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Distribution of spin-wave modes in an amorphous ferromagnetic chain

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The density of spin-wave states for an amorphous ferromagnetic chain has been calculated by using the coherent-potential approximation. A comparison with the recent exact numerical calculation of Huber shows very good agreement even near the high-energy band edge where the conventional mean-field theory is found to be inadequate.

I. INTRODUCTION

Recent inelastic-neutron-scattering experiments[1,2] at low temperatures have revealed the existence of spin waves in the linear-chain antiferromagnetic $(CD_3)_4NMnCl_3$ (TMMC). Although these spin-waves become overdamped and finally disappear at high temperatures, at low temperatures they are well defined for wavelengths shorter than the correlation length.

Studies of static and dynamic properties of such magnetic chains have been reported by many authors.[3] Most recently, Huber[4] has calculated the spin-wave densities of states for disordered one-dimensional ferromagnetic and antiferromagnetic chains with nearest-neighbor interactions, by using Sturm-sequence algorithm originally introduced by Dean.[5] He considered an amorphous ferromagnetic chain characterized by randomly distributed nearest-neighbor exchange interactions.

Dean's theory is a nonanalytic theory and the exact solutions for the density of states are obtained numerically. Recently, Montgomery, Krugler, and Stubbs[6] (MKS) and Foo and Bose[7] (FB) have proposed analytic theories for the amorphous ferromagnets. The MKS theory is basically a mean-field theory and is valid only in the weak-scattering limit. The FB theory is based on the coherent-potential approximation (CPA) which serves as an interpolating scheme between the weak and strong scattering limits. Both of the theories do not take into account the local spin fluctuations and the temperature effects and thus are valid only at very low temperatures. A comparison of the two theories reveals that the FB theory predicts a critical fluctuation Δ_c beyond which the ferromagnetic states become unstable—a result which cannot be reproduced by any mean-field theory. However, for the density of states of a three-dimensional lattice, the FB theory shows only a minor improvement over the MKS theory.

For the one-dimensional case, however, one expects the conclusion to be somewhat different. Since the density of states of a pure ferromagnetic chain has a singularity at the upper band edge and since the MKS theory is perturbative in nature, one would expect it to be in severe disagreement with exact solutions near this edge—a conclusion confirmed by Huber's calculations. When applied to an amorphous ferromagnetic chain the MKS theory shows a dip at $\omega/2\bar{J}S=4$ (upper energy edge), and a spurious peak at even higher energy. But the exact solution indicates a smooth density of states in this energy range.

The purpose of this paper is to point out that the FB theory is more accurate especially for the one-dimensional system. In Sec. II we shall outline the FB theory very briefly and discuss the outcome of our calculation for the one-dimensional case.

II. THEORY AND DISCUSSION

The Hamiltonian for an amorphous ferromagnet which is approximately represented by the "lattice model" is given by

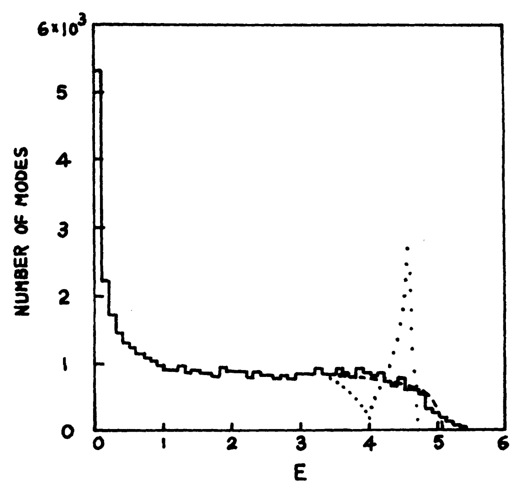

FIG. 1. Distribution of the spin-wave modes in an amorphous ferromagnetic chain where the fluctuation parameter $\Delta=0.5\,J_0$. Energy E is measured in units of J_0. The histogram corresponds to the exact numerical calculation of Huber for a chain of 50 000 spins. The dashed line represents our results obtained by using the CPA. The dotted line shows the results of the MKS theory.

$$H = \frac{1}{2} \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad , \tag{1}$$

where J_{ij} represents the nearest-neighbor exchange interaction, which is assumed to be randomly distributed between $J_0 - \Delta$ and $J_0 + \Delta$, where J_0 represents the mean value and Δ the fluctuation from the mean. In the spirit of the CPA, one can describe the actual amorphous system by an effective Hamiltonian which retains the symmetry of a perfect lattice, and is characterized by a yet-to-be-determined coherent exchange interaction J_c. Then one considers a single actual exchange interaction J_{ij} immersed in such an effective medium, and J_c is determined self-consistently by requiring that the net scattering from such a single scatterer J_{ij} must vanish on the average.

The t-matrix elements corresponding to J_{ij} can be shown to be

$$T_{ii} = -T_{ij} = \frac{J_{ij} - J_c}{1 - 2(J_{ij} - J_c)(G_0 - G_1)} \quad , \tag{2}$$

where G_0 and G_1 denote the diagonal and the first off-diagonal matrix elements of the coherent Green's function G_c. Taking the configurational average of the t matrix we have

$$\langle T_{ii} \rangle = \frac{1}{2\Delta} \int_{J_0-\Delta}^{J_0+\Delta} T_{ii}\, dJ = -\frac{\alpha}{2\Delta}\left(2\Delta + \alpha \ln \frac{J_0 - J_c - \alpha + \Delta}{J_0 - J_c - \alpha - \Delta}\right) \quad , \tag{3}$$

where

$$\alpha = \tfrac{1}{2}(G_0 - G_1)^{-1} \quad . \tag{4}$$

The value of J_c is determined by requiring $\langle T_{ii} \rangle = 0$, which leads to

$$J_c = J_0 - \alpha + \Delta \coth(\Delta/\alpha) \quad . \tag{5}$$

Then the spin-wave densities of states can be calculated via

$$\rho(E) = -(1/\pi)\, \mathrm{Im} G_0(E, J_c) \quad . \tag{6}$$

For a one-dimensional ferromagnet, the Green's functions have the form of

$$G_0 = [E(E - 4J_c)]^{-1/2}, \qquad G_1 = (1/2J_c)[1 - (E - 2J_c)G_0] \quad ; \tag{7}$$

using (5), (6), and (7) one can calculate the density of states for the amorphous linear ferromagnet in the CPA. The MKS theory can be derived from Eq. (4) by letting $\Delta \to 0$, then

$$J_M = J_0 + \Delta^2 / 3\alpha_M \quad ,$$

where α_M is evaluated from Eq. (4) by setting $J_c = J_0$. The criterion for the validity of the MKS theory is that $|\Delta^2/3\alpha_M| \ll J_0$. But at $E = 4J_0$, J_M approaches infinity or $|\Delta^2/3\alpha_M| \gg J_0$, and thus near this region the MKS theory becomes invalid.

The densities of states calculated via the FB theory is shown in Fig. 1.[8] The result shows that the FB theory is indeed in good agreement with the exact numerical solution.

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[8] In this calculation the functional form of the exchange interaction has been taken to be $J = J_0\,(1 - \frac{1}{2} + Y)$, where Y is a random number between 0 and 1. This functional dependence is the same as that of Huber except for a misprint of a factor of $\frac{1}{2}$ in the third term in Eq. (19) of his paper (Ref. 4).