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Theory of magnetophonon structure in the longitudinal magnetothermal emf

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The magnetophonon effect, as manifested in the longitudinal magnetothermal emf (Seebeck coefficient Q_{zz}), is examined analytically and numerically in the limit of no Landau-level broadening, for combined optic and acoustic-phonon scattering of electrons in nonpolar semiconductors. In addition to off-resonance maxima occurring at magnetic fields somewhat larger than those given by the Gurevich-Firsov resonance condition $N\omega_c = \omega_0$, $N = 1,2,\ldots$, where ω_c and ω_0 are the cyclotron and optic-phonon frequencies, discontinuities in the derivative of Q_{zz} with respect to magnetic field are found. The slope discontinuities lie precisely at $N\omega_c = \omega_0$ and at $(2n+1)\,\omega_c = 2\omega_0$, $n = 0,1,\ldots$, yielding additional structure characterized by $\partial|Q_{zz}|/\partial B_- > \partial|Q_{zz}|/\partial B_+$ at all temperatures and degrees of elastic scattering.

I. INTRODUCTION

In this paper we examine theoretically the magnetophonon effect as manifested in the longitudinal magnetothermal emf (magneto-Seebeck effect). The magnetic field dependence of any transport property in which inelastic electron scattering on optic phonons is significant is expected to show a characteristic oscillatory structure. Studies of these magnetophonon oscillations have, to date, been limited largely to the resistivity tensor, even though the first experimental report of magnetophonon oscillations[1] showed the effect in the longitudinal magneto-Seebeck coefficient Q_{zz} as well as in the longitudinal and transverse magnetoresistance. Magnetophonon studies have been reported on the magnetothermal emf only for n-InSb,[1-3] n-InAs,[4,5] and n-Ge.[6] The results can be summarized as follows: $|Q_{zz}|$ shows oscillatory structure whose amplitudes are comparable to or larger than those seen in the magnetoresistance. The data published to date do not permit one to say definitely whether the maxima or the minima are the most pronounced features, to be associated with the Gurevich-Firsov (GF) resonance condition[7]

$$N\omega_c = \omega_0, \quad N = 1, 2, \ldots, \tag{1}$$

where ω_c and ω_0 are, respectively, the cyclotron and optic-phonon frequencies. However, one can say that the maxima seem to be shifted systematically to somewhat higher magnetic fields than those given by Eq. (1), or the fields at which the transverse magnetoresistance maxima are found. The extrema also decrease in amplitude with decreasing mobility and with increasing temperature above a certain optimum temperature.

Two theoretical studies on the longitudinal magnetothermal emf have appeared to date. The first was by Pavlov and Firsov,[8] who used the same model as in the pioneering work of Gurevich and Firsov[7] on the magnetophonon effect in the magnetoresistance. This model consists of an isotropic polar nondegenerate semiconductor with a parabolic conduction band centered at $\vec{k} = 0$. Pavlov and Firsov treated combined scattering of electrons on acoustic and optic phonons, and neglected collisional broadening of the Landau levels. They deduced that maxima in $|Q_{zz}|$ should occur at or near the fields given by Eq. (1), for all temperatures and proportions of acoustic to optic phonon scattering. Recently, Barker[9] has used the Kubo-Luttinger response formalism to investigate the magnetophonon oscillations in Q_{zz}. Considering scattering on optic phonons in nonpolar semiconductors, in the limit of strong Landau-level broadening, his analysis indicates that $|Q_{zz}|$ would have maxima displaced toward higher fields, in qualitative agreement with the experiments. However, a deep minimum is experimentally observed at about $\omega_c \approx 1.5\omega_0^1$, which is not contained in Barker's results, and suggests that the strong damping approximations made were too severe for an accurate description at the higher fields.

In addition, one should expect on physical grounds[10-12] that a proper formulation would also contain structure at magnetic fields given by

$$(2n+1)\omega_c = 2\omega_0, \quad n = 0, 1, 2, \ldots \; . \tag{2}$$

The physical reason for this "pseudoresonance" structure is discussed below, and in detail in Refs. 11 and 12. References 8 and 9 do not contain this structure for the reason that optic-phonon emission processes are omitted from the analyses. Observed in the longitudinal magnetoresistance at the higher temperatures in high-mobility samples, the pseudoresonances in Q_{zz} have not been seen to date.

In this paper we present the results of a detailed study in which no approximations whatever are made within the model used, which is that of Pavlov and Firsov[8] except that we consider nonpolar semiconductors. As emphasized previously,[11,13] the nonpolar form of the scattering matrix elements greatly simplifies the mathematics, and together with the assumption of no level broadening—the opposite limit to that of Barker—allows a detailed examination of the predicted structure. Briefly, we find that there are maxima in $|Q_{zz}|$ as found by Pavlov and Firsov, but that they are systematically displaced to higher fields than those given by Eq. (1). The amount is typically several percent and depends both on temperature and amount of elastic scattering. The computed displacements are apparently not as great as observed experimentally, but show that the observed displacements are not due entirely to level-broadening effects. We also find the deep high-field minimum mentioned above. Our work also shows the existence of slope discontinuities in Q_{zz}, precisely at the Gurevich-Firsov fields, Eq. (1), and at the pseudoresonance fields, Eq. (2). These discontinuities are characterized by

$$\frac{\partial\,|Q_{zz}|}{\partial B_-} > \frac{\partial\,|Q_{zz}|}{\partial B_+} \tag{3}$$

at all temperatures and proportions of elastic scattering, where B_- and B_+ refer, respectively, to the derivative evaluated from the low-field and high-field sides of the field in question. Thus, additional structure, tending in the direction of maxima, occurs. However, this structure is less pronounced than the corresponding structure in the longitudinal magnetoresistance.[11] This probably explains why the pseudoresonances have not yet been observed in Q_{zz}, although as stated earlier, much less experimental work has been done on Q_{zz}, and the various derivative techniques[14-16] used so well on the magnetoresistance to reveal the structure more clearly have not yet been attempted for the longitudinal magnetothermal emf.

In Sec. II we give the theoretical description of the model used, evaluate the Seebeck coefficient, and work out the slope discontinuities at the GF and pseudoresonance fields. In Sec. III we present the results of computations in the form of two representative figures, and provide some further discussion.

II. THEORETICAL ANALYSIS

The model is that of an electron interacting with optic and acoustic phonons in a nondegenerate nonpolar semiconductor, having an isotropic, parabolic conduction band centered at $\vec{k}=0$ and characterized by an effective mass m^*. The interactions are of the deformation potential type, with squared matrix elements given by

$$|M_{\mathrm{op}}|^2 = E_{1\mathrm{op}}^2 \hbar\omega_0 / 2\rho\mu_l^2 , \tag{4}$$

$$|M_{\mathrm{ac}}|^2 = E_1^2 \hbar q / 2\rho\mu_l , \tag{5}$$

where $E_{1\mathrm{op}}$ and E_1 are, respectively, deformation potential energies for scattering on longitudinal-optic and acoustic phonons. The optic-phonon frequency ω_0 is taken as dispersionless, and linear dispersion ($\omega_q = q\mu_l$) is used for the acoustic phonons of wave number q. The sample mass density is ρ, and μ_l is the longitudinal sound velocity averaged over direction. Scattering on the acoustic phonons is considered to be elastic, and equipartition for the acoustic phonons is used.

Some aspects of this model perhaps deserve comment. The optic phonons are of course slightly dispersive. Although no magnetophonon calculations have yet taken this into account, one can see from the physical picture for the structure[11,12] that the effect will be to shift the magnetic field positions of the various features by a small amount; the optic-phonon dispersion will have no tendency to smear out the discontinuities. Nonparabolicity of the conduction band is often significant; its effect will be to split each kink calculated here into a group of closely spaced kinks. Level broadening will tend to smooth out each feature, and is more important at low magnetic fields and high temperatures. Finally, the structure calculated here for nonpolar materials should be qualitatively similar to that for polar materials. The only mathematical difference is that the scattering matrix element varies inversely as the phonon wave number, instead of the constant given by Eq. (4). Further, a comparison between our nonpolar results[11] and the recent results of Magnusson[17] for polar materials shows a close similarity of the magnetophonon structure in the longitudinal magnetoresistance.

The Hamiltonian for this electron-lattice system in parallel electric and magnetic fields in the z direction is

$$\mathcal{H} = \frac{p_x^2 + p_z^2 + (p_y + m^*\omega_c x)^2}{2m^*} + \mathcal{H}_L + V + eEz \tag{6}$$

in the Landau gauge, where $\omega_c = eB/m^*$ is the cyclotron frequency, $\mathcal{H}_L$ is the lattice Hamiltonian, and V represents the electron scattering mechanisms. The electron energy eigenvalues are

$$\mathcal{E}_n(k_z) = \hbar^2 k_z^2/2m^* + \hbar\omega_c(n+\tfrac{1}{2}), \quad n = 0, 1, \ldots \,. \tag{7}$$

The Boltzmann equation is adequate for this model. Its solution,[11] linearized in the electric field in order to describe the Ohmic regime, gives for the electron distribution function

$$f_n(k_z) = f_n^0(k_z) + eE\hbar^{-1}\tau_n(k_z)\,\frac{\partial f_n^0(k_z)}{\partial k_z} \,. \tag{8}$$

(This result is also obtained as a special case from a density matrix formalism developed by Arora and Miller.[18]) Here $f_n^0(k_z)$ is the Fermi function, which becomes in the nondegenerate case

$$f_n^0(k_z)=\{\exp[\beta(\mathcal{E}_n-\zeta)]+1\}^{-1}\to e^{\beta\zeta}e^{-\beta\mathcal{E}_n(k_z)} . \tag{9}$$

Also,

$$e^{\beta\zeta}=n_e\left(\frac{2\pi\beta\hbar^2}{m^*}\right)^{3/2}\frac{\sinh(\frac{1}{2}\beta\hbar\omega_c)}{\beta\hbar\omega_c} , \tag{10}$$

where ζ is the Fermi energy, $\beta=1/kT$, n_e is the electron concentration, and

$$\frac{1}{\tau_n(k_z)}=\frac{m^*\omega_c}{\pi\hbar^2}\sum_{\alpha,n'}\int_{-\infty}^{\infty}dk_{z'}\,|M_\alpha|^2 \times[\bar{n}_{q,\alpha}\,\delta_+ +(\bar{n}_{q,\alpha}+1)\,\delta_-] , \tag{11}$$

$$\bar{n}_{q,\alpha}=[\exp(\beta\hbar\omega_{q,\alpha})-1]^{-1} , \tag{12}$$

$$\delta_\pm=\delta[\mathcal{E}_n(k_z)-\mathcal{E}_{n'}(k_{z'})\pm\hbar\omega_{q,\alpha}] . \tag{13}$$

Spin degeneracy accounts for a factor of 2 in Eqs. (10) and (11), and α refers to the optic- or acoustic-phonon modes. The right-hand side of Eq. (11) reduces to the form shown for the present model because the apparent q dependence vanishes.

We now turn to the derivation of the expression for the longitudinal magneto-Seebeck coefficient $Q_{zz}(\vec{B})$. Under the combined action of a temperature gradient and an electric field, both in the z direction, the heat and electric currents are given, respectively, by

$$F_z=\gamma_{zz}E^*-\chi_{zz}\frac{\partial T}{\partial z} , \tag{14}$$

$$J_z=\sigma_{zz}E^*-\beta_{zz}\frac{\partial T}{\partial z} , \tag{15}$$

where $E^*=E+e^{-1}\partial\zeta/\partial z$. The Seebeck coefficient is defined by

$$E=Q_{zz}\frac{\partial T}{\partial z} , \tag{16}$$

with the subsidiary condition that $J_z=0$. To avoid a first-principles calculation of β_{zz}, since it is the coefficient of a temperature gradient, we make use of the Onsager (Kelvin) relation[1]

$$\gamma_{zz}(\vec{B})=T\beta_{zz}(\vec{B}) . \tag{17}$$

Thus

$$Q_{zz}=\frac{\beta_{zz}}{\sigma_{zz}}=\frac{\gamma_{zz}}{T\sigma_{zz}} . \tag{18}$$

To evaluate γ_{zz}, one determines[1] the heat current as

$$F_z=(n_e/m^*)\langle(\mathcal{H}_e-\zeta)p_z\rangle , \tag{19}$$

where $\mathcal{H}_e$ is the electron Hamiltonian, and $\langle\ \rangle$ denotes an average with respect to the distribution function (8). The electric current is given by

$$J_z=-n_e e\langle p_z\rangle/m^* . \tag{20}$$

Reduction gives

$$\sigma_{zz}=-eA\sum_n\int dk_z\tau_n(k_z)\left(\frac{\hbar k_z}{m^*}\right)^2\frac{\partial f_n^0}{\partial\mathcal{E}_n} , \tag{21}$$

$$\gamma_{zz}=\frac{\zeta\sigma_{zz}}{e}+eA\sum_n\int dk_z\left(\frac{\hbar k_z}{m^*}\right)^2\mathcal{E}_n\tau_n\frac{\partial f_n^0}{\partial\mathcal{E}_n} , \tag{22}$$

with

$$A=\frac{n_e e}{\pi}\left(\frac{2\pi\beta\hbar^2}{m^*}\right)^{1/2}\sinh\frac{\beta\hbar\omega_c}{2} . \tag{23}$$

The Seebeck coefficient thus is

$$Q_{zz}=\frac{\zeta(B)}{eT}-\frac{1}{eT}\frac{\sum_n\int dk_z k_z^2\mathcal{E}_n\tau_n f_n^0}{\sum_n\int dk_z k_z^2\tau_n f_n^0} . \tag{24}$$

The magnetophonon structure is thus determined by the relaxation time $\tau_n(k_z)$ and modified by the other factors in Eq. (24). We display $\tau_n(k_z)$ for the present model:

$$\frac{1}{\tau_n}=\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\frac{E_{1op}^2\bar{n}_0\hbar\omega_c\omega_0}{4\pi\rho\mu_l^2}\times\sum_{n'=0}^{\infty}\{[\mathcal{E}_z+\hbar\omega_c(n-n')+\hbar\omega_0]^{-1/2} +e^{\gamma}[\mathcal{E}_z+\hbar\omega_c(n-n')-\hbar\omega_0]^{-1/2} +C[\mathcal{E}_z+\hbar\omega_c(n-n')]^{-1/2}\} , \tag{25}$$

where

$$\mathcal{E}_z=\hbar^2k_z^2/2m^* , \tag{26}$$

$$\gamma=\hbar\omega_0/kT , \tag{27}$$

$$C=\frac{2}{\bar{n}_0\gamma}\left(\frac{E_1}{E_{1op}}\right)^2 , \tag{28}$$

and $\bar{n}_0$ is the thermal population of optic phonons, determined by Eq. (12). Terms of the form $x^{-1/2}$ are defined to be zero for $x\le 0$. The first term on the right-hand side of Eq. (25) describes optic-phonon absorption; the second term, optic-phonon emission; and the third term, both absorption and emission of acoustic phonons.

Inspection of these terms reveals the physical reasons for the various magnetophonon structural features (more details may be found in Refs. 11 and 12): The Pavlov-Firsov[8] maxima, which in fact are off-resonance features, and the strongly damped structure described by Barker,[9] are determined principally by the optic-phonon absorption term and are due to the onset of optic-phonon absorption processes ending at $k_z=0$ which become possible when the energy difference between two Landau levels becomes greater than the optic-phonon energy. The effect of these processes becomes more abrupt (though not discontinuous) as the temperature is lowered, because the Boltzmann factor favors the lowest-energy electrons. The discontinuities in slope occurring precisely at the GF and pseudoresonance fields are the result of a "limiting" or dominant scattering mechanism (a process ending at $k_z = 0$ which significantly affects the magnetic field dependence of σ_{zz} or γ_{zz}) changing from one type to another as the magnetic field increases through the value in question. The change is from optic-phonon absorption to elastic scattering at the GF fields, and from optic-phonon absorption to optic-phonon emission at the pseudoresonance fields.[11,12]

For easier comparison with experiment, we present the results as $\Delta Q_{zz}/Q_{zz}(0)$. For this, one needs $Q_{zz}(0)$, which is obtainable either from Eq. (24) as $B \to 0$, or directly from the $B=0$ counterparts to the preceding equations. The result is

$$Q_{zz}(0) = \frac{k}{e}\left(\frac{\zeta(0)}{kT} - \frac{F_1(\gamma, 0)}{F_0(\gamma, 0)}\right), \tag{29}$$

where

$$F_n(\gamma, 0) = \int_0^\infty dx\, \frac{e^{-x} x^{n+3/2}}{Cx^{1/2} + (x+\gamma)^{1/2} + e^{\gamma}(x-\gamma)^{1/2}}, \tag{30}$$

and $\zeta(0)$ is obtained from Eq. (10).

Finally, we calculate the values of the discontinuities in the magnetic field derivatives of $Q_{zz}(B)$. These are obtained readily from inspection of the results for the longitudinal magnetoresistance.[12] Both evaluation of the derivatives and numerical computation are greatly facilitated by using the transformation and resummation technique introduced previously.[11] One can then rewrite Eq. (24) in the form

$$Q_{zz}(B) = \frac{k}{e}\left(\frac{\zeta(B)}{kT} - \frac{F_1(\gamma, H)}{F_0(\gamma, H)}\right), \tag{31}$$

where

$$F_n(\gamma, H) = (\gamma H)^n \sum_{m=0}^{\infty} \int_0^1 dy\, e^{-\gamma H (y+m)} \times \frac{A_m(y)(y+m+\frac{1}{2})^n}{G_m(y;\gamma, H)}, \tag{32}$$

$$H = \omega_c/\omega_0, \tag{33}$$

$$A_m(y) = \sum_{n=0}^{m} (y+n)^{1/2}, \tag{34}$$

$$G_m(y;\gamma,H) = C\sum_{n=0}^{m}(y+n)^{-1/2} + \sum_{n=-\infty}^{m}(y+n+H^{-1})^{-1/2} + e^{\gamma}\sum_{n=0}^{m}(y+n-H^{-1})^{-1/2}. \tag{35}$$

We shall denote the derivative discontinuities by

$$\Delta_H A \equiv \frac{\partial A}{\partial H}\bigg|_{-} - \frac{\partial A}{\partial H}\bigg|_{+}, \tag{36}$$

where the $-$ and $+$ signs refer, respectively, to the derivative evaluated from the low-field and high-field sides of the magnetic field in question. Then from Eq. (31),

$$\Delta_H S = \Gamma[F_1(\Delta_H F_0)/F_0 - \Delta_H F_1], \tag{37}$$

where

$$S = Q_{zz}(B)/Q_{zz}(0) - 1, \tag{38}$$

$$\Gamma = k/[eF_0(\gamma, H)Q_{zz}(0)]. \tag{39}$$

$\Delta_H F_0$ was determined in Ref. 12, and $\Delta_H F_1$ may be determined from it by inspection. Thus one finally gets GF resonances ($H = 1/N$, $N = 1, 2, \ldots$)

$$\Delta_H S = \gamma H\,|\,\Gamma\,|\left(\sum_{m=0}^{\infty}(m+\tfrac{3}{2}-z)R_m(1;\gamma,H) + \sum_{m=N-1}^{\infty}(m+\tfrac{3}{2}-z)R_m(1;\gamma,H) - \sum_{m=0}^{N-1}(m+\tfrac{1}{2}-z)R_m(0;\gamma,H)\right); \tag{40}$$

and pseudoresonances [$H = 2/(2N+1)$, $N = 0, 1, \ldots$]

$$\Delta_H S = 2\gamma H\,|\,\Gamma\,|\sum_{m=N}^{\infty}(m+1-z)R_m(\tfrac{1}{2};\gamma,H), \tag{41}$$

where

$$z = F_1(\gamma, H)/[\gamma H F_0(\gamma, H)], \tag{42}$$

$$R_m(y;\gamma,H) = H^{-2}e^{-(m+y)\gamma H}A_m(y)/G_m(y;\gamma,H). \tag{43}$$

The term containing $R_m(0;\gamma,H)$ in Eq. (40) is identically zero except for pure optic-phonon scattering.[12]

To discover the signs of the derivative discontinuities, we note that in the quantum limit ($\gamma H \gg 1$) small values of y are favored in Eq. (32). Thus, retaining only the $m=0$ term, and setting $y+m+\frac{1}{2}$ equal to $\frac{1}{2}$ in F_1, we see that $z \approx \frac{1}{2}$. From Eqs. (40) and (42) it follows that all $\Delta_H S$ are positive in the quantum limit. At higher temperatures or lower fields, the value of z increases, but is not easy to estimate analytically; however, the following approximate argument can be used. From Eqs. (34) and (35), one sees that the ratio A_m/G_m is of the order of unity, so that the m dependence of R_m in Eq. (43) is principally $e^{-m\gamma H}$. With this form, the summation in Eqs. (40) and (41) can be carried out. Thus, for any ν and any integer N,

$$\sum_{m=N}^{\infty}(m+\nu-z)e^{-m\gamma H} = e^{-N\gamma H}\,\frac{e^{-\gamma H} + (N+\nu-z)(1-e^{-\gamma H})}{(1-e^{-\gamma H})^2}. \tag{44}$$

For γH sufficiently small, this expression is positive. Thus, although this approximate argument does not of course constitute a proof, we conclude that all $\Delta_H S$ are positive at most, if not all, values of T, B, and amount of elastic scattering. All of our numerical investigations further corroborate this.

III. RESULTS AND DISCUSSION

The results of a computer evaluation of Q_{zz} are presented in this section. Figure 1 shows the magnetophonon structure at three temperatures ($\hbar\omega_0/kT = 6$, 3, and 1.5) for pure optic-phonon scattering. The Pavlov-Firsov maxima dominate; they are seen to decrease in amplitude and shift a few percent toward higher fields as the temperature increases. Pseudoresonance "maxima" are observed, whose positions are indicated by arrows in the figure, and are slightly more pronounced at the higher temperatures. The slope discontinuity at the $N=1$ GF field can be seen in the inset, but is small at all temperatures. Figure 2 shows the magnetophonon features at $\hbar\omega_0/kT = 3$, and three proportions of elastic scattering [$b = (E_{1op}/E_1)^2 = 6$, 60, and ∞]. One observes that increasing elastic scattering causes the Pavlov-Firsov maxima to decrease in amplitude and shift toward higher fields by several percent at $b = 6$, which is a representative value[19] ($b = 3.5$ for p-Ge, a nonpolar material).

The two figures show that the discontinuous structure at the GF fields and pseudoresonance fields is considerably smaller than in the longitudinal magnetoresistance at the same temperatures and degrees of elastic scattering.[11] When the smoothing effects of level broadening are considered, it is clear that these structures will be correspondingly harder to observe experimentally. The structure at the GF fields will likely never be resolvable from the Pavlov-Firsov off-resonance maxima. The pseudoresonances are more likely to be observed, particularly the one at $\omega_c = 2\omega_0$ because of its isolation from the remaining structure. Finally, we call attention to the high-field minimum lying at about $\omega_c/\omega_0 = 1.2-1.4$ in the figures. Its amplitude and extermal position vary with temperature and elastic scattering in the same manner as the Pavlov-Firsov maxima.

In summary, our analysis of the longitudinal magneto-Seebeck effect in the limit of no Landau-level broadening in nonpolar semiconductors, shows the general magnetophonon structure which has been observed experimentally to date—including maxima shifted to the high-field sides of the GF resonance fields, the pronounced high-field minimum, and decreasing amplitudes with increasing temperature and increasing competition from elas-

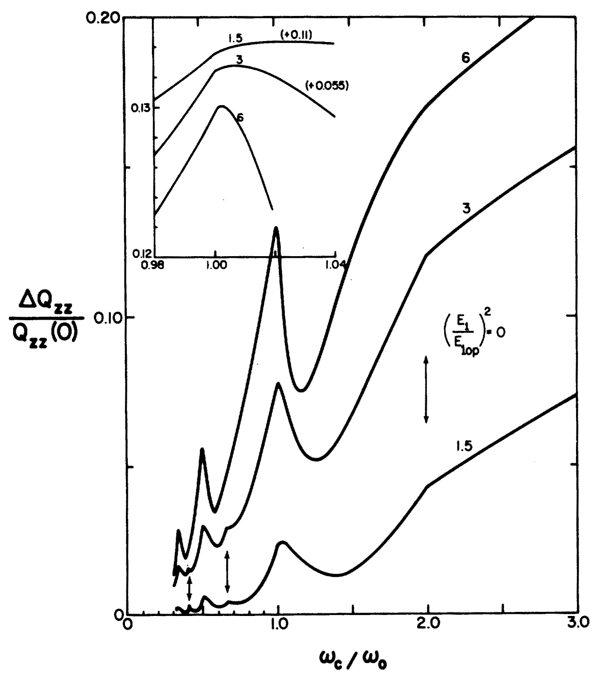

FIG. 1. Relative longitudinal magneto-Seebeck coefficient as a function of magnetic field for pure optic-phonon scattering of electrons at three temperatures. Numbers on curves are values of $\gamma = \hbar\omega_0/kT$. The arrows lie at the pseudoresonance fields. The inset shows the region near $\omega_c = \omega_0$ on an expanded scale; the $\gamma = 1.5$ and 3 curves are translated upward by the amounts shown in parentheses, to separate the curves.

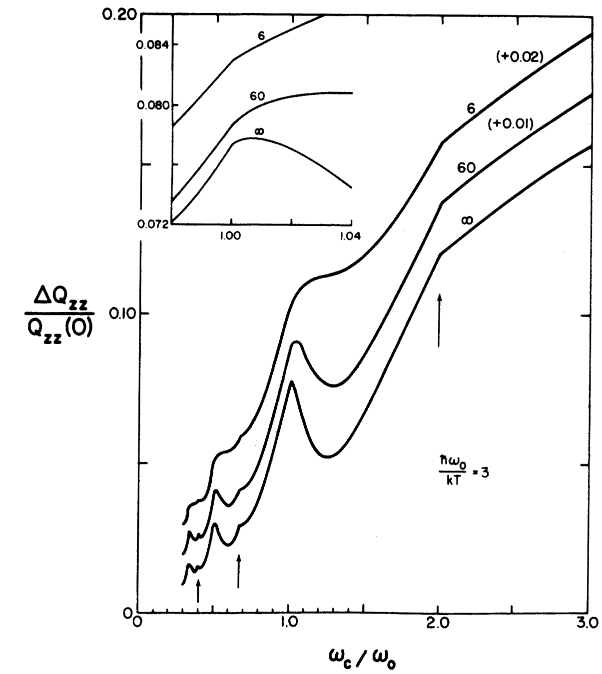

FIG. 2. Relative longitudinal magneto-Seebeck coefficient as a function of magnetic field, at a temperature corresponding of $\hbar\omega_0/kT = 3$, and three amounts of elastic scattering. Numbers on curves are values of $b = (E_{1op}/E_1)^2$; the $b = 6$ and 60 curves are translated upward by the amounts indicated in the parentheses, to separate the curves. The inset shows the region near $\omega_c = \omega_0$ on an expanded scale.

tic scattering. The shifts of the maxima are not as great as those observed, however, and level-broadening considerations, such as those of Barker,[9] likely have to be invoked to explain the differences. In addition, extra maxima are predicted, which take the form of kinks at the GF and pseudoresonance fields in the no-level-broadening limit. These have yet to be observed, although it may be worth noting that Puri and Geballe[1] stated that they observed small kinks in addition to the large Pavlov-Firsov maxima in their work on n-InSb. It is of interest to see whether the more-sensitive derivative measurement techniques[14-16] used with the magnetoresistance, but not used yet with the longitudinal magnetothermal emf, would reveal this structure.

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