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Landau domain structure. I. Theory*

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A new treatment is presented for the Landau domain structure that is realized when a magnetic field less than H_c is applied at a small angle to the large faces of a flat plate of a type-I superconductor. We work within the established framework of the Landau-Sharvin theory but attempt to take into account the nonlinear aspect of the magnetic interaction energy. The result is a new expression for the domain periodicity as a function of the fraction of the sample surface in the normal state, both of which quantities are direct experimental observables. Recent experimental work lends support to our result.

I. INTRODUCTION

This paper presents a new theoretical approach to the problem of the regular domain structure that can be realized in a type-I superconductor with the inclined-field technique.[1,2] Such a structure is the simplest and most regular that can be produced in the intermediate state and is the source of our most reliable experimental values for the fundamental surface-energy parameter Δ. The existing theoretical treatment is Sharvin's extension[1] of Landau's basic theory.[3] This treatment, the assumptions of which have been questioned in the past,[4,5] has nonetheless remained our only theoretical account of the problem and is indeed quite adequate to account for the observed domain periodicities as a function of H/H_c, where H is the applied field and H_c is the critical field. However, as discussed in the following paper,[6] it does not now seem adequate to account for some precise new data on the periodicity as a function of C_N^*, the surface fraction of material in the normal state.

Our treatment is essentially a modification of the Landau-Sharvin (LS) approach and so it is helpful to first discuss the relevant aspects of that theory. This is also done because we have found it convenient to introduce a simplified procedure for evaluating the thermodynamic potential. It is appropriate to demonstrate, as we shall do in Sec. IV, that our procedure does give numerical results very close to those of LS when applied to the thermodynamic potential chosen in that theory.

II. LANDAU THEORY

The Landau domain structure for the intermediate state of a flat superconducting plate in a uniform magnetic field H, applied perpendicular to the plate, is sketched in Fig. 1(a). In the bulk of the material a normal domain has width d which increases to d^* at the surface. If the spatial periodicity of the domain structure is a, then it is convenient to define the dimensionless ratios

$$C_N=\frac{d}{a}\;,\quad \Delta C_N=\frac{d^*-d}{a}\;,\quad C_N^*=\frac{d^*}{a}\;. \qquad (1)$$

Elementary arguments demonstrate that $C_N=H/H_c$ for a plate whose two other dimensions are very much larger than the indicated thickness l. Furthermore, using a boundary-condition argument, Landau suggested[3] a form for the profile of the normal domain near the surface and with it the dependence of ΔC_N upon C_N:

$$\Delta C_N=\frac{1}{\pi}\,(1-C_N^2)\tan^{-1}\!\left(\frac{2C_N}{1-C_N^2}\right). \qquad (2)$$

As far as we are aware, the only experimental test of this expression for ΔC_N is the microwave-absorption work of Wilkinson[7] on Al cylinders in a transverse field. That work gave rough agreement with Eq. (2) despite the fact that the domain structure for such an experimental geometry is probably much more complicated than shown in Fig. 1(a).

Landau's expression for the free energy due to the domain structure may be written with the coordinate system of Fig. 1(b) which exhibits the cross section of a single superconducting domain. The large sample surfaces are located in the $x=0$ and l planes while the origin is chosen as a point midway between two adjacent normal domains. The domain structure is periodic in the y direction, so in writing down the thermodynamic potential one only needs to be concerned with the single superconducting interface whose interaction with the x-y plane is the line, or profile, 0-a-b. Point a is the point of intersection of that profile, $y(x)$, with the y axis while b is some point deep in the superconductor where the profile has become essentially parallel to the x axis. If Φ is taken to represent the thermodynamic potential per unit plate area of the sample, then we get directly from Landau's paper[3] that

$$\Phi=\frac{H_c^2 l\Delta}{4\pi a}+\frac{H_c^2}{2\pi a}\int_a^b (w-y)\,dx+\frac{H}{2\pi a}\int_0^a H_y\, y\, dy$$

$$+\frac{HH_c}{2\pi a}\int_a^b (y\,ds - w\,dx)\ . \qquad (3)$$

In this expression H_y is the y component of the magnetic field at a point with coordinate y lying just outside the profile. The bulk width of the superconducting domain is represented by $2w$ while Δ denotes the surface-energy parameter and $ds=(dx^2+dy^2)^{1/2}$. The first term in the Landau potential is the well-known excess energy introduced by the existence of the normal-superconducting interface. It can easily be shown, and is in any case evident in the Landau result [Eq. (2)], that this term plays no role in establishing the relationship between ΔC_N and C_N. Its presence, however, is crucial in establishing the periodicity. The three integrals in Eq. (3) represent the relevant part of the magnetic contribution to the thermodynamic potential. Subtracted from the total magnetic energy is that contribution which would arise if the domains happened to be perfectly rectangular, i.e., not broadened at all. It will be shown explicitly in Sec. V that this portion does not influence any observable quantities. The first of the magnetic integrals represents the excess energy of normal material coexisting in thermal equilibrium with material in the superconducting state. Landau chose to represent the interaction energy between the external field and the total induced magnetic moment m of the sample by the term $-\frac{1}{2}mH$ which leads directly to the final two integrals. In these, the magnetic moment of both curved and straight portions of the superconducting domain is obtained from a consideration of the tangential component of the local field along the superconducting interfaces and the associated surface currents. The field magnitude everywhere along the normal-superconducting interface is

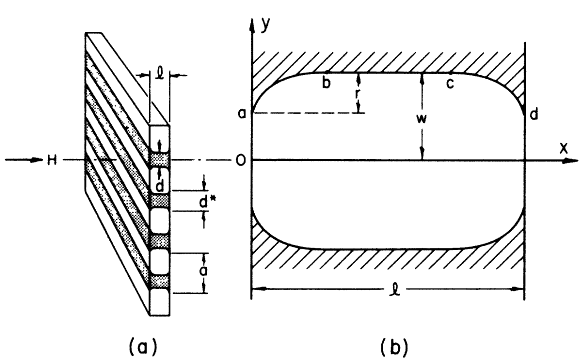

FIG. 1. (a) Landau intermediate state structure in a type-I superconductor. A magnetic field H is applied perpendicular to the plane of the plate and the normal (superconducting) regions are shown shaded (unshaded), respectively. (b) Coordinate system used for describing a single N-S interface (see text).

taken to be identically H_c. If Eq. (3) is now minimized[3] with respect to a, while holding the external field constant, one obtains the well-known Landau result for the periodicity, here denoted by $a_\perp$ to indicate that H is perpendicular to the plate,

$$a_\perp=[l\Delta/\phi(C_N)]^{1/2}\ , \qquad (4)$$

where ϕ is a dimensionless function tabluated by Lifshitz and Sharvin.[8]

III. SHARVIN'S EXTENSION (LS THEORY)

It is Sharvin's extension of the above theory that enables one to deal with the more important inclined-field geometry. Returning to Fig. 1(a) it is supposed that the external field has been rotated away from the direction shown, but maintained in the x-z plane so that $\vec{H}=(H_x,0,H_z)$. If β is in the angle of inclination of the field to the z axis, then $\tan\beta=H_x/H_z$. Experimentally, the domains are observed to lie along the z axis as drawn.

We recall that the thickness of the plate, l, is presumed very much smaller than both of its other dimensions. This implies that the solution to the z component of the magnetic field *everywhere* in the normal regions can reasonably be taken as H_z. The applied field normal to the plate is, of course, just H_x. Further, the relation $\mathrm{div}\vec{H}=0$ shows that the x component of the field along the interface at point b must be H_x/C_N, just as it is in the perpendicular case. Since the total field at point b must be H_c, one then has $(H_x/C_N)^2+H_z^2=H_c^2$. Setting $h=|\vec{H}|/H_c$ gives Sharvin's result[1] for the bulk fraction of the sample in the normal state:

$$C_N=\frac{H_x}{(H_c^2-H_z^2)^{1/2}}=\frac{h\sin\beta}{(1-h^2\cos^2\beta)^{1/2}}\ . \qquad (5)$$

Now, since the total field at any point along the profile is taken to be H_c and since the z component of the field is constant everywhere, it follows that the field tangent to the profile must be H_x/C_N everywhere from a to b. Sharvin therefore extended the Landau theory by replacing H by H_x and H_c by H_x/C_N in the final two integrals of Eq. (3). The first integral remains unchanged since it represents the condensation energy of the broadened normal regions. To complete the extension, the interaction energy between the z component of the induced magnetic moment and the external field must be included in the free energy.[9] Remembering the constancy of H_z everywhere in the normal domains, this contribution can be written

$$-\frac{H_z^2}{2\pi a}\int_a^b (w-y)\,dx\ . \qquad (6)$$

Adding Eq. (6) to the extension of Eq. (3) and employing Eq. (5), one thus obtains the LS thermodynamic potential

$$\Phi=\frac{H_c^2 l\Delta}{4\pi a}+\frac{H_x^2}{2\pi a C_N^2}\int_a^b (w-y)\,dx+\frac{H_x}{2\pi a}\int_0^a H_y y\,dy+\frac{H_x^2}{2\pi a C_N}\int_a^b (y\,ds-w\,dx)\ . \tag{7}$$

The general problem of deducing the relationships between observables from Eq. (7) is not a trivial one. We recall, however, that the profile of the normal domain near the plate surface was chosen by Landau to satisfy a set of boundary conditions, as indicated in Sec. II. These conditions amount essentially to ensuring that the local magnetic field magnitude is a constant along a-b [see Fig. 1(b)]. In the Landau theory, of course, the field is just H_c. In the inclined-field geometry, assuming the solution to the z component of field to be H_z everywhere in the normal regions, one finds that the local magnetic field magnitude along a-b, namely, $(H_c^2-H_z^2)^{1/2}$, is also a constant. Thus the same form for the profile, together with the same dependence of ΔC_N upon C_N, might be anticipated. [By placing certain constraints on the possible profile, one can derive the same result from Eq. (7) itself, as we shall show in a particular case in Sec. IV by constraining the profile to a very simple family of curves.] The LS theory proceeds on the basis that the broadening is indeed independent of the angle β. To establish the periodicity it is still necessary to evaluate the individual terms of Eq. (7). In order to evaluate the sort of integrals appearing in Eq. (7), Landau employed[3] an independent parameter, ξ, which locates points along the profile. He then introduced two functions to describe the profile a-b, $y=aY(C_N,\xi)$ and $x=aX(C_N,\xi)$, and a third to describe the profile, 0-a, $y=aF(C_N,\xi)$. For this choice of independent variables one also has, in the LS theory, $H_y=(H_x/C_N)g(C_N,\xi)$. From the previous considerations, all these expressions are taken to remain valid in the inclined-field situation, although Eq. (5) is, of course, employed to relate C_N to the external field. Hence, inserting the parametrized expressions into Eq. (7) and utilizing the geometric relation $2w=(1-C_N)a$, one obtains the LS thermodynamic potential in the form

$$\Phi=\frac{H_c^2 l\Delta}{4\pi a}+\frac{H_x^2 a}{2\pi C_N^2}\int_a^b (\omega-Y)\,dX+\frac{H_x^2 a}{2\pi C_N}\int_b^a gF\,dF+\frac{H_x^2 a}{2\pi C_N}\int_a^b (YL-\omega)\,dX\ , \tag{8}$$

where we have defined $\omega=\frac{1}{2}(1-C_N)$ and $L=[1+(dY/dX)^2]^{1/2}$. This expression can now be minimized with respect to a, holding H_x and C_N constant. The differentiation is trivial, since each integrand in Eq. (8) depends only upon C_N and ξ, and the resulting expression can be immediately solved for a giving

$$a=\left(\frac{L\Delta}{\phi(C_N)}\right)^{1/2}(C_N^2\cot^2\beta+1)^{1/2}\ , \tag{9}$$

where use has been made of Eq. (5). The function ϕ in this expression is in fact the same dimensionless function that appears in the Landau result [Eq. (4)] and is given explicitly by

$$\phi(C_N)=2\int_a^b(\omega-Y)\,dX+2C_N\int_b^a gF\,dF+2C_N\int_a^b(YL-\omega)\,dX\ . \tag{10}$$

Thus, Sharvin's result for the inclined-field periodicity is simply the Landau expression modified by the factor $(C_N^2\cot^2\beta+1)^{1/2}$.

IV. SIMPLIFIED PROCEDURE

As mentioned previously, it is useful at this point to demonstrate the validity of a simplified procedure for dealing with the sort of integrals that appear in Eq. (7).

In introducing these approximations one takes note of some features of the LS theory itself. In that treatment the profile $y(x)$ has an infinite slope at $x=0$ and also displays the feature $y(x=r)\cong w$ [see Fig. 1(b)]. The simplest curve displaying both these features is

$$y(x)=\begin{cases} w-r+[r^2-(x-r)^2]^{1/2}, & 0\le x\le r\\ w, & x\ge r\ ;\end{cases} \tag{11}$$

i.e., $y(x)$ consists of a circular arc of radius r centered at point $(r,\ w-r)$ followed by a line parallel to the x axis. Referring to Eqs. (7) and (8), it is further clear that an evaluation of the LS thermodynamic potential Φ entails a knowledge of the function g. Again, in the LS theory one has that $H_y(y=0)=0$ and $H_y(y=w-r)=H_x/C_N$. Clearly, any approximate H_y must at least have the functional form

$$H_y(y)=\frac{H_x}{C_N}\,g\!\left(\frac{y}{w-r}\right)\ , \tag{12}$$

where g is some monotonically increasing function of its argument having $g(0)=0$ and $g(1)=1$. The simplest function satisfying all of these constraints would appear to be

$$g(\theta)=\sin\tfrac{1}{2}\theta\pi\ , \tag{13}$$

for which

$$\int_0^1 g(\theta)\theta\,d\theta=4/\pi^2\ . \tag{14}$$

We further note the geometrical relations

$$\Delta C_N=2r/a\ ,\quad 2w=a(1-C_N)\ . \tag{15}$$

Now, it is clear that the value of ΔC_N must be

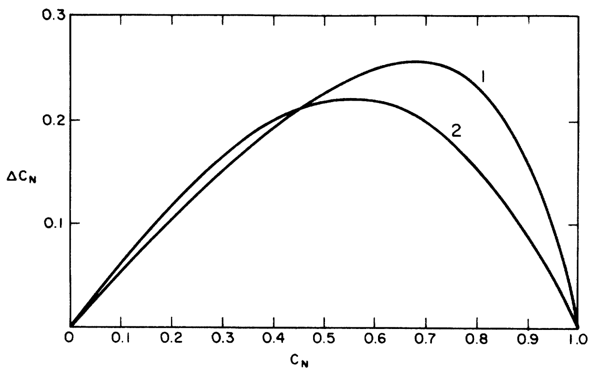

FIG. 2. Surface enhancement of the normal fraction, ΔC_N as a function of the bulk value, C_N. Curve 1: Result from simplified procedure [Eq. (17)]; curve 2: Landau's result [Eq. (2)].

unaffected by changes of a at constant H_x and C_N. (Such changes could be produced, for example, by changes in the plate thickness.) One must therefore have that $r = Ra$, where R may depend on H_x and C_N, but can *not* depend explicitly on a. Employing this relation, together with Eqs. (7), (11), (12), and (14) one obtains

$$\frac{8\pi^3 C_N \Phi}{\omega^2 H_x^2} = \frac{2\pi^2 C_N H_c^2 l\Delta}{\omega^2 H_x^2 a} + 16a - [32 + 4\pi^2 - 2\pi^3]\,\frac{R}{\omega}\,a + \left(\frac{4\pi^2 - \pi^3}{C_N} + 4\pi^2 + 16 - 2\pi^3\right)\left(\frac{R}{\omega}\right)^2 a, \tag{16}$$

where again $\omega = \frac{1}{2}(1 - C_N)$. At this point in the argument Φ is expressed as a function of the variables, a, H_x, C_N, and R. One can now proceed to minimize Φ with respect to R at constant a, H_x, and C_N, obtaining

$$\Delta C_N = \frac{C_N(1 - C_N)(16 + 2\pi^2 - \pi^3)}{\pi^2(4 - \pi) + 2C_N(8 + 2\pi^2 - \pi^3)} \;. \tag{17}$$

Although our Eq. (17) does not bear much formal resemblance to Eq. (2), it nevertheless gives numerical values fairly close to that expression as may be appreciated from Fig. 2. Note also that ΔC_N (and therefore R), has turned out to depend only on C_N and not on H_x.

One can also, of course, minimize the Φ of Eq. (16) with respect to a at constant H_x, C_N and R. In this manner one obtains, after some algebra,

$$a = 2\pi(C_N^2 \cot^2\beta + 1)^{1/2}\,\frac{(2l\Delta)^{1/2}}{(1 - C_N)\sqrt{C_N}} \times \left[16 - (32 + 4\pi^2 - 2\pi^3)\left(\frac{\Delta C_N}{1 - C_N}\right) + \left(\frac{4\pi^2 - \pi^3}{C_N} + 4\pi^2 + 16 - 2\pi^3\right)\left(\frac{\Delta C_N}{1 - C_N}\right)^2\right]^{-1/2} \;. \tag{18}$$

Note that this expression exhibits the characteristic square-root dependence on l and Δ which occurs in the LS expression. Further, the angular dependence is identical to that of LS [Eq. (9)]. Even the numerical results are very close to those given by the full LS treatment (see Fig. 3). Clearly, the procedure that we have discussed provides an excellent approximation to the formidable complexities of the full LS theory. Without some such procedure it would be difficult to make contact with experimental quantities in the theory that follows.

V. NEW TREATMENT OF LANDAU DOMAIN STRUCTURE IN AN INCLINED FIELD

It is our belief that the LS thermodynamic potential [Eq. (7)] is not sufficiently accurate to describe certain aspects of the inclined-field situation. Recall that its form was determined by representing the interaction energy between the external field $\vec{H} = (H_x, 0, H_z)$ and the sample magnetic moment $\vec{m} = (m_x, 0, m_z)$, as $-\frac{1}{2}m_x H_x - \frac{1}{2}m_z H_z$. This representation is only strictly correct[10] if the relationship between the magnetic moment and the field is a linear one. However, an elementary argument demonstrates that the relationship can-

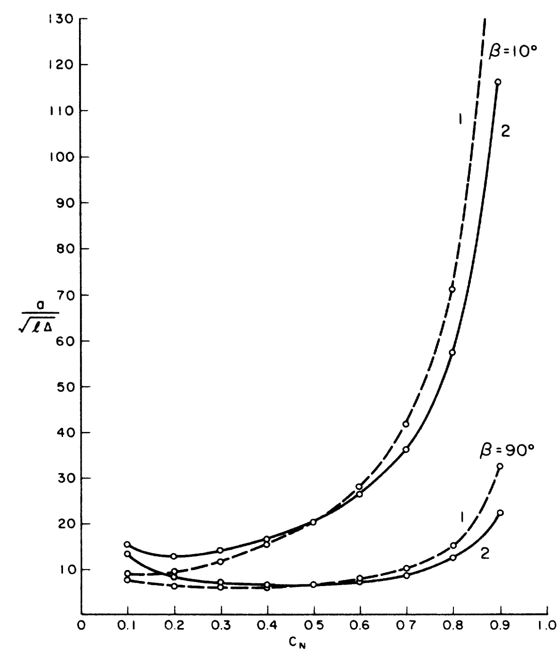

FIG. 3. Periodicity as a function of the bulk normal fraction, C_N. Curve 1: result from simplified procedure [Eq. (18)]; curve 2: LS theory [Eq. (9)].

not be linear, even in the ideally simple geometry of perfectly rectangular domains. The magnetic moment of a rectangular superconducting domain is proportional to its cross sectional area and also to the current flowing around its bounding walls. Using Sharvin's relation [Eq. (5)] gives us that

$$m_x \propto (H_x/C_N)(1-C_N) \propto (H_c^2 - H_z^2)^{1/2} - H_x$$

and

$$m_z \propto H_z(1-C_N) \propto H_z[1 - H_x/(H_c^2 - H_z^2)^{1/2}] \,.$$

One immediately sees that the dependence of m_z upon H_z is intrinsically nonlinear, even in the simplest case.

We therefore return to first principles and examine the fundamental differential form for the total free energy F of a magnetizable body located in a uniform externally applied field, $\vec{H}$. This can be stated[10] as

$$dF = -SdT - \vec{m}\cdot d\vec{H} \,, \qquad (19)$$

where S denotes the entropy and $\vec{m}$ the total magnetic moment of the sample, while T is the equilibrium temperature. Since the body considered here is a superconducting plate in a field $H < H_c$, one can explicitly recognize that it is in the intermediate state and write dF as a sum of contributions due to the normal (N) and superconducting (S) domains

$$dF = dF_N + dF_S \,. \qquad (20)$$

If each contribution is now integrated with respect to the applied field while holding the temperature constant, one obtains

$$F(T,\vec{H}) = -\int \vec{m}\cdot d\vec{H} + F_N(T,0) + F_S(T,0) \,, \qquad (21)$$

remembering that the total moment must vanish at zero field. The integration constants can be related to each other since

$$f_N(T,0) = f_S(T,0) + H_c^2/8\pi \,, \qquad (22)$$

where f_N and f_S are the free-energy densities. Letting V_N and V_S represent the total normal and superconducting phase volumes, respectively, we can write the free energy as

$$F(T,\vec{H}) = -\int \vec{m}\cdot d\vec{H} + Vf_S(T,0) + H_c^2 V_N/8\pi \,, \qquad (23)$$

where V denotes the plate volume. Consider the coordinate system of Fig. 1(b) where the normal-superconducting (N-S) interface is indicated by the line (profile) a-b-c-d. Points a and d are the points of intersection of that profile $y(x)$ with the sample faces, while b and c are points deep within the superconductor where the profile has become essentially parallel to the x axis. The total volume occupied by normal domains can be written as $C_N V$ plus a contribution due to the broadening near the surface. For each superconducting domain this contribution consists of the four

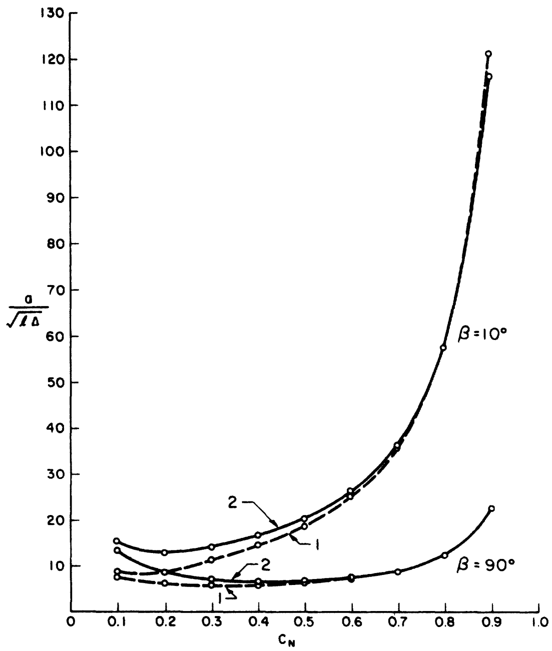

FIG. 5. Periodicity as a function of the bulk normal fraction. Curve 1: result obtained in this work [Eq. (35)]; curve 2: LS theory [Eq. (9)].

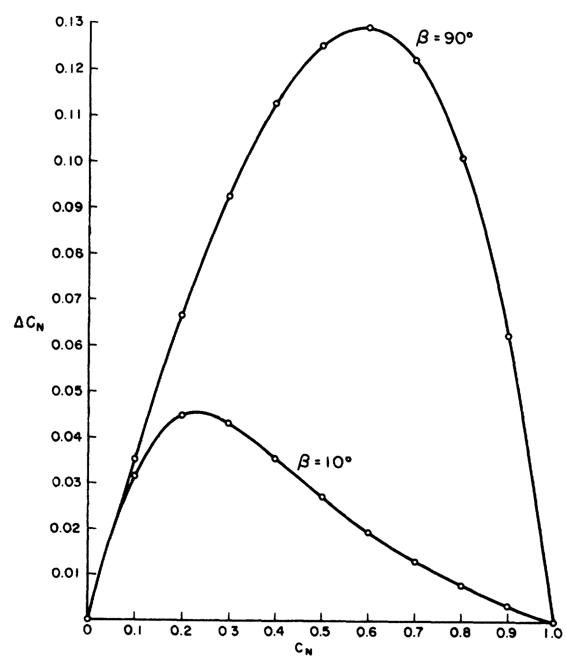

FIG. 4. Surface enhancement of the normal fraction, ΔC_N by Eq. (32).

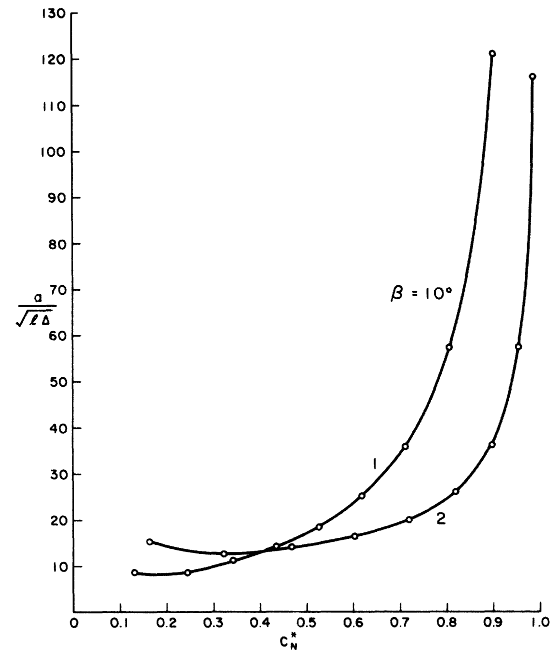

FIG. 6. Periodicity as a function of the *surface* normal fraction, C_N^*. Curve 1: result obtained in this work, using Eqs. (32) and (35); curve 2: LS theory using Eqs. (2) and (9).

corners, each of volume $l_z \int_a^b (w-y)dx$ where l_z is the plate length in the z direction. Letting l_y denote the plate length in the y direction and recognizing that the total number of superconducting domains is l_y/a, we therefore have that

$$V_N = C_N V + (4 l_y l_z / a) \int_a^b (w-y)dx \; . \tag{24}$$

A reasonable assumption of the LS theory is that the z component of the local field is everywhere H_z inside a normal domain. Elementary considerations then allow one to write down the components of the total magnetic moment of the plate in terms of integrals along the profile:

$$m_x = -\frac{l_y l_z}{\pi a}\int_0^a H_y y\,dy - \frac{l_y l_z}{2a}\int_a^d \frac{H_x y\,ds}{C_N} \; ,$$
$$m_z = -\frac{l_y l_z}{2\pi a}\int_a^d H_z y\,dx \; . \tag{25}$$

The algebra can be simplified to some extent if one examines the magnetic moment resulting from a hypothetical rectangular domain profile, say, $\vec{m}_R$. From Eq. (25) this is given by

$$\vec{m}_R = -l_y l_z \frac{H_x l w}{2\pi a C_N}\hat{x} - l_y l_z \frac{H_z l w}{2\pi a}\hat{z} \; . \tag{26}$$

Since $w = \frac{1}{2}a(1 - C_N)$ it is evident that $\vec{m}_R$ does not depend explicitly on the periodicity (or, of course, the profile). Thus, the simplifying addition of a term $\int \vec{m}_R \cdot d\vec{H}$ to the free energy can have no effect on any subsequent minimization with respect to the profile or the periodicity. Finally, of course, there is a contribution to the free energy not recognized by Eq. (23) which arises from the N-S interfaces and amounts to $H_c^2 l\Delta/4\pi a$ per unit area of the plate. Including this contribution, adding the term $\int \vec{m}_R \cdot d\vec{H}$, dropping constant terms and noting that $\int_a^d = 2\int_a^b + \int_a^c$, we employ Eqs. (23)–(26) to arrive at our form for the thermodynamic potential Φ per unit plate area of the sample,

$$\Phi = \frac{H_c^2 l\Delta}{4\pi a} + \frac{H_c^2}{2\pi a}\int_a^b (w-y)dx + \frac{1}{\pi}\int\left(\frac{1}{a}\int_0^a H_y y\,dy\right)dH_x$$
$$+\frac{1}{\pi}\int\left(\frac{H_x}{aC_N}\int_a^b (y\,ds - w\,dx)\right)dH_x$$
$$-\frac{1}{\pi}\int\left(\frac{H_z}{a}\int_a^b (w-y)dx\right)dH_z \; . \tag{27}$$

Clearly, a direct minimization of Eq. (27) is out of the question but one can make some progress by using the simplified procedure whose validity has been established in Sec. IV. Inserting Eqs. (11)–(14) into Eq. (27) and performing the boundary integrations one obtains, in this way,

$$\Phi = \frac{H_c^2 l\Delta}{4\pi a} + \frac{H_c^2 a}{2\pi}(1 - \tfrac{1}{4}\pi)R^2 + \frac{1}{\pi}\int \frac{4}{\pi^2 C_N}\left[R^2 - (1 - C_N)R + \tfrac{1}{4}(1 - C_N)^2\right]H_x a\,dH_x$$
$$+\frac{1}{\pi}\int \frac{1}{C_N}\left[R^2(1 - \tfrac{1}{2}\pi) + R(1 - C_N)(\tfrac{1}{4}\pi - \tfrac{1}{2})\right]H_x a\,dH_x - \frac{1}{\pi}\int R^2(1 - \tfrac{1}{4}\pi)H_z a\,dH_z \; , \tag{28}$$

where again the relations $2w = (1 - C_N)a$ and $r = Ra$ have been employed.

It is at once evident that the task of determining R and a is much more difficult now that the nonlinear aspect of the problem has been recognized. In order to minimize Φ one has first to evaluate the integrals but an exact evaluation involves knowing the solution to the problem. In this situation we content ourselves with an approximate approach that we believe does bring out the essential physical consequence of nonlinearity.

We first observe Eq. (9) has good experimental support, whereas the assumption that R $(= \frac{1}{2}\Delta C_N)$ is still given by Eq. (2) in the inclined-field case has had no previous experimental test. In addition, the results to be discussed in the following paper[6] do not support it. We therefore inquire what broadening would be displayed by a sample for which a was given by Eq. (9). In evaluating the required integrals of Eq. (28) one notes that in the linear solution for R we have $R=R(C_N)$. Further, one can rewrite Eq. (9), using Eq. (5), as

$$a(C_N, H_x)=(C_N H_c/H_x)a_\perp(C_N) . \tag{29}$$

It is evidentally simplest to hold C_N constant while evaluating the integrals, rather than the angle β. Using Eq. (5) we therefore transform to the same variables that were found convenient in Sec. IV, namely, a, H_x, C_N, and R, and evaluate the integrals holding R and C_N constant, obtaining

$$\Phi=\frac{H_c^2 l\Delta}{4\pi a}+\frac{H_c^2 a}{2\pi}(1-\tfrac{1}{4}\pi)R^2+\frac{4}{\pi^3 C_N}[R^2-(1-C_N)R+\tfrac{1}{4}(1-C_N)^2]aH_x^2$$
$$+\frac{1}{\pi C_N}[R^2(1-\tfrac{1}{2}\pi)+R(1-C_N)(\tfrac{1}{4}\pi-\tfrac{1}{2})]aH_x^2+\frac{R^2}{\pi}(1-\tfrac{1}{4}\pi)a\frac{H_x^2}{C_N^2} . \tag{30}$$

The domain broadening can now be obtained in exactly the same way as in Sec. IV by minimizing Eq. (30) with respect to R while holding a, H_x, and C_N fixed. This gives

$$R=\frac{H_x^2 C_N(1-C_N)(16+2\pi^2-\pi^3)}{H_c^2 C_N^2\pi^2(4-\pi)+H_c^2 C_N 4(8+2\pi^2-\pi^3)+H_x^2 2\pi(4-\pi)} . \tag{31}$$

If Eq. (5) is employed and the geometric relation $\Delta C_N=2R$, one finally obtains

$$\Delta C_N(C_N,\beta)=\frac{2C_N(1-C_N)(16+2\pi^2-\pi^3)}{\pi^2(4-\pi)(1+C_N^2\cot^2\beta)+4C_N(8+2\pi^2-\pi^3)+2\pi^2(4-\pi)} . \tag{32}$$

Note that broadening at small angles is reduced significantly from its value at $\beta=90°$ and that as $\beta\to 0$, $\Delta C_N\to 0$ for all C_N. This crucial reduction of the broadening at small angles is exhibited in Fig. 4 which displays our result at $\beta=90°$ and $10°$.

We can now proceed to show that our approximation scheme is reasonably self-consistent by inquiring what the periodicity would be for a sample which exhibited the broadening given by Eq. (32). We therefore substitute Eq. (32) into Eq. (28), noting that $\Delta C_N=2R$, and integrate this time holding a and C_N constant. The required integrals can be evaluated simply, giving,

$$\int H_x(\Delta C_N)dH_x=\frac{(1-C_N)H_c^2 C_N^3 Q}{2A^2}\left[\frac{A}{f^2}-\ln\left(1+\frac{A}{f^2}\right)\right],$$

$$\int H_x(\Delta C_N)^2 dH_x=\frac{(1-C_N)^2 H_c^2 C_N^4 Q^2}{2A^3}$$
$$\times\left[1+\frac{A}{f^2}-\frac{1}{1+A/f^2}-2\ln\left(1+\frac{A}{f^2}\right)\right] , \tag{33}$$

where we have utilized Eq. (5) and the definitions

$$f=(1+C_N^2\cot^2\beta)^{1/2} ,$$

$$A=2+\frac{4(8+2\pi^2-\pi^3)C_N}{\pi^2(4-\pi)} , \tag{34}$$

$$Q=\frac{2(16+2\pi^2-\pi^3)}{\pi^2(4-\pi)} .$$

If the thermodynamic potential is now minimized with respect to a, one obtains, after lengthy manipulations, the final result for the periodicity

$$a=\frac{f\sqrt{l\Delta}}{(1-C_N)\sqrt{C_N}}\left\{\frac{C_N Q^2 f^2}{2(f^2+A)^2}\left(1-\frac{\pi}{4}\right)+\frac{2}{\pi^2}+\frac{2Q^2C_N^2}{A^3}\left(\frac{1}{\pi^2}+\frac{1}{4}-\frac{\pi}{8}\right)\left[f^2+A-\frac{f^2}{1+A/f^2}-2f^2\ln\left(1+\frac{A}{f^2}\right)\right]\right.$$
$$\left.+\frac{QC_N}{A^2}\left(\frac{\pi}{4}-\frac{1}{2}-\frac{4}{\pi^2}\right)\left[A-f^2\ln\left(1+\frac{A}{f^2}\right)\right]+\frac{C_N Q^2}{2A^3}\left(1-\frac{\pi}{4}\right)\left[f^2+A-\frac{f^2}{1+A/f^2}-2f^2\ln\left(1+\frac{A}{f^2}\right)\right]\right\}^{-1/2} . \tag{35}$$

Note that this expression exhibits the characteristic square-root dependence on l and Δ and also the characteristic angular factor f, both of which occur in the LS expression and in Eq. (18). Indeed our result for the periodicity *as a function of* C_N is very close to that of the LS theory as may be appreciated from Fig. 5 where we have compared Eq. (35) to Eq. (9) for the two angles $90°$ and $10°$.

The distinction between the two approaches is evident, however, in Fig. 6 which compares the differing predictions for the periodicity as a function of C_N^*, the fraction of the sample surface in the normal state. The large difference stems essentially from the drastically reduced broadening predicted by our approach for small values of β.

VI. DISCUSSION

Since the experimental situation with regard to the predictions made in the previous section will be dealt with in considerable detail in the following paper,[6] the present discussion will be brief.

In the first place, our conclusion that the broadening is drastically reduced for small β does contradict the general argument given in Sec. III following the statement of Eq. (7). However, we do not believe that Landau's argument is rigorous since he simply chose a particular profile which satisfied the magnetostatic boundary conditions. No proof was supplied that the choice was unique. A recent investigation[11] started from the assumption that the porfile belonged to a particular family of curves. Landau's choice of profile turned out to be quite close to the best member of that family, selected on energy grounds, but there is no guarantee that such a result will be maintained in an inclined field and indeed we find that it is not.

It is an indication of the difficulty of the problem addressed by this paper that, even after employing our simplified procedure, the resulting equation [Eq. (28)] presents difficult problems. We have here obtained an approximate solution and indicated the consistency of our approximation scheme. While formal improvements in our treatment might be worthwhile, convincing experimental support for our results is reported in the following paper. In practice Eqs. (32) and (35) give numbers that are in good accord with all the experiments reported there and some others to be reported elsewhere.[12]

We conclude that the hitherto unrecognized nonlinear aspect of the Landau structure problem has important physical consequences. An approximate treatment has been developed in this paper which accounts very well for recent experimental data.

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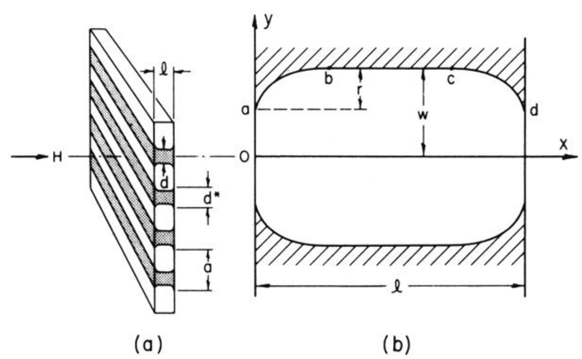

FIG. 1. (a) Landau intermediate state structure in a type-I superconductor. A magnetic field H is applied perpendicular to the plane of the plate and the normal (superconducting) regions are shown shaded (unshaded), respectively. (b) Coordinate system used for describing a single N-S interface (see text).