COMMENTS AND ADDENDA

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in Physical Review Letters and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in The Physical Review or Physical Review Letters; (2) addenda to papers previously published in The Physical Review or Physical Review Letters, in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section may be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.

Impurity corrections to magnon damping in ferromagnetic metals*

Victor Korenman†

Department of Physics and Astronomy and Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742

(Received 24 September 1973)

It is shown that impurity vertex corrections do not significantly modify the expression for magnon damping recently found by Korenman and Prange.

In a recent paper[1] Korenman and Prange found a contribution to magnon damping in ferromagnetic metals, involving a decay into electron-hole pairs of the same spin, mediated by the spin-orbit interaction with the lattice. The expression found for the frequency shift was (equation numbers preceded by an I are from Ref. 1)

$$\mathrm{Im}\Delta\Omega \propto \mathrm{Im}\sum_m \int d\vec{k}\, \frac{|\epsilon_m(\vec{k})|^2 \vec{q}\cdot\vec{V}_m(\vec{k})(dn/dE_m)}{\Omega-\vec{q}\cdot\vec{V}_m(\vec{k})+i/\tau}, \tag{I.35}$$

where $\vec{q}$ and Ω are the wave vector and frequency of the magnon, $\vec{V}_m(\vec{k})$ is the electron velocity, $n(E)$ is the Fermi function, τ is the electron lifetime, and $\epsilon_m(\vec{k})$ is a function of band index and wave vector, defined in Eq. (I.36) and to be discussed below.

The most striking consequence of Eq. (I.35) is the prediction of increased damping for increasing electron mean free path (ultimately saturating) in agreement with experiments on the purity and temperature dependence of the ferromagnetic-resonance linewidth in Ni[2] and Co.[3] A number of our colleagues have since remarked that this mean-free-path dependence was not fully demonstrated since we did not include impurity vertex corrections in our calculation, and these apparently have a drastic effect. It seems worthwhile to indicate here why impurity corrections do not, in fact, play a significant role in this calculation.

The problem arises since Eq. (I.35) is the imaginary part of a particle-hole bubble diagram, with vertex ϵ. Ignoring the momentum dependence of ϵ, were we to evaluate the same diagram including impurity corrections, we would find[4] (taking a spherical Fermi surface)

$$\mathrm{Im}(\Delta\Omega) \propto \Omega\tau[T(1-T)-U]/[(1-T)^2+U], \tag{1}$$

where

$$T \equiv (ql)^{-1}\tan^{-1}(ql),$$
$$U \equiv (\Omega\tau)^2/(1+q^2l^2)^2,$$

the mean free path is l, and we have assumed $\Omega\tau/ql \ll 1$.

For large ql, T and U become small and Eq. (1) reduces to $\Omega\tau T$ which is our original result Eq. (I.43). However, we have taken this result as correct for all values of ql, leading to the prediction of decreasing $\mathrm{Im}(\Delta\Omega)$ with decreasing ql, while the expression in Eq. (1) becomes large for small ql.

The crucial factor which determines which of these behaviors is correct is, of course, the momentum dependence of the vertex $\epsilon_m(\vec{k})$. In calculations of transport coefficients, such as the electrical conductivity, irregularities at small ql do not appear because the Fermi surface average of the corresponding vertex vanishes. Any irregular part of the coefficient would be, in fact, proportional to the square of that average.

We use a somewhat more general argument to show that no irregularity occurs in the present case. Equation (I.35) is the random-phase approximation to the imaginary part of the correlation function of the operator

$$O(q)\equiv\sum_m\int d\vec{k}\,\psi_m^\dagger(\vec{k}+\tfrac{1}{2}\vec{q})\,\epsilon_m(\vec{k})\,\psi_m(\vec{k}-\tfrac{1}{2}\vec{q})\;.$$

Now any singular small-q behavior of this correlation function will reflect diffusive behavior in the time evolution of $O(t)$, and that will occur only if $\delta O(q=0)$ is a conserved quantity, where δO is the deviation of O from equilibrium. If we can show that $\langle O(q=0)\rangle$ is fixed by the symmetry of the situation, then $\delta O(0)$ cannot have a conserved part, $O(t)$ will relax rapidly to its equilibrium value and diffusive behavior will not occur.

But, as noted after Eq. (I.40), $\epsilon_m(\vec{k})$ is the first-order variation of the exchange energy in band m at momentum $\vec{k}$, under a rotation of the applied magnetic field. Then $O(q=0)$ is the variation of the total Hartree-Fock exchange energy of the system under the same rotation. As a global scalar property of the system the total exchange energy can only have a second-order variation when the field is rotated from a symmetry direction of the crystal. Now ferromagnetic resonance experiments are generally done with the field in a symmetry direction, and we have implicitly assumed this by taking the field and total magnetization in the same direction. Then, in the context of our calculation, $\langle O(0)\rangle$ is constrained by symmetry to vanish and cannot have a partially conserved fluctuation. The correlation function of $O(q)$ does not show diffusive behavior, and impurity vertex corrections do not modify the mean free path dependence of magnon damping as derived in I.

If the external field is not in a symmetry direction the above argument is invalid, but we may note that magnetic anisotropy is a consequence of the spin-orbit interaction and argue that the Fermi-surface average of $\epsilon(\vec{k})$ is of second order in this interaction. This gives an irregularity at small ql whose strength is at most of fourth order in V_{so}, compared to the second-order term we have found. Such a term would give only a small correction. Changes in the damping with field direction have not been found[5] but the experiments to date have not been exhaustive.

We do not yet have a complete argument since we have only discussed impurity corrections to the correlation function found in I, while a calculation taking account of impurities from the start might lead to a quite different answer. Such a calculation (the same as done by Fulde and Luther[6] but including spin-orbit interaction in the basic Green's functions) presents the difficulty that the basic Green's functions are band functions while the impurity problem is only readily solved in a one-band approximation. To avoid this difficulty we have reformulated the magnon-damping problem using a projection-operator technique based on the work of Mori.[7] Even in the presence of impurities the damping is given by the correlation function of an operator such as O, but with ϵ replaced by a specific matrix element of the spin-orbit interaction. Again, when the field is in a symmetry direction, we can show that $\langle O(q=0)\rangle$ vanishes, so impurity corrections do not lead to diffusive behavior. Making a random-phase-like decomposition of the correlation function we regain the behavior found in I, though we have not been able to demonstrate the numerical identity of the two results. Away from a symmetry direction, the irregularity is again of at least fourth order. Details of this calculation will appear elsewhere.

---

*Supported in part by the U.S. Army Research Office, Durham, N.C., under Grant No. DA-ARO-D-21-124-71-G-114.

†Alfred P. Sloan Foundation Fellow.

[1]V. Korenman and R. E. Prange, Phys. Rev. B 6, 2769 (1972). Hereafter referred to as I. Equations from this work will be referred to with I preceding the equation number.

[2]S. M. Bhagat and L. L. Hirst, Phys. Rev. 151, 401 (1966).

[3]S. M. Bhagat and P. Lubitz, AIP Conf. Proc. 10, 125 (1973).

[4]See, for example, P. Fulde and A. Luther, Phys. Rev. 170, 570 (1968).

[5]J. R. Anderson, S. M. Bhagat, and F. L. Cheng, Phys. Status Solidi 45, 357 (1971).

[6]P. Fulde and A. Luther, Phys. Rev. 175, 337 (1968).

[7]H. Mori, Prog. Theor. Phys. 33, 423 (1965), and subsequent papers.