perp}} = \frac{h^2}{2m_{2\perp}} = -b. \qquad (59)$$

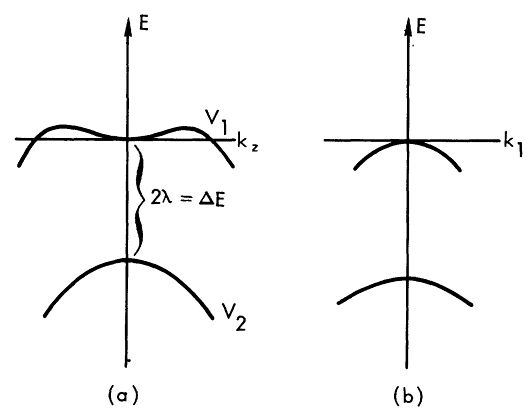

FIG. 2. Energy diagram for the tellurium crystal.

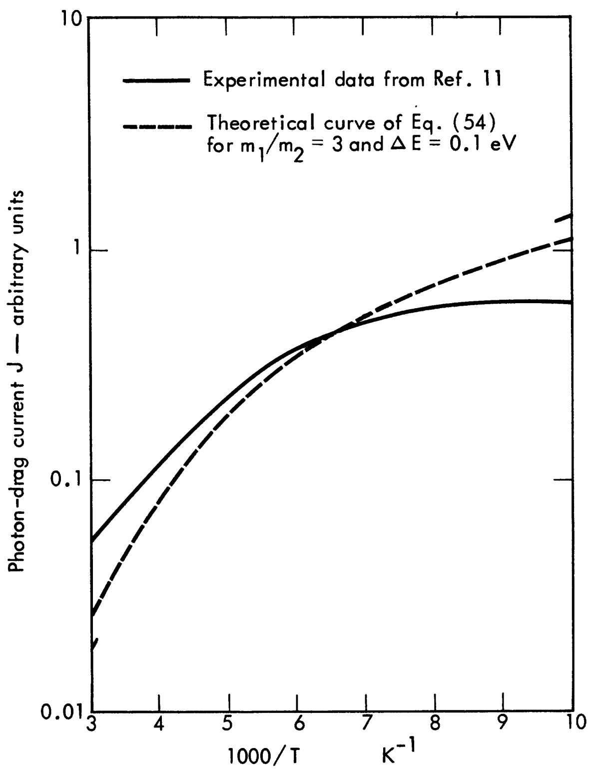

FIG. 3. Temperature dependence of the theoretical curve and the experimental curve of the photon drag current in p-type tellurium.

The theory developed in the present paper is not capable of handling crystal with the form of energy dispersion of that given in Eq. (56). Therefore, as a first-order approximation, we will use an average effective mass defined as the following:

$$m_1^x=\frac{1}{3}\left(\frac{2}{m_{1\perp}}+\frac{1}{m_{z1}}\right),\quad m_2^\alpha=\frac{1}{3}\left(\frac{2}{m_{2\perp}}+\frac{1}{m_{z2}}\right) . \tag{60}$$

Using the parameter from Eq. (56), we obtain

$$m_1\cong 0.29m_0 \ , \tag{61}$$

$$m_2\cong 0.1m_0 \ . \tag{62}$$

Using the effective masses given in Eqs. (61) and (62) together with values of the energy gap (ΔE) between the two valence bands, we obtain the current as a function of temperature as shown in Fig. 3. The experimental and theoretical curves were normalized at $T=150$ °K. As one can see from the comparison of the two curves, the theoretical curves agree quite well with the experimental curve. Actually, ΔE is a function of the temperature. At 77 °K, ΔE is about 0.11 eV; however, ΔE decreases with increasing temperature. Therefore, the value of ΔE in the temperature range of interest is less than 0.11 eV and we actually used 0.10 eV.

It must be pointed out that the theoretical curve given in Fig. 3 was based upon the assumption that all the acceptors in the crystal were ionized. This assumption can be seen to be a sound one by considering the following. The ionization of the acceptors is approximately in the 1–2-meV range. Within the temperature range given in Fig. 3, practically all acceptors are ionized. The impurity absorption due to acceptors will have no effect on photon drag. We have also made the approximation that the dominant scattering mechanism in the temperature range of interest is acoustic phonon scattering.

Since the average effective mass of the upper valence band is three times larger than that of the lower valence band, most of the photon-drag current arises from the lower band—a fact first pointed out by Panyakeow *et al.*[11] Mathematically, this is also true if one substitutes the effective mass given in Eqs. (61) and (62) into Eqs. (48) and (49). The terms with the effective mass m_2 dominate. This means that the band structure of the upper valence has very little effect on photon drag.

CONCLUSIONS AND DISCUSSION

In this paper we have derived the transport equation for the photon-drag effect arising from interband transition. In this derivation we make the approximation that in the steady state, the density matrix for the system can be written as a product. Furthermore, we assume that the phonon and the photon system density matrix was the same as before the interaction was turned on. This approximation has also been used by others in related work.[8] In deriving the current induced by the photon-drag effect, we also used the relaxation approximation for the terms in the transport equation arising from the electron-phonon interaction. For a polar crystal, the relaxation approximation is not meaningful in the temperature range in which the polar phonon energy is greater than the K_0T (e.g., $\hbar\omega_p > K_0T$). For $\hbar\omega_p < K_0T$, the approximation is meaningful.

In a photon-drag experiment, the light intensity is very high. Therefore, in the calculation of the photon-drag current (or voltage), the terms arising from spontaneous emission processes have been neglected.

The average matrix element [Eqs. (31) and (32)] was obtained by first expanding the Bloch wave function in a Taylor expansion in terms of the wave vector of the photon, and then averaging over the angle of the polarization of the photon and the momentum of the photons. In the averaging process the wave vector $\vec{q}$ was eliminated from the matrix elements. This explains why the matrix element given in Eqs. (31) and (32) is independent of the wave vector of the photons.

In developing Eqs. (23) and (25), we neglected the terms in Eq. (18) associated with interband transitions due to phonon scattering. This assumption is good if the gap between the valence bands or the conduction bands is greater than the phonon energy. Hence, for germaniumlike crystals in which the two valence bands touch at $k=0$, one must take into consideration all terms in Eq. (18) involving the phonon interaction.

In order to use Eqs. (43)–(55b) we must replace the steady-state function (f) with the thermal distribution function (f^0) because, in the present formulation, the steady-state distribution cannot be determined. However, the effect of replacing f by f^0 in these equations is the same as neglecting the second-order effect introduced by the terms δf, where $f=f^0+\delta f$. Therefore, replacing f in Eqs. (43)–(55b) by the thermal distribution function should be a good approximation. This approximation has also been used by others in photon-drag work.

Comparing the experimental and theoretical curves of Fig. 3, the theory seems to match the temperature dependence of the photon-drag effect in tellurium quite well.

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