PHYSICAL REVIEW B VOLUME 9, NUMBER 7 1 APRIL 1974

Impurity-interaction effects in dilute silver-manganese alloys

D. A. Tindall and M. H. Jericho*
Physics Department, Dalhousie University, Halifax, Nova Scotia, Canada

D. Jha
Physics Department, Patna University, Patna, India

(Received 5 November 1973)

We report measurements down to 60 mK of the electrical resistivity of three AgMn alloys containing nominally 1.12-, 0.12-, and 0.005-at.% Mn. As the temperature is lowered, the resistivity reaches a Kondo-type minimum and then rises logarithmically to a maximum at a temperature $T_{\max}$, which is approximately proportional to concentration ($\sim 25c$ for c in at.%): Below $T_{\max}$ the resistivity decreases very linearly and then (at least for the 1.12-at.% alloy) it abruptly flattens out. It is believed that ordering of the Mn spins gives rise to the decrease in resistivity below $T_{\max}$, and we have assumed that it is possible to write the resistivity as $\rho=\rho_s F$, where ρ_s is the spin resistivity and F takes the effects of ordering into account by way of an internal magnetic field distribution. It is shown that the data can be successfully interpreted with a spin-scattering term of the form $\rho_s = A - B\ln(T^2+W_0^2c^2)^{1/2}$, where A and B are dependent on concentration and $W_0 = 8.5$ K/at.%. The form of $F(T)$ was one used previously with some success in this system, viz., $F(T)\sim\tan^{-1}(a_2T/c)$, with $a_2 = 0.1$ at.% K^{-1}. Evidence is also presented to show that it might be possible to construct a universal curve for the AgMn alloy system if $[\rho(T)-\rho(0)]/c$ is plotted as a function of $T/T_{\max}$. The resistivity of the 0.12-at.% alloy is linear below 0.5 K with a large temperature coefficient (0.12 K^{-1}) and we suggest it may make a useful thermometer in this regime, especially for monitoring temperatures in the dilution refrigerator.

I. INTRODUCTION

In a previous paper,[1] hereafter referred to as I, results on the electrical resistivity and thermal conductivity of a number of AgMn alloys down to 0.3 K were presented. The alloys showed the well-known resistance-minimum phenomenon which, for the more concentrated samples, was followed at lower temperatures by a maximum in the resistance. For temperatures significantly below $T_{\max}$, the temperature for the maximum, the resistance was proportional to temperature for some of the more concentrated samples. These features were fairly successfully interpreted in terms of a phenomenological model based on earlier work by Blandin,[2] Béal,[3] and Harrison and Klein.[4] The basic assumption of the model is that in Ag the Mn atoms have a fully developed magnetic moment and that the impurities interact via the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction to produce an antiferromagnetically ordered spin system below a certain temperature. In the ordered state only a small number of impurities are correlated in any given region in the alloy and the magnetic field that exists at a given impurity site is a random variable due to the random distribution of the impurities and is described by a probability distribution function $P(g\mu_BH)$: $P(g\mu_BH)dH$ is the probability that the magnetic energy of a given impurity due to the internal field H caused by all the other moments will be between $g\mu_BH$ and $g\mu_B(H+dH)$, where g is the Lande g factor and μ_B the Bohr magneton. When the sample temperature is such that $k_BT \gg g\mu_BH$, the model assumes that the electron-impurity scattering is elastic and the resistivity is essentially unaffected by interactions and is given by a suitable resistivity expression for the Kondo effect. For $k_BT \ll g\mu_BH$, Zeeman splitting of the impurity states is large so that spin-flip scattering of the conduction electrons becomes an inelastic process and the Kondo effect is quenched. The lowest temperature reached in I was 0.3 K and the following questions remained unanswered: (i) Does a very dilute alloy such as alloy No. 6 of I, which contained 50 ppm of Mn, also show a resistance maximum if cooled to a sufficiently low temperature? (ii) Down to what temperature is the resistance of the more concentrated alloys linear in T? These questions are of interest, particularly if viewed in connection with experimental results on other alloy systems. Laborde and Radhakrishna,[5] for example, have measured the electrical resistivity of AuFe alloys down to 36 mK and report for very low concentrations a concentration dependence of the resistance maximum faster than the first power of the concentration and, in fact, do not find a maximum down to their lowest temperature for a 50-ppm Fe in Au alloy.

The extent of the linear region of the temperature-dependent resistivity for the more concentrated alloys is also of interest, since it gives some information about the shape of the probability function near $H=0$, as discussed previously.[1,6]

To answer these questions and to hopefully reach

TABLE I. Manganese concentrations and some data on the alloys.

Alloy No.	Nominal[a] concentration (at.%)	T_{max}[b] (K)	T_{max}/c[b] (K/at.%)	$\rho(0)$[c] fitted ($\mu\Omega$ cm)	$\rho(0)$ fitted[c] / c ($\mu\Omega$ cm/at.%)	$\rho(T_{\text{max}})$ ($\mu\Omega$ cm)	Calculated height of resistivity peak ($0.2c$ $\mu\Omega$ cm)	From Fig. 5[d] Calculated residual resistivity $\rho^*(0)$ ($\mu\Omega$ cm)	$\rho^*(0)/c$ ($\mu\Omega$ cm/at.%)
1	1.12	(22)	(20)	1.6108	1.44	...	...	...	...
2	0.332	7.4	22	0.473	1.42	0.5385	0.0664	0.472	1.42
3	0.125	3.3	26	0.1749	1.40	0.2003	0.0250	0.175	1.40
4	0.0558	1.9	34	0.0795	1.42	0.0920	0.0112	0.0808	1.45
6	0.005	0.18	36	(0.0081)	(1.62)	0.00997	0.0010	0.00897	1.79
	0.006		30	(0.0078)	(1.30)		0.0012	0.00877	1.46

[a] Two concentrations are shown for alloy No. 6: this is explained in the text.

[b] The value in parentheses for alloy No. 1 (which has no maximum in the experimental range) is that used in plotting Fig. 5.

[c] Values in parentheses are uncertain because of the large extrapolation involved.

[d] The measured heights above $\rho(0)$ of the resistivity peaks for the more concentrated alloys can be expressed closely as $0.20c$ $\mu\Omega$ cm: thus we can calculate a residual resistivity $\rho^*(0)=\rho(T_{\text{max}})-0.2c$ on the assumption that the scaling law suggested by Fig. 5 is valid.

a better understanding of the resistivity of the AgMn system we have re-examined, down to about 60 mK, three of the samples previously reported in I.

In Sec. II we give a discussion of the experimental details and the electrical resistivity results on the alloys are given in Sec. III. In Sec. IV we attempt to compare the results reported here as well as those on the other alloys from I, with the phenomenological model, although the approach adopted here is somewhat different than that of I.

II. EXPERIMENTAL DETAILS

We have extended the electrical resistivity measurements on three AgMn alloys (Nos. 1, 3, and 6 of I) from 0.3 to 0.06 K by cooling the samples in an Oxford Instruments Mark-III dilution refrigerator. A detailed description of the sample preparation was given in I: Suffice it to say here that the samples were annealed strips of dilute Mn in Ag alloys which measured approximately 2×0.2 mm in cross section with 45 mm between the potential probes. Table I shows some of the sample characteristics.

The sample holder consisted of a copper tube (15 cm long, 2 cm in diameter) screwed into the base of the mixing chamber of the dilution refrigerator; around this was closely wound thin insulated-copper wire, the purpose of which was to thermally bond the sample without connecting it electrically to the cryostat. The sample was greased onto this layer of wire and covered with a strip of bakelite that was screwed at each end to the tube. Four superconducting electrical leads (copper-clad Nb_3Zr) were soldered to the contacts on the sample and a length of about 5 cm of copper was removed from each lead in order to reduce the heat load on the mixing chamber. The leads were thermally anchored by GE7031 varnish at the mixing chamber, the still (0.6 K) and the pumped-helium vessel (1.2 K) before being run up a tube inside the helium bath.

On the lower end of the sample-mounting tube were attached a number of germanium thermometers (cryo-resistors, Cryocal Inc., Riviera Beach, Fla.) and a balanced mutual inductance containing cerium magnesium nitrate (CMN) for use as a susceptibility thermometer. The measurement of the CMN thermometer was made with a mutual inductance bridge (Cryotronics, Inc., High Bridge, N. J., Model No. ML17B, operating at 17 Hz) and an oscilloscope as a null detector. Low-resistance leads were used in the circuit to minimize effects due to changing lead resistances. The CMN thermometer was calibrated during each run in the range 4.5–1.5 K against a calibrated germanium thermometer and then the Curie law was used to measure temperatures below 1.5 K. It was found that the gradient of mutual inductance with respect to reciprocal temperature was very constant from run to run but that the intercept changed slightly (~ 1%), presumably due to small changes in the relative positions of the coils.

A carbon thermometer (Speer, 220 Ω), mounted in a holder made from copper wire, was thermally bonded to the lower end of the sample and the CMN thermometer was used to calibrate this during each run. The purpose of this procedure was to monitor the sample temperature, and in particular to provide a check upon any heating of the sample by the measuring current. Also, in later parts of the experiment, its resistance was sensed by a resistance thermometer bridge (Oxford Instrument Co., Oxford, England), the output of which controlled an amplifier feed-back current into a noninductively wound heater on the mixing chamber, in order to stabilize the temperature of the sample.

A four-probe technique was used to measure the electrical resistance of the sample. The measuring currents (which were reversed to eliminate thermal emfs in the circuit) generally used were 30 mA for alloy 6 and 2 mA for alloys 1 and 3. The heat generated in the samples was thus approximately 10 nW and in order to further check that this was not heating the samples unduly the measuring current was doubled and the resistance remeasured: It was always found that this did not significantly affect the value and so we conclude that resistive-heating effects were negligible with the measuring currents used.

The sample voltage was measured with a cryogenic relay modulator and lock-in amplifier (PAR Model No. HR8 with type-B preamplifier) as described by Jericho and March[7] with a typical sensitivity of about 1 nV. This enabled the sample resistance to be determined with an error of about 0.1%. The shape factor of the samples was determined with a micrometer and travelling microscope to an accuracy of 2%.

III. RESULTS

Figure 1 shows the measurements made on the three alloys investigated by us: subsequent figures show both our data and those of I. The resistivity of alloy No. 1 is linear with a slope of 30 nΩ cm K^{-1} from about 2 to 0.25 K and then flattens off rather abruptly below this temperature. At first sight this levelling off might suggest that the measuring current caused progressively more significant sample heating at the lowest temperatures and that the actual sample temperature was higher than shown. Resistivity determinations at several sample current settings gave the same result, however, so that the low-temperature levelling for alloy No. 1 is a genuine effect.

The results for alloy No. 3 show that for a manganese concentration of about 0.1 at.% the resistivity is a linear function of temperature from 0.5 K down to the lowest temperature obtainable in our dilution refrigerator (0.06 K). The temperature coefficient of resistance in this region is 0.12 K^{-1}.

The results for alloy No. 6, which contains about 50 ppm of Mn, show that the resistivity increases approximately logarithmically as the temperature is lowered but then goes through a maximum near 0.18 K. This is quite interesting and shows that the impurity-impurity interaction in this system is very long range (the mean impurity separation in this alloy is about 15 lattice spacings) and even more dilute alloys might be expected to show ordering effects.

With our low-temperature data it is possible to obtain reasonable estimates of $\rho(0)$ for alloys 1 and 3. From the flat region for alloy No. 1, $\rho(0)/c = 1.44$ μΩ cm/at.%, while a linear extrapolation of the results for No. 3 suggests $\rho(0)/c = 1.40$ μΩ cm/at.%. This small discrepancy is entirely consistent with the 2% error in the shape factor of the samples, especially when one considers that the value used in I for alloy No. 2 was $\rho(0)/c = 1.42$ μΩ cm/at.%.

The data for all the alloys are summarized in Table I. For alloy No. 6 two concentrations are shown because of the rather large uncertainty in its calculated concentration (50 ppm) due to the very dilute nature of this alloy. The two concentration values differ by only 10 ppm and, as we shall see later, 60 ppm seems more consistent with the logarithmic slope at high temperature.

IV. DISCUSSION

We will discuss the data with the help of the phenomenological model that was developed in I. There the basic idea was that the impurity interactions lead to an antiferromagnetic ordering of the impurity spins, which in turn produces a magnetic field at an impurity site. The random distribution of impurities then leads to a distribution of local

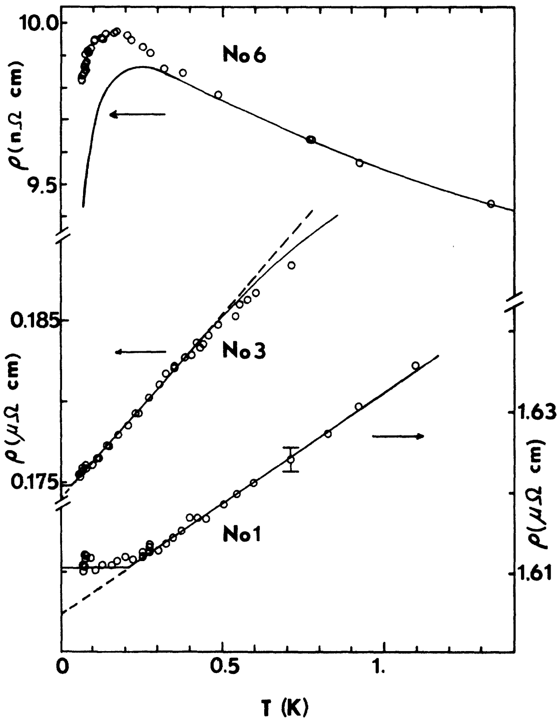

FIG. 1. Electrical resistivity as a function of temperature below 1.5 K. Solid lines show the behavior according to Eq. (8) with parameters as given in Table II; the dashed lines show where the theoretical curve deviates significantly from a straight line for the more concentrated alloys. The error bar drawn for alloy No. 1 corresponds to a resistivity error of ±1 part in 1000, as do the sizes of the circles for the others.

fields and the results in I were fairly successfully interpreted with a Lorentzian distribution of local fields:

$$P(g\mu_B H) = \Delta/\pi[\Delta^2 + (g\mu_B H)^2],$$

where Δ is the width of the distribution and is proportional to the impurity concentration, c. In calculating the temperature dependence of the resistivity, and hence the effect of the magnetic ordering, the electron-impurity scattering is assumed to be elastic for $g\mu_B H < k_B T$, so that for this field range the resistivity is described by the zero-field expression for the magnetic resistivity ρ_s. For $g\mu_B H > k_B T$ the magnetic-energy-level separation of the impurities is sufficiently large so that the conduction-electron scattering becomes an inelastic process and the resistivity in this limit is therefore assumed to have a small value ρ_0, which will be set equal to zero in our analysis. If ρ_I is the contribution to the resistivity from nonmagnetic scattering, then the temperature-dependent resistivity can be written

$$\rho(T) = (\rho_I + \rho_0) + [\rho_s(T) - \rho_0] F(k_B T/\Delta), \qquad (1)$$

where

$$F(k_B T/\Delta) = (2/\pi)\tan^{-1}(a_2 T/c).$$

As $T \to 0$ K, $F(k_B T/\Delta) \to 0$, so that $\rho(0) \to (\rho_I + \rho_0)$.

The increase in resistivity above the value at 0 K, $\delta\rho = \rho(T) - \rho(0)$, is thus expressed as a product of two functions. The first one gives essentially the temperature dependence of the resistivity in the absence of internal fields, while $F(k_B T/\Delta)$ takes care of the effects of internal fields. In I, $[\rho_s(T) - \rho_0]$ was assumed to be given by the second-order Born perturbation calculation for the spin scattering. Matho and Béal-Monod[8] used an expression of the same general form as Eq. (1) [see Eq. (78) of Ref. 8], but their expression was obtained from a pair-interaction model for impurity spins. It is interesting to note, however, that their function $L_{ex}(\bar{n}) = (1 - e^{-\bar{n}})/\bar{n}$, where $\bar{n} = T_2 c/T$ is almost equal to our $F(a_2 T/c)$ over the whole range of T/c if we set $(2/\pi)(a_2 T/c) = T/cT_2$. A comparison of the numerical value of a_2 with T_2 will be made in Sec. V. We have tried to fit an expression of the form of Eq. (1) to the results for alloys 1, 3, and 6, as well as the data of the rest of the AgMn alloys of I, over the whole temperature range for which results are now available. We find that the position of the resistivity maxima as well as the general shape of the curves near the maxima can be well described with a suitable choice of a_2 if ρ_s is of the form A-$B\ln T$. This form for ρ_s cannot, however, also explain the large extent of the region over which the resistivity is a linear function of temperature. Alloy No. 1, for example, is linear (to within our experimental error) between about 0.2 and 2 K, while alloy No. 3 is linear from about 0.5 to below 0.06 K. For low T and large c, Eq. (1) does not only contain a term proportional to T but also a term proportional to $T\ln T$. It is the presence of this latter term which made a fit to the data of alloys No. 1 and No. 3 impossible down to the lowest temperature.

The approach of the resistivity of alloy No. 1 to a constant below 0.2 K suggests that the distribution function for internal fields is not a maximum for zero field, but that it must show a considerable dip for low fields. We tried to incorporate this into our model by assuming that $P(g\mu_B H)$ is Lorentzian except in a region of width $g\mu_B H = \eta\Delta$. Δ is the width of the distribution and proportional to concentration, while $\eta = a_2 T_h/c_1$, where T_h is the temperature (0.24 K) at which the resistivity starts to level off for alloy No. 1; c_1 is the concentration of alloy No. 1. We therefore have reason to believe that the distribution resembles Fig. 8 of I, i.e., that the distribution function has a hole near the origin and that the width of this hole is proportional to concentration. The function F in Eq. (1) for the modified distribution is then

$$F\left(\frac{a_2 T}{c}\right) = \frac{\tan^{-1}(a_2 T/c) - \tan^{-1}\eta}{\frac{1}{2}\pi - \tan^{-1}\eta} \qquad (2)$$

This expression shows that since T_h is about 0.2 K for alloy No. 1, the effect of the hole in the distribution will be important only for this alloy.

In addition to being able to give the positions of the resistivity maxima fairly well, this expression for F also generates a $\ln T$ dependence of the function ρ_s to temperatures well below the resistance maximum, as can be seen in Fig. 2. In this figure we have plotted the following function vs $\ln T$:

$$G(T) \equiv \frac{\rho(T) - \rho(0)}{c\,F(a_2\,T/c)'} = \frac{\rho_s(T) - \rho_0}{c}.$$

To calculate $G(T)$, a_2 was taken to be 0.1, and $\rho(0)$ was obtained from the extrapolated resistivity curves for alloy No. 1 and No. 3 and the nominal concentrations of Mn. This gave $\rho(0)/c = 1.42$ μΩ cm. The magnitude of G for the more dilute samples such as Nos. 4 and 6 is concentration dependent, although the linearity of G in a $\ln T$ plot is relatively insensitive to the choice of c. For alloy No. 4, for example, $G(T)$ varies as $\ln T$ from 10 K to well below the temperature of the resistance maximum. $\rho_s(T)$ for the alloys is thus fairly well described by a function of the form A-$B\ln T$ over a large temperature range. This simple form seems to break down, however, as the region of linear temperature dependence of the resistivity is approached. Although the errors in G become large at the lowest temperatures, it is quite evident from Fig. 2 that G becomes essentially temperature independent for alloys No. 2 and 3, while G is practically constant

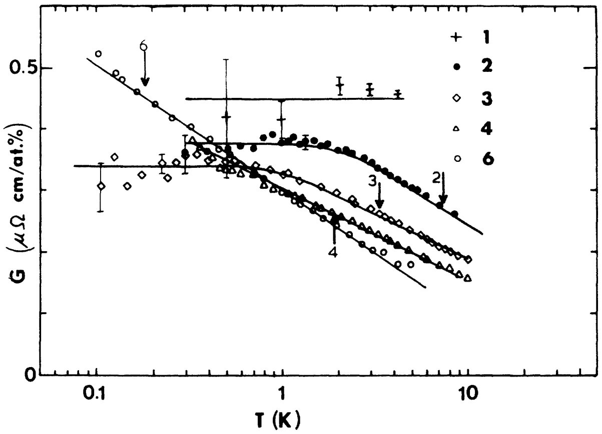

FIG. 2. Temperature dependence of the magnetic scattering term $G \equiv [\rho(T) - \rho(0)]/cF$ showing extension of the logarithmic behavior down to below $T_{\max}/3$: the arrows indicate the values of $T_{\max}$ for the various alloys. To avoid confusion, error bars (corresponding to ±1 part in 1000 in the resistivity) are drawn for only a representative selection of points: the errors are too small to show once the curves reach the negative-slope logarithmic region.

below 4 K for alloy No. 1.

At first it appears that the initial $\ln T$ dependence of ρ_s followed by a levelling off indicates that the spin scattering term might be described by an expression of the Hamann type suitably modified to include phase shifts,[9] so that the low-temperature plateau would be interpreted as the spin-compensated region. We have tried to fit such an expression to the results for a whole range of values of spin, Kondo temperature, phase shift, and unitarity limits, but we were unable, with such an expression, to account for the very abrupt change in slope displayed by the results in Fig. 2. We have, therefore, hesitated to apply such an equation to our data to determine these various parameters for this alloy system. It is of course possible that the particular form of G or ρ_s in Fig. 2 is spurious and simply a result of a poor choice for F. If ρ_s is indeed described by an equation of the Hamann type then F would have to have a much more complicated temperature dependence in the low-temperature limit in order to give a resistivity that is linear in T. In addition to being simple, the fact that our form for F can linearize the resistivity results on a $\ln T$ plot to well below $T_{\max}$ with the adjustment of only one parameter (i.e., a_2), suggests to us that our choice for F is a reasonable approximation.

If we therefore assume that F has the form given in Eq. (2), then we have to explain the rather abrupt change of slope in Fig. 2 of some of the higher concentration alloys. We find that an equation of the form

$$\rho_s = A - B\ln(T^2 + W^2)^{1/2}, \tag{3}$$

where W is a constant for a particular alloy, can account for the shape of the curves for alloys Nos. 2 and 3 very well, as shown by the solid lines in Fig. 2. Such an expression was first proposed by Suhl[10] in 1968 to account for impurity-interaction effects on the spin scattering. Recently Matho and Béal-Monod[8] and also Tsay and Klein[11] have proposed similar modifications of the $\ln T$ term. The values for W required to fit the data of alloy Nos. 2 and 3 are $W_2 = 2.1$ and $W_3 = 1.09$, respectively. It thus appears that W is approximately proportional to impurity concentration and thus we shall set $W = W_0 c$, where W_0 is a constant to be determined from the data. From the high- and low-temperature sections of the curves in Fig. 2 we can also then determine A and B. Evaluation of these parameters suggests that both are concentration dependent and of the form

$$A = 0.3c + 0.66c^2$$

and (4)

$$B = 0.07c + 0.16c^2$$

for c in at.%. Hence it appears that A and B have a similar dependence on concentration, so that we may instead write

$$A = (\alpha + \beta c)c, \quad B = (\alpha + \beta c)Dc, \tag{5}$$

where $\alpha = 0.30$ μΩ cm/at.%, $\beta = 0.66$ μΩ cm/(at.%)^2 and $D = 0.24$. The quadratic terms are thus only significant for the more concentrated samples.

Hence we have the following form for G:

$$G = \rho_s/c = (\alpha + \beta c)[1 - D\ln(T^2 + W_0^2c^2)^{1/2}]. \tag{6}$$

Table II summarizes the values of the parameters used to fit the data. We may rearrange this equation to give

$$\frac{2}{D}\left(\frac{G}{\alpha+\beta c} - 1\right) + 2\ln c = -\ln\left[\left(\frac{T}{c}\right)^2 + W_0^2\right], \tag{7}$$

so that a plot of the left-hand side (which we denote by g) vs T/c should result in a universal curve for the AgMn alloy system. Such a plot is shown in Fig. 3. The solid curve which is a plot of the

TABLE II. Values of parameters fitted to experimental points.

Parameter	Value
a_2	0.1 at.%/K
W_0	8.5 K/at.%
$\frac{1}{2}D$	0.12
α	0.30 μΩ cm/at.%
β	0.66 μΩ cm/(at.%)^2
$\rho(0)/c$	1.42 μΩ cm/at.%

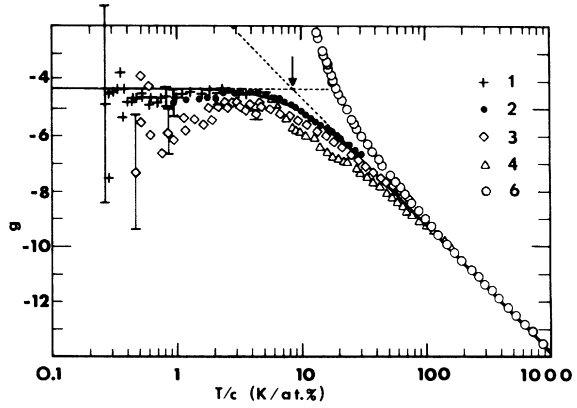

FIG. 3. g [Eq. (7)] plotted as a function of T/c for all the alloys. The solid curve is $-\ln[(T/c)^2+W_0^2]$ where $W_0=8.5$ and the dashed lines show the two asymptotes to this curve which intersect at $T/c=W_0$ as illustrated by the arrow. Representative error bars once again show the effect of ±1 part in 1000 error in the resistivity.

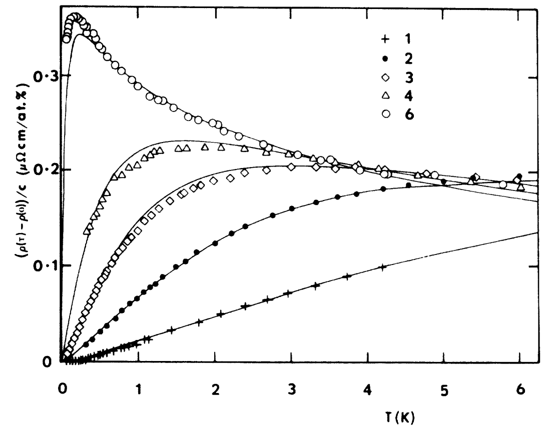

FIG. 4. Temperature dependence of the resistivity per impurity: Solid lines show the theoretical behavior according to Eq. (8) with parameters given in Tables I and II. An error of ±1 part in 1000 in the total resistivity corresponds approximately to the size of the symbols used for alloy No. 2.

right-hand side of Eq. (7) with $W_0=8.5$ K/at.% is seen to give a fair description of the results over several decades of T/c. The results for alloy No. 6 are difficult to plot on Fig. 3, since the positions of the points are very sensitive to the choice of $\rho(0)$ for this concentration. The results for alloy No. 6 follow the general trend of the other samples if the concentration is taken to be 0.006 at.% instead of the nominal 0.005 at.%. We, therefore, feel that the resistivity of AgMn alloys is well represented over a large range of temperatures and concentration by the following expression, which is shown in Fig. 4 along with the experimental points:

$$\frac{\rho(T)-\rho(0)}{c}=(\alpha+\beta c)[1-(\tfrac{1}{2}D)\ln(T^2+W_0^2c^2)]\,F\left(\frac{a_2T}{c}\right), \tag{8}$$

where $F(a_2T/c)$ is given by Eq. (2). Interactions between impurities thus lead to an ordering of the impurity moments which gives rise to a distribution of internal magnetic fields, which in turn tend to quench the elastic spin flip scattering of the conduction electrons. In addition, however, it is also necessary to postulate a modification of the second-order Born-perturbation result of the elastic spin-flip scattering and to modify ρ_s in the manner described. Eq. (8) now gives a reasonably good description of the temperature dependence of the resistivity above the maxima, the positions of the maxima as well as the long linear regions below the maxima for the more concentrated alloys.

Figure 5 shows a rather interesting scaling property of the alloys: We plot the temperature-dependent resistivity per impurity as a function of the logarithm of the temperature scaled to the maximum ($T_{\max}$). In order to plot the points for alloy No. 1 (which does not have a maximum in the range of the data) we have taken $T_{\max}=22$ K in order to agree with alloys 2 and 3 in the region of overlap. For the three most concentrated alloys there is no problem in obtaining $\rho(0)$ and the three sets of points agree remarkably well. However, in the case of Nos. 4 and 6, there is difficulty in determining $\rho(0)$ precisely, as noted previously. Consequently, in Fig. 5 we have adjusted $\rho(0)$ for these samples until their maxima were aligned with those of the three concentrated alloys. Table I shows the values obtained by this procedure and compares them with estimates derived earlier. For alloy No. 4 the two estimates of $\rho(0)/c$ are quite consistent with the possible error in concentration and shape factor. For No. 6 the agreement appears

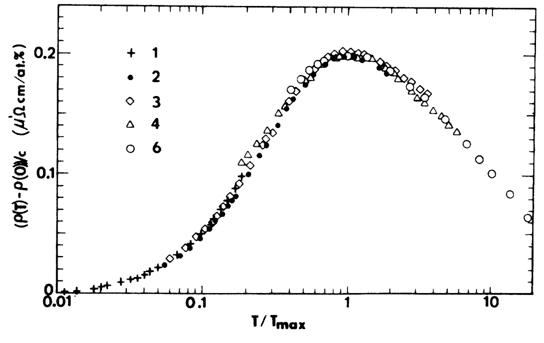

FIG. 5. Temperature dependent resistivity per impurity as a universal logarithmic function of the temperature scaled to the maximum ($T_{\max}$). Table I gives the values of $\rho(0)$ used in this figure.

less good, but the fitted $\rho(0)$ values are unreliable because of the large extrapolation over a steep region of the curve that is involved and, furthermore, the large uncertainty in concentration renders exact comparison difficult. It does appear, however, that 0.006% is probably closer to the true value, as was indicated previously by the logarithmic slope of this alloy in the high temperature regime.

Unfortunately, the expression for $T_{\max}$ obtained by differentiation of Eq. (8) is complicated, and it is not clear that the right-hand side of (8) should be a function of $T/T_{\max}$ only. Numerical evaluation in fact shows that this is indeed not the case. However, it is apparent from the agreement between our model resistivity and the data that Eq. (8) does possess this scaling property over the range of temperature and concentration for which we have data. In order to decide between our model resistivity and the type of variation suggested by Fig. 5, we would need data to lower temperatures for the two dilute alloys and to higher temperatures for No. 1. However, these data are difficult to obtain and interpret: Lower temperature measurements on alloy No. 4 would almost certainly not decide between the two possibilities, as Table I shows that they give substantially the same $\rho(0)$; also, measurements on No. 6 would have to be carried down to very low temperatures (less than 10 mK) and this region is inaccessible to us. In addition, higher temperature measurements on alloy No. 1 would be beset by problems of interpretation: Since Matthiessen's rule has been shown to be invalid, it would be difficult to eliminate the phonon contribution to the resistivity which would become progressively more important as one approached $T_{\max}$.

V. CONCLUDING REMARKS

The answer to the first question posed in Sec. I is yes: even an alloy containing as little as 50 ppm of Mn (where the impurities are separated by an average of about 15 lattice spacings) shows a pronounced maximum in its resistivity near 0.18 K. It is apparent from Table I that the value of $T_{\max}/c$ increases as the concentration is reduced; this is unlike the behavior of AuFe reported by Laborde and Radhakrishna,[5] where $T_{\max}$ in the more dilute alloys occurs at temperatures lower than an extrapolation of $T_{\max}/c = \text{constant}$ from higher concentrations would predict. For the AgMn system then, much lower concentrations and lower temperatures are required to observe the disappearance of the resistance maximum.

The second question has been answered to a large extent: alloy No. 1 (1.12 at.%) shows a very linear variation of resistivity from about 2.5 to 0.24 K, at which temperature (T_h) it quite abruptly levels out; alloy No. 3 (0.125%), on the other hand, shows no levelling off of the resistivity at our lowest temperature (60 mK). If T_h is proportional to concentration, as we assumed in our analysis, then one would not expect a levelling for alloy No. 3 until about 30 mK, which is consistent with our findings. We could not see any deviation from linearity between 60 mK and 0.5 K for the 0.125-at.% alloy, and its temperature coefficient in this region is 0.12 K^{-1}, which might make it suitable as a thermometer material for the millidegree temperature range. In fact, after measuring the resistivity of this alloy we used it to measure its temperature to an accuracy of better than 5 mK at 0.4 K (this corresponds to an accuracy of 1 part in 2000 in the resistance measurement).

Our interpretation of the resistivity curves is based on the existence of a distribution of magnetic fields at impurity sites. The shape of the resistivity curves then puts certain restrictions on the shape of this distribution function. The extended linear region in the resistivity below the maximum implies a distribution function that is relatively flat in a region of width $k_B T$. For if $P(g\mu_B H)$ is nearly constant, then the temperature dependent part of the magnetic scattering in the low-temperature limit is

$$\lim_{T\to 0} \rho_s(T) = 2P(0) k_B T (A - B \ln W).$$

We therefore expect a linear temperature dependence of the resistivity. In the linear region a slope that is essentially concentration independent, then, requires that the maximum of the probability distribution varies as $1/c$. The rapid change of the resistivity for alloy No. 1 at T_h to a constant value then further implies that near $H = 0$ the distribution function falls to a very low value (probably zero) and the width of this hole and hence the value of T_h must decrease with concentration. The detailed shape of the distribution function near $H = 0$ is not ascertainable from our resistivity measurements. From the form of the resistivity expression we also expect to find that below the resistance maximum $\delta\rho/c$ is proportional to T/c. Such a scaling was already demonstrated by Jha[12] and it represents additional evidence for the reduced diagram representation for impurities interacting via the RKKY interaction suggested by Souletie and Tournier.[13] In view of this it would be interesting to establish that $\delta\rho/c$ also scales in the manner suggested by Fig. 5. It should be stressed again, however, that a rather different (but not inconsistent) choice of $\rho(0)$ for alloy No. 6 is required for this picture to be valid, so further measurements on this alloy to much lower temperatures (less than 10 mK) are really required to see which value of $\rho(0)$ is in fact correct. At the moment Fig. 5 should perhaps be considered as a piece of speculation which is, however, consistent with the facts as we know them.

As pointed out in Sec. IV, our parameter a_2 is related to parameter T_2 of Matho and Béal Monod.[8] Allowing for the fact that our impurity concentrations are measured in at.%, we find that $a_2 \equiv 100\pi/2T_2$. Their T_2 value of 1700 K then yields an $a_2 = 0.092$, which is within 10% of our value.

It is also interesting to compare our results with a resistivity calculation for interacting magnetic impurities by Larkin *et al.*[14] They find that for impurities interacting via the RKKY interaction the resistivity can be written as

$$\rho = \rho_0 + \rho_s(1 - a_s c V_0/T),$$

where ρ_s is linear in the impurity concentration and represents the resistance due to exchange scattering of the electrons. V_0 measures the strength of the impurity interaction which is of the form

$$V(r) = V_0[\cos(2p_0 r)/r^3] \text{ for } p_0 r \gg 1.$$

The a_s are spin-dependent constants. For large values of a_2T/c our resistivity equation can be written $\rho(T) = \rho_I + \rho_s(1 - 2c/\pi a_2 T)$, which has the same form as the expression of Larkin *et al.*[14] If an impurity has $s = \frac{5}{2}$, for which $a_s = 2.35$, then $V_0 = 850$ K for interacting manganese impurities in a silver matrix.

ACKNOWLEDGMENTS

We thank Dr. Paton and other members of the Dalhousie Physics Department for many valuable discussions. One of us (D. A. T.) would like to thank the Trustees of the Killam Fund of Dalhousie University for a Fellowship, during the tenure of which this work was carried out.

---

*Work supported by grants from the National Research Council of Canada.

[1] D. Jha and M. H. Jericho, Phys. Rev. B 3, 147 (1971) (referred to as I).

[2] A. Blandin, J. Phys. Rad. 20, 160 (1959).

[3] M. T. Béal, J. Phys. Chem. Solids 25, 543 (1964).

[4] R. J. Harrison and M. W. Klein, Phys. Rev. 154, 540 (1967).

[5] O. Laborde and P. Radhakrishna, Solid State Commun. 9, 701 (1971).

[6] J. W. Loram, T. E. Whall, and P. J. Ford, Phys. Rev. B 3, 953 (1971).

[7] M. H. Jericho and R. H. March, Rev. Sci. Instrum. 38, 428 (1967).

[8] K. Matho and M. T. Béal-Monod, Phys. Rev. B 5, 1899 (1972).

[9] See, for example, K. H. Fisher, in *Proceedings of the Eleventh International Conference on Low Temperature Physics*, edited by J. F. Allen, D. M. Finlayson, D. M. McCall (University of St. Andrews, St. Andrews, Scotland, 1969).

[10] H. Suhl, Phys. Rev. Lett. 20, 656 (1968).

[11] Y. C. Tsay and M. W. Klein, Phys. Rev. B 7, 352 (1973).

[12] D. Jha, thesis (Dalhousie University, Halifax, N. S., 1970) (unpublished).

[13] J. Souletie and R. Tournier, J. Low Temp. Phys. 1, 95 (1969).

[14] A. I. Larkin, V. I. Mel'nikov and D. E. Khmel'nitskii, Zh. Eksp. Teor. Fiz. 60, 846 (1971) [Sov. Phys.-JETP 33, 458 (1971)].