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Plasma oscillations of a two-dimensional electron gas in a strong magnetic field*

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The quantum-mechanical response of a two-dimensional electron gas in the presence of a strong dc magnetic field is calculated in the random-phase approximation. The results are used to discuss the magnetoplasma oscillations of a two-dimensional electron gas. The general dispersion relation is derived, and numerical results are presented for both long and short wavelengths.

I. INTRODUCTION

The study of a two-dimensional electron gas (2 DEG) has aroused considerable interest during the past few years. This interest is in part due to the technological importance of the metal-oxide-semiconductor (MOS) system in which many experimental studies have been carried out, and to similarities between some properties of the 2 DEG and those of thin films and surfaces. In addition, however, the 2 DEG is of quite fundamental interest since it represents a many-body system in which the effective electron density can be experimentally varied over a wide range.

By the application of a very strong electric field normal to the surface of a semiconductor, electrons or holes can be confined to the surface. If the electric field is strong enough, the motion perpendicular to the surface will be quantized. At very low temperature when electron scattering is unimportant, such a system will behave as a 2 DEG. This behavior has been confirmed by observation of the oscillatory magnetoconductance in inverted silicon surfaces.[1,2] It was also found that surface quantization into Landau levels depends only on the normal component but not the tangential component of the magnetic field.[2] Very recently, other experiments have been carried out in order to study the dependence of the effective mass[3] and the quasiparticle g factor[4] on the surface electron density of a 2 DEG. It is worth mentioning that in the inversion layers of MOS structures, the carrier concentration can easily be controlled and can be varied over a wide range by simply changing the applied electric field. Such a system is very useful for the study of many-body effects.

The study of plasma oscillations in a 2 DEG is still in an early stage. The first theoretical discussion was given by Stern.[5] In his paper, the response of the electron gas to a longitudinal electric field was calculated in the self-consistent-field approximation. With this result, the plasmon dispersion for a 2 DEG imbedded in a three-dimensional dielectric was obtained. In the past few years, the problem of surface plasmons and especially, surface magnetoplasmons in a semi-infinite solids and layer structures has attracted the interest of many authors.[6] It is interesting to ask what kind of magnetoplasma waves can exist in a 2 DEG. In this paper, we generalize Stern's work by allowing an external dc magnetic field normal to the plane of the 2 DEG. In the presence of the applied field, the electrons become quantized in the Landau levels and a new parameter ω_c, the cyclotron frequency, is introduced. The plasma oscillations are no longer purely longitudinal as they were in the absence of the applied field. How the magnetic field affects the plasmon dispersion, and what the behavior of the dispersion curves is in the long- and short-wavelength limits are studied in this paper.

In Sec. II, we calculate the quantum-mechanical response of a 2 DEG in the presence of a strong dc magnetic field in the random-phase approximation. For simplicity, the effect of collisions of the electrons is neglected. The calculation is similar to that presented by Quinn *et al.*[7] and in a later paper by Greene *et al.*[8] for the three-dimensional case. As in that case, the components of the conductivity tensor are found to consist of two terms, namely, an oscillatory term and a secular term. The former gives rise to quantum oscillations and the latter is simply the result one would obtain in the semiclassical limit. In Sec. III, we present the derivation of the plasma dispersion relation of a 2 DEG imbedded in a dielectric in the presence of a perpendicular applied magnetic field. The dispersion relation written in terms of the polarizability tensor is completely general. In the limit of zero applied field, the longitudinal mode reduces to the plasmon mode obtained by Stern. In Sec. IV, the magnetoplasma oscillations of the 2 DEG are investigated carefully in both the long-wavelength and the short-wavelength limits. In the long-wavelength limit, we make series expansions of the Bessel functions. For short wavelengths, the asymptotic expressions of the Bessel functions are used. For simplicity, we consider here only the nonoscillatory terms of the polarizability tensor. Therefore, the dispersion curves presented in this section are valid only in the semiclassical limit.

II. QUANTUM MAGNETOPOLARIZABILITY TENSOR

In this section, we evaluate the response function of a 2 DEG to an external disturbance in the random-phase approximation. Our treatment is quite similar to that presented by Greene *et al.*,[8] but for simplicity, electron collisions are neglected. The result obtained for the polarizability tensor can be expressed as a sum of terms involving Bessel functions. It consists of two parts: an oscillatory part whose amplitude decreases exponentially with increasing temperature and a nonoscillatory part which is insensitive to temperature. The oscillatory part gives rise to quantum oscillations and will not be discussed in any detail in this paper. The nonoscillatory part of the polarizability tensor, also known as the semi-classical result, will be used later in Sec. IV in calculating the magnetoplasma dispersion relation for a plane of electrons imbedded in a dielectric.

We consider a 2 DEG to occupy the plane $z=0$ of a Cartesian coordinate system in the presence of a dc magnetic field $\vec{B}_0$ oriented in the z direction. If such a system is perturbed by a small electromagnetic disturbance, a self-consistent electromagnetic field will be set up. This field can be described by a scalar potential $\phi(\vec{r},t)$, and a vector potential $\vec{A}(\vec{r},t)$. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ associated with the self-consistent field are related to the potentials by the equations

$$\vec{E}=-\vec{\nabla}\phi-\frac{1}{c}\frac{\partial\vec{A}}{\partial t} \tag{1}$$

and

$$\vec{B}=\vec{\nabla}\times\vec{A} . \tag{2}$$

The Hamiltonian for an electron of charge $-e$ and mass m^* is

$$H=(1/2m^*)[\vec{p}+(e/c)\vec{A}_0+(e/c)\vec{A}]^2-e\phi . \tag{3}$$

Here $\vec{p}$ is the canonical momentum and $\vec{A}_0=(0, B_0x, 0)$ is the vector potential associated with the dc magnetic field $\vec{B}_0$. In the linear response theory, the Hamiltonian H may be written in the form

$$H=H_0+H_1 , \tag{4}$$

where

$$H_0=\tfrac{1}{2}m^*\vec{v}_0^{\,2} \tag{5}$$

is the Hamiltonian of an electron in the absence of the self-consistent field. $\vec{v}_0$ is the velocity operator given by

$$\vec{v}_0=(1/m^*)[\vec{p}+(e/c)\vec{A}_0] . \tag{6}$$

The quantity H_1 is given by

$$H_1=(e/2c)(\vec{v}_0\cdot\vec{A}+\vec{A}\cdot\vec{v}_0)-e\phi \tag{7}$$

to first order in the self-consistent field.

The eigenfunction and eigenvalues of H_0 are well known. They can be written

$$|\nu\rangle=|nk\rangle=L^{-1/2}e^{iky}u_n(x+\hbar k/m^*\omega_c) \tag{8}$$

and

$$E_\nu=E_n=(n+\tfrac{1}{2})\hbar\omega_c . \tag{9}$$

In writing these equations we have imposed periodic boundary conditions with period L in the x and y directions. In Eqs. (8) and (9), $\omega_c=eB_0/m^*c$ is the cyclotron frequency, u_n is a normalized harmonic-oscillator wave function, and $k=(2\pi/L)\times$integer. Furthermore, in order to restrict the center of the wave packet to lie inside the square of edge L, it is necessary to impose the condition

$$0\le\hbar k/m^*\omega_c\le L . \tag{10}$$

The response of the system to the disturbance can be obtained by solving for the density matrix which satisfies the equation of motion

$$\frac{\partial f}{\partial t}+\frac{i}{\hbar}[H,f]=0. \tag{11}$$

In Eq. (11), the effect of collisions of electrons has been neglected. In solving Eq. (11), we make the assumption that

$$f=f_0+f_1 , \tag{12}$$

where

$$f_0(H,\mu)=(1+e^{(H-\mu)/kT})^{-1} \tag{13}$$

is the thermal equilibrium density matrix for the electron gas in the absence of the disturbance. f_1, which is to first order in $\vec{A}$ and ϕ, is small compared to f_0. By linearizing Eq. (11) and by taking matrix elements on both sides, we obtain

$$\langle\nu|f_1|\nu'\rangle=\Lambda_{\nu'\nu}\langle\nu|H_1|\nu'\rangle , \tag{14}$$

where

$$\Lambda_{\nu'\nu}=\frac{f_0(E_{\nu'})-f_0(E_\nu)}{E_{\nu'}-E_\nu-\hbar\omega} . \tag{15}$$

The current and charge densities are given by the trace of the product of the density matrix and the current and charge density operators

$$\vec{j}(\vec{r}_0,t)=\mathrm{Tr}\{-\tfrac{1}{2}e[\vec{v}_0+(e/m^*c)\vec{A}(\vec{r},t)]\delta(\vec{r}-\vec{r}_0)f+\mathrm{H.c.}\} \tag{16}$$

and

$$\rho(\vec{r}_0,t)=\mathrm{Tr}[-e\delta(\vec{r}-\vec{r}_0)f] , \tag{17}$$

where H. c. denotes the Hermitian conjugate of the preceding operator. By charge conservation, ρ is related to $\vec{j}$ by

$$\vec{\nabla}\cdot\vec{j}+\frac{\partial\rho}{\partial t}=0 . \qquad (18)$$

Therefore, it is sufficient to evaluate the current $\vec{j}(\vec{r}_0, t)$. Including only terms linear in the self-consistent field, Eq. (16) becomes

$$\vec{j}(\vec{r}_0, t)=-2e\sum_\nu\langle\nu|[\tfrac{1}{2}\vec{v}_0\delta(\vec{r}-\vec{r}_0)+\tfrac{1}{2}\delta(\vec{r}-\vec{r}_0)\vec{v}_0]f_1|\nu\rangle-\frac{2e^2}{m^*c}\sum_\nu\langle\nu|\vec{A}(\vec{r},t)\delta(\vec{r}-\vec{r}_0)f_0|\nu\rangle . \qquad (19)$$

The factor 2 in front of the summation signs is due to the two spin states. We recall that f_1 is linear in the disturbance $\vec{A}$ and ϕ. Suppose $\vec{A}$ and ϕ have a single Fourier component, namely,

$$\vec{A}(\vec{r}, t)=\vec{A}(\vec{q},\omega)\,e^{i(\omega t-\vec{q}\cdot\vec{r})} ,\quad \phi(\vec{r}, t)=\phi(\vec{q},\omega)\,e^{i(\omega t-\vec{q}\cdot\vec{r})} . \qquad (20)$$

It is easy to see that the current $\vec{j}$ must have the same Fourier component, i.e.,

$$\vec{j}(\vec{r}, t)=\vec{j}(\vec{q},\omega)\,e^{i(\omega t-\vec{q}\cdot\vec{r})} . \qquad (21)$$

In order to obtain $\vec{j}(\vec{q},\omega)$, we express the two-dimensional δ function of Eq. (19) in terms of its Fourier transform

$$\delta(\vec{r}-\vec{r}_0)=L^{-2}\sum_{\vec{q}'}e^{i\vec{q}'\cdot(\vec{r}-\vec{r}_0)} , \qquad (22)$$

and take the Fourier transform of Eq. (19) making use of Eq. (14). For the 2 DEG, $\vec{j}(\vec{q},\omega)$ is found to be

$$\vec{j}(\vec{q},\omega)=\frac{Ne^2}{m^*c}[-\vec{A}(\vec{q},\omega)-\overleftrightarrow{I}\cdot\vec{A}(\vec{q},\omega)+\vec{K}\phi(\vec{q},\omega)] , \qquad (23)$$

where N is the density of electron, i.e., the number of electrons per unit area.

In Eq. (23), the tensor $\overleftrightarrow{I}$ and the vector $\vec{K}$ are defined by

$$\overleftrightarrow{I}=\frac{2m^*}{N}\sum_{\nu\nu'}\Lambda_{\nu'\nu}\langle\nu'|\vec{V}(\vec{q})|\nu\rangle\langle\nu'|\vec{V}(\vec{q})|\nu\rangle^* , \qquad (24)$$

$$\vec{K}=\frac{2m^*c}{N}\sum_{\nu\nu'}\Lambda_{\nu'\nu}\langle\nu'|\vec{V}(\vec{q})|\nu\rangle\langle\nu'|e^{i\vec{q}\cdot\vec{r}}|\nu\rangle^* , \qquad (25)$$

where the operator $\vec{V}(\vec{q})$ is given by

$$\vec{V}(\vec{q})=\tfrac{1}{2}e^{i\vec{q}\cdot\vec{r}}\vec{v}_0+\tfrac{1}{2}\vec{v}_0e^{i\vec{q}\cdot\vec{r}} . \qquad (26)$$

It is not difficult to demonstrate that Eq. (23) is gauge invariant. Therefore, we can choose a particular gauge such that

$$\phi=0 . \qquad (27)$$

For this particular gauge, the electric field $\vec{E}$ is related to the vector potential $\vec{A}$ by

$$\vec{E}=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}=-\frac{i\omega}{c}\vec{A} . \qquad (28)$$

By making use of Eqs. (27) and (28), Eq. (23) can be written in the following convenient form:

$$\vec{j}(\vec{q},\omega)=\overleftrightarrow{\sigma}(\vec{q},\omega)\cdot\vec{E}(\vec{q},\omega)=i\omega\overleftrightarrow{\chi}(\vec{q},\omega)\cdot\vec{E}(\vec{q},\omega) , \qquad (29)$$

where $\overleftrightarrow{\chi}$ and $\overleftrightarrow{\sigma}$ are given by

$$\overleftrightarrow{\chi}(\vec{q},\omega)=-(i/\omega)\overleftrightarrow{\sigma}(\vec{q},\omega)=-(Ne^2/m^*\omega^2)[\overleftrightarrow{1}+\overleftrightarrow{I}(\vec{q},\omega)] . \qquad (30)$$

It is obvious that $\overleftrightarrow{\chi}$ is the polarizability tensor and $\overleftrightarrow{\sigma}$ is the conductivity tensor.

Due to the rotational symmetry of the system, we can choose the direction of the propagation vector $\vec{q}$ to lie along the y axis. This will simplify the problem with no loss of generality. The only nonzero matrix elements of $\vec{V}(\vec{q})$ and e^{iqy} are

$$\langle n', k+q|V_x(q)|n,k\rangle=i\omega_c\frac{\partial f_{n'n}(q)}{\partial q} , \qquad (31)$$

$$\langle n', k+q|V_y(q)|n,k\rangle=(\omega_c/q)(n'-n)f_{n'n}(q), \qquad (32)$$

$$\langle n'\ k+q|e^{iqy}|n,k\rangle=f_{n'n}(q), \qquad (33)$$

where $f_{n'n}(q)$ stands for the integral

$$f_{n'n}(q)=\int_{-\infty}^{\infty}dx\,u_{n'}\left(x+\frac{\hbar q}{m^*\omega_c}\right)u_n(x) . \qquad (34)$$

$u_n(x)$ is the well-known simple-harmonic-oscillator wave function. Substituting Eqs. (31) and (32) into Eq. (24), it is easy to see that

$$I_{yy}=\frac{2m^*}{N}\sum_k\sum_{n'n}\Lambda_{n'n}\frac{\omega_c^2}{q^2}(n'-n)^2f_{n'n}^2(q), \qquad (35)$$

$$I_{xx}=\frac{2m^*}{N}\sum_k\sum_{n'n}\Lambda_{n'n}\omega_c^2\left(\frac{\partial f_{n'n}(q)}{\partial q}\right)^2 , \qquad (36)$$

and

$$I_{xy}=-I_{yx}=\frac{2m^*}{N}\sum_k\sum_{n'n}\Lambda_{n'n}\left(i\frac{\omega_c^2}{q}(n'-n)\times f_{n'n}\frac{\partial f_{n'n}(q)}{\partial q}\right) . \qquad (37)$$

In order to obtain the polarizability tensor or the conductivity tensor, we have to evaluate the matrix $\overleftrightarrow{I}$ defined by Eqs. (35)–(37). This can be done by following almost exactly the procedure given in Appendix C of the paper by Greene *et al.*[8] We therefore merely give the final result, referring the reader to that paper for the details of the derivation. We find that

$$I_{yy}=-1-2\frac{\Omega^2}{X^2}\left(1+\sum_{\alpha=-\infty}^{\infty}\frac{\Omega^2}{\alpha^2-\Omega^2}J_\alpha^2(X)\right)-\delta\Omega^4\frac{1}{X}\frac{\partial}{\partial X}\sum_{\alpha=-\infty}^{\infty}\frac{1}{\alpha^2-\Omega^2}J_\alpha^2(X) , \qquad (38)$$

$$I_{xy}=\frac{i}{2\Omega}\frac{1}{X}\frac{\partial}{\partial X}[X^2(1+I_{yy})] , \qquad (39)$$

and

$$I_{xx} = -1 - 2 \sum_{\alpha=-\infty}^{\infty} \frac{\Omega^2}{\alpha^2 - \Omega^2} [J'_\alpha(X)]^2 - \delta\left(1 + \sum_{\alpha=-\infty}^{\infty} \frac{\Omega^2(\Omega^2 - X^2)}{\alpha^2 - \Omega^2} \frac{1}{X} \frac{\partial}{\partial X} J^2_\alpha(X)\right). \quad (40)$$

In these equations

$$\delta = \frac{\pi}{\beta\mu} \sum_{l\neq 0} \frac{(-)^l \cos(2\pi l \mu/\hbar\omega_c)}{\sinh(2\pi^2 l/\beta\hbar\omega_c)}, \quad (41)$$

$\Omega = \omega/\omega_c$, and $X = qv_F/\omega_c$. The functions $J_\alpha(X)$ and $J'_\alpha(X)$ denote the Bessel function of order α and its first derivative, respectively. The terms proportional to δ are the oscillatory terms. They become exponentially small as $\beta = 1/kT$ is decreased to temperature for which $2\pi^2 kT \gg \hbar\omega_c$. The nonoscillatory terms, or semiclassical polarizability will be used later in evaluating the dispersion relation of magnetoplasma waves.

III. DERIVATION OF THE DISPERSION RELATION

In this section, we present the derivation of the dispersion relation of a magnetoplasma wave in a 2 DEG imbedded in a dielectric (dielectric constant $= \epsilon_0$). The method used is similar to that used for the case of surface plasmons or surface polaritons in semi-infinite solids.[6] We consider a system of electron gas confined in the plane $z = 0$ of a Cartesian coordinate. The dc magnetic field $\vec{B}_0$ is oriented in the z direction. The dispersion relation is obtained by solving Maxwell's equations inside the dielectric and by matching the standard boundary conditions at the surface $z = 0$. For convenience, let us divide space into two regions. Subscript 1 will be used to denote quantities in region 1 with $z > 0$, and subscript 2 for region 2 with $z < 0$. Let $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$ be the electric field and the magnetic field associated with the plasma oscillations. For a wave propagating in the y direction, we may simply consider $\vec{E}$ and $\vec{B}$ to be proportional to $e^{i\omega t - iq_y y}$, where ω is the angular frequency and q_y is the wave vector. $\vec{E}$ and $\vec{B}$ must satisfy Maxwell's equations

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \quad (42)$$

$$\vec{\nabla} \times \vec{B} = \frac{\epsilon_0}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}(x, y, t)\, \delta(z). \quad (43)$$

In Eq. (43), $\vec{j}\,\delta(z)$ is the surface current which is induced by the self-consistent field. Likewise, the self-consistent field itself is determined by the surface current and the surface charge with the proper boundary conditions satisfied at the plane $z = 0$. Eliminating $\vec{B}$ from Eqs. (42) and (43), we obtain

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \epsilon_0 \frac{\omega^2}{c^2} \vec{E} - i \frac{4\pi\omega}{c^2} \vec{j}\,\delta(z). \quad (44)$$

For $z \neq 0$, the last term in Eq. (44) vanishes. By solving that wave equation, we get the solution

$$\vec{E}(x, y, z) = \vec{E}_0\, e^{i\omega t - iq_y y \mp iq_\perp z}, \quad (45)$$

with

$$q_y^2 + q_\perp^2 = \epsilon_0\, \omega^2/c^2. \quad (46)$$

It is easy to see from Eq. (46) that when $q_y^2 > \epsilon_0\omega^2/c^2$, $q_\perp^2$ becomes negative. In other words, $q_\perp$ itself is imaginary. In this region of the $\omega - cq_y$ plane, we can have waves with either exponentially growing or exponentially decaying amplitude. This is the region where one may find surface waves. Of course, the waves with amplitude exponentially growing away from the surface $z = 0$ must be disregarded. Hence we must have

$$\vec{E}_1(\vec{r}, t) = \vec{E}_1 e^{-\beta z} e^{i\omega t - iq_y y}, \quad z > 0$$

and (47)

$$\vec{E}_2(\vec{r}, t) = \vec{E}_2 e^{\beta z} e^{i\omega t - iq_y y}, \quad z < 0,$$

where

$$\beta^2 = q_y^2 - \epsilon_0\, \omega^2/c^2 > 0. \quad (48)$$

To take into account the surface current, let us go back to Eq. (44). By taking the curl operator twice on the electric field $\vec{E}$, we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = (q_y^2 E_x - E_x'', \; -iq_y E_z' - E_y'', \; q_y^2 E_z - iq_y E_y'). \quad (49)$$

Here we have used a prime to denote the first derivative with respect to z, and a double prime the second derivative. Knowing that the amplitude of the electric field decreases exponentially in both directions away from the plane $z = 0$, it is easy to see that for $z \neq 0$,

$$\begin{pmatrix} E_x' \\ E_y' \\ E_z' \end{pmatrix} = \mp\beta \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \quad \text{for } z \gtrless 0, \quad (50)$$

$$\begin{pmatrix} E_x'' \\ E_y'' \\ E_z'' \end{pmatrix} = \beta^2 \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}. \quad (51)$$

Substituting Eq. (49) into (44) and making use of Eqs. (50) and (51), we obtain (for $z \neq 0$)

$$\begin{pmatrix} q_y^2 - \beta^2 - \frac{\epsilon_0\omega^2}{c^2} & 0 & 0 \\ 0 & -\beta^2 - \frac{\epsilon_0\omega^2}{c^2} & \pm iq_y\beta \\ 0 & \pm iq_y\beta & q_y^2 - \frac{\epsilon_0\omega^2}{c^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0. \quad (52)$$

In order to have nontrivial solution, the determinant of the matrix must vanish. This gives the secular equation

$$\epsilon_0(q_y^2-\beta^2-\epsilon_0\omega^2/c^2)^2=0. \qquad (53)$$

The wave solution is found to be

$$\vec{E}_{1,2}(\vec{r},t)=\begin{pmatrix} E_x \\ E_y \\ \mp i\frac{q_y}{\beta}E_y \end{pmatrix} e^{\mp\beta z}e^{i\omega t-iq_y y}, \qquad (54)$$

where the ∓ signs are used corresponding to medium 1 ($z>0$) and medium 2 ($z<0$), respectively. E_x and E_y can be chosen arbitrarily. We emphasize that Eqs. (50) and (51) are valid everywhere except at $z=0$. From the behavior of the electric fields given in Eq. (47) or (54), it is easy to note that E_x and E_y are continuous but E_x' and E_y' and E_z are discontinuous at $z=0$. Therefore E_x'', E_y'', and E_z' will have δ-function singularities at $z=0$, i.e.,

$$E_x''=\beta^2E_x-2\beta E_x(0)\delta(z),$$
$$E_y''=\beta^2E_y-2\beta E_y(0)\delta(z),$$

and (55)

$$E_z'=\mp\beta E_z+2E_z(0^+)\delta(z)$$
$$=\mp\beta E_z-2i(q_y/\beta)E_y\delta(z).$$

We substitute Eqs. (55) and (49) into Eq. (44). By picking up only those terms with δ-function singularities in Eq. (44), we obtain

$$i\omega\left(\frac{\beta}{2\pi}\frac{c^2}{\omega^2}E_x, -\frac{\epsilon_0}{2\pi\beta}E_y\right)=\vec{j}=i\omega\overleftrightarrow{\chi}\cdot\vec{E}, \qquad (56)$$

where $\overleftrightarrow{\chi}$ is the polarizability tensor which we have discussed in great detail in Sec. II. Equation (56) can be rewritten in the following matrix form:

$$\begin{pmatrix} \chi_{xx}-\frac{\beta}{2\pi}\frac{c^2}{\omega^2} & \chi_{xy} \\ \chi_{yx} & \chi_{yy}+\frac{\epsilon_0}{2\pi\beta}\end{pmatrix}\begin{pmatrix}E_x\\E_y\end{pmatrix}=0. \qquad (57)$$

By putting the determinant of the matrix equal to zero, we readily obtain the dispersion relation

$$\left(\chi_{xx}-\frac{\beta}{2\pi}\frac{c^2}{\omega^2}\right)\left(\chi_{yy}+\frac{\epsilon_0}{2\pi\beta}\right)-\chi_{xy}\chi_{yx}=0. \qquad (58)$$

Recall that $\beta=(q_y^2-\epsilon_0\omega^2/c^2)^{1/2}$. Equations (58) gives the dispersion relation of the plasma oscillations of a plane of electrons imbedded in a dielectric with a normal magnetic field. It is easy to see that in the limit of zero magnetic field, χ_{xy} and χ_{yx} vanish and the result reduces to $\epsilon_0+2\pi\beta\chi_{yy}=0$, the result obtained by Stern.[5]

The generalization of Eq. (58) for a plane of electrons imbedded in two dielectrics with different dielectric constants ϵ_1 and ϵ_2 is straightforward. For that case, the dispersion relation is found to be

$$\left(\chi_{xx}-\frac{\beta_1+\beta_2}{4\pi}\frac{c^2}{\omega^2}\right)\left[\chi_{yy}+\frac{1}{4\pi}\left(\frac{\epsilon_1}{\beta_1}+\frac{\epsilon_2}{\beta_2}\right)\right]-\chi_{xy}\chi_{yx}=0, \qquad (58a)$$

where

$$\beta_1=(q^2-\epsilon_1\omega^2/c^2)^{1/2} \text{ and } \beta_2=(q^2-\epsilon_2\omega^2/c^2)^{1/2}.$$

Further study of the dispersion relation [Eq. (58)] in both the long- and the short-wavelength limits will be given in Sec. IV. We will not consider any further the more general case with two different dielectrics as described by Eq. (58a).

IV. MAGNETOPLASMA OSCILLATIONS IN THE LONG- AND SHORT-WAVELENGTH LIMITS

In this section we study the magnetoplasma oscillations of 2 DEG in both the long- and short-wavelength limits using the results we obtained for the dispersion relation and the polarizability tensor $\overleftrightarrow{\chi}$. For simplicity, we take into account only the nonoscillatory part (n.o.) of $\overleftrightarrow{\chi}$, the oscillatory part (osc.) of $\overleftrightarrow{\chi}$ being disregarded. Therefore, the results obtained in this section are valid only in the semiclassical limit. For convenience, we given here once again the expressions for the polarizability tensor $\overleftrightarrow{\chi}$ which we have obtained in Sec. II:

$$\chi_{yy}=-(\lambda/\Omega^2)(1+I_{yy}),$$
$$\chi_{xx}=-(\lambda/\Omega^2)(1+I_{xx}), \qquad (59)$$
$$\chi_{xy}=-\chi_{yx}=-(\lambda/\Omega^2)I_{xy}.$$

The matrix elements of $\overleftrightarrow{I}$ are given by

$$1+I_{yy}(\text{n.o.})=-2\frac{\Omega^2}{X^2}\left(1+\sum_{\alpha=-\infty}^{\infty}\frac{\Omega^2}{\alpha^2-\Omega^2}J_\alpha^2\right),$$
$$1+I_{xx}(\text{n.o.})=-2\sum_{\alpha=-\infty}^{\infty}\frac{\Omega^2}{\alpha^2-\Omega^2}(J_\alpha')^2, \qquad (60)$$
$$I_{xy}=-I_{xy}=i\frac{1}{2\Omega X}\frac{\partial}{\partial X}[X^2(1+I_{yy})],$$

where $\Omega=\omega/\omega_c$ and $X=qv_F/\omega_c$. In Eq. (59) we have introduced the parameter λ defined by

$$\lambda=Ne^2/m^*\omega_c^2. \qquad (61)$$

In the following, we will discuss the dispersion relation given by Eq. (58) in the long-wavelength limit $qv_F\ll\omega_c$ and in the short-wavelength limit $qv_F\gg\omega_c$.

A. Long-wavelength limit

In the long-wavelength limit where $qv_F\ll\omega_c$ (v_F being the Fermi velocity), we make series expansion of the Bessel functions $J_n(X)$,

$$J_n(X)=\left(\frac{X}{2}\right)^n\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(n+k)!}\left(\frac{X}{2}\right)^{2k}\ ,\quad X\ll 1 \tag{62}$$

for $n=0, 1, 2, \ldots$; and

$$J_{-n}(X)=(-1)^nJ_n(X)\ . \tag{63}$$

By substituting Eqs. (62) and (63) into Eq. (60), it is straightforward to show that

$$1+I_{yy}=-\frac{4\Omega^2}{X^2}\left\{\frac{1}{1-\Omega^2}\left(\frac{X}{2}\right)^2+\frac{\Omega^2}{4-\Omega^2}\frac{1}{4}\left(\frac{X}{2}\right)^4+\frac{\Omega^2}{9-\Omega^2}\frac{1}{36}\left(\frac{X}{2}\right)^6+\cdots+\frac{\Omega^2}{\alpha^2-\Omega^2}\left[\frac{1}{\alpha!}\left(\frac{X}{2}\right)^{\alpha}\right]^2+\cdots\right\}\ , \tag{64}$$

$$1+I_{xx}=-4\left\{\frac{\Omega^2}{1-\Omega^2}\frac{1}{4}+\frac{\Omega^2}{4-\Omega^2}\frac{1}{4}\left(\frac{X}{2}\right)^2-\frac{1}{2}\left(\frac{X}{2}\right)^2+\cdots+\frac{\Omega^2}{\alpha^2-\Omega^2}\left[\frac{1}{2}\frac{1}{(\alpha-1)!}\left(\frac{X}{2}\right)^{\alpha-1}\right]^2+\cdots\right\}\ , \tag{65}$$

and

$$I_{xy}=-i\Omega\left[\frac{1}{1-\Omega^2}+\frac{\Omega^2}{4-\Omega^2}\frac{1}{2}\left(\frac{X}{2}\right)^2+\cdots+\frac{\Omega^2}{\alpha^2-\Omega^2}\alpha\left(\frac{1}{\alpha!}\right)^2\left(\frac{X}{2}\right)^{2\alpha-2}+\cdots\right]\ . \tag{66}$$

Although the symbol (n.o.) denoting the nonoscillatory part of Υ is omitted in the above three equations, we should bear in mind that we are looking only in the semiclassical limit in this section. It is easy to see from Eqs. (64)–(66) that when Ω is not very close to an integer, the first term of each of these infinite series is most important because $X=qv_F/\omega_c$ is small compared to 1. However, when Ω is sufficiently close to an integer α, then the particular term proportional to $(\alpha^2-\Omega^2)^{-1}$ will give the major contribution to the sum. The criterion may be stated as follows: If Ω is very close to an integer α such that

$$|\Omega^2-\alpha^2|<\frac{\alpha^2-1}{[(\alpha-1)!]^2}\left(\frac{X}{2}\right)^{2\alpha-2}\ ,\quad \alpha=2, 3, 4, \ldots, \tag{67}$$

the αth term with denominator $\alpha^2-\Omega^2$ is dominant. On the other hand, if the inequality given by Eq. (67) is not true for all $\alpha\geq 2$, then the first term in each series, i.e., the term proportional to $(1-\Omega^2)^{-1}$ will give major contribution.

Let us first consider $\Omega=\omega/\omega_c$ not too close to any integer ≥ 2, then χ_{yy}, χ_{xx}, and χ_{xy} are given by

$$\chi_{yy}=\frac{\lambda}{1-\Omega^2}+O(X^2)\ ,$$
$$\chi_{xx}=\frac{\lambda}{1-\Omega^2}+O(X^2)\ , \tag{68}$$
$$\chi_{xy}=\chi_{yx}=\frac{i}{\Omega}\frac{\lambda}{1-\Omega^2}+O(X^2)\ .$$

If the small quantity $O(X^2)$ is neglected, then Eq. (68) is obviously the result one would obtain for a classical system of noninteracting electrons. The dispersion relation Eq. (58) becomes

$$\left(\frac{\lambda}{1-\Omega^2}+\frac{\epsilon_0}{2\pi\beta}\right)\left(\frac{\lambda}{1-\Omega^2}-\frac{\beta}{2\pi}\frac{c^2}{\omega^2}\right)=\left(\frac{\lambda}{1-\Omega^2}\frac{1}{\Omega}\right)^2\ . \tag{69}$$

We find it convenient to introduce the parameter a by

$$a=2\pi\lambda\omega_c^2/\epsilon_0=2\pi Ne^2/m^*\epsilon_0\ . \tag{70}$$

Then Eq. (69) can be expressed in the following simple form:

$$\epsilon_0 a-\frac{\omega^2\epsilon_0}{\beta}+\beta c^2-\frac{c^2}{a}(\omega^2-\omega_c^2)=0\ . \tag{71}$$

When $c\to\infty$, Eq. (71) agrees with that obtained by Horing and Yildiz.[9] But in their work, they have neglected the effect of retardation and used the local dielectric constant. Equation (71) is a quadratic equation whose solution is

$$\beta=\left(q^2-\frac{\epsilon_0\omega^2}{c^2}\right)^{1/2}=-\frac{1}{2}\left(\frac{a\epsilon_0}{c^2}-\frac{\omega^2-\omega_c^2}{a}\right)+\left[\frac{1}{4}\left(\frac{a\epsilon_0}{c^2}-\frac{\omega^2-\omega_c^2}{a}\right)^2+\epsilon_0\frac{\omega^2}{c^2}\right]^{1/2}\ . \tag{72}$$

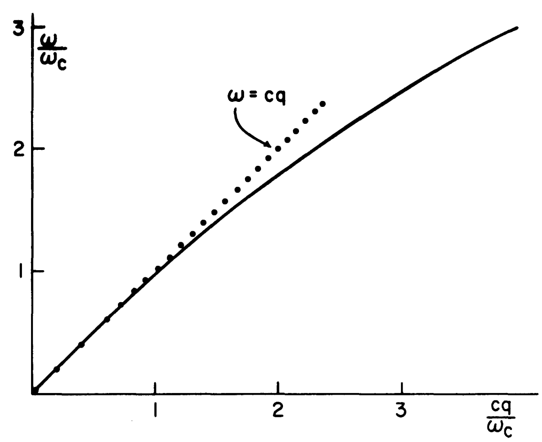

FIG. 1. Plasmon dispersion curve for a 2 DEG is plotted as ω vs cq in the long-wavelength limit. This curve is valid when ω is not close to $\alpha\omega_c$ where α is an integer greater or equal to 2. The parameters are $N=2\times10^{12}$ cm^{-2}, $B_0=2.22$ kG, $\epsilon_0=1$, and $m^*=0.195m_0$. These give a cyclotron frequency of 2×10^{11} sec^{-1}. For semiconductors like Si with spin degeneracies 2 and valley degeneracies 2 along the [100] direction, v_F is found to be 1.5×10^7 cm/sec. Since $\epsilon_0=1$, our numerical calculations are for the case of a 2 DEG imbedded in vacuum.

Here, $a/c\omega_c$ is a dimensionless quantity. When $a \gg c\omega_c$, Eq. (72) can be approximated by

$$q^2=\epsilon_0\omega^2/c^2+\omega^4/a^2 \ . \qquad (73)$$

This is exactly the result obtained by Stern[5] for zero magnetic field. In Fig. 1, we plot the dispersion curve given by Eq. (72) as ω vs cq. The parameters are chosen such that $N=2\times10^{12}$ cm^{-2}, $B_0=2.22$ kG, $\epsilon_0=1$, and $m^*=0.195m_0$. The cyclotron frequency is found to be $\omega_c=2\times10^{11}$ sec^{-1}. Since $\epsilon_0=1$, our numerical calculations are for the (unphysical) case of a 2 DEG imbedded in vacuum. For low frequencies, the dispersion curve approaches the light line $\omega=cq$. As ω increases, cq increases. The dispersion curve begins to bend to the right-hand side where the phase velocity ω/q is smaller than the speed of light c. The effect of the magnetic field on this dispersion curve Eq. (72) is very small.

When $\Omega=\omega/\omega_c$ is sufficiently close to an integer $\alpha\geq2$, such that Eq. (67) is satisfied, we have, from Eqs. (64)–(66),

$$\chi_{yy}\cong\lambda/Q+\lambda/(1-\Omega^2) \ ,$$
$$\chi_{xx}\cong(\alpha^2/\Omega^2)\frac{\lambda}{Q}+\frac{\lambda}{(1-\Omega^2)} \ , \qquad (74)$$

and

$$\chi_{xy}\cong i[(\alpha/\Omega)\lambda/Q+(1/\Omega)\lambda/(1-\Omega^2)] \ ,$$

where

$$1/Q=[\Omega^2/(\alpha^2-\Omega^2)](1/\alpha!)^2(X/2)^{2\alpha-2} \ . \qquad (75)$$

We note that $1/Q$ diverges as Ω approaches α. Substituting $\overleftrightarrow{\chi}$ into the dispersion relation Eq. (58), we easily get

$$\left(\frac{\lambda}{Q}+\frac{\lambda}{1-\Omega^2}+\frac{\epsilon_0}{2\pi\beta}\right)\left(\frac{\alpha^2}{\Omega^2}\frac{\lambda}{Q}+\frac{\lambda}{1-\Omega^2}-\frac{\beta}{2\pi}\frac{c^2}{\omega^2}\right)=\left(\frac{\alpha}{\Omega}\frac{\lambda}{Q}+\frac{1}{\Omega}\frac{\lambda}{1-\Omega^2}\right)^2 . \qquad (76)$$

Those terms proportional to Q^{-2} cancel. The terms proportional to Q^{-1} are

$$\frac{\lambda}{2\pi Q}\left(\frac{\epsilon_0}{\beta}-\beta\frac{c^2}{\omega^2}-\frac{2a\epsilon_0}{\omega(\omega+\omega_c)}\right) . \qquad (77)$$

The root at $\omega=\alpha\omega_c$, where $\alpha\geq2$ can easily be obtained by putting the large parentheses of Eq. (77) equal to zero. This gives

$$\beta=\left(q^2-\frac{\epsilon_0\omega^2}{c^2}\right)^{1/2}=-\frac{\alpha a\epsilon_0}{(\alpha+1)c^2}+\left(\frac{\alpha^2a^2\epsilon_0^2}{(\alpha+1)^2c^4}+\epsilon_0\frac{\omega^2}{c^2}\right)^{1/2} . \qquad (78)$$

For $a\gg c\omega$, Eq. (78) can be approximated by

$$q^2=\epsilon_0\frac{\omega^2}{c^2}+\frac{\omega^4}{a^2}\left(\frac{\alpha+1}{2\alpha}\right)^2 , \quad \text{at } \omega=\alpha\omega_c \ . \qquad (79)$$

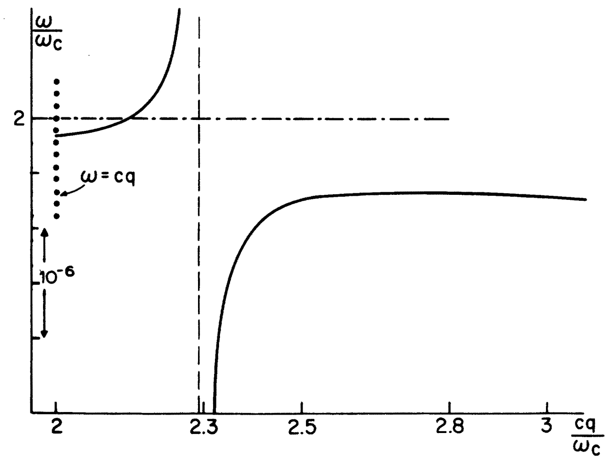

FIG. 2. Plasmon dispersion curve in a 2 DEG is plotted for frequency ω close to $2\omega_c$ in the long-wavelength limit to illustrate the splitting of the dispersion curve at a frequency very close to but slightly less than $2\omega_c$. Similar behavior is found for $\omega=\alpha\omega_c$ for $\alpha=3,4,5,\ldots$. The same parameters are used here as in Fig. 1.

In order to obtain the dispersion curve for ω close to $\alpha\omega_c$, it is convenient to introduce the parameter p defined by

$$1/Q=p/(1-\alpha^2) \ . \qquad (80)$$

Using Eqs. (75) and (80), we see that

$$\Omega^2-\alpha^2\cong(1/p)\alpha^2(\alpha^2-1)(1/\alpha!)^2(X/2)^{2\alpha-2} , \qquad (81)$$

for $X\ll1$, and Ω close to α. p describes how close Ω is to α. When $|p|$ is very large, ω is very close to $\alpha\omega_c$. As $|p|$ decreases, ω gets further and further away from $\alpha\omega_c$. Substituting Eq. (80) into Eq. (74), we obtain

$$\chi_{yy}=\chi_{xx}=[\lambda/(1-\alpha^2)](1+p) \ ,$$
$$\chi_{xy}=-\chi_{yx}=i[\lambda/(1-\alpha^2)](\alpha^{-1}+p) \ . \qquad (82)$$

For convenience, we introduce a tensor $\overleftrightarrow{Z}$ defined by

$$\overleftrightarrow{Z}=(2\pi\omega_c/c)\overleftrightarrow{\chi} \ . \qquad (83)$$

With Eq. (83), the dispersion relation Eq. (58) can be written

$$(Z_{yy}+\epsilon_0\omega_c/c\beta)(\Omega^2Z_{xx}-c\beta/\omega_c)=\Omega^2Z_{xy}Z_{yx} \ , \qquad (84)$$

which is a quadratic equation in $c\beta/\omega_c$ written

$$\left(\frac{c\beta}{\omega_c}\right)^2+2\frac{c\beta}{\omega_c}Z_p-\epsilon_0\Omega^2\frac{Z_{xx}}{Z_{yy}}=0 \ , \qquad (85)$$

where

$$Z_p=\tfrac{1}{2}(\epsilon_0-\Omega^2Z_{yy}Z_{xx}+\Omega^2Z_{xy}Z_{yx})/Z_{yy} \ . \qquad (86)$$

Solving Eq. (85), we obtain

$$\frac{c\beta}{\omega_c}=\frac{c}{\omega_c}\left(q^2-\frac{\epsilon_0\omega^2}{c^2}\right)^{1/2}=-Z_p+\left(Z_p^2+\epsilon_0\Omega^2\frac{Z_{xx}}{Z_{yy}}\right)^{1/2} . \qquad (87)$$

Equation (87) is valid for Ω very close to α. Therefore, we may put $\omega = \alpha\omega_c$ everywhere in Eq. (87). For any given p, Eq. (87) gives a value of q. When p and q are both given, one can easily find the frequency ω from Eq. (81). Hence by varying the parameter p, we can obtain the dispersion curves near $\omega = \alpha\omega_c$. To illustrate the result, we present the dispersion curve near $\omega = 2\omega_c$ in Fig. 2. At frequency very close to but slightly less than $2\omega_c$, we find splitting of the dispersion curve due to the magnetic field. As a matter of fact, splittings are found for all $\omega \approx \alpha\omega_c$ (α being integer larger or equal to 2). No splitting is found for $\omega \approx \omega_c$. When ω is less than and not too close to $2\omega_c$, the dispersion curve is quite similar to that given in Fig. 1. As q increases, ω increases until it reaches its maximum value and then decreases gradually. As we will see later, when q becomes sufficiently large, ω will come close to ω_c and oscillate. Another curve starts at the light line $\omega = cq$ with frequency ω slightly less than $2\omega_c$. As q increases, ω also increases. The dispersion curve approaches asymptotically to the curve we presented in Fig. 1. In Fig. 2, the upper curve actually cuts the line $\omega = 2\omega_c$. At the intersection of the dispersion curve and the light line, we have $\beta = 0$. This implies that $\chi_{xx} = 0$. From Eq. (74) this gives a frequency

$$\omega \cong \alpha\omega_c[1 - \tfrac{1}{2}(\alpha^2 - 1)(1/\alpha!)^2(\alpha v_F/2c)^{2\alpha-2}], \quad \alpha \geq 2 . \tag{88}$$

B. Short-wavelength limit

In the short-wavelength limit where $X = qv_F/\omega_c \gg 1$, we use the asymptotic expansion of the Bessel functions, i.e.,

$$J_\alpha(X) = (2/\pi X)^{1/2} \cos[X - \tfrac{1}{2}\pi(\alpha + \tfrac{1}{2})] , \tag{89}$$

$$J_\alpha^2(X) = (1/\pi X)[1 + (-1)^\alpha \sin 2X] . \tag{90}$$

Substituting Eq. (90) into Eqs. (59) and (60), it is easy to show that

$$\chi_{yy} = (\lambda/X^2)A , \tag{91}$$

where

$$A = -\frac{X^2}{\Omega^2}(1 + I_{yy}) = 2 + \frac{2\Omega^2}{\pi X}\sum_{\alpha=-\infty}^{\infty} \frac{1}{\alpha^2 - \Omega^2}[1 + (-1)^\alpha \sin 2X] . \tag{92}$$

We then make use of the following identities:

$$\sum_{\alpha=-\infty}^{\infty} \frac{1}{\alpha^2 - \Omega^2} = -\frac{\pi}{\Omega}\cot\pi\Omega , \qquad \sum_{\alpha=-\infty}^{\infty} \frac{(-1)^\alpha}{\alpha^2 - \Omega^2} = -\frac{\pi}{\Omega}\csc\pi\Omega . \tag{93}$$

Substituting Eq. (93) into Eq. (92) we easily obtain

$$A = 2 - (2\Omega/X)(\cot\pi\Omega + \csc\pi\Omega \sin 2X) . \tag{94}$$

In a similar way, we can show that

$$\chi_{xx} = (\lambda/\Omega^2)B, \quad \chi_{xy} = -\chi_{yx} = i(\lambda/\Omega X)C , \tag{95}$$

where B and C are given by

$$B = -(2\Omega/X)\cot\pi\Omega + (2\Omega/X)(\sin 2X + X^{-1}\cos 2X)\csc\pi\Omega , \qquad C = (\Omega/X^2)(\cot\pi\Omega + \csc\pi\Omega \sin 2X) - (2\Omega/X)\csc\pi\Omega\cos 2X . \tag{96}$$

With Eqs. (91) and (95), the dispersion relation can be written

$$(A + X^2\epsilon_0/2\pi\beta\lambda)[B - (\beta/2\pi)c^2/\omega_c^2\lambda] = C^2 . \tag{97}$$

With the parameters which we have been using, namely, $N = 2\times10^{12}$ cm^{-2}, $v_F = 1.5\times10^7$ cm/sec, $B_0 = 2.22$ kG, $\omega_c = 2\times10^{11}$ sec^{-1}, $\epsilon_0 = 1$, and $m^* = 0.195m_0$, it is easy to see that

$$X^2\epsilon_0/2\pi\beta\lambda \cong 1.5\times10^{-4}X \quad \text{and} \quad (\beta/2\pi)c^2/\omega_c^2\lambda \cong 600X \tag{98}$$

for $cq \gg \omega$. In order to solve Eq. (97) with Eq. (98) we consider a trial solution such that when X is very large, $\Omega = \omega/\omega_c$ is very close to an integer α. The parameter $X = qv_F/\omega_c$ is assumed to have the value from 9 to 20. We make the expansion $\Omega(X) = \alpha + \delta(X)$ with $\delta(X) \ll 1$. A study of Eq. (97) shows that the parameters B and C are small compared to $90X$. Therefore, without too much error, we can write

$$(A + 1.5\times10^{-4}X) = (B - 600X)^{-1}C^2 = 0 . \tag{99}$$

In solving Eq. (99) together with Eq. (94), we further neglect the term $1.5\times10^{-4}X$ which is small compared to 2. Then it is straightforward to show that

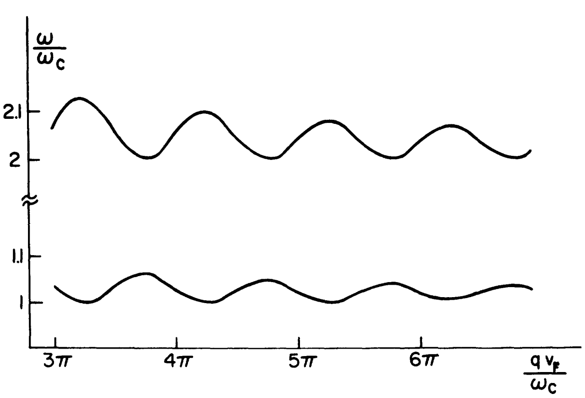

FIG. 3. Plasmon dispersion curve in a 2 DEG is plotted in the short-wavelength limit showing oscillatory behavior of the dispersion curve. Same parameters are used here as in Fig. 1.

$$\Omega = \alpha + \delta_1 - \tfrac{1}{2}(\alpha/X)\pi\delta_1^2[1-(1/\pi X)(1\pm\sin 2X)]^{-1}, \quad (100)$$

where

$$\delta_1 = (\alpha/\pi X)(1\pm\sin 2X)[1-(1/\pi X)(1\pm\sin 2X)]^{-1}. \quad (101)$$

In Eqs. (100) and (101), the upper sign is used when α is even and the lower sign is used when α is odd. Terms of order δ_1^3 or higher are neglected in Eq. (100) for simplicity. To illustrate the dispersion curves in the short-wavelength limit, we present in Fig. 3 a plot of the solution of Eq. (100) giving Ω as a function of X. The large values of X (from 3π to 7π) make possible the asymptotic expansion of the Bessel functions. The dispersion curve is found to oscillate with X and the amplitude of the oscillations decreases as X increases. As a matter of fact, the lower curve of Fig. 2 and the lower curve of Fig. 3 are two limits of the same curve. Similarly, the upper curve of Fig. 2 will go up rapidly as X increases. After reaching its maximum frequency very near but slightly less than $3\omega_c$, the frequency gradually decreases and finally goes to the upper curve of Fig. 3 in the short-wavelength limit.

V. CONCLUSION

In this paper, we have studied the plasma oscillations of a two-dimensional electron gas in a normal dc magnetic field. In Sec. II we have calculated the quantum-mechanical response of a two-dimensional electron gas in the presence of a strong magnetic field. We have obtained an expression of the polarizability tensor in terms of the well-known Bessel functions. The result is found to consist of two parts: the oscillatory part and the nonoscillatory part. The semiclassical nonoscillatory part was used later in Sec. IV in discussing the dispersion curves of the plasma oscillations. Quantum effects are described by the oscillatory part of the polarizability tensor; they were not discussed in this paper. In Sec. III, we have derived the plasma dispersion relation of a two-dimensional electron gas imbedded in a three-dimensional dielectric. Our result, given in Eqs. (58) and (58a), is a generalization of that obtained by Stern[5] to include the effect of an applied magnetic field. In Sec. IV we have investigated the magnetoplasma oscillations of a two-dimensional electron gas in both the long- and short-wavelength limits. For very long wavelengths, we found interesting splittings of the dispersion curves at frequencies slightly less that $\alpha\omega_c$ for $\alpha \geq 2$. Whenever there is a splitting of the dispersion curve, the upper curve starts at the light line $\omega = cq$ and goes up very rapidly as q increases. The lower curve bends over and the frequency gradually decreases (the group velocity being negative). In the short-wavelength limit, the dispersion curves display oscillatory behavior as shown in Fig. 3.

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