Theory of surface polaritons in anisotropic dielectric media with application to surface magnetoplasmons in semiconductors*

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A theory is presented for surface polaritons associated with the planar surface of a semi-infinite anisotropic dielectric medium. Retardation is included. In general, two attenuating components with different attenuation constants must be superposed within the medium in order to satisfy the boundary conditions, and the macroscopic electric field vector does not lie in the sagittal plane. For special cases, however, only one attenuating component is required, and the electric vector does lie in the sagittal plane. The theory is applied to the specific case of surface magnetoplasmons in a semiconductor for magnetic fields either perpendicular or parallel to the surface. In the latter case, propagation directions parallel and perpendicular to the magnetic field are considered. Possibilities for the experimental observation of the effects predicted are discussed.

I. INTRODUCTION

Polaritons are photons coupled to the elementary excitations of a crystal such as plasmons, phonons, and magnons and have been extensively investigated both theoretically[1] and experimentally[2] for bulk crystals. The interaction between the photon and the elementary excitation is particularly significant when retardation is important—i.e., the wave vector $\vec{k}$ is on the order of the frequency ω divided by the speed of light c.

Recently, considerable interest has arisen in surface polaritons in which the coupled excitation is localized near the surface of a crystal. Kliewer and Fuchs[3] have treated the case of photons coupled to long-wavelength optical phonons in ionic crystals with an isotropic dielectric tensor and have derived the dispersion relation for surface polaritons. The same problem has been treated by Ruppin and Englman.[4] These theoretical results have been extended to the case of uniaxial insulating crystals by Lyubimov and Sannikov.[5] Ibach[6] has observed surface optical phonons in ZnO in the unretarded regime using the inelastic scattering of low-energy electrons.

Surface plasmons have been discussed theoretically by Ritchie[7] and by Ferrell[8] and can be observed experimentally by inelastic electron scattering.[9] Certain optical techniques for observing surface plasmons depend upon surface roughness or a grating ruled on the surface to provide the additional momentum required so that the surface plasmon can couple to the electromagnetic field. Thus, Teng and Stern[10] have used this technique to determine the dispersion relation for surface plasmons in aluminum by measuring the radiation emitted from a grating bombarded by 10-keV electrons. On the other hand, Marschall, Fischer, and Queisser[11] have measured the optical (infrared) reflectivity of a surface of n-type indium antimonide upon which a grating has been ruled and determined the surface-plasmon dispersion relation.

Another and perhaps more versatile technique is attenuated total reflection (ATR) which has been employed by Otto[12] to study surface plasmons in metals. Ruppin[13] suggested that this technique can be used to study surface optical phonons, and experimental observations have been made, for example, by Marschall and Fischer[14] for gallium phosphide and by Bryksin, Gerbshtein, and Mirlin[15] for NaCl.

The interaction of surface plasmons and surface optical phonons in polar semiconductors has been treated theoretically by Kheifets,[16] Chiu and Quinn,[17] and Wallis and Brion.[18] Experimental work bearing on this topic has been reported by Anderson, Alexander, and Bell[19] and Reshina, Gerbshtein, and Mirlin.[20] The influence of a magnetic field on surface plasmons has been discussed in the zero-retardation limit by Pakhomov and Stepanov,[21] Abdel-Shahid and Pakhomov,[22] and Chiu and Quinn.[23] Chiu and Quinn[24] and Quinn and Chiu[25] have recently studied this problem including retardation. These magnetic field investigations have been extended to include interaction with surface optical phonons by Quinn and Chiu,[25] by Chiu and Quinn[26,27] and Brion, Wallis, Hartstein, and Burstein.[28]

The problems of surface polaritons associated with surface optical phonons in noncubic materials, with surface magnetoplasmons, and with surface magnons all involve an anisotropic dielectric or permeability tensor. The surface-optical-phonon case involves a symmetric dielectric tensor and has been treated by the present authors[29] and, for uniaxial crystals, by Lyubimov and Sannikov.[5] However, the treatment in Ref. 29 is not complete in that allowance is not made for the possibility that more than one decay constant may be required to describe the surface wave. The surface-magnetoplasmon (gyrodielectric-media) and surface-magnon (gyromagnetic-media) cases involve antisymmetric off-diagonal parts to the dielectric and magnetic permeability tensors. The gyromagnetic case has been treated by Hartstein *et al.*[30] in a recent publication. An interesting result was found for certain configurations of magnetic field, surface, and wave vector $\vec{k}$—namely, nonreciprocity between $-\vec{k}$ and $+\vec{k}$. Furthermore, under certain conditions, the surface polariton does not persist out to $\vec{k}=-\infty$, so the surface excitation does not exist in the absence of the photon field, and the surface polariton may be called a "photon-induced" surface polariton. Photon-induced surface polaritons have been discussed theoretically by Bryksin, Mirlin, and Reshina[31] and references cited therein and by Harstein *et al.*[32] Experimental observations of photon-induced surface polaritons in α-quartz have been reported by Falge and Otto.[33]

In a previous paper[34] surface polaritons in gyrodielectric media were investigated theoretically for the particular case of surface magnetoplasmons in semiconductors with the magnetic field parallel to the surface and the wave vector perpendicular to the magnetic field. Photon-induced surface polaritons were found under appropriate conditions. In the present paper, we give a general theory of surface polaritons in an anisotropic dielectric medium. The theory is then applied to the specific case of surface magnetoplasmons in n-type InSb. Results for various orientations of the magnetic field relative to the surface and of the wave vector relative to the magnetic field are presented. As we shall see, our results differ in certain significant details from those of Chiu and Quinn.[24]

II. GENERAL ANISOTROPIC DIELECTRIC MEDIUM

Let us consider a semi-infinite general anisotropic dielectric medium, which is described by a dielectric tensor $\epsilon_{ij}(\omega)$. We assume that the dielectric tensor at a given point in the medium is independent of the proximity of the point to the surface. This is a reasonable assumption for wavelengths much larger than a lattice spacing. Furthermore, we neglect the wave-vector dependence of the dielectric tensor. In semiconductor problems, this is generally satisfactory for excitations whose wavelength is large compared to the Thomas-Fermi length and to the cyclotron-orbit radius or to the carrier mean free path. For n-type InSb with carrier concentration of 10^{18} cm^{-3} and mobility of 3×10^{4} cm^{2} V sec^{-1}, the Thomas-Fermi length is ~70 Å, the mean free path is ~6000 Å; and in a magnetic field of 25 kG, the orbit radius is ~800 Å. All of these distances are small compared to the infrared wavelengths of interest, ~10^{5} Å. We also neglect the interaction of the magnetoplasmons with optical phonons. This is reasonable for carrier concentrations $>10^{18}$ cm^{-3} such as were employed experimentally by Marschall, Fischer, and Queisser.[11]

We choose the system of coordinates shown in Fig. 1. The surface lies in the y-z plane, and the x direction is perpendicular to the surface of the material, penetrating into it. The wave vector of the macroscopic electric field lies in the y-z plane and makes an angle φ with the z axis. The magnetic field lies in the x-y plane and makes an angle θ with the x axis.

We start from Maxwell's equations. After eliminating the magnetic field variables, we arrive at the following equation for the macroscopic electric field:

$$\nabla\times\nabla\times\vec{E}+\frac{1}{c^2}\frac{\partial^2\vec{D}}{\partial t^2}=0. \qquad (1)$$

We seek a solution to Eq. (1) for $x>0$ of the form

$$\vec{E}=\vec{E}^{0}e^{-\alpha x}e^{i(k_y y+k_z z-\omega t)}. \qquad (2)$$

The form of this solution specifies an electric field which is exponentially damped as one leaves the surface of the material, and which is a running wave along the surface. Substituting this solution into Eq. (1), we arrive at a set of three homogeneous algebraic equations for E_x^0, E_y^0, and E_z^0.

Elimination of E_x^0 yields the following system of two homogeneous algebraic equations for E_y^0 and E_z^0:

$$A_{yz}(\alpha)E_y^0+B_{yz}(\alpha)E_z^0=0, \qquad (3a)$$

$$B_{zy}(\alpha)E_y^0+A_{zy}(\alpha)E_z^0=0, \qquad (3b)$$

where

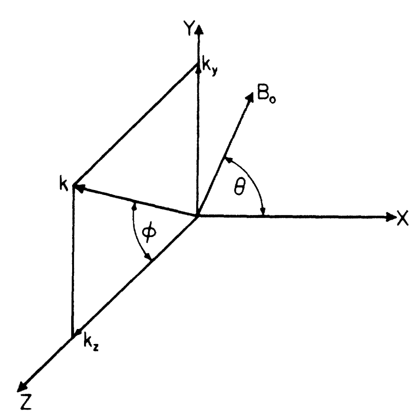

FIG. 1. Diagram showing the relationships of the magnetic field and wave vector. The surface is the y-z plane.

$$A_{\mu\nu}(\alpha)=\left(k_\nu^2-\frac{\omega^2}{c^2}\epsilon_{\mu\mu}-\alpha^2\right)\kappa_x^2+k_\mu^2\alpha^2 -ik_\mu\alpha(\epsilon_{x\mu}+\epsilon_{\mu x})\frac{\omega^2}{c^2}-\epsilon_{x\mu}\epsilon_{\mu x}\frac{\omega^4}{c^4}, \quad (4a)$$

$$B_{\mu\nu}(\alpha)=-\left(k_\mu k_\nu+\frac{\omega^2}{c^2}\epsilon_{\mu\nu}\right)\kappa_x^2+k_\mu k_\nu\alpha^2 -i\alpha(k_\nu\epsilon_{\mu x}+k_\mu\epsilon_{x\nu})\frac{\omega^2}{c^2}-\epsilon_{\mu x}\epsilon_{x\nu}\frac{\omega^4}{c^4}, \quad (4b)$$

$$\kappa_x^2=k^2-(\omega^2/c^2)\epsilon_{xx}, \quad (5)$$

and $k^2=k_y^2+k_z^2$.

To effect a nontrivial solution to these two homogeneous equations, we set their determinant of coefficients to zero. This procedure yields an equation $F(\alpha, k, \omega)=0$, which has the roots $\alpha_i=\alpha_i(k, \omega)$. Of these roots, only those with positive real parts make suitable attenuation constants for surface waves. We shall see that, in general, two such roots are required to satisfy the electromagnetic boundary conditions at the vacuum-dielectric interface.

The equation $F(\alpha, k, \omega)=0$ may be put in the form

$$F(\alpha, k, \omega)=\epsilon_{xx}\alpha^4+a(k, \omega)\alpha^3+b(k, \omega)\alpha^2+c(k, \omega)\alpha+d(k, \omega)=0, \quad (6)$$

where

$$a(k, \omega)=-ik_y(\epsilon_{xy}+\epsilon_{yx})-ik_z(\epsilon_{xz}+\epsilon_{zx}), \quad (7a)$$

$$b(k, \omega)=(\omega^2/c^2)(\epsilon_{xx}\epsilon_{yy}+\epsilon_{xx}\epsilon_{zz}-\epsilon_{yx}\epsilon_{xy}-\epsilon_{zx}\epsilon_{xz})-[k^2\epsilon_{xx}+k_y^2\epsilon_{yy}+k_z^2\epsilon_{zz}+k_yk_z(\epsilon_{yz}+\epsilon_{zy})], \quad (7b)$$

$$c(k, \omega)=(i\omega^2/c^2)[k_y(\epsilon_{zx}\epsilon_{yz}+\epsilon_{xz}\epsilon_{zy}-\epsilon_{zz}\epsilon_{xy}-\epsilon_{zz}\epsilon_{yx})+k_z(\epsilon_{xy}\epsilon_{yz}+\epsilon_{yx}\epsilon_{zy}-\epsilon_{yy}\epsilon_{xz}-\epsilon_{yy}\epsilon_{zx})] +ik^2[k_y(\epsilon_{xy}+\epsilon_{yx})+k_z(\epsilon_{xz}+\epsilon_{zx})], \quad (7c)$$

$$d(k, \omega)=k^2[k_y^2\epsilon_{yy}+k_z^2\epsilon_{zz}+k_yk_z(\epsilon_{yz}+\epsilon_{zy})]+(\omega^2/c^2)k^2(\epsilon_{yz}\epsilon_{zy}-\epsilon_{yy}\epsilon_{zz}) +(\omega^2/c^2)[k_z^2(\epsilon_{zx}\epsilon_{xz}-\epsilon_{xx}\epsilon_{zz})+k_y^2(\epsilon_{yx}\epsilon_{xy}-\epsilon_{xx}\epsilon_{yy})+k_zk_y(\epsilon_{zx}\epsilon_{xy}+\epsilon_{yx}\epsilon_{xz}-\epsilon_{xx}\epsilon_{yz}-\epsilon_{xx}\epsilon_{zy})] +(\omega^4/c^4)[\epsilon_{xx}\epsilon_{yy}\epsilon_{zz}-\epsilon_{xx}\epsilon_{yz}\epsilon_{zy}+\epsilon_{zx}(\epsilon_{xy}\epsilon_{yz}-\epsilon_{yy}\epsilon_{xz})+\epsilon_{yx}(\epsilon_{zy}\epsilon_{xz}-\epsilon_{xy}\epsilon_{zz})]. \quad (7d)$$

This dispersion relation reduces to that of Ref. 29 for $k_z=0$ and $\epsilon_{ij}=\epsilon_{ji}$.

Let Eq. (6) have two roots with positive real parts. Call these roots α_1, α_2. Then the homogeneous equations for E_y^0 and E_z^0 may be solved twice; once with the substitution $\alpha=\alpha_1$ into the determinant of coefficients, and again with the substitution $\alpha=\alpha_2$. Let $E_y^0(\alpha_i)$, $E_z^0(\alpha_i)$ be the solution corresponding to the root α_i. Then by inspection of Eqs. (3) we determine that

$$E_y^0(\alpha_i)=K_iA_{zy}(\alpha_i), \quad (8a)$$

$$E_z^0(\alpha_i)=-K_iB_{zy}(\alpha_i), \quad i=1, 2, \quad (8b)$$

where the quantities K_i are amplitudes to be determined from the boundary conditions. Maxwell's equations then yield

$$E_x^0(\alpha_i)=K_iC(\alpha_i), \quad (9)$$

where

$$C(\alpha_i)=(1/\kappa_x^2)\{[ik_y\alpha_i+(\omega^2/c^2)\epsilon_{xy}]A_{zy}(\alpha_i) -[ik_z\alpha_i+(\omega^2/c^2)\epsilon_{xz}]B_{zy}(\alpha_i)\}. \quad (10)$$

The general solution to Eq. (1) can be written as the linear superposition

$$E_x=[K_1C(\alpha_1)e^{-\alpha_1x}+K_2C(\alpha_2)e^{-\alpha_2x}]e^{i(k_yy+k_zz-\omega t)}, \quad (11a)$$

$$E_y=[K_1A_{zy}(\alpha_1)e^{-\alpha_1x}+K_2A_{zy}(\alpha_2)e^{-\alpha_2x}]e^{i(k_yy+k_zz-\omega t)}, \quad (11b)$$

$$E_z=-[K_1B_{zy}(\alpha_1)e^{-\alpha_1x}+K_2B_{zy}(\alpha_2)e^{-\alpha_2x}]e^{i(k_yy+k_zz-\omega t)}. \quad (11c)$$

We turn now to the region $x<0$ outside the dielectric medium. We seek a solution to Eq. (1) of the form

$$(E_x, E_y, E_z)=(E_x^e, E_y^e, E_z^e)e^{\alpha_0x}e^{i(k_yy+k_zz-\omega t)}. \quad (12)$$

Maxwell's equations then lead to the result that

$$E_x^e=-(ik_yE_y^e+ik_zE_z^e)/\alpha_0, \quad (13)$$

where $\alpha_0^2=k^2-(\omega^2/c^2)$.

The boundary conditions at the vacuum-dielectric medium interface require the continuity of the tangential components of $\vec{E}$ and $\vec{B}$ and the normal components of $\vec{D}$ and $\vec{B}$ ($\mu=1$). Utilizing Eqs. (8) and (9), the boundary conditions on $\vec{E}$ and $\vec{D}$ become

$$-(ik_yE_y^e+ik_zE_z^e)/\alpha_0=\epsilon_{xx}[C(\alpha_1)K_1+C(\alpha_2)K_2] +\epsilon_{xy}[A_{zy}(\alpha_1)K_1+A_{zy}(\alpha_2)K_2] -\epsilon_{xz}[B_{zy}(\alpha_1)K_1+B_{zy}(\alpha_2)K_2], \quad (14a)$$

$$E_y^e=A_{zy}(\alpha_1)K_1+A_{zy}(\alpha_2)K_2, \quad (14b)$$

$$E_z^e=-B_{zy}(\alpha_1)K_1-B_{zy}(\alpha_2)K_2. \quad (14c)$$

If one uses the Maxwell equation $i(\omega/c)\vec{B} = \nabla\times\vec{E}$, the boundary conditions on $\vec{B}$ can be written

$$(k_z E_y^e - k_y E_z^e) = k_z[A_{zy}(\alpha_1)K_1 + A_{zy}(\alpha_2)K_2] + k_y[B_{zy}(\alpha_1)K_1 + B_{zy}(\alpha_2)K_2], \tag{15a}$$

$$(k_z/\alpha_0)(k_y E_y^e + k_z E_z^e) - \alpha_0 E_z^e = ik_z[C(\alpha_1)K_1 + C(\alpha_2)K_2] - [\alpha_1 B_{zy}(\alpha_1)K_1 + \alpha_2 B_{zy}(\alpha_2)K_2], \tag{15b}$$

$$-\alpha_0 E_y^e + (k_y/\alpha_0)(k_y E_y^e + k_z E_z^e) = ik_y[C(\alpha_1)K_1 + C(\alpha_2)K_2] + [\alpha_1 A_{zy}(\alpha_1)K_1 + \alpha_2 A_{zy}(\alpha_2)K_2]. \tag{15c}$$

Equations (14) and (15) constitute a set of six linear homogeneous algebraic equations in the four unknowns K_1, K_2, E_y^e, and E_z^e. Clearly, there are redundancies. Equation (15a) follows from Eqs. (14b) and (14c). The other redundancy is more difficult to isolate. All we need, however, is four independent equations. We chose three of these to be Eqs. (14a), (14b), and (14c). The fourth equation we obtain by multiplying Eq. (15b) by ik_y and Eq. (15c) by ik_z and subtracting to give

$$\alpha_0(-k_z E_y^e + k_y E_z^e) = k_y[\alpha_1 B_{zy}(\alpha_1)K_1 + \alpha_2 B_{zy}(\alpha_2)K_2] + k_z[\alpha_1 A_{zy}(\alpha_1)K_1 + \alpha_2 A_{zy}(\alpha_2)K_2]. \tag{16}$$

Equations (14) and (16) now constitute our boundary conditions. In order for these equations to have a nontrivial solution, the determinant of coefficients must vanish. This condition gives the dispersion relation for surface polaritons which can be reduced to the form

$$\left\{\alpha_0\epsilon_{xx}\left[\left(ik_y\alpha_1 + \frac{\omega^2}{c^2}\epsilon_{xy}\right)A_{zy}(\alpha_1) - \left(ik_z\alpha_1 + \frac{\omega^2}{c^2}\epsilon_{xz}\right)B_{zy}(\alpha_1)\right] + \kappa_x^2[(ik_y + \alpha_0\epsilon_{xy})A_{zy}(\alpha_1) - (ik_z + \alpha_0\epsilon_{xz})B_{zy}(\alpha_1)]\right\}(\alpha_0 + \alpha_2)[k_y B_{zy}(\alpha_2) + k_z A_{zy}(\alpha_2)]$$
$$-\text{(corresponding term with } \alpha_1,\ \alpha_2 \text{ interchanged)} = 0. \tag{17}$$

An alternative expression for the surface-polariton dispersion relation can be obtained by making the replacements $A_{zy}(\alpha_i)\to B_{yz}(\alpha_i)$ and $B_{zy}(\alpha_i)\to A_{yz}(\alpha_i)$ and using Eqs. (14b), (14c), (15b), and (15c). The result is

$$\left\{\left[-\frac{ik_z}{\kappa_x^2}\left(ik_y\alpha_1 + \frac{\omega^2}{c^2}\epsilon_{xy}\right) + \left(\frac{k_y k_z}{\alpha_0}\right)\right]B_{yz}(\alpha_1) + \left[\frac{ik_z}{\kappa_x^2}\left(ik_z\alpha_1 + \frac{\omega^2}{c^2}\epsilon_{xz}\right) - \frac{k_z^2}{\alpha_0} + \alpha_0 + \alpha_1\right]A_{yz}(\alpha_1)\right\}$$
$$\times\left\{\left[\frac{ik_y}{\kappa_x^2}\left(ik_y\alpha_2 + \frac{\omega^2}{c^2}\epsilon_{xy}\right) - \frac{k_y^2}{\alpha_0} + \alpha_0 + \alpha_2\right]B_{yz}(\alpha_2) - \frac{ik_y}{\kappa_x^2}\left(ik_z\alpha_2 + \frac{\omega^2}{c^2}\epsilon_{xz}\right) - \frac{k_y k_z}{\alpha_0}\ A_{yz}(\alpha_2)\right\}$$
$$-\text{(corresponding term with } \alpha_1,\ \alpha_2 \text{ interchanged)} = 0. \tag{18}$$

We shall use either Eq. (17) or (18) as seems most convenient for the particular case under consideration.

The amplitudes K_1, K_2, E_y^e, and E_z^e can now be calculated by solving the linear homogeneous equations. From Eqs. (14) and (16), we find

$$K_1 = K(\alpha_0 + \alpha_2)[k_y B_{zy}(\alpha_2) + k_z A_{zy}(\alpha_2)], \tag{19a}$$

$$K_2 = -K(\alpha_0 + \alpha_1)[k\ B_{zy}(\alpha_1) + k_z A_{zy}(\alpha_1)], \tag{19b}$$

where K is an arbitrary constant. E_y^e and E_z^e follow from Eqs. (14b) and (14c).

The problem has now been formally solved. The dispersion relation, as well as the electric fields and attenuation constants are known as functions of frequency or wave vector. It should be noted that our treatment assumes that the dielectric tensor has a step-function behavior, so that it can be valid only for wavelengths large compared to the lattice spacing. We also neglect nonlocal effects and damping effects due to scattering processes.

III. SURFACE MAGNETOPLASMONS IN SEMICONDUCTORS

We now apply the results for the general anisotropic dielectric medium obtained in the previous section to the particular case of surface polaritons associated with surface magnetoplasmons in semiconductors. The geometrical configuration is shown in Fig. 1. The semiconductor is assumed to be semi-infinite and to fill the half-space given by $x \geq 0$. The external magnetic field $\vec{B}_0$ is taken to lie in the x-y plane and to make an angle θ with the x axis. The two-dimensional wave vector $\vec{k}$ describing the propagation of the surface wave parallel to the surface is taken to make an angle φ with the z axis.

Let us consider a semiconductor which is n- or p-type and in which the free charge carriers occupy an energy band which is simple and parabolic. We neglect effects due to damping ($\omega_c\tau \gg 1$), to spatial dispersion, and to the interaction of magnetoplasmons and optical phonons. The dielectric tensor can then be written in the form[21,22]

$$\epsilon(\omega) = \begin{bmatrix} \sin^2\theta\epsilon_1 + \cos^2\theta\epsilon_3 & \sin\theta\cos\theta(\epsilon_3 - \epsilon_1) & -i\sin\theta\epsilon_2 \\ \sin\theta\cos\theta(\epsilon_3 - \epsilon_1) & \cos^2\theta\epsilon_1 + \sin^2\theta\epsilon_3 & i\cos\theta\epsilon_2 \\ i\sin\theta\epsilon_2 & -i\cos\theta\epsilon_2 & \epsilon_1 \end{bmatrix}, \tag{20}$$

where $\epsilon_1 = \epsilon_\infty\{1 + [\omega_p^2/(\omega_c^2 - \omega^2)]\}$, $\epsilon_2 = \epsilon_\infty\omega_c\omega_p^2/\omega(\omega^2 - \omega_c^2)$, $\epsilon_3 = \epsilon_\infty[1 - (\omega_p^2/\omega^2)]$, ω_c is the cyclotron frequency defined by $\omega_c = eB_0/m^*c$, m^* is the effective mass, ω_p is the plasma frequency defined by $\omega_p^2 = 4\pi Ne^2/m^*\epsilon_\infty$, ϵ_∞ is the background dielectric constant, and N is the free-carrier concentration. Using the general results of Sec. II and the expressions for the components of the dielectric tensor, we can obtain the dispersion curves, attenuation constants, and electric field components for surface polaritons associated with surface magnetoplasmons in semiconductors. We now consider several specific cases for the orientation of the external magnetic field $\vec{B}_0$ relative to the surface and to the wave vector $\vec{k}$.

A. $\vec{B}_0$ perpendicular to the surface

For this case, $\sin\theta = 0$, and we have $\epsilon_{xy} = \epsilon_{xz} = \epsilon_{yx} = \epsilon_{zx} = 0$. The problem possesses cylindrical symmetry about the external magnetic field, so the results are independent of φ. For simplicity we take $\varphi = 0$, so that $\vec{k}$ is in the z direction. Equation (6), which determines the decay constants α_i, reduces to the biquadratic equation

$$\epsilon_3\alpha^4 - (\epsilon_1\kappa_3^2 + \epsilon_3\kappa_1^2)\alpha^2 + \kappa_3^2[\epsilon_1\kappa_1^2 + (\omega^2/c^2)\epsilon_2^2] = 0, \tag{21}$$

where $\kappa_i^2 = k^2 - (\omega^2/c^2)\epsilon_i$. The solutions for α^2 are

$$\alpha^2 = k^2\left(\frac{\epsilon_1 + \epsilon_3}{2\epsilon_3}\right) - \frac{\omega^2}{c^2}\epsilon_1 \pm \left[k^4\left(\frac{\epsilon_1 - \epsilon_3}{2\epsilon_3}\right)^2 - \frac{\kappa_3^2(\omega^2/c^2)\epsilon_2^2}{\epsilon_3}\right]^{1/2}. \tag{22}$$

In general, both solutions must be utilized to satisfy the boundary conditions, as discussed in Sec. II. In order to have a bonafide surface polariton, it is necessary that both α_1 and α_2 be real and positive or complex with positive real parts. Under some circumstances, however, one α may have a positive real part, but the other α may be pure imaginary. We then have a superposition of an attenuated or surface component and a propagating or bulk component. This type of wave is known[35] as a pseudosurface wave. Specific criteria for the onset of pseudosurface waves will be given later. It must be pointed out that the propagating component normal

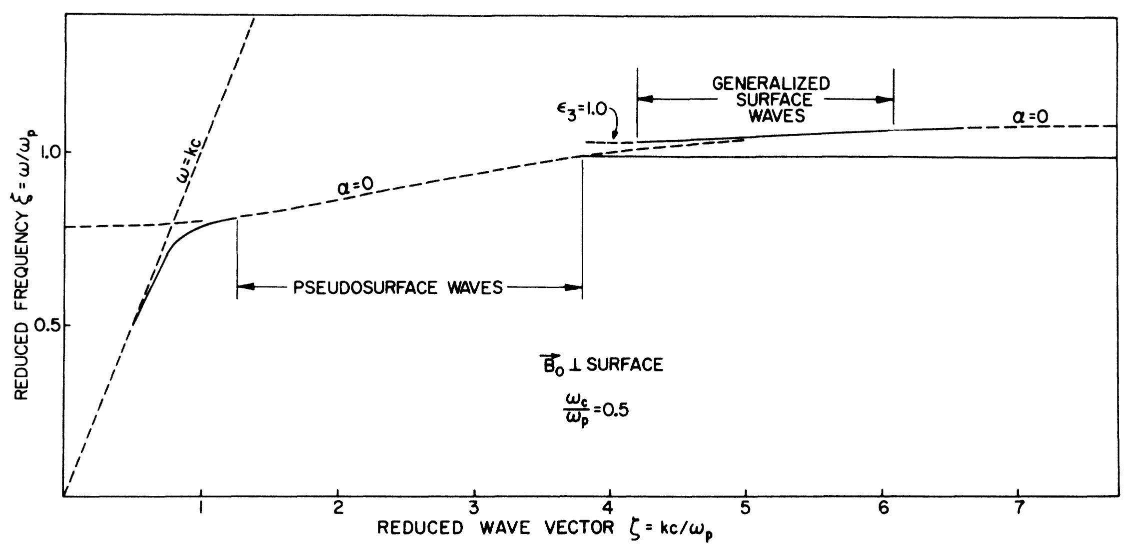

FIG. 2. Surface polariton dispersion curve for n-InSb with $\vec{B}_0$ perpendicular to the surface and $\omega_c = 0.5\omega_p$. The dashed curve $\alpha = 0$ is a bulk polariton dispersion curve.

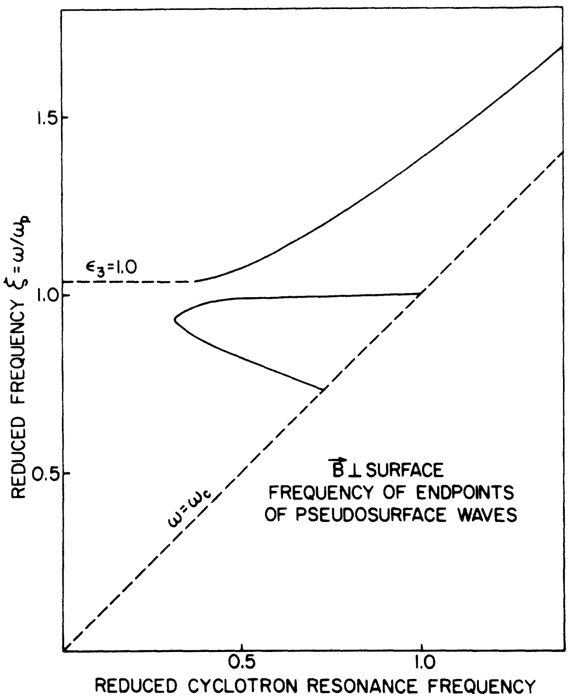

FIG. 3. Frequencies of endpoints of pseudosurface waves vs magnetic field for n-InSb with $\vec{B}_0$ perpendicular to the surface.

to the surface produces a loss of energy from the surface into the interior of the crystal, and hence the wave must attenuate in the direction of propagation parallel to the surface—i.e., the wave vector $\vec{k}$ must be complex. We have not taken this into account, so our treatment does not apply to the pseudosurface waves.

The dispersion relation given by Eq. (17) can be reduced for the configuration under discussion to the form

$$(\kappa_3^2/\alpha_0\epsilon_3)[\kappa_1^2+\alpha_1\alpha_2+\alpha_0(\alpha_1+\alpha_2)] + \alpha_1\alpha_2(\alpha_1+\alpha_2)+\alpha_0(\alpha_1^2+\alpha_1\alpha_2+\alpha_2^2)-\alpha_0\kappa_1^2=0\,. \quad (23)$$

This relation has already been derived by Chiu and Quinn.[24,25] Equations (22) and (23) have been solved for the dispersion curves using a high-speed computer. Typical results are shown in Fig. 2 for the case of n-InSb with $\omega_c/\omega_p=0.5$. Since $\vec{k}$ is perpendicular to $\vec{B}_0$ and the direction of $\vec{k}$ is immaterial, k appears only as even powers in Eqs. (22) and (23), and the curves for positive and negative k are identical. The dispersion curve starts on the light line $\omega=kc$ at $\omega=\omega_c$, rises just to the right of the light line, flattens out, and then after a gap, approaches an asymptotic value for large $\vec{k}$. For $\omega<\omega_c$, helicons propagate and there are no surface waves.

The asymptotic frequency of the surface magnetoplasmon is specified by the equation

$$\epsilon_3(\epsilon_1/\epsilon_3)^{1/2}=-1\,, \quad (24)$$

which has already been obtained by Pakhomov and Stepanov[21] and by Abdel-Shahid and Pakhomov.[22] The asymptotic decay constant is given by

$$\alpha=k(\epsilon_1/\epsilon_3)^{1/2}\,. \quad (25)$$

We see immediately from Eqs. (24) and (25) that ϵ_1

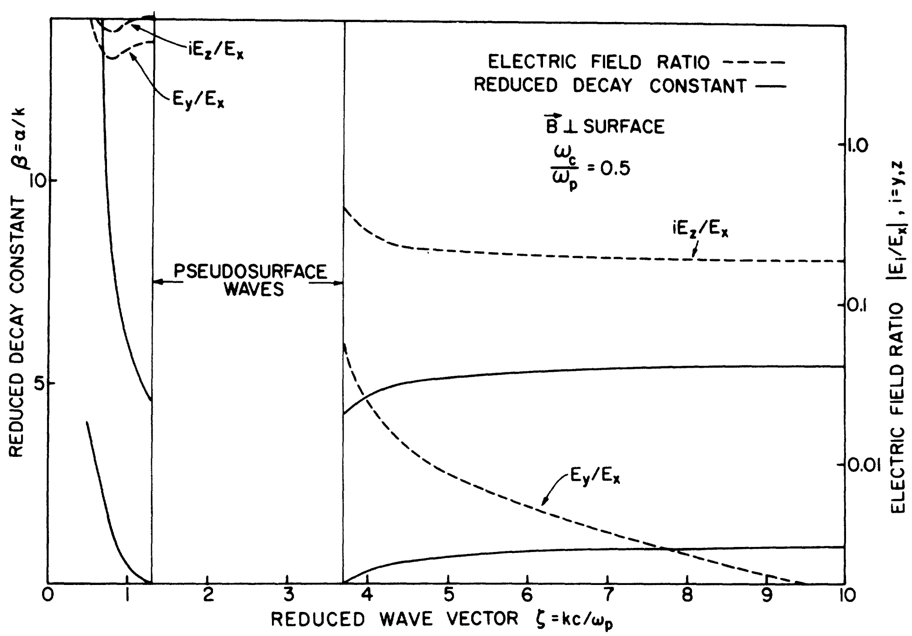

FIG. 4. Electric field ratios and reduced decay constants vs wave vector for n-InSb with $\vec{B}_0$ perpendicular to the surface and $\omega_c=0.5\omega_p$.

and ϵ_3 must both be negative in order to have a bonafide surface wave in the large-k limit. This in turn can be so only if $\omega_c < \omega_p$.

One also sees in Fig. 2 that there is a second surface branch which lies in a frequency range above ω_p. This branch never reaches the light line $\omega = ck$ and has no asymptotic limit. In fact, it starts at the frequency where $\epsilon_3 = 1$ and terminates when it intersects the bulk-magnetoplasmon dispersion curve specified by $\alpha = 0$. Chiu and Quinn[24] did not find this branch because they assumed $\epsilon_\infty = 1$ and under this assumption $\epsilon_3 = 1$ only at infinite ω.

For the case under consideration $\omega_c/\omega_p = 0.5$, pseudosurface waves exist in the region indicated in Fig. 2. The criterion for the onset of pseudosurface waves is that one of the α's be zero—i.e., the surface-wave dispersion curve intersects the bulk-magneto plasmon dispersion curve. Then from Eqs. (22) and (23) one can show that the onset frequencies are specified by the equation

$$\epsilon_1(\epsilon_v - 1)^{1/2} + \left(\frac{\epsilon_v(\epsilon_1 + \epsilon_3)}{\epsilon_3} - 2\epsilon_1\right)^{1/2} - \frac{\epsilon_2^2}{\epsilon_1} = 0, \qquad (26)$$

where

$$\epsilon_v = (\epsilon_1^2 - \epsilon_2^2)/\epsilon_1 . \qquad (27)$$

Equation (26) has been solved for the onset frequencies as functions of magnetic field. The results are plotted in Fig. 3 for n-InSb. One sees that no pseudosurface waves exist below $\omega_c/\omega_p = 0.32$. Above the latter value, pseudosurface waves exist over a range of frequencies whose width increases with increasing ω_c. The frequency of the lower boundary of the pseudosurface wave region decreases until it reaches ω_c. The frequency of the upper boundary increases until it becomes equal to ω_p. The upper curve in Fig. 3 specifies the right-hand end-point of the upper branch in Fig. 2. The upper branch exists for n-InSb only for $\omega_c/\omega_p \geq 0.4$. Quinn and Chiu[25] did not report this upper branch for n-GaAs, probably because this branch does not exist for the values of the parameters they considered.

In the configuration under discussion ($\vec{B}_0$ perpendicular to the surface), we have seen that, in general, two solutions corresponding to two different attenuation constants must be superposed in order to satisfy the boundary conditions. Furthermore, the electric vector does not lie in the sagittal plane (the plane defined by the normal to the surface and the direction of propagation) except in the unretarded limit $k \gg \omega_p/c$. The electric vector sweeps out an ellipse which contains the direction of propagation and is inclined to the normal to the surface. The tilting of the electric vector out of the sagittal plane for finite k can be attributed to the photon content of the mode. We have calculated the decay constants and electric field components for the case of n-InSb with $\omega_c/\omega_p = 0.5$. The results for the reduced decay constants $\beta_i = \alpha_i/k$, $i = 1, 2$ are plotted against wave vector in Fig. 4. At the starting point of the surface wave on the light line at $\omega = \omega_c$, $\beta_0 = 0$ and $\beta_2 = \infty$, while β_1 is finite. In the limit of large wave vector, the amplitude of the solution involving β_2 goes to zero, so that in the unretarded limit, only the solution involving β_1 is needed. The results for the electric vector components are plotted against wave vector in Fig. 4

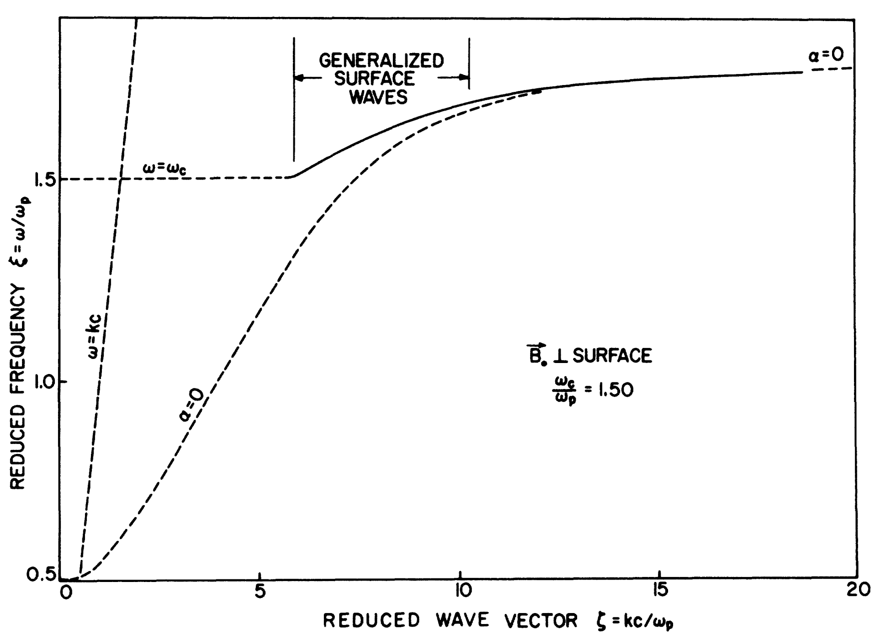

FIG. 5. Surface polariton dispersion curve for n-InSb with $\vec{B}_0$ perpendicular to the surface and $\omega_c = 1.5\,\omega_p$. The dashed curve $\alpha = 0$ is a bulk polariton dispersion curve.

in terms of the ratios iE_z/E_x and E_y/E_x. (The z direction has been taken to be the direction of propagation.) For $k > \omega_c/c$, the wave is elliptically polarized with the plane of the ellipse inclined to the sagittal plane. In the limit $k \gg \omega_p/c$, the surface wave becomes an elliptically polarized wave with the electric vector in the sagittal plane. The ratio iE_z/E_x approaches the value $1/\beta_2$.

Although no true surface waves exist in the asymptotic limit $k\to\infty$ for $\omega_c > \omega_p$, they may exist at smaller k. This is illustrated in Fig. 5 for n-InSb with $\omega_c/\omega_p = 1.5$. A true surface wave branch exists over a finite range in k and is the analogue of the upper branch in Fig. 2 for $\omega_c/\omega_p = 0.5$. The right-hand end-point of the surface branch in Fig. 5 again corresponds to the intersection of the surface branch with the bulk branch. The left-hand end-point occurs when $\omega = \omega_c$ rather than when $\epsilon_3 = 1$, since the first criterion is now the more restrictive of the two. If $\epsilon_\infty = 1$, the upper surface branch no longer appears.

B. $\vec{B}_0$ parallel to the surface, $\vec{k}\perp\vec{B}_0$

The configuration discussed in this section is particularly simple, since Maxwell's equations decouple even when retardation is included. The electric vector is always confined to the sagittal plane and only one decay constant is required to describe the surface wave in the active medium. A preliminary account of this case has already been reported[34] by the present authors, while the ferromagnetic case has been treated in Ref. 30. The effect of interaction with optical phonons has been considered by Brion *et al.*[28] and by Chiu and Quinn.[25-27]

From Maxwell's equations we find that the decay constant is specified by

$$\alpha^2 = k^2 - (\omega^2/c^2)\epsilon_v \tag{28}$$

where ϵ_v is the so-called "Voigt" dielectric constant given by Eq. (27). The dispersion relation can be obtained from Eq. (17) as has been done by Chiu and Quinn[24,25]; however a simpler expression results if one uses the equation

$$\nabla\cdot\vec{D} = 0. \tag{29}$$

Within the dielectric medium, one can take the electric vector of the form given by Eq. (2) with $k_y = 0$ and $k_z = k$ and obtain after substitution in Eq. (29),

$$E_x/E_z = (ik\epsilon_{xx} - \alpha\epsilon_{xz})/(ik\epsilon_{xz} + \alpha\epsilon_{xx}). \tag{30}$$

In the vacuum, from Eq. (29) one obtains

$$E_x^e/E_z^e = -ik/\alpha_0. \tag{31}$$

The continuity of the z component of $\vec{E}$ and of the x component of $\vec{D}$ give

$$E_x/E_z = -(ik + \alpha_0\epsilon_{xz})/\alpha_0\epsilon_{xx}. \tag{32}$$

The boundary conditions for the magnetic vector yield nothing new. Combining Eqs. (30) and (32), we obtain the dispersion relation in the simple form

$$\alpha + \alpha_0\epsilon_v + ik(\epsilon_{xz}/\epsilon_{xx}) = 0. \tag{33}$$

One notes immediately that this dispersion relation is nonreciprocal—i.e., positive and negative values of the wave vector $\vec{k}$ are not equivalent.

Calculations of the dispersion relation for this configuration have been carried out for n-InSb. The case $k > 0$ with $\omega_c/\omega_p = 0.5$ is plotted in Fig. 6. A

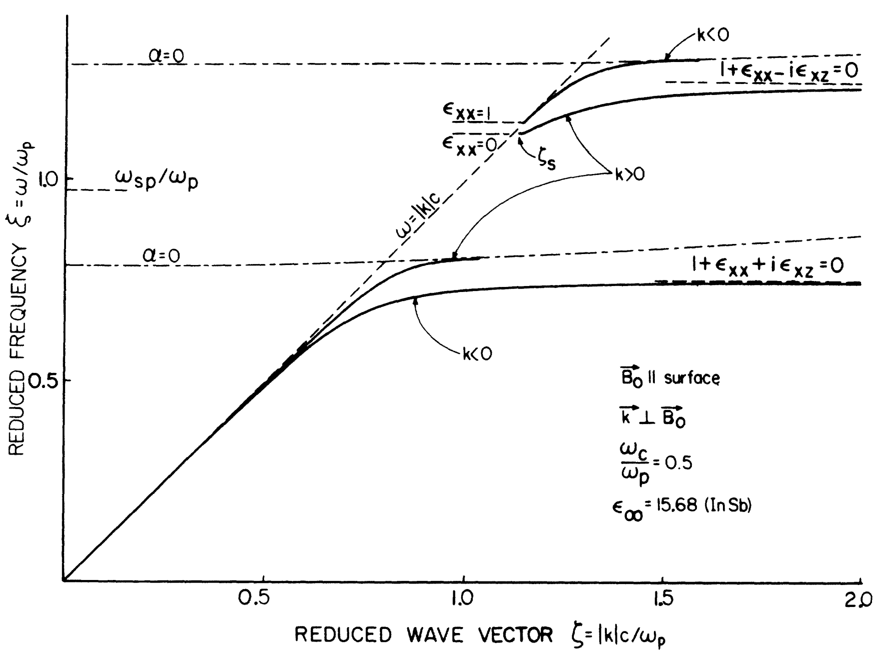

FIG. 6. Surface polariton dispersion curves for n-InSb with $\vec{B}_0$ parallel to the surface, $\vec{k}\perp\vec{B}_0$, and $\omega_c = 0.5\,\omega_p$. The dashed curves $\alpha = 0$ are bulk polariton dispersion curves. ω_{sp} is the zero-field unretarded surface-plasmon frequency defined by $\omega_p(1+1/\epsilon_\infty)^{-1/2}$. ζ_s is defined in the text.

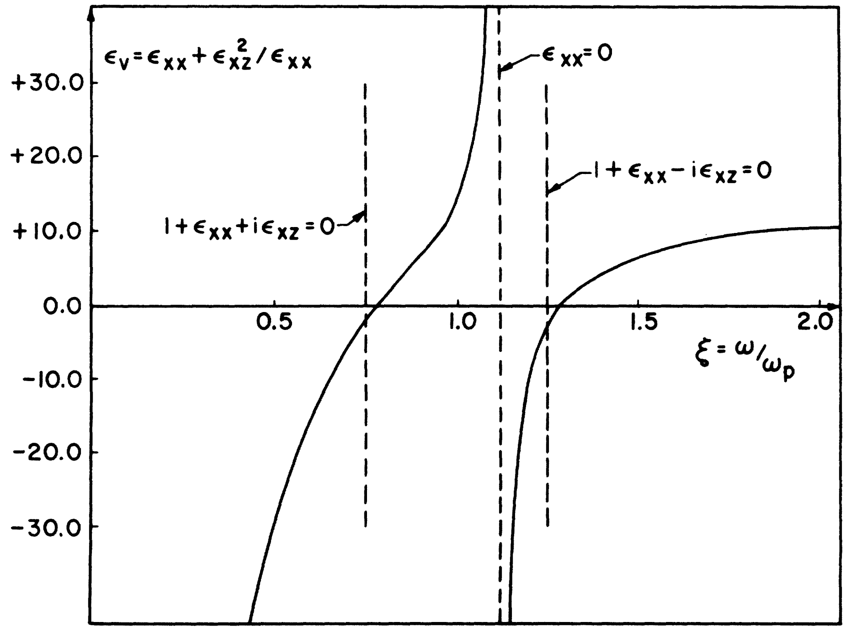

FIG. 7. Dielectric constant ϵ_v vs frequency for n-InSb.

very interesting feature is immediately apparent, namely, the dispersion curve consists of two parts with a gap between them. The lower portion starts from the origin (in contrast to the case with $\vec{B}_0$ perpendicular to the surface), rises just to the right of the light line $\omega = kc$, bends over, and terminates when the curve intersects the dispersion curve for bulk magnetoplasmons (bulk polaritons) defined by $\alpha^2 = k^2 - \omega^2\epsilon_v/c^2 = 0$. The upper branch starts on the line defined by $\epsilon_{xx} = 0$, rises, and then approaches the asymptotic frequency for unretarded surface magnetoplasmons found by Pakhomov and Stepanov[21] and defined by the equation $1 + \epsilon_{xx} - i\epsilon_{xz} = 0$. In the large-wave-vector unretarded limit, the electric vector for the upper branch executes a circular motion in the sagittal plane. At the small wave vector extremity of this branch, the electric vector is plane polarized perpendicular to the surface. Note that this branch stops before it reaches the light line.

The reduced wave vector at which the upper branch starts is specified by the equation

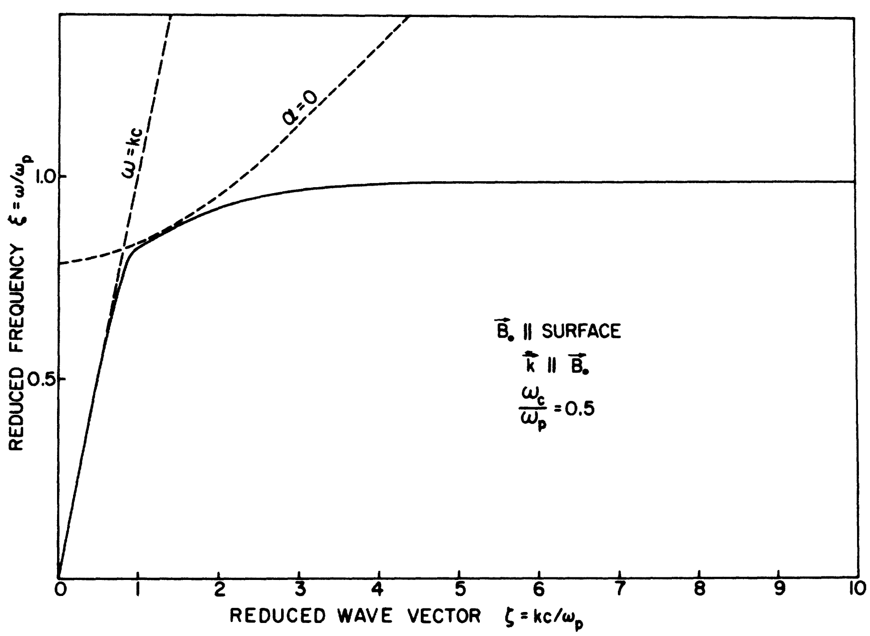

FIG. 8. Surface polariton dispersion curve for n-InSb with $\vec{B}_0$ parallel to the surface, $\vec{k} \parallel \vec{B}_0$, and $\omega_c = 0.5\ \omega_p$. The dashed curve $\alpha = 0$ is a bulk polariton dispersion curve.

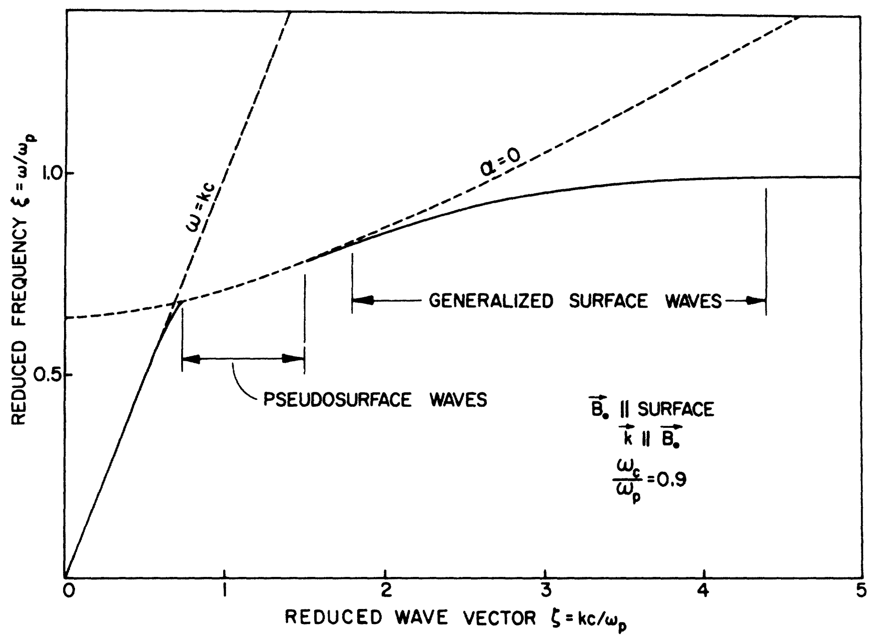

FIG. 9. Surface polariton dispersion curve for n-InSb with $\vec{B}_0$ parallel to the surface, $\vec{k} \parallel \vec{B}_0$, and $\omega_c = 0.9\,\omega_p$. The dashed curve $\alpha = 0$ is a bulk polariton dispersion curve.

$$\zeta_s^2 = \frac{1+\eta^2}{1-(1+\eta^2)/\epsilon_\infty^2\eta^2}\,, \tag{34}$$

where $\zeta = |k|c/\omega_p$ and $\eta = \omega_c/\omega_p$. If ζ_s^2 is to be finite and positive, then ϵ_∞ and η must satisfy the inequality

$$\epsilon_\infty > (1+\eta^2)^{1/2}/\eta = (\omega_c^2+\omega_p^2)^{1/2}/\omega_c\,. \tag{35}$$

That a gap can exist in the dispersion curve is evident from a consideration of Fig. 7 where ϵ_v is plotted against ξ. There is a region below the line $\epsilon_{xx} = 0$ where ϵ_v is large and positive, so that α^2 is negative for finite wave vectors and no surface wave exists. If the unretarded surface-magnetoplasmon frequency lies *above* the line $\epsilon_{xx} = 0$, as it does for InSb with $\eta = 0.5$, then a surface wave exists both above and well below this line, and a gap must exist just below this line. To have a surface-wave branch above the line $\epsilon_{xx} = 0$, the ratio ω_c/ω_p must exceed a critical value specified by Eq. (35). For InSb, this critical value is 0.064.

For a value of ϵ_∞ just below the right-hand side of Eq. (35), the upper branch lies below the line $\epsilon_{xx} = 0$ and to the right of the lower bulk dispersion curve. As ϵ_∞ decreases, the gap rapidly closes, and only a single branch remains.

Turning now to the case of $k < 0$, we plot in Fig. 6 the surface-polariton dispersion curves for n-type InSb with $\eta = 0.5$ and $k < 0$. It is clear that the situation is qualitatively different from that where $k > 0$. We now have a complete lower branch running from the origin out to the asymptotic value specified by the equation $1 + \epsilon_{xx} + i\epsilon_{xz} = 0$ given by Pakhomov and Stepanov.[21] In addition, however, we have an upper branch which starts at the light line where $\epsilon_{xx} = 1$, rises to the right of the light line, and ends when it meets the upper bulk-polariton dispersion curve, $\alpha = 0$. The upper branch for $k < 0$ exists whenever $\epsilon_\infty > 1$ and $\eta > 0$. It may be remarked that, for both $k > 0$ and $k < 0$, there are frequencies at which both a bulk wave and a surface wave can propagate, a situation which does not exist in the absence of a magnetic field.

C. $\vec{B}_0$ parallel to the surface, $\vec{k} \parallel \vec{B}_0$

This configuration is more like that discussed in Sec. III A ($\vec{B}_0$ perpendicular to the surface) than that discussed in Sec. II B ($\vec{B}_0$ parallel to the surface, $\vec{k} \perp \vec{B}_0$). For general values of k, the electric vector traces out an ellipse which contains the normal to the surface and is inclined to the direction of propagation. Solutions corresponding to two decay constants must be superposed in order to satisfy the boundary conditions.

The equation which determines the decay constants α_i [Eq. (6)] becomes

$$\epsilon_1\alpha^4 - [(\epsilon_1+\epsilon_3)\kappa_1^2 + (\omega^2/c^2)\epsilon_2^2]\alpha^2 + \epsilon_3[\kappa_1^4 - (\omega^4/c^4)\epsilon_2^2] = 0\,. \tag{36}$$

The solutions are

$$\alpha_{1,2}^2 = \kappa_1^2\left(\frac{\epsilon_1+\epsilon_3}{2\epsilon_1}\right) + \frac{\omega^2}{c^2}\frac{\epsilon_2^2}{2\epsilon_1} \pm \left\{\left[\kappa_1^2\left(\frac{\epsilon_1-\epsilon_3}{2\epsilon_1}\right) + \frac{\omega^2}{c^2}\frac{\epsilon_2^2}{2\epsilon_1}\right]^2 + k^2\frac{\omega^2}{c^2}\frac{\epsilon_3}{\epsilon_1}\frac{\epsilon_2^2}{\epsilon_1}\right\}^{1/2}. \tag{37}$$

The dispersion relation can be obtained from either Eq. (17) or (18). The simpler relation seems to follow from the latter equation which gives

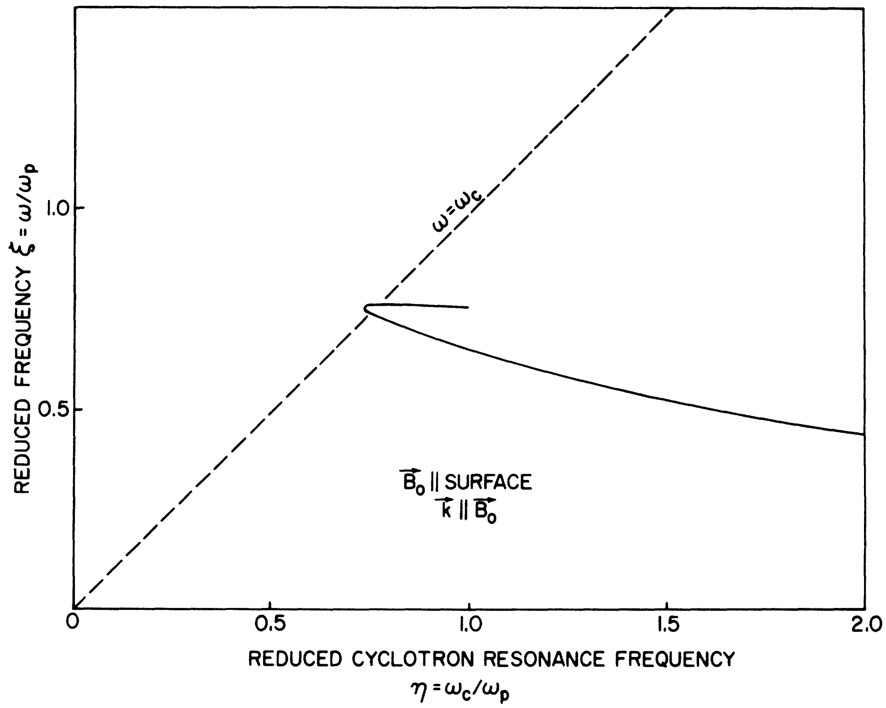

FIG. 10 Frequencies of endpoints of pseudosurface waves vs magnetic field for n-InSb with $\vec{B}_0$ parallel to the surface and $\vec{k} \parallel \vec{B}_0$.

$$(\alpha_0+\alpha_1+\alpha_2)\alpha_1\alpha_2\epsilon_1+[\alpha_0(\alpha_1+\alpha_2)+\alpha_1^2+\alpha_1\alpha_2+\alpha_2]\alpha_0\epsilon_1\epsilon_3+\alpha_0\kappa_1^2\epsilon_3(1-\epsilon_3)=0. \quad (38)$$

This relation is equivalent to but a little simpler than the corresponding dispersion relation given by Chiu and Quinn.[24,25] As in the case of $\vec{B}_0$ perpendicular to the surface, k appears only as even powers in Eqs. (37) and (38), so reciprocity holds.

We have obtained numerical solutions to Eqs. (37) and (38) using a high-speed computer for the case of n-InSb with $\omega_c/\omega_p = 0.5$. The dispersion curve, which is plotted in Fig. 8, starts at the origin, rises just to the right of the light line, and then flattens out to an asymptotic value given by the equation

$$\epsilon_1(\epsilon_3/\epsilon_1)^{1/2} = -1 \quad (39)$$

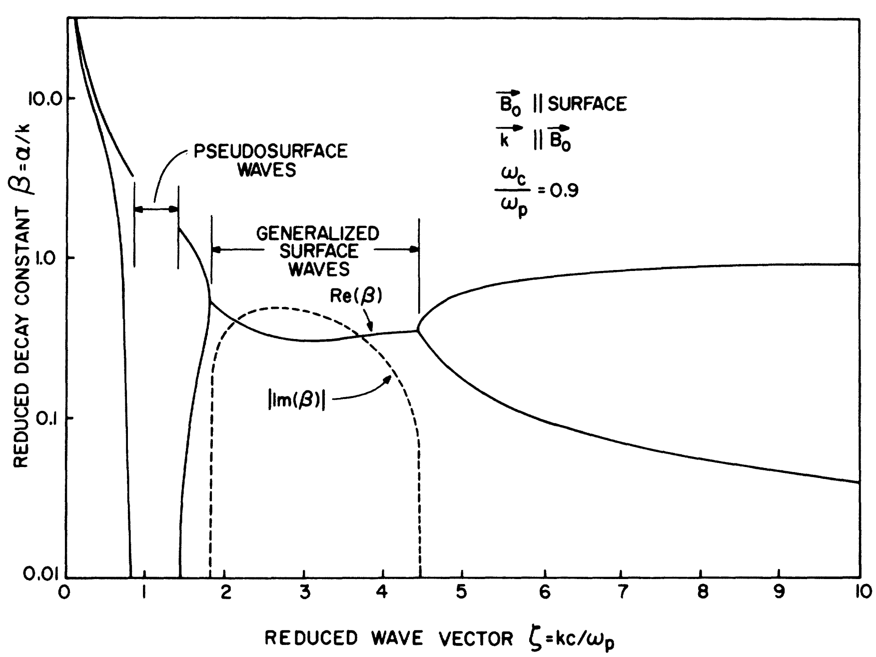

FIG. 11. Reduced decay constants versus wave vector for n-InSb with $\vec{B}_0$ parallel to the surface, $\vec{k} \parallel \vec{B}_0$, and $\omega_c = 0.9\ \omega_p$.

obtained by Pakhomov and collaborators.[21,22]

In the asymptotic limit, the surface wave is circularly polarized in the sagittal plane, and only one decay constant is required. The asymptotic decay constant is given by

$$\alpha = k(\epsilon_3/\epsilon_1)^{1/2} . \quad (40)$$

Just as in the case $\vec{B}_0$ perpendicular to the surface, both ϵ_1 and ϵ_3 must be negative in order to have a bonafide surface wave in the large-k limit—i.e., $\omega_c < \omega_p$.

Note in Fig. 8 the manner in which the surface wave dispersion curve is repelled by the bulk-magnetoplasmon dispersion curve $\alpha = 0$. No pesudo-surface-wave region exists in the present case. Also no upper branch analogous to that in Fig. 2 exists. If we increase ω_c/ω_p to 0.9, however, we get a pseudosurface-wave region as shown in Fig. 9, but again no upper branch. An equation specifying the end points of the pseudosurface-wave region can be obtained in a manner analogous to that employed for $\vec{B}_0$ perpendicular to the surface. The result is

$$[\epsilon_1(\epsilon_1 \pm \epsilon_2 - 1)]^{1/2}[\epsilon_2^2 \pm \epsilon_2(\epsilon_1 + \epsilon_3)]^{1/2} + \epsilon_2(\epsilon_2 \pm \epsilon_1 \pm 1) = 0 . \quad (41)$$

A plot of the end points as a function of ω_c/ω_p is given in Fig. 10.

For the case plotted in Fig. 9, $\omega_c/\omega_p = 0.9$, both decay constants are real and positive at large wave vector. However, there is a region from $\zeta = 1.8$ to $\zeta = 4.4$ where the decay constants are complex conjugates, and the surface polariton is a generalized surface wave. The end points of a region of generalized surface waves are characterized by $\alpha_1 = \alpha_2 = \alpha$. This means that the discriminant of Eq. (37) must be zero, and one gets

$$k^2 = (\omega^2/c^2)\epsilon_1(\epsilon_v - \epsilon_3)/(\epsilon_1 - \epsilon_3) \quad (42)$$

and

$$\alpha^2 = (\omega^2/c^2)(\epsilon_3/\epsilon_1)\epsilon_2^2/(\epsilon_1 - \epsilon_3) . \quad (43)$$

These equations together with the dispersion relation with $\alpha_1 = \alpha_2 = \alpha$ determine the end points.

The reduced decay constants β_i are plotted against reduced wave vector in Fig. 11 for the case $\omega_c/\omega_p = 0.9$. In the generalized-surface-wave region the real and imaginary parts of β are plotted. Note that as $\zeta \to \infty$, one of the β's approaches unity while the other approaches a small but nonzero value.

The electric field component ratios are plotted against wave vector in Fig. 12. As $\zeta \to \infty$, the ratio $-iE_z/E_x \to 0$ and $iE_y/E_x \to 1/\beta_2$. The electric vector in this limit lies in the sagittal plane and executes a circular motion. For finite ξ, the motion is elliptical, the ellipse containing the normal to the surface and being inclined to the direction of propagation.

Dispersion curves similar to those shown in Figs. 8 and 9 have been obtained by Quinn and Chiu[25] for n-type GaAs. Their results are complicated somewhat by a choice of parameters which lead to strong interaction with surface optical phonons. However, they find that a gap appears in the dispersion curve for ω_c/ω_p greater than a critical value, in agreement with our results for n-type InSb.

IV. DISCUSSION

The calculations reported in the present paper have been carried out with the aid of a number of

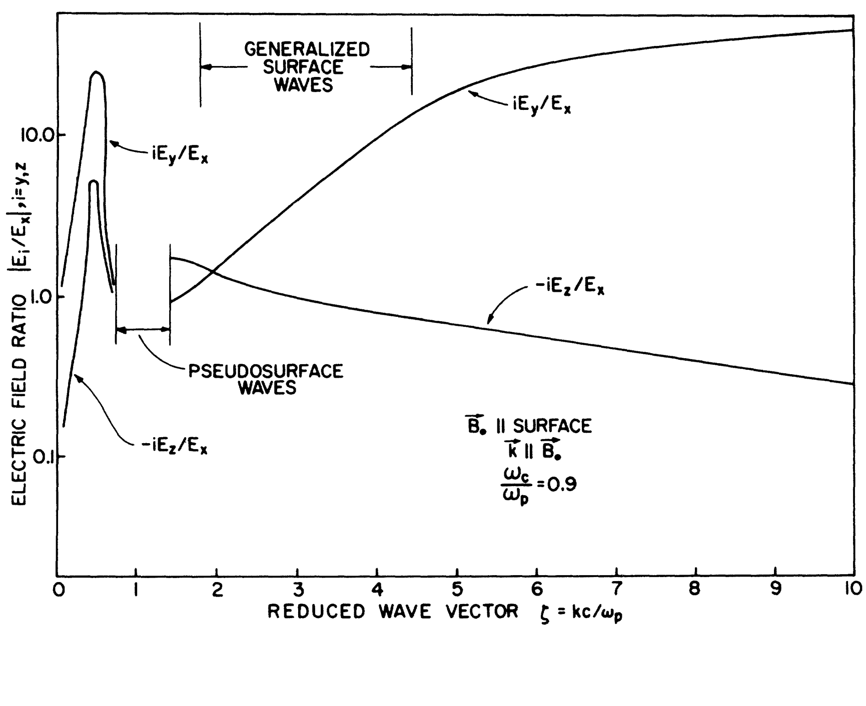

FIG. 12. Electric field ratios vs wave vector for n-InSb with B_0 parallel to the surface, $\vec{k} \parallel \vec{B}_0$, and $\omega_c = 0.9\ \omega_p$.

approximations. It would be interesting to study the effect of removing these approximations. In making a detailed comparison between theory and experiment, one should certainly include damping associated with the scattering of the charge carriers by lattice vibrations or imperfections. It would be interesting to see under what circumstances effects arising from spatial dispersion can be observed. The assumption that the dielectric tensor is a simple step function at the surface needs modification if depletion or accumulation layers are present. The effect of these layers on the surface polariton dispersion curves has already been investigated by the present authors.[36] Another approximation in the present work is the neglect of the interaction between magnetoplasmons and optical phonons. This is justified in n-type InSb for carrier concentrations $> 10^{18}/\mathrm{cm}^3$ such as in the experiments of Marschall, Fischer, and Queisser.[11] Some theoretical results including the interaction with optical phonons have been reported recently by Quinn and Chiu[25] for n-type GaAs. We prefer to focus attention on the phenomena exhibited by the magnetoplasmon-photon system alone and avoid complications associated with optical phonon interactions.

In the present investigation the detailed ω-vs-$\vec{k}$ curves have not been determined in the pseudosurface wave region. This is simply due to the additional complication of having to deal with a complex wave vector. However, it should be emphasized that the pseudosurface waves are well-defined modes of the system. In fact, the dispersion curves in the pseudosurface-wave region are simply the continuations of the dispersion curves from the ordinary-surface-wave regions. Any technique that is used to detect the modes in one region will work effectively in the other. The situation is similar to that for pseudosurface elastic waves.[35]

The results of the present work are applicable to materials such as n-type InSb with simple spherical energy bands. It would be of interest to consider n-type Ge or PbTe which have multiple anisotropic energy surfaces. One may expect additional surface polariton branches for appropriate orientations of the magnetic field arising from the presence of subgroups of carriers with different effective masses. An investigation of p-type InSb or Bi should reveal additional phenomena arising from several types of carriers.

The experimental observation of the surface polariton dispersion curves discussed in the present paper for n-type InSb should be possible using superconducting or Bitter magnets giving fields in the 50-kG range. One wants to operate under conditions such that $\omega_c\tau > 1$. The simplest configuration is probably that with $\vec{B}_0$ parallel to the surface, $\vec{B}_0 \parallel \vec{k}$. Other configurations will probably require somewhat more complicated mirror arrangements or ports normal to the magnet axis, but are nevertheless feasible.

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