# COMMENTS AND ADDENDA

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## Influence of the $Z_1^3$ contribution to stopping power on the evaluation of mean excitation potentials and shell corrections*

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(Received 15 May 1973)

The careful experimental evaluations of mean excitation potentials and of shell corrections need to be reexamined in view of the recent theoretical evaluation of the $Z_1^3$ contribution to the stopping power of matter for charged particles. We present here stopping power formulas including the $Z_1^3$ contribution and apply these to recalculate the excitation potentials and shell corrections derived from very precise measurements of stopping power for some elements in $20 \le Z_2 \le 30$. Including the $Z_1^3$ contribution does not appear to produce a significant change (considering current experimental accuracy) in the mean excitation potentials for elements in this range, but important changes are found in both the shape and magnitude of the shell-correction curves.

### I. INTRODUCTION

Recently, very careful measurements of stopping powers for 5–12-MeV protons and deuterons[1] have been used to obtain very exact values for excitation potentials and shell corrections for the elements $Z_2 = 20–30$.[2] Evidence has been accumulating for several years now indicating the presence of a contribution to stopping power at low energies (e.g., protons of energy $\lesssim 10$ MeV) which depends on the sign of the charge of the incident particle. Theoretical calculations of the $Z_1^3$ contribution to stopping power[3-5] ($Z_1$ is the charge number of the incident particle) and ranges[5,6] of charged particles are now available and can be incorporated in the experimental evaluation of mean excitation potentials and shell corrections. We will reexamine the data presented in Ref. 2 to illustrate the influence of the $Z_1^3$ effect in these measurements.

### II. MEAN EXCITATION POTENTIALS

For a heavy particle of charge $Z_1|e|$, and velocity $v_1 \equiv \beta c$ (energy $E_1$), moving in a material with atomic number $Z_2$ and atomic density $n_2$, the stopping power, including the $Z_1^3$ contribution, may be written in the form [Ref. 4, Eq. (12)]

$$\frac{mc^2\beta^2}{4\pi(Z_1e^2)^2 n_2 Z_2}\left(-\frac{dE_1}{dx}\right) = L(\beta, Z_2) + D(\beta, Z_1, Z_2). \tag{1}$$

The term $D(\beta, Z_1, Z_2)$ which arises from the $Z_1^3$ contribution to the stopping power is given by[4,6]

$$D(\beta, Z_1, Z_2) \equiv \frac{Z_1}{Z_2^{1/2}} \frac{F(1.8/x^{1/2})}{x^{3/2}}, \tag{2}$$

where $x \equiv \beta^2/\alpha^2 Z_2$ and $\alpha = e^2/\hbar c$. The function $F$ is defined and plotted in Ref. 4 and is tabulated in Ref. 7. The dimensionless function $L(\beta, Z_2)$ is given by the Bethe expression

$$L(\beta, Z_2) = f(\beta) - \ln I - C/Z_2, \tag{3}$$

where $I$ is the mean excitation potential of the target, $C/Z_2$ represents the shell corrections, and

$$f(\beta) \equiv \ln[2mc^2\beta^2/(1-\beta^2)] - \beta^2. \tag{4}$$

The experimental stopping-power measurements[1,2] may be written in the reduced form $L'(\beta, Z_1, Z_2)$ defined by

$$L'(\beta, Z_1, Z_2) \equiv f(\beta) - \ln I' - (C/Z_2)'. \tag{5}$$

For agreement between theory and experiment we should have

$$L'(\beta, Z_1, Z_2) = L(\beta, Z_2) + D(\beta, Z_1, Z_2). \tag{6}$$

TABLE I. Shell corrections evaluated from the stopping power data of Ref. 1. The theoretical $Z_1^3$ contribution to the stopping power and shell corrections for Ca, Ti, Fe, and Cu are listed for protons of energy 2.50–12.00 MeV. The mean excitation potential used in Eq. (8) is given by $I_{asy}$ from Table I of Ref. 2.

| | Ca ($Z_2=20$) | | Ti ($Z_2=22$) | | Fe ($Z_2=26$) | | Cu ($Z_2=29$) | |
|---|---|---|---|---|---|---|---|---|
| $E$ (MeV) | $10^2D$ | $C/Z_2$ | $10^2D$ | $C/Z_2$ | $10^2D$ | $C/Z_2$ | $10^2D$ | $C/Z_2$ |
| 2.50 | 5.03 | 0.195 | 5.16 | 0.199 | 5.66 | 0.219 | 5.45 | 0.221 |
| 3.00 | 4.40 | 0.191 | 4.52 | 0.196 | 4.67 | 0.217 | 4.81 | 0.222 |
| 3.50 | 3.84 | 0.183 | 3.99 | 0.186 | 4.18 | 0.216 | 4.28 | 0.218 |
| 4.00 | 3.45 | 0.176 | 3.56 | 0.177 | 3.81 | 0.210 | 3.86 | 0.212 |
| 4.50 | 3.10 | 0.168 | 3.25 | 0.168 | 3.45 | 0.200 | 3.53 | 0.205 |
| 5.00 | 2.80 | 0.160 | 2.93 | 0.161 | 3.16 | 0.191 | 3.29 | 0.199 |
| 5.50 | 2.58 | 0.154 | 2.68 | 0.154 | 2.88 | 0.182 | 3.01 | 0.193 |
| 6.00 | 2.35 | 0.146 | 2.49 | 0.149 | 2.68 | 0.174 | 2.80 | 0.187 |
| 6.50 | 2.18 | 0.140 | 2.30 | 0.143 | 2.47 | 0.167 | 2.60 | 0.182 |
| 7.00 | 2.03 | 0.135 | 2.14 | 0.139 | 2.31 | 0.161 | 2.42 | 0.177 |
| 7.50 | 1.91 | 0.130 | 1.99 | 0.134 | 2.17 | 0.157 | 2.27 | 0.172 |
| 8.00 | 1.78 | 0.127 | 1.86 | 0.129 | 2.04 | 0.152 | 2.15 | 0.168 |
| 8.50 | 1.67 | 0.122 | 1.75 | 0.126 | 1.91 | 0.147 | 2.03 | 0.163 |
| 9.00 | 1.58 | 0.118 | 1.65 | 0.123 | 1.82 | 0.143 | 1.91 | 0.160 |
| 9.50 | 1.49 | 0.116 | 1.57 | 0.119 | 1.72 | 0.140 | 1.81 | 0.155 |
| 10.00 | 1.42 | 0.113 | 1.49 | 0.117 | 1.63 | 0.137 | 1.72 | 0.152 |
| 10.50 | 1.34 | 0.110 | 1.42 | 0.113 | 1.55 | 0.134 | 1.64 | 0.149 |
| 11.00 | 1.28 | 0.107 | 1.36 | 0.112 | 1.48 | 0.131 | 1.57 | 0.144 |
| 11.50 | 1.22 | 0.105 | 1.29 | 0.109 | 1.42 | 0.129 | 1.50 | 0.140 |
| 12.00 | 1.17 | 0.103 | 1.24 | 0.106 | 1.36 | 0.127 | 1.44 | 0.138 |

If we require that the measured shell corrections $(C/Z_2)'$ are the same as the shell corrections that appear in the Bethe expression $C/Z_2$ then the mean excitation potential measured on the basis of Eq. (5), $I'$, is related to the $I$ in Eq. (1) by

$$\ln I' = \ln I - D(\beta, Z_1, Z_2). \tag{7}$$

To extract $I'$ values from the experimental $L'(\beta, Z_1, Z_2)$'s, we consider the "asymptotic fitting" procedure of Ref. 2. The quantity $f(\beta) - L'(\beta, Z_1, Z_2) - (C/Z_2)_{\text{theor}} = \ln I'$ is plotted versus proton energy where $(C/Z_2)_{\text{theor}}$ are the shell corrections calculated by Bonderup[8] or Walske.[9,10] Although the curves using the two different theoretical shell corrections have different shapes they approach a common horizontal asymptote at high energies which gives the value of $\ln I'$. The consistent placement of the asymptote is assured by requiring that the ratio of the distances of the two curves from the asymptote at 12 MeV is directly proportional to the ratio of the slopes at that point. If we take values of $\ln I'$ from Figs. 1 and 2 of Ref. 2, calculate $\ln I$ by Eq. (7) [values of $D(\beta, Z_1, Z_2)$ are shown in Table I], and do the asymptotic fitting procedure, we find values of $I$ for Ca and Cu within the error limits stated in Ref. 2. Since the $Z_1^3$ contribution to stopping power does not appear to produce very significant changes in the $I$ values in this $Z_2$ range we will use the values in Table I of Ref. 2 determined by asymptotic fitting $I_{asy}$ in our discussion of shell corrections.[11] As we shall see later,

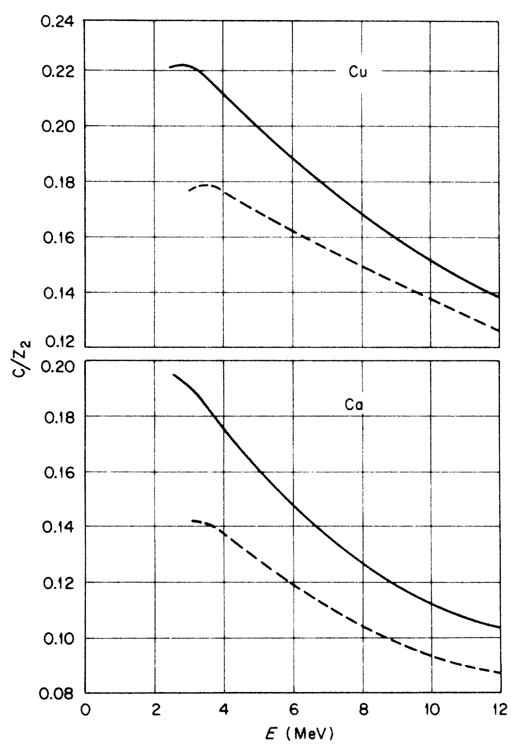


FIG. 1. Comparison of the shell corrections derived from experimental stopping powers with the $Z_1^3$ contribution accounted for (solid curves) with the results of Ref. 2 (dashed curves) for Ca ($Z_2=20$, $I_{asy}=193.6$ eV) and Cu ($Z_2=29$, $I_{asy}=320.8$ eV) for protons of energy 2.50–12.00 MeV.

changes in these mean excitation potentials due to a more careful "asymptotic fitting," or any other method, may be included very easily in the shell correction.

## III. SHELL CORRECTIONS

Given the mean excitation potentials calculated with the $Z_1^3$ contribution accounted for (the $I$'s above), the shell corrections may be evaluated from experimental data. From the stopping powers tabulated in Ref. 1 we find $L'(\beta, Z_1, Z_2)$. The shell corrections are then given by Eqs. (3) and (6) as

$$C/Z_2 = f(\beta) - L'(\beta, Z_1, Z_2) - \ln I + D(\beta, Z_1, Z_2) . \qquad (8)$$

The $Z_1^3$ contributions calculated according to Eq. (2) and the shell corrections evaluated using Eq. (8) are listed in Table I for protons of 2.50–12.00 MeV in four materials. For the mean excitation potentials we used the $I_{asy}$'s from Table I of Ref. 2 for this $Z_2$ range since no significant changes were found in Sec. II. At the lowest energies considered here, the $Z_1^3$ contribution to the stopping power is only ~2%, but this produces changes of 20–30% in the values derived for the shell corrections.

The results for the shell corrections with the $Z_1^3$ contribution to stopping power accounted for are compared in Fig. 1 (solid curves) with the shell corrections reported in Ref. 2 (dashed curves) for Ca ($Z_2 = 20$) and Cu ($Z_2 = 29$). A significant change in the shape of the curves has been introduced by including the $Z_1^3$ effect. Note that any errors involved in the evaluation of $I_{asy}$ will not alter the shape of these curves but will simply shift them up or down as we can see from Eq. (8).

## IV. CONCLUSION

We have discussed the influence of the $Z_1^3$ effect on the evaluation of mean excitation potentials and shell corrections from experimental data. Significant changes are found in the shell corrections in the limited $Z_2$ range considered here. This underscores the importance of including the $Z_1^3$ contribution to stopping power when experimental data are analyzed to obtain precision values for shell corrections. A more careful reanalysis may reveal small but significant changes in $I$ values for $20 \leq Z_2 \leq 30$ which can be readily incorporated in these calculations. Also, the changes in mean excitation potentials may be more significant as we go to larger values of $Z_2$.

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*Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corp.

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