Effects of Fermi-surface anisotropy on cyclotron waves in metals*

D. R. Fredkin and R. Freedman†
Physics Department, University of California, San Diego, La Jolla, California 92037
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We have shown that the spread in cyclotron frequencies of electrons on different orbits of a nonspherical closed Fermi surface leads to a collisionless damping of cyclotron waves for a range of wave numbers that depends on the geometry of the surface, and also to a shift, from the isotropic limit, in the onset of the waves. Explicit calculation of the dispersion relation, in the long-wavelength limit and in the absence of interactions, is performed for a metal whose Fermi surface has a small cubic distortion (as, for example, the heavy alkali metals, Cs and Rb). We find that Fermi-surface anisotropy produces effects similar in both magnitude and quality to those caused by Fermi-liquid interactions in a metal with an essentially spherical Fermi surface (as, for example, Na and K).

I. INTRODUCTION

The recent application of the Landau-Silin[1] phenomenological theory of Fermi liquids to conduction electrons in metals has resulted in a fruitful confrontation between theory and experiment. From the spin waves[2] one can obtain from experiment the first two Legendre coefficients of the spin-dependent part of the Landau correlation function for the alkali metals sodium and potassium. More recently,[3] it has been discovered that electron correlations in g-anisotropic metals may observably affect the conduction-electron-spin resonance which allows one to estimate a many-body parameter B (similar to B_0 for an isotropic system) as well as the rms spread in g values over the Fermi surface for metals in which spin waves have not yet been observed (e.g., aluminum, copper and silver). The above phenomena yield information about the electron-electron exchange interaction in metals. There is yet another phenomena which is sensitive to the direct electron-electron interaction in metals. These are the cyclotron waves (also called high-frequency or magnetoplasma waves) which propagate in the presence of a static dc field H_0 (taken to be along the z axis) oriented in the plane of the sample surface.[4] For simple metals these waves propagate in the vicinity of the fundamental Azbel'-Kaner cyclotron-resonance frequency and its harmonics so that $\omega \sim n\omega_c$ ($n = 1, 2, \ldots$), where ω is the frequency of the applied rf field and $\omega_c = eH_0/m^*c$ (m^* is the effective mass) is the cyclotron frequency. Although the existence of these waves is well understood on the basis of the free-electron theory it has been found that in the long-wavelength limit $qR \ll 1$ ($R = V_F/\omega$, V_F is the Fermi velocity and q is the wave vector of the rf field) electron correlations manifest themselves and modify the dispersion relations for these waves.[5] These results are valid for the simple metals sodium and potassium whose Fermi surfaces are known from de Haas–van Alphen studies to be very nearly spherical.[6] For these metals it is found that for the so-called "ordinary wave" in the collisionless limit $\omega\tau = \infty$ (τ is a measure of the mean time between electron-impurity collisions) that the onset of the wave occurs at $\omega = n\omega_c(1 + A_{n+1})$ instead of at $\omega = n\omega_c$, as predicted by free electron theory. Here the Landau parameters $\{A_n\}$ are the Legendre coefficients of the spin-independent part of the Landau correlation function. These shifts in the $q \to 0$ intercepts of the dispersion curves offer the possibility of determining from experiment the parameters $\{A_n\}$ for $n \geq 2$.

It is of obvious interest to know to what extent the above results for cyclotron waves are still valid if one allows for anisotropy of the Fermi surface. Ideally one would like to have a theory which takes into account both electron correlations and anisotropy. In this paper we take a more modest approach and neglect correlations altogether. Furthermore, we restrict our calculations to the long-wavelength limit $qR \ll 1$, and we only consider electron trajectories which are closed in k space. The results for cyclotron waves cited in the preceding paragraph are valid for the bulk metal. We also concern ourselves only with the response of the electrons in the bulk and ignore completely such interesting questions as the coupling of the incident rf fields to the metal surface and the intensity of the transmitted (or reflected) fields. These questions can only be answered by a complete solution of the boundary value problem.

Using Maxwell's equations and assuming a perturbing field of the form $\vec{E} = \vec{E}_0 e^{i(\vec{q}\cdot\vec{r} - \omega t)}$, we find

$$\vec{q} \times (\vec{q} \times \vec{E}_0) + (\omega/c)^2 \overleftrightarrow{\epsilon} \cdot \vec{E}_0 = 0 , \tag{1}$$

where we have introduced the dielectric tensor

$$\epsilon_{\alpha\beta}(\vec{q}, \omega, H_0) = \delta_{\alpha\beta} + (4\pi i/\omega) \times \sigma_{\alpha\beta}(\vec{q}, \omega, H_0) , \tag{2}$$

where $\sigma_{\alpha\beta}(\vec{q}, \omega, H_0)$ is the wave-vector-, frequency-, and field-dependent conductivity tensor. For Eq.

(1) to have a nontrivial solution requires the vanishing of the determinant

$$\begin{vmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy}-(qc/\omega)^2 & 0 \\ 0 & 0 & \epsilon_{zz}-(qc/\omega)^2 \end{vmatrix} = 0 , \qquad (3)$$

where we have taken $\vec{q} \parallel \hat{x}$. We have used the symmetry of the magnetoconductivity tensor to set $\epsilon_{xz} = \epsilon_{yz} = \epsilon_{zx} = \epsilon_{zy} = 0$. (This is correct if the x-y plane is a reflection plane for the point group of the crystal, as, for example, when the dc field is in a $\langle 100 \rangle$ direction of a cubic crystal, and also when we consider a polycrystalline sample with crystallite size small compared to q^{-1}, so that $\sigma_{\alpha\beta}$ is understood to be an average over crystalline orientations.) Clearly, Eq. (3) has two solutions, each of which corresponds to a possible mode of propagation. The mode which is excited in an experiment depends on the polarization of the applied rf field. In this paper we will discuss the "ordinary wave" which is a purely transverse wave whose dispersion relation is given by solutions of

$$(qc/\omega)^2 = 1 + (4\pi i/\omega)\sigma_{zz}(\vec{q}, \omega, H_0) . \qquad (4)$$

The experimental conditions under which cyclotron waves are observed lead to a simplification of the above equation. The term $(4\pi i/\omega)\sigma_{zz} \sim O(\omega_p/\omega)^2 \sim 10^{10}$ (for $\omega \sim 10^{11}$ rad/sec) where $\omega_p = (4\pi ne^2/m^*)^{1/2}$ is the plasma frequency. Therefore we can neglect the first term in Eq. (4) which comes from the displacement current in Maxwell's equations. Furthermore, in the regime where cyclotron waves are observed $(qc/\omega)^2 \sim 10^4$ so that solutions of (4) are given to $O(qc/\omega_p)^2 \sim 10^{-6}$ by the zeros of the conductivity,

$$\sigma_{zz}(\vec{q}, \omega, H_0) = 0 . \qquad (5)$$

In Sec. II of this paper we consider a closed single-sheet Fermi surface and show that a spread in cyclotron frequencies over the Fermi surface leads to a collisionless damping of cyclotron waves for a range of wave numbers corresponding to the spread in frequencies. Mathematically the spread in cyclotron frequencies manifests itself by causing branch cuts in the conductivity. We study the behavior of the dispersion relation near the endpoints of the branch cuts and conclude that the collisionless damping results in a shift, from the isotropic limit, in the onset of the wave. In Sec. III we consider a model Fermi surface which is a sphere with a small cubic distortion (this should be a fair qualitative description of the heavy alkali metals cesium and rubidium[7]). For this model we calculate the dispersion relation for the first harmonic in detail to first order in the distortion. The results of this calculation completely confirm the general results of Sec. II. Moreover, we find that the effects on the dispersion relation of a small amount of Fermi-surface anisotropy simulates both qualitatively and quantitatively the effects of correlations in an isotropic metal.

II. GENERAL THEORY FOR CLOSED ORBITS

We calculate $\sigma_{zz}(\vec{q}, \omega, H_0)$ by solving the linearized Boltzmann transport equation

$$\frac{\partial \delta f}{\partial t} + \vec{V}_k \cdot \frac{\partial \delta f}{\partial \vec{r}} + \frac{e}{c}(\vec{V}_k \times \vec{H}_0) \cdot \frac{\partial \delta f}{\partial \vec{k}} = -\frac{\partial f_0}{\partial \mathcal{E}_k} e\vec{E} \cdot \vec{V}_k - \frac{\delta f}{\tau} , \qquad (6)$$

where $\delta f(\vec{k}, \vec{r}, t) = f(\vec{k}, \vec{r}, t) - f_0(\mathcal{E}_k)$ is the deviation of the electron distribution function from equilibrium [$f_0(\mathcal{E}_k)$ is a Fermi function], and $\vec{V}_k = \nabla_k \mathcal{E}(\vec{k})$ is the electron velocity which for most metallic Fermi surfaces will be highly anisotropic. We have treated collisions[8] in the relaxation time approximation by introducing the phenomenological scattering time τ. Equation (6) is readily solved by a method due to Chambers,[9]

$$\delta f(\vec{k}, \vec{r}, t) = -e\frac{\partial f_0}{\partial \mathcal{E}_k}\int_{-\infty}^{t} dt' \, e^{-(t-t')/\tau} \vec{E}(\vec{r}(t'), t') \cdot \vec{V}_k(t') ,$$

where the integral is over the electron trajectory in phase space. The current density is given by the usual integral over the distribution function

$$\vec{J}(\vec{r}, t) = 2e \int \frac{d^3k}{(2\pi)^3} \vec{V}_k \, \delta f(\vec{k}, \vec{r}, t) ,$$

from which we easily obtain the z component of the current density

$$J_z(\vec{r}, t) = 2e \int_{k_M}^{k_M} dk_z \, m_c \int_0^{2\pi} d\varphi \, V_z(\varphi) \times \int_{-\infty}^{t} dt' [E_0 V_z(t') \times e^{i[qx(t') - \omega t'] - (t-t')/\tau}] , \qquad (7)$$

where $|k_z| \le k_M$ on the Fermi surface and we have taken $\vec{E} = \vec{E}_0 e^{i(qx-\omega t)}$ to be along the z axis and the wave vector $\vec{q}$ to be along the x axis. We have introduced the following standard coordinates[10] $(\mathcal{E}, k_z, \varphi)$, where

(i) $\mathcal{E}$ is the energy,

(ii) k_z is the component of $\vec{k}$ along $\vec{H}_0$, and

(iii) φ is a phase variable specifying the position of an electron along an orbit and is defined by

$$\varphi = (1/m_c) \int dk/V_\perp ,$$

where dk is an element of arc length along the trajectory; $V_\perp$ is the component of $\vec{V}$ perpendicular to $\vec{H}_0$; m_c is the cyclotron mass for the trajectory and is given by

$$m_c = \frac{1}{2\pi}\oint \frac{dk}{V_\perp} = \frac{1}{2\pi}\frac{\partial A(\mathcal{E}, k_z)}{\partial \mathcal{E}}, \tag{8}$$

where $A(\mathcal{E}, k_z)$ is the area in $\vec{k}$ space enclosed by the orbit. In arriving at Eq. (7) we have written the $\vec{k}$ space volume element in terms of these coordinates $d^3k = m_c\, d\mathcal{E}\, dk_z\, d\varphi$ and have used the low-temperature property of the Fermi function $\partial f_0/\partial \mathcal{E} \approx -\delta(\mathcal{E} - \mathcal{E}_F)$ to do the integration over the energy so that all quantities appearing in Eq. (7) are to be evaluated on the Fermi surface. If we make a change of variables in Eq. (7) and use the relation

$$x(t-t') - x(t) = -(1/m_c\omega_c) \times [k_y(t-t') - k_y(t)],$$

which comes from integrating the equation of motion $\dot{\vec{k}} = (e/c)(\vec{V}_k \times \vec{H}_0)$ along the trajectory, then we easily find

$$\sigma_{zz}(\vec{q}, \omega, H_0) = \frac{2e^2}{(2\pi)^3}\int_{-k_M}^{k_M} dk_z \frac{m_c}{\omega_c} \times \int_0^{2\pi} d\varphi\, V_z(\varphi) \int_0^{\infty} d\varphi' \left[V_z(\varphi - \varphi') \times \exp\left(-\frac{iq}{m_c\omega_c}\right)[k_y(\varphi - \varphi') - k_y(\varphi) - \lambda\varphi'] \right], \tag{9}$$

where $\omega_c(k_z) = eH_0/m_c(k_z)c$ is the cyclotron frequency of the orbit and we have defined $\lambda = (1/\omega_c\tau) \times (1 - i\omega\tau)$. For an arbitrary Fermi surface, $k_y(\varphi)$ can be obtained by solving the coupled equations

$$\frac{\partial k_x}{\partial \varphi} = m_c V_y$$

and

$$\frac{\partial k_y}{\partial \varphi} = -m_c V_x .$$

The physics of the problem is contained in $\sigma_{zz} \times (\vec{q}, \omega, H_0)$ and we now focus our attention on this object. For fixed k_z we expand the z component of the velocity in a Fourier series in the variable φ

$$V_z(\varphi) = \sum_{n=-\infty}^{\infty} V_n(k_z)\, e^{in\varphi}, \tag{10}$$

and similarly

$$k_y(\varphi) = \sum_{n=-\infty}^{\infty} K_n(k_z)\, e^{in\varphi}. \tag{11}$$

In the long wavelength limit

$$(q/m_c\omega_c)[k_y(\varphi - \varphi') - k_y(\varphi)] \sim O(qV_F/\omega_c) \ll 1,$$

we can expand the exponential in Eq. (9) and write an equation of the following form:

$$\sigma_{zz}(\vec{q}, \omega, \eta) = \sigma(0)\,[1 - C(\eta)(qR)^2], \tag{12}$$

where $\sigma(0)$ is the field-independent infinite-wavelength conductivity, $C(\eta)$ is a dimensionless function of the variable, $\eta = \omega_{AK}/\omega$, where ω_{AK} is the cyclotron frequency of the electrons on the extremal Azbel'-Kaner orbit. The dimensionless variable η has no real physical significance and is introduced here to provide a correspondence with the experimental situation where the magnitude of H_0 is varied (note that $\eta \propto H_0$). Physically, equation (12) can contain no term linear in qR because the response of the electrons cannot depend on the direction of the incident field.

Using the above results and performing some simple manipulations, we easily find

$$C(\eta)R^2 = -\left(2\Big/\int_{-k_M}^{k_M} dk_z\, m_c V_0^2\right)\sum_{n=1}^{\infty}\int_{-k_M}^{k_M} \frac{dk_z\,(m_c/m_{AK}^2\omega^2)|K_n^2|\,V_0^2}{(m_c^2/n^2m_{AK}^2)(1 + i/\omega\tau)^2 - \eta^2}, \tag{13}$$

where $m_{AK} = eH_0/\omega_{AK}c$ is the Azbel'-Kaner cyclotron mass. We have restricted ourselves to Fermi surfaces for which V_z is a constant of the motion so that in Eq. (10) we have taken $V_n = 0$ for $n \neq 0$. An analytical example of such a surface is

$$\mathcal{E}(\vec{k}) = \sum_{i=1}^{3} \mathcal{F}_i(k_i) \equiv \mathcal{E}_F ,$$

where the $\mathcal{F}_i$ are arbitrary functions of the Cartesian components of $\vec{k}$. More generally, V_z will be a constant of the motion for any surface for which k_z in the energy-momentum relation is not coupled to k_x or k_y. The neglect of V_n for $n \neq 0$ is, perhaps, justified for real crystals when the z axis is a symmetry direction, since, if the z axis is an N-fold rotation axis, $V_n = 0$ unless n is an integer multiple of N.

We can, at this point, make some general statements about cyclotron wave propagation in the collisionless limit $\omega\tau \to \infty$. As k_z varies from $-k_M$ to k_M, $m_c(k_z)/m_{AK}$ varies over some finite range

$$1 \leq m_c/m_{AK} \leq \eta_1$$

(or, perhaps, $\eta_1 \leq m_c/m_{AK} \leq 1$; we consider only the first alternative because no new ideas appear in the other case). We can read off from (13) that $C(\eta)$ has branch cuts whenever

$$1/n < |\eta| < \eta_1/n, \quad n = 1, 2, \ldots$$

and the nth and $(n+1)$th cuts overlap when

$$n > 1/(\eta_1 - 1).$$

$C(\eta)$ is an analytic function of η with no singularities besides the branch cuts just mentioned. From (5) and (11), we have the dispersion relation

$$(qR)^2 = 1/C(\eta) , \tag{14}$$

provided that (14) leads to $|qR| \ll 1$. From (13), we see that we may expect wave propagation for $\eta > \eta_1$, where $C(\eta) > 0$, that Landau- (collisionless) damped wave propagation will occur for $1 < \eta < \eta_1$, and that in general there may be undamped wave propagation for

$$\eta_1/(n+1) < \eta \ll 1/n$$

and Landau-damped wave propagation when

$$1/n < |\eta| < \eta_1/n .$$

The origin of the Landau damping is simply resonance of the wave with a harmonic of the cyclotron motion of electrons on some orbit.

We anticipate some singular behavior of $qR(\eta)$ when η is near a branch point. We shall make some plausible assumptions about the Fermi surface in order to study the behavior of $C(\eta)$ in the neighborhood of $\eta = 1$, η_1. At the left-hand endpoint, $\eta = 1$ and the major contribution to $C(\eta)$ comes from values of $k_z \approx 0$, which, we suppose, is the extremal (Azbel'-Kaner) orbit. As $k_z \to 0$, we assume the following limiting behavior:

$$V_0^2 \propto k_z^2 , \quad K_1^2 \approx \text{constant} ,$$

and

$$m_c(k_z)/m_{AK} = 1 + bk_z^2 ,$$

where b is a positive constant. If we put the above *Ansatz* into Eq. (13) and do the integral, we find

$$C(\eta) = -|C(1)|\,[1 - \beta(1-\eta^2)^{1/2} + \cdots] , \tag{15}$$

for $|\eta - 1| \ll 1$, where β is a positive constant. At the right-hand endpoint of the cut, $\eta = \eta_1$ and the major contribution to $C(\eta)$ comes for $k_z \approx k_M$. As $k_z \to k_M$, we suppose that

$$V_0^2 \approx \text{constant} ,$$

$$K_1^2 \propto (k_M - k_z) ,$$

and

$$m_c(k_z)/m_{AK} \approx \eta_1 - \alpha(k_M - k_z) ,$$

where α is a positive constant. If we put the above *Ansatz* into Eq. (14) and do the integral, we find

$$C(\eta) = C(\eta_1)\,[1 - \gamma(\eta^2 - \eta_1^2) \times \ln(\eta^2 - \eta_1^2) + \cdots] \tag{16}$$

for $|\eta - \eta_1| \ll 1$, where γ is a positive constant. Combining Eq. (14) with (15) and (16), we obtain the complex dispersion relations

$$qR = i\,|C(1)|^{1/2} \times [1 + \tfrac{1}{2}\beta(1-\eta^2)^{1/2} + \cdots] \tag{17}$$

for $|\eta - 1| \ll 1$, and

$$qR = -C(\eta_1)^{-1/2}\,[1 + \tfrac{1}{2}\gamma(\eta^2 - \eta_1^2) \times \ln(\eta^2 - \eta_1^2) + \cdots] \tag{18}$$

for $|\eta - \eta_1| \ll 1$. These equations contain some useful information. Equation (17) tells us that as $\eta \to 1$, from the low-field side of the Azbel'-Kaner cyclotron frequency, qR is pure imaginary so that no wave exists. As the field is increased and η passes through $\eta = 1$ we see that the imaginary part of qR decreases (though remains finite) and that qR develops a small real part which starts off with infinite slope at $\eta = 1$. From Eq. (18) we see that as $\eta \to \eta_1$, from the low-field side, the imaginary part of qR approaches zero. On the other hand, the real part of qR is finite as $\eta \to \eta_1$ and has infinite slope at $\eta = \eta_1$. The effect of the Landau damping is to cause a shift, from $\eta = 1$, in the onset of the wave. This is because a wave is not well defined until $\mathrm{Re}qR \gtrsim \mathrm{Im}qR$ and this occurs somewhere on the high-field side of $\eta = 1$.

III. MODEL FERMI SURFACE

In Sec. II we studied the effects of Fermi-surface anisotropy from a general point of view. Here we consider a particular Fermi surface and calculate the dispersion relation in detail to $O(qR)^2$. Consider the surface

$$\mathcal{E}(\vec{k}) = \frac{k^2}{2m^*} + \frac{\epsilon}{2m^* k_F^2}\sum_{\alpha=1}^{3} k_\alpha^4 \equiv \text{constant} , \tag{19}$$

when m^* is an effective band mass, ϵ is a small positive dimensionless parameter, and $k_F = (3\pi^2 n)^{1/3}$ when n is the density of conduction electrons. The surface in Eq. (19) describes a sphere with a small cubic distortion.

In Appendix A we calculate $\omega_c(k_z)$ for the surface given in Eq. (19) for the dc field $\vec{H}_0$ having an arbitrary orientation with respect to the symmetry axis of the surface. For $\vec{H}_0$ along a $\langle 100\rangle$ symmetry direction we find

$$\omega_c = \omega_c^{(0)}(1 + \tfrac{3}{2}\epsilon\sin^2\theta) , \tag{20}$$

where θ is the usual polar angle in spherical coordinates. Using equations (13) and (20) we easily find

$$C(\eta) = (2\epsilon\eta^2)^{-1}\left[(a^2 - \tfrac{2}{3}) + \tfrac{1}{2}a(a^2-1)\ln\frac{a-1}{a+1}\right] , \tag{21}$$

where

$$a^2 = [\eta^2 - (1 + i/\omega\tau)^2]/3\epsilon\eta^2 ,$$

and in obtaining Eq. (21) we have written $2iK_1 = k_F\sin\theta$, $K_n = 0$ for $n \neq 1$, to within corrections of $O(\epsilon^2)$. In the limit $\omega\tau \to \infty$, $C(\eta)$ has a branch cut for $1 \leq \eta \leq 1 + 3/2\epsilon$ corresponding to the spread in cyclotron frequencies. Combining Equations (5), (12) and (21), we obtain the dispersion relation

$$qR = (2\epsilon\eta^2)^{1/2}\,[(a^2 - \tfrac{2}{3}) + \tfrac{1}{2}a(a^2-1)$$

$$\times \ln(a-1)/(a+1)]^{-1/2} . \qquad (22)$$

We have computed qR as a function of η for various values of ϵ and $\omega\tau$. Some results of these computations are shown in Figs. 1–3. In Fig. 1 we show results for the $\omega\tau \to \infty$ limit. The dotted line is the ImqR for $\epsilon = 0.05$ and represents the collisionless damping discussed in Sec. II. The ReqR is also shown in Fig. 1 and has the behavior discussed at the end of Sec. II; note the infinite slope at both endpoints of the branch cut. The curve marked $\epsilon = 0.001$ is for a metal having an essentially spherical Fermi surface and for which interactions have been neglected. Finally, the curve marked $A_2 = -0.02$ is the dispersion relation at the first harmonic for a simple metal for which interactions have been included by solving the Landau-Silin kinetic equation.[5] In Figs. 2 and 3 we show the ReqR and ImqR, respectively, for $\omega\tau = 10$. From Fig. 1 we see that Re$qR \approx$ ImqR at a frequency shifted from the Azbel'-Kaner cyclotron frequency by an amount comparable to the shift produced by an $A_2 = -0.02$. Figures 2 and 3 show that even when collisions are included, the effects of anisotropy on the dispersion relation simulate the effects of interactions.

Since it is difficult to work with single-crystal alkali samples, cyclotron-wave experiments are generally done with polycrystalline samples so that the dc field is not along a symmetry direction. It is natural to ask whether or not the above effects will be "averaged out" in a polycrystalline sample. In an attempt to answer this question we have calculated $C(\eta,\hat{n})$, where $\hat{n}$ is a unit vector specifying the orientation of $\vec{H}_0$. We have studied[11] the analytic behavior of $\langle C(\eta,\hat{n})\rangle$ which is obtained by averaging $C(\eta,\hat{n})$ over all orientations. It is found that $\langle C(\eta,\hat{n})\rangle$ for $\omega\tau \to \infty$ has a branch cut for $1 \le \eta \le 1+2\epsilon$ so that the effect of the averaging is to increase the range of frequencies for which there will be collisionless damping. This is understandable since the averaging allows all possible orbits to be sampled. This result suggests that not only will the above effects be present in a polycrystalline sample but may well be more pronounced.

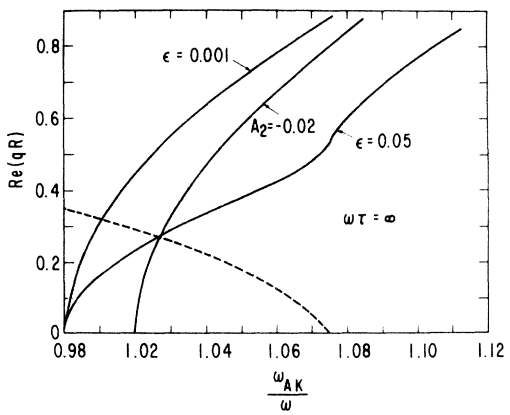

FIG. 1. Solid curves marked $\epsilon = 0.001$ and $\epsilon = 0.05$ are plots of the real part of Eq. (22) for $\omega\tau = \infty$. The dashed line is a plot of the imaginary part of Eq. (22) for $\epsilon = 0.05$ and $\omega\tau = \infty$ and represents the collisionless damping discussed in the text. The curve marked $A_2 = -0.02$ is a plot of the $\omega\tau = \infty$ dispersion relation for a metal with a spherical Fermi surface and for which interaction have been included (Ref. 5).

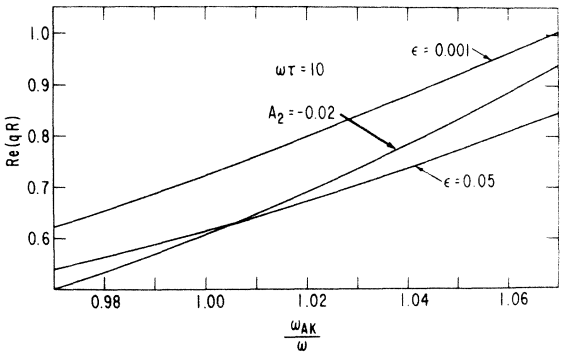

FIG. 2. Curves marked $\epsilon = 0.001$ and $\epsilon = 0.05$ are plots of the real part of Eq. (22) for $\omega\tau = 10$; a typical experimental value for the alkali metals. The curve marked $A_2 = -0.02$ is plot of the real part of the dispersion relation for $\omega\tau = 10$ in a metal having a spherical Fermi surface and for which interaction have been included (Ref. 5).

IV. CONCLUSIONS

The spread in cyclotron frequencies for a metal having a nonspherical Fermi surface leads to a collisionless damping of cyclotron waves (for the polarization $\vec{E} \parallel \vec{H}_0$). This damping can cause effects

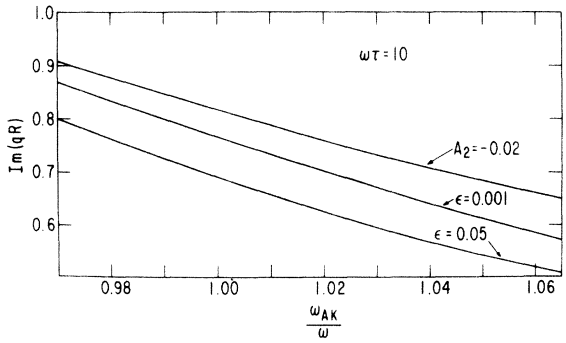

FIG. 3. Curves marked $\epsilon = 0.001$ and $\epsilon = 0.05$ are plots of the imaginary part of Eq. (22) for $\omega\tau = 10$. The curve marked $A_2 = -0.02$ is a plot of the imaginary part of the dispersion relation for $\omega\tau = 10$ in a metal having a spherical Fermi surface and for which interactions have been included (Ref. 5).

on the dispersion relation similar to the effects caused by electron-electron interactions in metals with spherical Fermi surfaces. From our calculations we conclude that the Landau parameters $\{A_n\}$ may not be accurately determined from cyclotron-wave experiments even when there is only a small amount of anisotropy.

APPENDIX A

The cyclotron frequency of electrons on the Fermi surface may be written

$$\omega_c(k_z) = 2\pi m^* \omega_c^{(0)} \left(\frac{\partial A}{\partial \mathcal{E}}\right)^{-1}_{\mathcal{E}=\mathcal{E}_F} , \tag{A1}$$

where $\omega_c^{(0)} = eH_0/m^*c$ and $A(k_z, \mathcal{E})$ is the area enclosed in $\vec{k}$ space by the orbit. Let a dc field $\vec{H}_0 = H_0\hat{n}$ have an arbitrary orientation with respect to the symmetry axis of a crystal; then we can write

$$A(k_\parallel, \mathcal{E}) = \int d^3k\, \eta([\mathcal{E} - \mathcal{E}(\vec{k})]\, \delta(\vec{k}\cdot\hat{n} - k_\parallel)) , \tag{A2}$$

where $k_\parallel$ is the component of $\vec{k}$ along the field, $\mathcal{E}$ is the energy, and $\eta(\mathcal{E} - \mathcal{E}(\vec{k}))$ is the unit step function defined by

$$\eta(x) = 1 , \quad x \gtrless 0$$

and

$$\eta(x) = 0 , \quad x < 0 .$$

It is useful to have the following integral representations:

$$\eta(\mathcal{E} - \mathcal{E}(\vec{k})) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} \frac{dS}{S} e^{[\mathcal{E}-\mathcal{E}(\vec{k})]S} , \quad a > 0$$

which is the inversion integral of the Laplace transform of the unit step function, and

$$\delta(k_\parallel - \vec{k}\cdot\hat{n}) = (1/2\pi) \int_{-\infty}^{\infty} dt\, e^{i(\vec{k}\cdot\hat{n} - k_\parallel)t} ,$$

which is the usual integral representation of the one-dimensional Dirac δ function. As before, we consider

$$\mathcal{E}(\vec{k}) = \frac{k^2}{2m^*} + \lambda \sum_{\alpha=1}^{3} k_\alpha^4 , \tag{A3}$$

with $\lambda = \epsilon/2m^*k_F^2$. If we substitute the above integral representation into (A2) and take the derivative with respect to $\mathcal{E}$, we obtain

$$\frac{\partial A}{\partial \mathcal{E}} = \frac{1}{2\pi i} \cdot \frac{1}{2\pi} \int_{a-i\infty}^{a+i\infty} dS\, e^{\mathcal{E}S} \int_{-\infty}^{\infty} dt\, e^{-ik_\parallel t} \times \int d^3k\, e^{i\vec{k}\cdot\hat{n}t - \mathcal{E}(\vec{k})S} . \tag{A4}$$

It is convenient to define the integral

$$F(t, \lambda, \hat{n}, S) = \int d^3k\, e^{i\vec{k}\cdot\hat{n}t - \mathcal{E}(\vec{k})S} ,$$

which becomes, upon substituting for $\mathcal{E}(\vec{k})$ from (A3), expanding the exponential to $O(\epsilon)$, and performing the integration

$$F(t, \lambda, \hat{n}, S) = \left(\frac{2\pi m^*}{S}\right)^{3/2} e^{-m^*t^2/2S} \times \left(1 - \frac{9\lambda m^{*2}}{S} + \frac{6\lambda m^{*3}t^2}{S^2} - \frac{\lambda m^* t^4 \sigma}{S^3}\right) , \tag{A5}$$

where we have introduced $\sigma = \sum_{\alpha=1}^{3} \hat{n}_\alpha^4$, $\hat{n}_\alpha$ being the Cartesian components of $\hat{n}$. We are led to consider the following integral:

$$G(\lambda, \sigma, S) = \int_{-\infty}^{\infty} dt\, e^{-ik_\parallel t} F(t, \lambda, \hat{n}, S) ,$$

which is readily integrated to give

$$G(\lambda, \sigma, S) = (2\pi)^2 m^* e^{-Sk_\parallel^2/2m^*} \times \left(\frac{1}{S} + \frac{6\lambda m^* k_\parallel^2}{S}(\sigma - 1) - \frac{3\lambda m^{*2}}{S^2}(\sigma + 1) - \lambda\sigma k_\parallel^4\right) . \tag{A6}$$

From (A4) and the above equation we have

$$\frac{\partial A(k_\parallel, \mathcal{E})}{\partial \mathcal{E}} = \frac{1}{2\pi i} \frac{1}{2\pi} \int_{a-i\infty}^{a+i\infty} dS\, e^{\mathcal{E}S}\, G(\lambda, \sigma, S) ,$$

which becomes, after substituting from (A6) and doing the integral by contour integration,

$$\frac{\partial A}{\partial \mathcal{E}} = 2\pi m^* [1 + 6\lambda m^* k_\parallel^2(\sigma - 1) - 3\lambda m^{*2}(\mathcal{E} - k_\parallel^2/2m^*)(\sigma + 1)] . \tag{A7}$$

If we evaluate (A7) at $\mathcal{E} = \mathcal{E}_F$, we find

$$\left(\frac{\partial A}{\partial \mathcal{E}}\right)_{\mathcal{E}=\mathcal{E}_F} = 2\pi m^* [1 + 3\epsilon x^2(\sigma - 1) - \tfrac{3}{4}\epsilon(1 - x^2)(\sigma + 1)] , \tag{A8}$$

where to within corrections of $O(\epsilon^2)$ we have written $k_\parallel = k_F x$ with $x = k\cdot\hat{n} = \cos\theta$. Finally, combining Eqs. (A1) and (A8), we find

$$\omega_c = \omega_c^{(0)} [1 + \tfrac{3}{4}\epsilon(1 - x^2)(\sigma + 1) - 3\epsilon x^2(\sigma - 1)] . \tag{A9}$$

In (A9) we note that the orientational dependence of the field is contained in $\sigma = \sum_{\alpha=1}^{3} \hat{n}_\alpha^4$ while the variable x serves to locate a particular orbit.

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†Present address: Xerox Palo Alto Research Center, Palo Alto, Calif. 94304.

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