Universal range-velocity and stopping-power equations for fission fragments and partially stripped heavy ions in solid media

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The applicability of Bohr's stopping-power formula for range and stopping-power calculations has been investigated by a detailed analysis of the available range data for fission products in various solid media (Be, C, Al, Si, Cu, Zr, W, Au, and U). In the case of heavy media ($Z \geq 45.5$), the result of the analysis is in agreement with the Thomas-Fermi statistical approach for obtaining the effective quantum number for the outer orbital electrons and also with Bohr's original estimate of the effective charge of a fast-moving heavy ion. In the case of light media, an empirical function $n(U_s) = 0.28Z^{2/3}U_s/V_0$ is found necessary in place of $n(U_s) = Z^{1/3}U_s/V_0$, where $n(U_s)$ is the number of orbital electrons in a medium of atomic number Z whose velocities are less than a given velocity U_s and $V_0 = e^2/\hbar$. The incorporation of these findings into Bohr's stopping-power formula leads to universal stopping-power and range-velocity equations valid for partially stripped ions ($Z > 30$) in different solid media, including compound media of known molecular formula. Available experimental data on stopping powers and ranges of heavy ions in various solid media and also the equilibrium charges of very light ions in gaseous media are in excellent agreement with the corresponding calculated values.

I. INTRODUCTION

A precise knowledge of the stopping powers or range-energy relationship for fission products in various media is of considerable interest both from the practical as well as theoretical points of view. Among the practical applications, one may mention calculation of the fission-fragment energy-deposition efficiency,[1] which is essential in designing fission-electric cells. More important, however, is the theoretical insight which might be gained regarding the mechanism of energy loss of heavy ions, the velocities of which are such that they always retain a core of orbital electrons and for which no adequate stopping-power equation exists in literature. Bethe's[2] stopping-power equation is not easily applicable in the case of partially stripped heavy ions and the equation of Lindhard *et al.*[3] predicts stopping powers which are rarely in agreement with the experimental ones.[4-8] The available range-energy relations, on the other hand, are either purely empirical ones[9] or are empirical modifications[4,10-12] of the equation of Lindhard.[3] A very fruitful line of approach was indicated by Niday,[13] who showed that a stopping power formula due to Bohr[14] may be modified slightly to yield an accurate range-velocity relationship without any loss of significance of Bohr's[14] original assumptions. Niday's[13] equation, which was deduced for calculating the ranges of fission products in uranium, has been extended in the present work so as to include all solid media both for range and stopping-power calculations.

II. SEMIEMPIRICAL DERIVATION

A. Heavy solid media

The starting point is the stopping-power formula of Bohr[14]:

$$\frac{\Delta E}{\Delta X} = nB_\epsilon\left(\sum_s \ln(\eta_s^2\,[\kappa/\eta_s]^{-1}) + \sum_s \ln(\eta_s^2[\kappa]^{-2})\right), \qquad (1)$$

where $\Delta E/\Delta X$ is the energy lost per unit path length by a heavy ion of charge ze and velocity V, and n is the number of atoms of the medium per unit volume. The other symbols are defined as follows:

$$B_\epsilon = 2\pi z^2 e^4/mV^2\,, \qquad (2)$$

$$\eta_s = 2V/U_s\,, \qquad (3)$$

$$\kappa = 2z\,V_0/V\,, \qquad (4)$$

where m and e are the electronic mass and charge, respectively, U_s is the orbital velocity of the sth electron in the atom of the medium, and V_0 stands for $e^2/\hbar$. In Eq. (1) the terms within the square brackets, if less than unity, should be replaced by unity. The summation of the logarithmic terms should include the velocities of only those electrons which interact with the moving ion at any given velocity V. Because of the continuous capture of electrons by the decelerating ion, z in Eqs. (2) and (4) has to be replaced by an "effective charge" Z^{eff}. The evaluation of Z^{eff} and the summation of the logarithmic terms in Eq. (1) was done by Bohr[14] in the case of a "heavy" ion passing through a "heavy" medium on the basis of two assumptions. The first is that the number of orbital electrons

$n(U_s)$, whose velocities are less than a given velocity U_s, is

$$n(U_s)=\nu_s U_s/V_0 , \quad (5)$$

where ν_s is the "effective" quantum number for the outer orbital electrons. If a "heavy" atom is defined as one with atomic number Z large enough for the Thomas-Fermi statistical approach to be applicable, then ν_s may be replaced by $Z^{1/3}$. Hence

$$n(U_s)=Z^{1/3} U_s/V_0 . \quad (6)$$

The second assumption is that the number of electrons lost by a heavy ion at a velocity V is equal to the number of its orbital electrons with velocity less than V. Hence, from Eq. (6)

$$Z^{\text{eff}}=n(U_s=V)=Z^{1/3} V/V_0 . \quad (7)$$

A subsequent estimate of Bohr and Lindhard[15] is

$$Z^{\text{eff}}=kZ^{1/3} V/V_0 , \quad (8)$$

where $k=1.5$ in solid media and $k=1$ in gaseous media. With the use of Eq. (8) in Eq. (1) and after making appropriate provision for the nonparticipation of many of the orbital electrons of the heavy medium,[14] one obtains the following range-velocity relation:

$$\rho\, dx=\frac{A_1A_2\, dV}{127.3\times10^{11}Z_2^{1/3}[4.7622(kZ_1^{1/3})^{5/3}+kZ_1^{1/3}]} , \quad (9)$$

where A_1 and Z_1 are the mass number and atomic number of the moving ion, while ρ, A_2, and Z_2 stand, respectively, for the density, mass number, and atomic number of the medium. The total range or the "integrated range" might be obtained if one could integrate the right-hand side of Eq. (9) down to zero velocity. However, Eq. (9) is not strictly valid at $V<V_0$, since at these velocities the ion is practically neutral and the stopping would be due mainly to screened-field-type interactions.[14] Niday[13] advanced the plausible argument that as far as the projected range in the direction of the moving ion is concerned, which corresponds to the experimentally determined range, it is sufficient, as a first approximation, to evaluate the integral between the initial ion velocity V_i and V_0. At $V<V_0$ there would be insignificant addition to the projected range because of the very large-angle scatterings associated with nuclear collisions. Hence, the experimentally determined range R may be related to the initial velocity V_i by the following expression:

$$R=\frac{A_1A_2(V_i-V_0)}{127.3Z_2^{1/3}[4.7622(kZ_1^{1/3})^{5/3}+kZ_1^{1/3}]} , \quad (10)$$

where both V_i and V_0 are expressed in units of 10^8 cm/sec and all the other symbols have been defined earlier; R is in units of mg/cm^2. Niday[13] showed that with $k\approx1$, the ranges of various fission products in uranium are predicted with fair accuracy by Eq. (10). Since $k\sim1$ is different from the value suggested by Bohr and Lindhard,[15] we have analyzed all available range data for fission products in heavy solid media in order to investigate if there is any systematic medium-dependent variation in the value of k. The procedure we have used is as follows. For a given fission product of mass number A_1 of known range R, the initial velocity V_i is first calculated. For this purpose, the precursor fragment mass number A_1' is first determined by means of the experimentally measured prompt-neutron number as a function of the fragment mass number. From the experimentally measured fission-fragment kinetic energies existing in literature and A_1', the values of V_i have been calculated. Since the prompt neutrons are almost isotropically emitted from the moving fragments, the velocity of the fission product of mass number A_1 is identical with the velocity of the precursor fragment of mass number A_1'. The atomic number Z_1 of the fission product of mass number A_1 is given by the "most probable charge" $Z_p(A_1)$, which has been calculated from Mukherji's[16] prescription: for light fission products

$$Z_p(A_1)=Z_c-\frac{A_c-A_1'}{2.587} ; \quad (11)$$

for heavy fission products

$$Z_p(A_1)=\frac{A_1'}{2.587} , \quad (12)$$

where A_c and Z_c are the mass and the atomic numbers of the parent-compound fissile nucleus. Using appropriate values of V_i, Z_1, and A_1 in Eq. (10), the values of k for different fission products in a particular medium have been obtained. The range values have been taken from Niday[13] for uranium, Alexander and Gazdik[9] for gold, Almodovar *et al.*[17] and Hontzeas and Blok[12] for tungsten, and Smith and Frank[18] for zirconium. In all cases, the precursor fragment mass number A_1' has been obtained with the prompt-neutron emission data of Apalin *et al.*[19] The fragment kinetic energies have been taken from Schmitt *et al.*[20] Table I shows the arithmetic mean values of k obtained with different fission products in the same medium together with

TABLE I. Mean values of k in different heavy media.

Medium	Atomic number (Z_2)	k mean
Zirconium	40	0.976 ± 0.05
Tungsten	74	1.0 ± 0.003
Gold	79	0.980 ± 0.03
Uranium	92	1.06 ± 0.03

the maximum deviation of an individual value from the mean value. Since there are finite errors associated with the measured range values and the kinetic energies, it is possibly justifiable to assume that in heavy solid media and possibly in all solid media, k may be considered as unity. This supports Bohr's[14] original assumption that $Z^{eff} = Z^{1/3}V/V_0$ independent of the medium being penetrated. Hence, for ranges in all heavy media, we have

$$R = \frac{A_1 A_2 (V_i - V_0)}{127.3 Z_2^{1/3}(4.7622 Z_1^{5/9} + Z_1^{1/3})} . \qquad (13)$$

B. Light solid media

In the case of light solid media, the main problem is the summation of the logarithmic terms in Eq. (1). Since the minimum value of Z_1 in the case of fission products is close to 40, the use of the Thomas-Fermi statistical approach is possibly justifiable and Eq. (7) is usable. However, for media like C or Al, Eq. (6) is certainly not applicable. Bohr[14] used Eq. (6) to evaluate the summation terms in Eq. (1). In our analysis we have proceeded with the assumption that in the case of a "light" atom ($Z \leq 40$) one may write

$$n(U_s) = f(Z) U_s / V_0 , \qquad (14)$$

where $f(Z)$ is a function of Z whose form has to be determined. Using Bohr's[14] procedure of replacing the summation terms in Eq. (1) by integrals and obtaining an expression for $dn(U_s)$ from Eq. (14), we get

$$\int_{U_s=0}^{U_s=U_s'} \ln(\eta_s^2[\kappa/\eta_s]^{-1})\, dn(U_s) + \int_{U_s=0}^{U_s=U_s'} \ln(\eta_s^2[\kappa]^{-2})\, dn(U_s) = \frac{2f(Z_2)V}{V_0}\{3[2Z_1^{1/3}]^{-1/3} + [2Z_1^{1/3}]^{-1}\} , \qquad (15)$$

where Z_1 and Z_2 refer to the atomic numbers of the "heavy" ion and the "light" medium, respectively. The upper limits of integration, U_s', correspond to those values of U_s which make the logarithmic terms zero. The use of U_s' thus automatically eliminates consideration of all those orbital electrons of the medium that do not participate in the stopping process at a given velocity V. After substitution of the values of the physical constants and the replacement of the summation terms by Eq. (15), one obtains the following expression from Eq. (1):

$$\frac{dE}{dX} = 1.327 \frac{f(Z_2)}{A_2}(4.7622 Z_1^{5/9} + Z_1^{1/3}) V , \qquad (16)$$

where dE/dX is in units of MeV/(mg/cm^2). Further, by the use of Niday's[13] assumptions, the range-velocity equation becomes

$$R = \frac{A_1 A_2 (V_i - V_0)}{127.3 f(Z_2)(4.7622 Z_1^{5/9} + Z_1^{1/3})} . \qquad (17)$$

In order to evaluate $f(Z_2)$, we have followed the same procedure as in the case of evaluating k in heavy media. Range data for various fission products in a particular medium, along with the corresponding values of V_i and Z_1 [$= Z_p(A_1)$], were put into Eq. (17) to obtain the individual values of $f(Z_2)$ in one medium. Ranges in Al were taken from Alexander and Gazdik,[9] Aras *et al.*,[10] Brown and Oliver,[21] and Nakahara *et al.*[11]; in C from Chinaglia *et al.*[22]; and in Be and Si from Demichelis *et al.*[23] Only one datum on range in Cu is available from Segré and Wiegand[24] for the *average* fission product from the thermal-neutron fission of ^{235}U. Since the fission products in equal and maximum yields have mass and atomic numbers 140, 90 and 54.70, 39.44, respectively, the values of A_1 and and Z_1 for the *average* fission product may be taken as 119.5 and 47.07, respectively. Using similarly an average velocity V_i, $f(Z_2)$ for Cu has been calculated from Eq. (17). The arithmetic mean value of $f(Z_2)$ together with its maximum deviation from an individual value for any particular medium is shown in Table II. We have also included the case of zirconium[18] among those of the light media to approximately locate the dividing line between the "light" and the "heavy" media. From the plot of $f(Z_2)$ against $Z_2^{2/3}$ shown in Fig. 1, it is clear that $f(Z_2)$ may be represented very accurately by

$$f(Z_2) = 0.28 Z_2^{2/3} . \qquad (18)$$

Further, from $n(U_s) = Z_2^{1/3} U_s / V_0 = 0.28 Z_2^{2/3} U_s / V_0$, we find that $Z_2 = 45.5$ constitutes the border line between the light and heavy media. For "light" atoms, one may write

$$n(U_s) = 0.28 Z^{2/3} U_s / V_0 , \qquad (19)$$

$$Z^{eff} = 0.28 Z^{2/3} V / V_0 . \qquad (20)$$

TABLE II. Mean $f(Z_2)$ values for different light media.

Medium	Atomic number (Z_2)	$f(Z_2)$
Hydrogen	1	0.275 ± 0.012
Helium	2	0.347 ± 0.009
Beryllium	4	0.710 ± 0.03
Carbon	6	0.879 ± 0.07
Nitrogen	7	1.080 ± 0.02
Neon	10	1.166 ± 0.02
Aluminium	13	1.582 ± 0.02
Silicon	14	1.597 ± 0.02
Argon	18	2.182 ± 0.04
Copper	29	2.64
Zirconium	40	3.23 ± 0.33

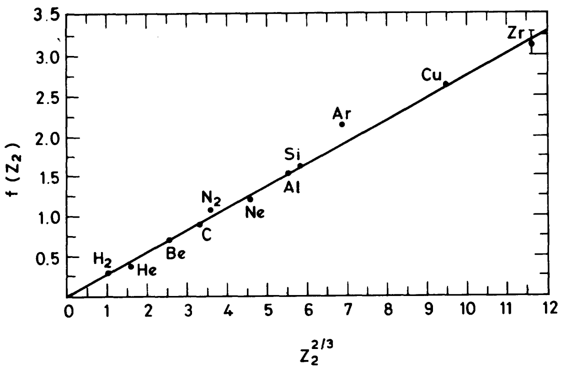

FIG. 1. Plot of $f(Z_2)$ vs $Z_2^{2/3}$ for various media.

Equations (16) and (17) may now be written in the form of comprehensive stopping-power and range-velocity equations in solid media:

$$\frac{dE}{dX} = 1.327\frac{f(Z_2)}{A_2}\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}\, V\,, \quad (21)$$

$$R = \frac{A_1 A_2 (V_i - V_0)}{127.3 f(Z_2)\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}}\,, \quad (22)$$

where A_1, Z_1 and A_2, Z_2 are the mass and atomic numbers of the ion and the medium, respectively, V_i is the initial velocity of the ion, and $V_0 = e^2/\hbar$. V, V_i, and V_0 are to be expressed in units of 10^8 cm/sec. $f(Z_1)$ or $f(Z_2)$ or in general $f(Z)$ is given by

$$f(Z) = 0.28 Z^{2/3} \quad \text{for } Z < 45.5\,, \quad (23)$$

$$f(Z) = Z^{1/3} \quad \text{for } Z > 45.5\,. \quad (24)$$

C. Light gaseous media

Range data for products from the fission of ^{233}U in the light gases H_2, He, N_2, Ne, and Ar have been taken from Petrzhak *et al.*[25] and were subjected to the analysis outlined for the light solid media. The initial velocity V_i and the precursor fragment mass A_1' have been calculated with the data from Pleasonton[26] and Apalin *et al.*[19] The arithmetic mean values of $f(Z_2)$ thus obtained are shown in Fig. 1. Except in the case of Ar, $f(Z_2)$ for these media are well given by Eq. (18). In the case of hydrogen and helium this agreement is fortuitous but fortunate in the sense that it would be valuable in calculating ranges in compound media like Mylar, which contain a number of hydrogen atoms, as is shown in Sec. II E.

D. Ranges of accelerated heavy ions

Although there should be no difference between a fission product and an accelerated heavy ion as far as Eq. (22) is concerned, it would be interesting to verify this. Bridwell and Moak[27] have reported the measured ranges of ^{127}I and ^{79}Br accelerated to different energies in the form of $R(E) - R(10)$, where $R(E)$ is the range corresponding to energy E and $R(10)$ is the range corresponding to $E = 10$ MeV. Figure 2 shows these values along with our calculated ones. In all cases, the agreement seems to be excellent.

E. Compound media

The stopping-power equation given by Eq. (21) may be written

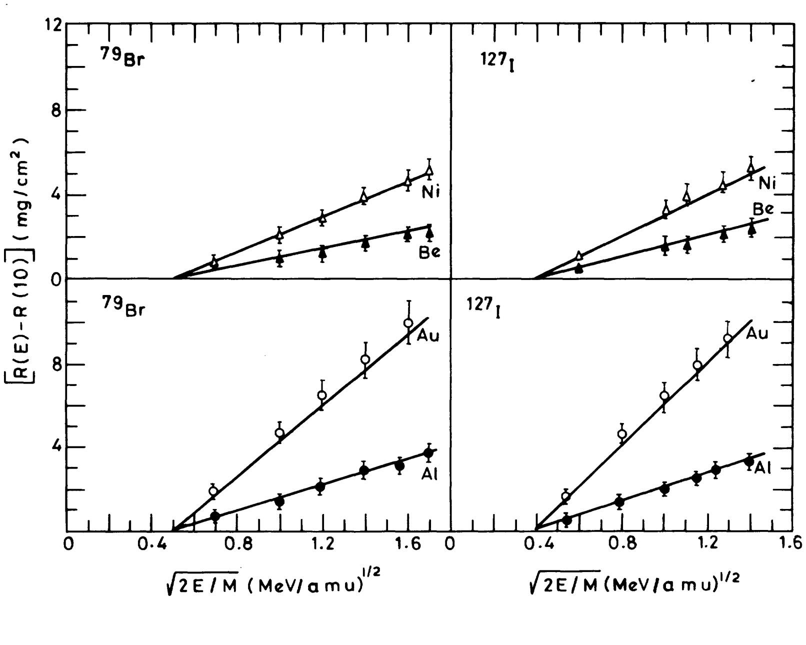

FIG. 2. Plot of $R(E)$-$R(10)$ against $(2E/M)^{1/2}$ for ^{79}Br and ^{127}I ions in Be, Al, Ni, and Au. $R(E)$ and $R(10)$ are the ranges corresponding to initial energies E MeV and 10 MeV, respectively. ⌶ and ⌶, experimental values from Bridwell and Moak, Ref. 23; —, calculated values.

TABLE III. Calculated and experimental ranges in mg/cm^2 in compound media.

Fission product	Collodion calc.	Collodion exp.	Mylar calc.	Mylar exp.	Al calc.	Al exp.	CsI calc.	CsI exp.
Average fission product $A_1=119.5$ $Z_1=47.07$ $V_i=11.75$	2.609	2.6[a]	...	...	3.65	3.70[a]	...	...
Median light $A_1=99$ $Z_1=39.44$	...	...	2.756	2.57 ± 0.05[b]	...	...	7.84	7.6[c]
Median heavy $A_1=140$ $Z_1=54.7$	...	...	2.20	2.11 ± 0.04[b]	...	...	6.08	6.6[c]

[a]Segrè and Wiegand (Ref. 24). [b]Cumming and Crespo (Ref. 4). [c]Suzor (Ref. 28).

$$\frac{dE}{dX}=\alpha n f(Z_2)\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}\,V\,, \quad (25)$$

where α is a constant and n is the number of atoms of the medium per unit volume. If a compound is made up of X_i number of atoms of the ith atomic species per molecule then the actual stopping power would be the sum of the stopping powers due to all the individual atoms in each molecule. Hence,

$$\frac{dE}{dX}=\frac{\alpha\rho N}{A_2}\left(\sum_i X_i f(Z_i)\right)\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}\,V, \quad (26)$$

where ρ is the density of the medium, N is the Avogadro number, A_2 is the mass number of the molecule $[A_2=\sum_i X_i A_i]$; Z_i and A_i are the atomic number and mass number of the ith atomic species. Hence, for a compound medium, the stopping-power and range-velocity equations are

$$\frac{dE}{dX}=1.327\frac{f(Z_2)}{A_2}\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}\,V\,, \quad (27)$$

$$R=\frac{A_1A_2(V_i-V_0)}{127.3 f(Z_2)\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}}\,, \quad (28)$$

where $f(Z_2)=\sum_i X_i f(Z_i)$ and $A_2=\sum_i X_i A_i$ and $f(Z)$ is defined by Eqs. (23) and (24).

The calculation of ranges in Mylar ($C_{10}H_8O_4$), Collodion ($C_{12}H_{17}O_{16}N_3$) and UF_4 would involve some assumption regarding $f(Z_i)$. For C and U as also for H_2, $f(Z_i)$ is known. Since N_2 also conforms to $f(Z)=0.28Z^{2/3}$, we have made the reasonable assumption that this would also hold in the cases of O_2 and F_2. Using the appropriate mass and atomic numbers and the initial velocity for the *average* fission product from the thermal-neutron fission of ^{235}U as indicated in Sec. II B, the range in Collodion has been calculated by means of Eq. (28) and

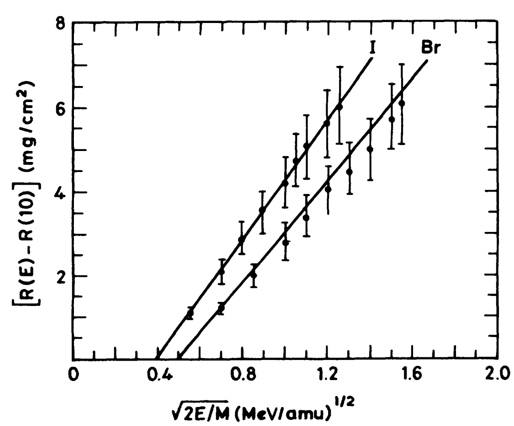

FIG. 3. Plot of $R(E)$-$R(10)$ vs $(2E/M)^{1/2}$ for ^{79}Br and ^{127}I ions in UF_4. $R(E)$ and $R(10)$ are the ranges corresponding to initial energies E MeV and 10 MeV, respectively. The symbols with the error bars show the experimental values, while the straight lines represent the calculated values (——).

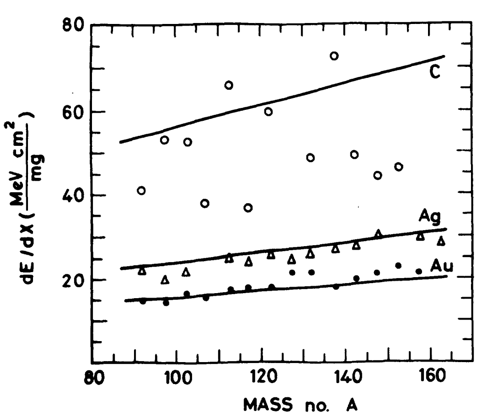

FIG. 4. Plot of the stopping powers (dE/dX) vs the mass number (A) of fission products in C, Ag, and Au. The experimental values are from Bridwell and Walters, Ref. 7. The various symbols show the experimental values, while the straight lines represent the calculated values. The experimental errors are not shown in the figure.

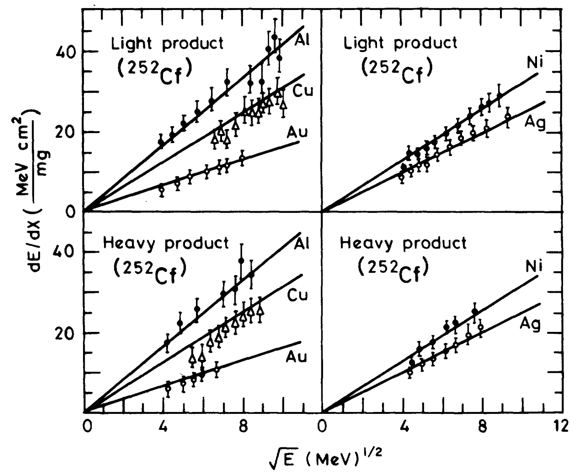

FIG. 5. Plot of the stopping powers (dE/dX) vs $\sqrt{E}$ for the median heavy and median light fission products from the spontaneous fission of ^{252}Cf in Al, Ni, Cu, Ag, and Au. The symbols with the error bars show the experimental values, while the straight lines represent the calculated values. The experimental values in Al, Ni, Ag, and Au are from Kahn and Forgue (Ref. 6) and in Cu from Müller and Gönnenwein (Ref. 8).

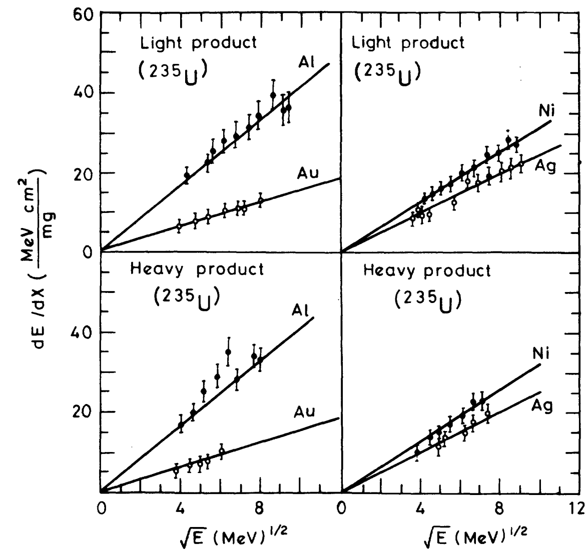

FIG. 6. Plot of the stopping powers (dE/dX) vs $\sqrt{E}$ for the median heavy and median light fission products from the thermal neutron fission of ^{235}U in Al, Ni, Ag, and Au. The symbols with the error bars show the experimental values taken from Kahn and Forgue (Ref. 6) and the straight lines represent the calculated values.

is shown in Table III along with the corresponding experimental value of Segrê and Wiegand.[24] The calculated and experimental[24] ranges for the *average* fission product in Al are also shown in the same table as an additional verification of the correctness of our procedure for calculating A_1, Z_1, and V_i of the *average* fission product. Table III also lists the calculated values of the ranges of fission products of mass numbers 99 and 140 in CsI and in Mylar along with the corresponding experimental values of Suzor[28] and of Cumming and Crespo.[4] In the case of UF_4, Fig. 3 shows the plot of the calculated values of $R(E) - R(10)$ against $(2E/M)^{1/2}$ along with the corresponding experimental values,[27] where $R(E)$ and $R(10)$ correspond to ranges at initial ion energy of E and 10 MeV, respectively, and M is the mass number of the heavy ion.

F. Simpler form of stopping-power equation

A simpler, but somewhat less precise, stopping-power equation is obtainable from Bloch's equation[29] which, in the classical limit, i.e., $Z^{eff}e^2/\hbar V \gg 1$ or $f(Z_1) \gg 1$, reduces[30] to Bohr's[31] earlier equation:

$$\frac{dE}{dX} = \frac{4\pi e^4 (Z^{eff})^2}{mV^2} n Z_2 \ln \frac{1.123 m V^3}{Z^{eff} e^2 \bar{\omega}} , \qquad (29)$$

where $\bar{\omega}$ represents the geometric mean value of the cyclic frequency of the electron in an atom of atomic number Z_2 averaged over *all* the orbital electrons. Because of the nonparticipation of many of the orbital electrons of a heavy medium in interactions with partially stripped heavy ions and because of a continuous increase in the number of such nonparticipating electrons with decreasing ion velocity, it is not meaningful to use Eq. (29) with $\bar{\omega}$ in the logarithmic term. Instead, one may retain the summation sign:

$$\frac{dE}{dX} = \frac{4\pi e^4 (Z^{eff})^2 n}{mV^2} \sum_s \ln \frac{1.123 m V^3}{Z^{eff} e^2 \omega_s} , \qquad (30)$$

where the summation has to be carried over all the

TABLE IV. Stopping power of the median light fission product from the spontaneous fission of ^{252}Cf in U–Pd alloy. The experimental value is from Kahn and Forgue (Ref. 6).

A_1	Z_1	t (mg/cm²) (foil thickness)	E_i (MeV)	E_e (MeV)	ΔE (MeV)	$\frac{1}{2}(E_i+E_e)$ (MeV)	$\frac{\Delta E}{\Delta X}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right)$ (exp.)	$\frac{\Delta E}{\Delta X}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right)$ (calc.)
106.44	42.55	0.73	34.2	25.7	8.5	29.9	11.64	12.27

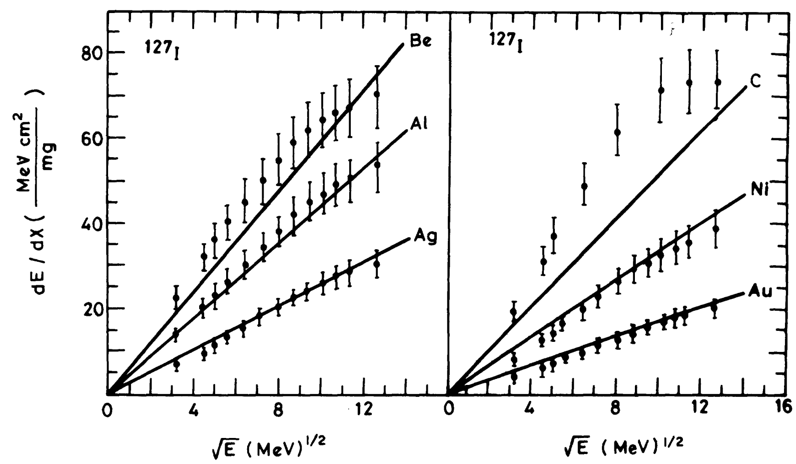

FIG. 7. Plot of the stopping powers (dE/dX) of ^{127}I ions vs $\sqrt{E}$ in Be, C, Al, Ni, Ag, and Au. The experimental values from Moak and Brown (Ref. 32) are shown by the symbols with the error bars, while the straight lines represent the calculated values.

s orbital electrons of the medium and ω_s is the cyclic frequency of the sth electron. The logarithmic term may be simplified by putting $Z^{\text{eff}} = f(Z_1)\, V/V_0$, $I_s = \hbar\omega_s = \frac{1}{2} m U_s^2$, and $V_0 = e^2/\hbar$, where e and m are the electronic charge and mass, I_s is the ionization potential of the sth electron with orbital velocity U_s and n is the number of atoms of the medium per unit volume. Following Bohr's[14] procedure, the summation may be replaced by integration, with the upper limit $U_s = U_s'$ when U_s' makes the logarithmic term zero. Thus,

$$\sum_s \ln \frac{2(1.123)V^2}{f(Z_1)U_s^2} = \int_{U_s=0}^{U_s=U_s'} \left(\ln \frac{2.246V^2}{f(Z_1)U_s^2} \right) dn(U_s), \tag{31}$$

where $dn(U_s) = f(Z_2) dU_s / V_0$ from Eq. (14). Equation (30) now becomes

$$\frac{dE}{dX} = \frac{8(2.246)^{1/2}\pi e^4 n}{m V_0^3} f(Z_2)[f(Z_1)]^{3/2}\, V, \tag{32}$$

which, in turn, leads to the following range-velocity equation:

$$R = \frac{A_1 A_2 (V_i - V_0)}{127.3 f(Z_2)\{6[f(Z_1)]^{3/2}\}}, \tag{33}$$

in which the symbols are identical with those in Eq. (22). Since Eq. (22) predicts heavy ion ranges with accuracy a comparison between Eqs. (22) and (33) shows that the latter would be useful if

$$\frac{4.7622[f(Z_1)]^{5/3} + f(Z_1)}{6[f(Z_1)]^{3/2}} \approx 1\,. \tag{34}$$

Actual computation of the above ratio shows that for all ions with $Z_1 < 45.5$ it remains very close to 1.05 while for $Z_1 > 45.5$ it varies from 1.055 to 1.07. Thus adjustment of Eqs. (32) and (33) by a factor of 1.06 leads to the following simple equations, which are about $\pm 1\%$ less precise than Eqs. (21) and (22):

$$\frac{dE}{dX} = 8.4 \frac{f(Z_2)}{A_2} [f(Z_1)]^{3/2}\, V, \tag{35}$$

$$R = \frac{A_1 A_2 (V_i - V_0)}{809 f(Z_2)[f(Z_1)]^{3/2}}\,. \tag{36}$$

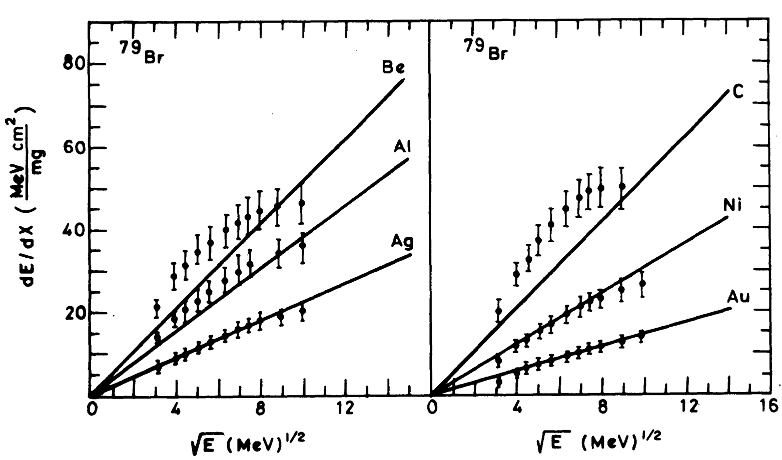

FIG. 8. Plot of the stopping powers (dE/dX) of ^{79}Br ions vs $\sqrt{E}$ in Be, C, Al, Ni, Ag, and Au. The experimental values from Moak and Brown (Ref. 32) are shown by the symbols with the error bars, while the straight lines represent the calculated values.

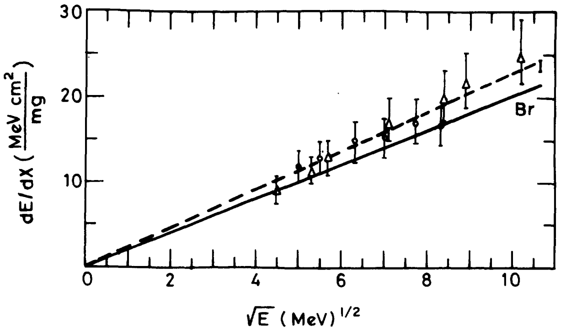

FIG. 9. Plot of the stopping powers (dE/dX) of ^{79}Br and ^{127}I ions vs $\sqrt{E}$ in UF_4. The experimental points from Bridwell and Moak (Ref. 27) are shown by the symbols for ^{127}I and for ^{79}Br. The calculated values are shown by the straight lines (—— for ^{79}Br and --- for ^{127}I).

Further, Eqs. (21) and (22) may be somewhat simplified by noting that

$$\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}\approx 20+0.5Z_1 \quad \text{for } Z_1>45.5, \tag{37}$$

$$\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}\approx 0.938Z_1 \quad \text{for } Z_1<45.5, \tag{38}$$

with the introduction of ~1% error.

III. STOPPING POWERS

A. Fission fragments

Figure 4 shows the stopping powers of different fission products from the spontaneous fission of ^{252}Cf calculated with Eq. (21) along with the corresponding experimental values of Bridwell and Walters[7] (using thick foils of Ag and Au). All calculated and experimental values are at $V=1.38\times10^9$ cm/sec in the case of C, $V=1.35\times10^9$ cm/sec in the case of Ag, and $V=1.32\times10^9$ cm/sec in the case of Au. The experimental errors, which are of the order of 25–30% in the cases of Ag and Au and is somewhat higher in the case of C, are not shown in the figure. Kahn and Forgue[6] have measured the energy degradation of the median light and the median heavy fission products from the spontaneous fission of ^{252}Cf and the thermal-neutron fission of ^{235}U, using foils of different metals of varying thicknesses. Using these authors'[6] values for the mass and atomic numbers of the fission products, we have obtained the experimental stopping powers at different energies in the following manner. From the incident and emergent energies E_i and E_e of the fission products before and after passing through a foil of thickness t, the experimental stopping power is obtained as $(E_i-E_e)/t$. The corresponding calculated values have been obtained from Eq. (21) using a value of V corresponding to the mean energy $\frac{1}{2}(E_i+E_e)$. Figures 5 and 6 show these values in Al, Ni, Ag, and Au. In a similar manner the energy-degradation data of Müller and Gönnenwein[8] have been treated to obtain the experimental and

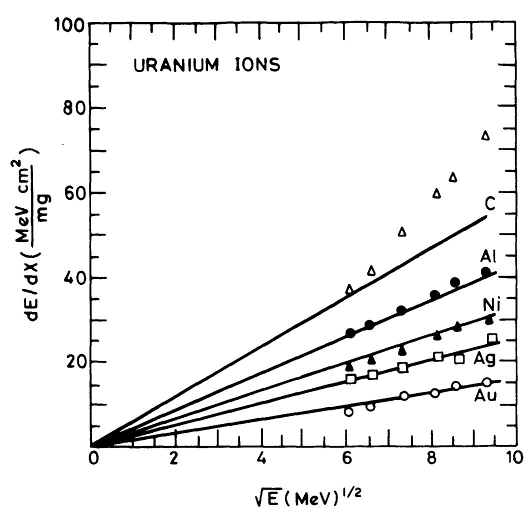

FIG. 10. Plot of the stopping powers (dE/dX) of ^{238}U ions vs $\sqrt{E}$ in C, Al, Ni, Ag, and Au. The symbols show the experimental values from Brown and Moak (Ref. 5) and the straight lines show the corresponding calculated values. The experimental errors (±10%) are not shown.

TABLE V. Directly measured equilibrium charges ($\bar{q}_e$) in gaseous media for various ions along with the calculated values of the effective charge Z^{eff} [$=f(Z)V/V_0$] at different energies.

Ions	Velocity (10^8 cm/sec)	He	N_2	Ar	Kr	Z^{eff}(calc.)
^{7}Li[a]	4.01	1.12	1.25	1.27	1.14	1.07
	5.69	1.52	1.73	1.82	1.66	1.52
	8.03	2.17	...	...	2.28	2.14
	8.06	...	2.38	2.50	...	2.15
	11.64	2.81	2.80	2.77	2.64	3.11
^{11}B[a]	2.75	1.13	1.23	1.20	1.05	1.03
	3.79	1.35	1.56	1.57	1.45	1.42
	7.00	2.55	...	2.85	2.80	2.63
	7.87	2.85	3.03	3.10	3.02	2.95
^{14}N[a]	2.60	1.2	1.3	1.3	1.0	1.22
	4.11	1.6	2.0	1.9	1.6	1.93
	5.69	2.4	2.7	2.8	2.6	2.67
	7.02	2.9	3.5	3.6	3.6	3.29
	7.91	3.5	3.8	4.2	3.8	3.72
	9.15	4.0	4.4	4.5	4.4	4.30
	11.64	4.8	4.9	4.9	4.6	5.47
^{16}O[b]	6.29	...	...	3.5	...	3.23
	8.55	...	...	4.8	...	4.39
	10.21	...	...	5.2	...	5.24
^{20}Ne	2.65[a]	1.6	1.7	1.7	1.5	1.58
	4.04[a]	2.2	2.2	2.5	2.2	2.41
	5.45[b]	...	...	3.3	...	3.25
	5.52[a]	2.7	3.5	3.5	3.3	3.29
	6.33[a]	3.4	4.0	...	...	3.77
	8.25[b]	...	...	4.9	...	4.92
	9.94[b]	...	...	6.0	...	5.93

[a]Nikolaev *et al.* (Ref. 36).

[b]Hubbard and Lauer (Ref. 37).

TABLE VI. Directly measured equilibrium charges of ^{27}Al ions in $N_2(\bar{q}_e)$ along with the calculated values of Z^{eff} [$=f(Z)V/V_0$] at different energies.

Energy (MeV)	0.80	1.14	1.40	1.72	2.09	2.35	2.65	3.03	3.35	3.72	4.00
$\bar{q}_e$ [a]	1.80	2.06	2.29	2.52	2.78	2.91	3.08	3.32	3.50	3.67	3.85
Z^{eff} (calc.)	1.69	2.02	2.24	2.48	2.74	2.90	3.08	3.29	3.46	3.65	3.78

[a] G. Ryding *et al.* (Ref. 38).

calculated stopping powers for the median light and median heavy fission products from the spontaneous fission of ^{252}Cf. Figure 5 includes these results only in the case of Cu; for the other metallic media the agreement between the experimental and calculated values are excellent but are not shown in the figure. Only one experimental datum on dE/dX for the median light fission product from ^{252}Cf could be obtained from the data of Kahn and Forgue[6] in the case of U-Pd alloy, which is shown in Table IV. The alloy containing 20-wt.% U and 80-wt.% Pd has the molecular formula $UPd_{8.88}$ and using Eq. (27) the stopping power has been calculated and is shown in Table IV.

B. Accelerated heavy ions

Figures 7 and 8 show the experimental values of the stopping powers of accelerated ^{127}I and ^{79}Br ions as measured by Moak and Brown[32] at various energies along with the corresponding calculated values with Eq. (21) in Be, C, Al, Ni, Ag, and Au. In Fig. 9 we have shown the values of the stopping powers of ^{127}I and ^{79}Br ions in UF_4 at various energies as calculated with Eq. (27) along with the corresponding measured values from Bridwell and Moak.[27] Figure 10 shows the experimental values of the stopping powers of accelerated ^{238}U ion in C, Al, Ni, Ag, and Au from Brown and Moak[5] and the corresponding values calculated with Eq. (21). The experimental values have ±10% error which are not shown in the figure.

IV. DISCUSSION

A. Limits of applicability

Equations (21) and (22) are based on the integrations, represented by Eq. (15), performed with $\kappa > 1$. For partially stripped ions, this imposes the condition

$$2f(Z_1) > 1 \tag{39}$$

for the applicability of Eqs. (21) and (22). On the other hand, Eqs. (35) and (36), based on purely classical treatment,[13] require $\kappa \gg 1$ or $2f(Z_1) \gg 1$ for their validity. Only in the case of ^{238}U ion for which $2f(Z_1) \approx 9$, this condition may be considered as fulfilled. In practice, however, the calculated ranges of the light fission products as also the stopping powers of ^{79}Br ion agree with the corresponding experimental data and we have assumed $Z_1 = 30$ as the approximate lower limit for the use of Eqs. (35) and (36). In the case of the media, the lower limits of Z_2 are obtainable from Eqs. (15) and (31). If U_s' is the value of U_s which makes the logarithmic term zero and if the orbital velocity of the K-shell electron in the atom of the medium is U_K, then the result represented by Eq. (15) becomes physically meaningless if $U_s' > U_K$. Use of this criterion gives the condition

$$V \leq U_K\,[\tfrac{1}{4}f(Z_1)]^{1/3} \tag{40}$$

as essential for the validity of Eqs. (21) and (22). Similarly, Eqs. (35) and (36) would be valid if $V \leq U_K[f(Z_1)/2.246]^{1/2}$, which follows from Eq. (31). Equation (40) is barely satisfied in the case of fission products in carbon as the medium. The fit of our range-velocity equation in the case of Be may be either fortuitous or due to the relatively small contribution of the higher values of U_s to the total integral. In the case of hydrogen and helium the agreement appears to be purely fortuitous. In the case of heavy media, U_K may greatly exceed the ion velocity at which the ion becomes completely stripped of its orbital electrons. Because of the limitation[15] that Eq. (9) would not be valid if $Z^{eff} > \frac{1}{2}Z_1$, the permissible upper limit for the ion velocity is

$$V \leq \tfrac{1}{2}Z_1 V_0[f(Z_1)]^{-1} . \tag{41}$$

B. Deviation from Niday's approach

Niday,[13] as a first approximation, integrated the right-hand side of Eq. (9) between the limits V_i and

TABLE VII. Directly measured equilibrium charges of ^{23}Na, ^{31}P, and ^{40}Ar ions in $N_2(\bar{q}_e)$ along with the calculated values of Z^{eff} [$=f(Z)V/V_0$].

Ions	Velocity (10^8 cm/sec)	$\bar{q}_e$	Z^{eff} (calc.)
^{23}Na [a]	4.5	3.1	2.86
^{31}P [a]	4.05	3.0	3.16
^{40}Ar [a]	4.05	3.2	3.57

[a] Nikolaev *et al.* (Ref. 36).

TABLE VIII. Directly measured[a] equilibrium charges of ^{32}S ions in air ($\bar{q}_g$) and carbon ($\bar{q}_s$) along with the calculated values of Z^{eff} [$=f(Z)V/V_0$] at different energies.

Energy (MeV)	$\bar{q}_g$ (air)	$\bar{q}_s$ (carbon)	Z^{eff} (calc.)
3.70	3.45	5.85	3.84
6.40	4.65	6.85	5.05
8.40	5.16	7.45	5.78
10.00	5.60	7.80	6.31
12.70	6.50	8.45	7.11
17.00	7.50	9.09	8.23
20.00	8.06	9.45	8.92
22.70	8.50	9.71	9.51
23.80	8.65	9.87	9.73
26.40	8.90	10.14	10.26
27.20	8.80	10.25	10.41
29.90	9.28	10.46	10.91
32.60	9.58	10.66	11.40

[a]Betz *et al.* (Ref. 39).

V_0 on the assumption that at $V < V_0$ the moving ion undergoes large-angle scatterings and the resultant penetration in the beam direction is negligible. As a refinement to this, Niday[13] took the lower limit as V_c instead of V_0, where V_c is the ion velocity at which the screening radius becomes equal to the collision diameter. The introduction of V_c in Eq. (10) was done mainly to get the best agreement between the experimental and calculated values of ranges in uranium. We have also carried out calculations with V_c as the lower limit and obtained values of k varying from 1.07 to 1.12 as one goes from the heavy to the light media (i.e., from uranium to carbon and beryllium). Although this would cause a better agreement between the calculated and experimental values of the stopping powers in C and Be, it would also cause the excellent agreements in the cases of Al, Cu, Ag, Ni, and Au to disappear. Since the spirit of this work is semiempirical parametrization, we have kept V_0 as the lower velocity limit for the sake of simplicity as well as to optimize the agreement between the calculated and the experimental values of both ranges and stopping powers.

C. Stopping powers in C and Be

The calculated stopping powers in carbon and beryllium have been consistently lower than the corresponding experimental values. On the other hand, the ranges of fission products in both C and Be and the ranges of accelerated heavy ions in Be, calculated with Eq. (22), are in excellent agreement with the corresponding experimental values. Without attempting to resolve this inconsistency we wish to point out that both Bridwell and Moak[27] and Booth and Grant[33] have mentioned that oxidation during thin-film deposition by vacuum evaporation in the cases of both Be and C and the tenacious retention of moisture in the case of C may lead to unassignable errors in the measured stopping powers.

D. Effective charge

The excellent agreement of the calculated values of the stopping powers of various ions (^{79}Br, ^{127}I, all fission products, and ^{238}U) with the corresponding experimental values in different solid media, except C and Be, seems to provide very strong, although indirect, evidence in favor of our assumption that $Z^{eff} = f(Z_1)\, V/V_0$. The fact that the light gases, as shown in Fig. 1, also obey Eq. (18), possibly indicates that the above relation is independent of the physical state of the medium. Betz,[34] in his review work, has agreed with the conclusion of Betz and Grodzins[35] that *inside* a solid medium,

TABLE IX. Directly measured equilibrium charges of ^{35}Cl and ^{79}Br ions in gases ($\bar{q}_g$) and carbon ($\bar{q}_s$) along with the calculated values of Z^{eff} [$=f(Z)V/V_0$] at different energies.

Ions	Energy (MeV)	$\bar{q}_g$ in various gaseous media: H_2	He	N_2	O_2	Ar	Kr	$\bar{q}_s$ C	Z^{eff} (calc.)
^{35}Cl[a]	2.00	2.15	2.36	2.76	2.78	2.68	2.33	4.61	2.81
	4.00	3.26	3.13	3.86	3.74	3.87	3.54	6.06	3.97
	6.00	4.20	3.87	4.60	4.46	4.76	4.46	6.69	4.87
	8.00	5.08	4.59	5.34	5.18	5.54	5.56	7.56	5.62
	10.00	5.78	5.21	5.77	5.66	6.01	6.16	8.09	6.28
	12.00	6.41	5.78	6.54	6.33	6.77	6.80	8.53	6.88
^{79}Br[a,b]	2.00	1.46	2.25	2.20	2.42	1.96	1.60	5.74	3.02
	4.00	2.01	2.64	3.10	3.27	2.97	2.45	7.73	4.28
	6.00	2.71	3.17	4.08	4.06	3.82	3.42	8.95	5.24
	8.00	3.29	3.75	4.85	4.80	4.70	4.23	9.69	6.05
	10.00	3.95	4.32	5.44	5.45	5.43	5.04	10.40	6.77
	12.00	4.70	4.83	6.03	6.09	6.08	5.94	11.20	7.41
	14.00	5.29	5.38	6.44	6.58	6.66	6.55	···	8.01

[a]Wittkower and Ryding (Ref. 40).
[b]Experimental values have uncertainty of ±1 charge unit.

the equilibrium charge is possibly the same as that in a gaseous medium, even though the equilibrium charge of an ion after it emerges from a solid medium ($\bar{q}_s$) may be appreciably greater than the equilibrium charge on emerging from a gaseous medium ($\bar{q}_g$). In Tables V–IX we have shown some experimental values of $\bar{q}_g$ and $\bar{q}_s$ from some authors[36–40] along with our calculated values using Eq. (20). In the cases of the very light ions, the agreement between $\bar{q}_g$ and the calculated Z^{eff} is surprisingly good. In this connection we wish to make some comments regarding the procedure followed by some authors[4,5,33,41,42] to obtain an estimate of Z^{eff} from their measured stopping powers by means of Bethe's[2] equation, as shown below:

$$\frac{dE}{dX}=\frac{4\pi e^4(Z^{eff})^2 n}{mV^2}Z_2\ln\frac{2mV^2}{I}, \qquad (42)$$

where I stands for the geometric mean of the ionization potential of all the orbital electrons in the medium and all the other symbols have been defined earlier. Assuming that the logarithmic term is solely velocity dependent, for a given medium, these authors have estimated Z^{eff} on the basis

$$\frac{(Z^{eff})^2_{HI}}{(Z^{eff})^2_p}=\frac{(dE/dX)_{HI}}{(dE/dX)_p}, \qquad (43)$$

where the subscripts HI and p refer to the heavy ion and the proton, respectively. Bohr[14] pointed out that Eq. (42) is applicable if $2Z^{eff}e^2/\hbar V \ll 1$ or $2f(Z_1)\ll 1$, a condition which is not always fulfilled in the case of partially stripped heavy ions. On the other hand, if $2f(Z_1)\gg 1$, which is marginally valid for most heavy ions, one may use for stopping powers Eq. (32), which is derived from Eq. (30). Since Eq. (32) predicts stopping powers that are in fair agreement with the experimental values, a comparison between Eqs. (42) and (30) shows that the assumption of the logarithmic term being simply velocity dependent may not be strictly valid. The values of Z^{eff} obtained with Eq. (43) may, therefore, be somewhat different from the actual values inside a solid medium.

IV. SUMMARY

A. Range-velocity equation

In all solid media, the following range-velocity equations [our Eqs. (22) and (36), respectively] predict ranges with an average deviation of ~5% from the experimental ranges:

$$R=\frac{A_1A_2(V_i-V_0)}{127.3f(Z_2)\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}},$$

$$R=\frac{A_1A_2(V_i-V_0)}{809f(Z_2)[f(Z_1)]^{3/2}},$$

where A_1, A_2 and Z_1, Z_2 are the mass numbers and the atomic numbers of the ion and the medium, respectively. V_i is the initial velocity of the ion and $V_0=e^2/\hbar$ (=2.18×10^8 cm/sec). Both V_i and V_0 are to be expressed in units of 10^8 cm/sec. $f(Z)$ is given by $f(Z)=0.28Z^{2/3}$ for $Z<45.5$ and $f(Z)=Z^{1/3}$ for $Z>45.5$.

B. Stopping-power equation

From Eqs. (21) and (35) we found that

$$\frac{dE}{dX}=1.327\frac{f(Z_2)}{A_2}\{4.7622[f(Z_1)]^{5/3}+f(Z_1)\}V,$$

$$\frac{dE}{dX}=8.4\frac{f(Z_2)}{A_2}[f(Z_1)]^{3/2}V,$$

where V is the velocity in units of 10^8 cm/sec at which the stopping power is to be calculated. The other symbols have already been defined.

C. Compound media

The equations given above are applicable in the case of compound media if A_2 and $f(Z_2)$ are replaced by $\sum_i X_iA_i$ and $\sum_i X_i f(Z_i)$, where the compound is made up of X_i atoms of the ith atomic species having mass number and atomic number A_i and Z_i, respectively.

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