Voltage steps in the current-induced resistive state of thin-film type-I superconductors*

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We report the observation of voltage steps in the current-induced resistive state in superconducting strips of lead and indium. The strips were 1.7–6 mm long, 0.1–8.0 μm thick, and 100–300 μm wide. The voltage steps are attributed to the nucleation of trains of flux tubes moving rapidly from the edge to the center of the strip. In the center, opposite tubes arriving from opposite edges annihilate each other. The moving flux-tube trains are, obviously, the outgrowth to the case of a three-dimensional type-I superconductor of the dynamic behavior of the weak-link Josephson junction, and, in particular, of the Anderson-Dayem bridge. From a simple eddy-current-damping model we have estimated the number of flux tubes existing simultaneously in a single flux-tube train.

I. INTRODUCTION

In recent experiments we have shown[1] that the current-induced resistive state in thin-film type-I superconductors has dynamic character, with arrays of flux tubes moving rapidly from the edge to the center of the superconducting strip. In the center, flux tubes of opposite sign, generated at the opposite edges of the strip, annihilate each other. The nucleation of the individual flux-tube trains has been demonstrated in high-resolution magneto-optical experiments.[2,3] The abrupt nucleation of the moving flux-tube trains with increasing electrical transport current results in detailed structure in the voltage-current characteristic, as seen from the derivative of the $V(I)$ curves.[1]

In the following, we report the observation of regular voltage steps in the current-induced resistive state of superconducting lead and indium films. These steps can be understood by the nucleation of individual flux-tube trains moving rapidly from the edge to the center of the strips. The observation of these voltage steps is complicated because of the high sensitivity of the behavior to small changes in sample properties, caused by thermal cycling to room temperature, strain, oxidation, etc. This high sensitivity to slight variations in sample characteristics is, of course typical of a nucleation phenomenon. Sometimes, the step structure in the $V(I)$ curve may be obscure because of instabilities in the magnetic-flux structure generated by the current. Fluctuations in the magnetic structure are accompanied by relatively large electrical noise power and suppress the structural features of the voltage-current behavior in a time-averaging measurement. We have attempted, so far without clear success, to induce resistive voltage steps by locally weakening the superconducting microstrips. Preliminary results of the present investigation were reported elsewhere.[4,5]

Step structure in the voltage-current characteristic has been observed previously for "one-dimensional" type-I superconducting whiskers[6,7] and thin-film microbridges.[8] Most recently, steps in the I-V curves were reported for long, very thin microbridges of tin, and were explained in terms of spatially localized quantum phase-slip centers.[9,10] It appears that our present results are the outgrowth to the three-dimensional case of these experiments on one-dimensional systems, in which the sample geometry is equal to or smaller than the characteristic lengths of the superconductor.

II. EXPERIMENTAL

Our samples were strips of lead or indium, 1.7–6 mm long, 0.1–8.0 μm thick, and 100–300 μm wide. They were vacuum deposited (starting pressure about 10^{-6} Torr) on glass substrates using Pb and In with 99.9999% purity and tunable masks with two razor-blade sections for defining the sample edges. Electrical current and voltage leads were soldered with indium to rather wide sections at both ends of the strips (see insert of Fig. 1). In a number of specimens separate voltage tabs were deposited over the strips in the usual four-probe arrangement. The wide end sections and the separate voltage tabs were deposited after the center strip and always consisted of lead with a thickness larger than that of the center strip.

We have attempted in various ways to induce resistive voltage steps in a controlled fashion by locally weakening the microstrips. For this purpose, we have scratched a small notch into the edge of the strip or illuminated a section of 10–20 μm width extending all the way across the strip with light of about 1-W/cm^2 intensity. We also placed overlays from nonsuperconducting metals across the strips. These overlays were 10–20 μm wide, about 0.5 μm thick, and consisted of gold, copper, or permalloy. For the lead strips, the "overlays" actually were deposited first, to minimize the effect of Pb oxidation on the contact. However, the various methods for weakening the superconductor did not generate reproducible voltage steps. Magneto-optically, at increasing current, the first flux-

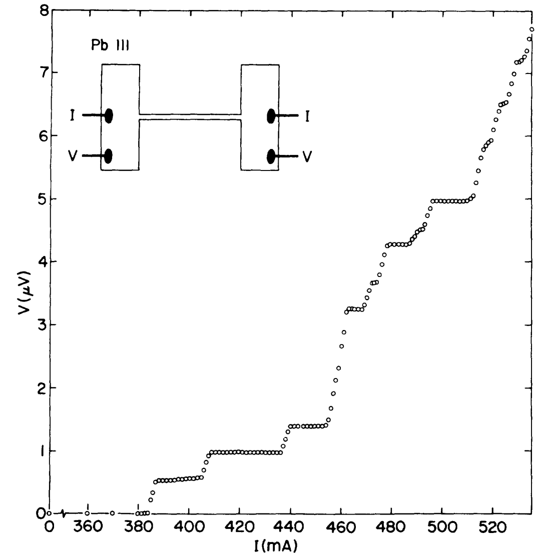

FIG. 1. Voltage vs current in a Pb microstrip at 4.2 K at the onset of the current-induced resistive state (sample Pb-111).

tube array was observed as expected at the constriction where a notch had been scratched into the edge. However, no distinct voltage step could be detected for this configuration.

The magneto-optical observation of a lead film with 5.0-μm thickness carrying a 0.5-μm gold overlay indicated that in the neighborhood of the overlay the proximity of the gold had no effect on the location where the flux-tube arrays were nucleated. We note that the weakening effect of a normal metal in close proximity to the superconductor increases relatively as the film thickness of the superconductor decreases. Therefore, the negative result of the magneto-optical experiment on the 5.0-μm Pb film with the gold overlay may not be extrapolated to our Pb specimens which are much thinner than 5 μm. Unfortunately, one reaches the magneto-optical limit of resolution for flux structures in Pb films slightly thinner than 1 μm. In the first five columns of Tables I and II we list the characteristics of the specimens studied in the present experiments. For the Pb films the resistance ratio $R(295\ K)/R(4.2\ K)$ was obtained by applying a magnetic field of 1 kOe perpendicular to the films at 4.2 K.

Our measurements were performed in zero applied magnetic field, the only field present being that of the electric transport current. Direct current was applied to the strips using a Fluke Model No. 382A voltage/current source. During the experiments the samples were in direct contact with liquid helium. As a protection against destruction through thermal runaway, a constantan-wire shunt with 30–60-mΩ resistance was attached parallel to the strips. The shunt resistance was always several orders of magnitude larger than the sample resistance in the current range investigated. Voltage was detected with a resolution of 10^{-2} μV. In some cases the electrical noise power was measured simultaneously with the voltage steps using the same instrumentation as previously.[1]

III. RESULTS

A typical step structure in the $V(I)$ curve is shown in Fig. 1 for sample Pb-111. The voltage steps have about 0.5-μV magnitude and take place over about 3-mA current range. We believe the voltage steps are associated with the nucleation of individual trains of flux tubes moving rapidly from the edge to the center of the strip. Each flux-tube train very likely consists of opposite flux tubes originating at the opposite edges and annihilating each other in the center of the strip. Following the nucleation of a new train, the rate at which the flux tubes of this array are generated at the edge and their velocity in moving toward the center of the strip increase rapidly with increasing current. This rapid increase continues until saturation occurs either for the flux-tube nucleation rate or their traveling speed through the superconductor. A further increase in current then has only little influence on the dynamics of this flux-tube train. In this way, the voltage steps can be qualitatively understood. It is important that within this model each new voltage step corresponds to the nucleation of an additional flux-tube train. Usually, at increasing current the first voltage steps are very distinct. As the current increases and more flux-tube arrays become nucleated, further nucleation occurs after

TABLE I. Lead specimens (d is sample thickness; w is sample width).

Sample	Overlay	d (μm)	w (μm)	$R(295\ K)/R(4.2\ K)$	T (K)	I^* (A)	ΔV_{obs} (nV)	a (μm)	ΔV_{calc} (nV)	Factor n
Pb-111	none	3.2	100	634	4.2	0.4	500	1.0	35	14
Pb-131	none	3.1	105	252	4.2	0.3	300–400	1.0	56	~6
PbAu-9	gold	1.0	160	256	4.2	0.6	1000–2000	0.6	60	~25
PbAu-10	gold	1.0	160	241	4.2	0.6	1000–2000	0.6	58	~25
Pb-145	none	0.09	177	37	4.2	0.085	1200	(0.03)	(1.4)	(850)

smaller and smaller current increments, and the current range of the voltage plateau diminishes. For the higher currents in Fig. 1 the step structure nearly disappears because of the rapid sequence with which new flux-tube trains are nucleated at increasing current. The large step at about 460 mA may be associated with the nucleation of two or three flux-tube trains at nearly the same current level. As indicated in Fig. 1, stationary voltage levels were also obtained on the ramp of each step. The structure in the $V(I)$ curve was reproducible during the same low-temperature run and was the same or very similar at increasing and decreasing current. Current hysteresis was more pronounced following the application of a current much larger than that at which the first voltages appeared.

In Fig. 2 we show the gradual development of the structure in the $V(I)$ curve of an indium strip as the temperature is reduced from the transition temperature. Apparently, with decreasing temperature, the voltage steps become more distinct.

Figure 3 shows the voltage-current characteristic for an indium strip, displaying some current hysteresis. For increasing and decreasing current, the voltage levels are very similar. Interestingly, at about 490 mA for decreasing current, apparently the resistance switched briefly to a higher level before the "regular" level was adopted again. Such regions with negative differential resistivity are not uncommon for similar conditions.[11]

Finally, in Fig. 4 we show the electrical noise power at 320 Hz together with the $V(I)$ behavior of a lead strip at 4.2 K. Apparently, the onset of each voltage step coincides with a distinct peak in the electrical noise power, the first voltage step showing by far the largest noise-power value. The data in Fig. 4 suggest strong electric and magnetic fluctuations at the onset of the first resistive voltage steps. A strong peak in the electrical noise power at the onset of the current-induced resistance had also been observed in our previous experiments.[1]

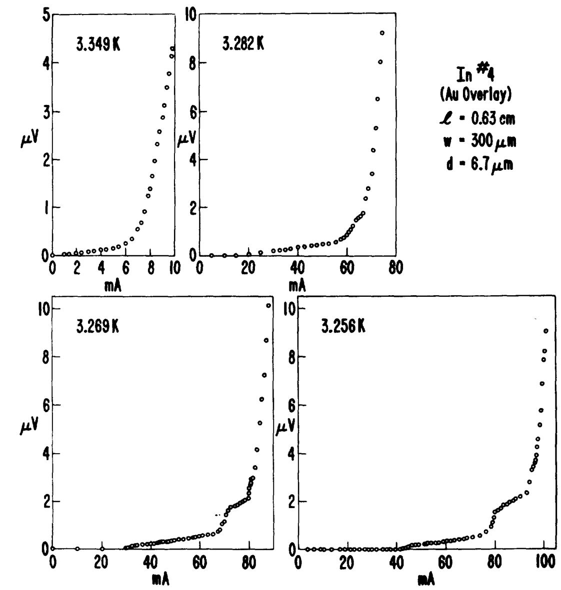

FIG. 2. Voltage vs current for an In microstrip at different temperatures (sample In-4).

IV. DISCUSSION

So far, our attempts for controlling the appearance of the resistive voltage steps (notch, light irradiation, and normal overlay) have been unsuccessful. For obtaining a distinct voltage step, one clearly requires perfect periodicity of flux-tube nucleation at the sample edge and a stable path for the flux tubes to travel to the center of the strip. Fluctuations in the nucleation rate or in the location of the traveling path can easily obscure the step structure in the voltage. It is likely that a change in the traveling path and in the nucleation conditions with current accounts for our inability to observe voltage steps in the specimens where a small notch had been scratched into the sample edge. It is possible that local heating, through energy dissipation of the moving flux tubes, has a stabilizing influence on the location of the path for

TABLE II. Indium specimens (d is sample thickness; w is sample width).

Sample	Overlay	d (μm)	w (μm)	$R(295\text{ K})/R(4.2\text{ K})$	T (K)	I^* (A)	ΔV_{obs} (nV)	a (μm)	ΔV_{calc} (nV)	Factor n
In-5	gold	8	300	96	2.11	1.1	150–250	1.5	26	~8
In-6	copper	8	300	98	2.61	0.66	150–300	1.5	17	~13
In-7	permalloy	8	300	137	2.11	1.5	200–400	1.5	21	~14
In-4	gold	6.7	300	49	2.13	0.6	500–800	1.5	36	~18
In-11	none	1.3	190	79	2.07	0.4	500	0.6	45	11
In-8	gold	0.75	300	32	2.05	0.4	1000–2000	0.5	22	~70
In-9	copper	0.75	310	30	2.06	0.45	1400	0.5	19	74
In-10	permalloy	0.75	310	47	2.05	0.5	1000–3000	0.5	18	~110

the flux tubes (in striking analogy with lightning).

Closely coupled with our inability to successfully control the appearance of voltage steps is the very high sensitivity of the detailed voltage-current behavior to slight variations in sample characteristics. Here, lead appears to change its behavior due to thermal cycling, etc., much easier than indium.

Recently, we have investigated in detail the thermodynamic aspects of magnetic-flux penetration into type-I superconductors.[12] A flat superconducting strip carrying an increasing electrical transport current first shows a narrow region containing magnetic flux along both edges of the strip (edge structure). In this state, the strip still has zero electrical resistance. Only when flux tubes separate from the edge structure and travel across the strip does electrical resistance appear. The motion of flux tubes toward the center of the strip is hindered by a barrier in the Gibbs free energy. The irreversible penetration of flux tubes into the strip is possible only when this energy barrier is removed by raising the electrical current to the critical level.

As pointed out in Sec. III, the voltage steps can be understood in terms of the saturation of the rate with which flux tubes in a given train are nucleated at the edge and transported to the center of the strip. Referring to the case of Fig. 1, the voltage steps of about 0.5 μV, together with the Josephson relation, indicate that in each array flux quanta are traversing the strip at the rate of 2.5×10^8 sec^{-1}. In a lead film of 3.2 μm thickness, one expects flux tubes containing about 60 flux

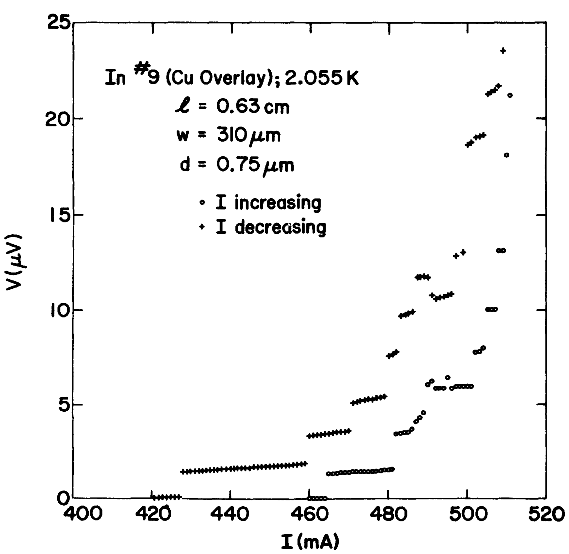

FIG. 3. Voltage vs current for an In strip at 2.055 K at increasing and decreasing current (sample In-9).

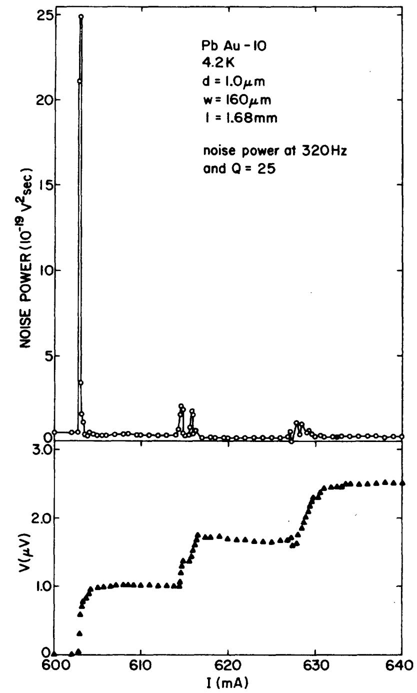

FIG. 4. Voltage and electrical noise power vs current in a Pb strip at 4.2 K (sample PbAu-10).

quanta.[13] The voltage steps in Fig. 1 then correspond to the passage of flux tubes in each array at the rate of 4×10^6 sec^{-1}. This flux-tube nucleation rate is consistent with the fact that the superposition of an alternating current of a few mA rms and a frequency up to 500 kHz had no other effect than the expected broadening of the dc current range over which the voltage steps took place.

We have analyzed our results in terms of the following eddy-current-damping model. The average flux-tube velocity v_φ can be obtained from the balance between the Lorentz force and the viscous damping force per unit length of flux tube

$$j\Phi/c=\eta v_\varphi \; . \qquad (1)$$

Here, j is the electrical current density averaged across the strip and Φ is the magnetic flux per tube. For eddy-current damping, the coefficient η is

$$\eta=\sigma_n H_c \Phi/c^2 \; , \qquad (2)$$

where σ_n is the normal electrical conductivity and H_c is the critical field. The time-averaged voltage $\overline{V}$ associated with a moving flux-tube array is given by the Josephson relation

$$\overline{V}=\Phi/c\tau \; , \qquad (3)$$

where τ is the relaxation time for the nucleation of a flux tube. The flux tube nucleation rate can be written as

$$1/\tau = n(2v_\varphi/w) , \quad (4)$$

w being the width of the strip. In Eq. (4) we assume the nucleation of pairs of opposite flux tubes at the opposite edges. The factor n indicates the number of pairs of opposite flux tubes existing simultaneously in a single train. Combining Eqs. (1)–(4) one finds

$$\overline{V} = (2na^2\pi/\sigma_n w^2 d) I . \quad (5)$$

Here, a is the flux tube radius, d is the thickness of the strip, and I is the electrical current.

In columns 6–8 of Tables I and II we have listed the observed voltage steps ΔV_{obs} together with the current I^* and the temperature at which these were measured. The voltage steps, calculated from Eq. (5), are listed in column 10. In calculating these values, we have set $n = 1$. The flux tube diameter a given in column 9, was estimated from previous experiments.[2,13] The calculated voltage steps ΔV_{calc}, assuming $n = 1$, are seen to be much smaller than the experimental values. In column 11 we have listed the number n of pairs of opposite flux tubes in a single array, which yields agreement between Eq. (5) and the experimental observation. For the strips with approximately the same width, it is seen that the quantity n increases with decreasing film thickness. The flux-tube diameter increases with the square root of the film thickness. One would expect the distance between two successive flux tubes in an array, which is determined by their mutual repulsion, to increase with the flux-tube diameter. Therefore, the variation of n with film thickness appears quite reasonable.

A quantitative theory of these phenomena still has to be worked out. We definitely oversimplified the situation by assuming a constant electrical current density across the strip. In particular, the mechanism which determines the saturation of the flux-tube nucleation rate in a single train remains unresolved.

The moving flux-tube trains are, obviously, the outgrowth to the case of a three-dimensional type-I superconductor of the dynamic behavior of the weak-link Josephson junction, and, in particular, of the Anderson-Dayem bridge.[14] In our strips, the flux tubes play the role of the single flux quanta in the traditional Josephson junction. The oscillatory behavior of the Josephson junction, i.e., phase slippage induced by a radio-frequency field and emission of radio-frequency radiation, can also be expected for our systems. However, the Josephson relation, which regulates these phenomena, must be modified in our case through a factor given by the number of flux quanta per flux tube.

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