PHYSICAL REVIEW B VOLUME 9, NUMBER 6 15 MARCH 1974

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Evidence for a mobility edge in inversion layers

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(Received 12 November 1973)

The published transport properties of carriers in silicon inversion layers suggest the existence of a mobility edge separating conducting from localized states at low temperature. Further experimental tests of the mobility-edge model are proposed.

I. INTRODUCTION

The idealized model of an inversion layer at a semiconductor-insulator interface postulates that carriers move in a potential well bounded on one side by a large, smooth barrier which keeps them from entering the insulator and on the other side by a smooth, monotonically increasing potential representing the band bending. The potential is assumed to be independent of the coordinates parallel to the interface. Electronic states in such a barrier form a series of subbands, only one of which is occupied at low temperatures. The carrier concentration can be varied continuously by varying the electric field at the interface or the voltage V_g, called the gate voltage, between the semiconductor and a gate electrode deposited on the insulating layer.

A number of experiments[1–6] have shown that the mobility of the first carriers to enter the inversion layer is considerably lower than the peak mobility, an effect which is especially pronounced at low temperatures. We shall examine this behavior to see how well it agrees with the notion of a mobility edge[7] separating localized from conducting states, and shall propose a number of experiments to test the mobility edge model.

In three-dimensional (3D) systems, a mobility edge can arise when sufficiently large potential fluctuations are present. The lowest-energy states correspond to large potential wells which are relatively unlikely and therefore widely spaced on the average. Carriers in these states carry current by hopping processes at low temperatures. At higher energies, the barriers between adjacent wells are reduced and carriers are free to move through the system, with the potential fluctuations acting as a scattering mechanism to limit the mobility. The mobility edge, presumably sharp at low temperatures, is an energy separating localized from conducting states. The conductivity of EuS in which the electron concentration is varied from sample to sample by changing the stoichiometry is consistent with a mobility-edge model.[8] Another semiconductor with similar behavior is compensated GaAs, in which the degree of compensation of donor and acceptor impurity concentrations is adjusted by electron bombardment.[9]

Potential fluctuations arise in inversion layers, which have a quasi-two-dimensional character, primarily because of the presence of changes in the adjacent insulator, at the semiconductor-insulator interface, and in the semiconductor itself. Interface roughness may also contribute to these fluctuations.[10] Estimates of the fluctuations associated with the charge distribution have been made by a number of authors.[11–14] There appears to be no reason why the transition from localized to conducting states seen in 3D systems should not also be seen in inversion layers.

II. RELEVANT EXPERIMENTS

We now list a number of experimental results reported by many authors which tend to support a mobility-edge model. Alternative explanations are considered in Sec. III.

(i) Activated mobilities like those reported for 3D semiconductors with potential fluctuations[8,9,15] have been reported in Si inversion layers. Fang and Fowler[1] found that the mobility of electrons is activated at low temperatures, with an activation energy in one sample equal to 20 meV when the inversion-layer electron concentration $N_{\rm inv}$ equals 2×10^{11} cm^{-2} and falling to zero when $N_{\rm inv}\sim10^{12}$ cm^{-2}. Chen and Muller[5] found that the mobility very close to threshold, the point at which conduction in the inversion layer becomes measurable, is activated for both n-type and p-type layers. They deduced activation energies of the order of 0.1 eV from measurements near room temperature.

(ii) The mobility of inversion-layer electrons at low temperatures is very small near threshold, and then rises rapidly.[1,4] Extrapolation of the conductance leads to an apparent threshold voltage which is higher than the threshold at higher temperatures. This shift can be accounted for by supposing that the threshold shift represents carriers which have gone into localized states. The magnitude of the shift, and therefore the number of such localized states,[16] is well correlated with the magnitude of the oxide charge density.

(iii) When the conductance[17] or capacitance[18,19] of an inversion layer is measured versus carrier concentration in a fixed magnetic field at low temperatures, oscillations are observed which represent the successive filling and emptying of Landau levels, with spin splitting and valley splitting also resolved as the magnetic field is increased. But the lowest magnetic quantum level is often broadened, absent, or otherwise different from the higher ones, consistent with a localized character for the states at low energy which prevents them from forming Landau orbits. The rapid rise in capacitance as the inversion layer is occupied has been found by Voshchenkov[19] to occur at gate voltages which shift rapidly with increasing magnetic field. He has invoked localized states to explain the shift, but his model assumes that localized states and mobile states coexist over a range of energies rather than being separated by a mobility edge.

(iv) Capacitance studies by Pals[20] suggest that best agreement between theory and experiment for a particular sample can be obtained if the lowest energy levels are broadened by about 5 meV. The broadening is presumed to arise from potential fluctuations associated with oxide and interface charges. Note that the density of states in the lowest subband for a Si (100) surface is 1.6×10^{11} cm^{-2} meV^{-1}. Thus a 5-meV range of localized states corresponds roughly to 8×10^{11} states/cm^2.

(v) A pronounced feature of the transport properties of inversion layers is the sharp structure seen near threshold in the field-effect mobility[1,21] $\mu_{\rm FE}=C^{-1}(d\sigma/dV_g)$, where C is the capacitance per unit area and σ is the channel conductivity. It appears that this structure cannot be explained by a carrier-concentration-independent density of states and mobility edge, but requires that the extent of the localized-state region shrink as the inversion layer is occupied. Such an effect is consistent with the screening of Coulomb potentials by the inversion-layer carriers.[4,14,22] A similar effect is expected in compensated GaAs,[9] but did not appear to be necessary to explain the experimental results in EuS.[8]

III. DISCUSSION

None of the experiments we have described can determine the form of the density of states unless a quantitative model is applied. It cannot be claimed that the results cited above support the mobility-edge model over a model in which there is a gap in the density of states separating localized from conducting states. Nor can one rule out the possibility that the low-lying states are strongly scattered,[2,23] rather than localized. The analogy with the 3D results suggests, however, that the mobility-edge model is more plausible when there are substantial densities of oxide charges and other potential fluctuations.

Several experiments which can shed additional light on the question readily suggest themselves. Most of the present experiments have been carried out on samples with relatively good interfaces, because the fluctuations which give rise to the mobility edge behavior are generally undesirable. Thus tests of the model are likely to require measurements on samples which would normally be rejected. One should look for the temperature dependence of the mobility at low temperatures near threshold in such samples to see if it follows the $\exp[-(T_0/T)^{1/3}]$ dependence expected for extended range hopping between localized states in a two-dimensional system at low temperatures.[24,25] One should also look at the dependence of the mobility on the electric field F between source and drain electrodes in such layers, to see if it has the exponential dependence $\mu\sim e^{eaF/KT}$ found in 3D systems and to determine the characteristic length a given by this dependence[15]; existing high-field experiments[26] appear to have been carried out under conditions in which free carrier heating results from the application of a field, and do not bear on the proposed model. And one should look at the magnetoconductance oscillations near threshold as a function of gate voltage and magnetic field, to see if the missing peaks are quantitatively consistent with the presence of a region of localized states. Another test of the mobility edge model is a comparison of the increase in low-temperature conductance above threshold with the predictions of two-dimensional percolation models.[27] The possibility that interface states[28] may influence the results of experiments like those we have proposed must be considered.

All of these experiments could usefully be studied in samples with varying oxide charge density. This should be considerably easier than corresponding measurements in 3D samples, because experimental techniques for varying the oxide charge density near the semiconductor-insulator interface, perhaps even reversibly, already exist.[29,30] The possibility that the charges exist in patches could be a complication.[13,31,32]

One experiment which can eliminate the influence of conventional bound states involves samples in which the oxide charges have the same sign as the inversion layer carriers, producing repulsive rather than attractive potentials. There is evidence that such samples also have low mobilities near threshold at low temperatures, supporting a mobility edge or "Swiss cheese" model over a model involving bound states.[33]

Quantitative interpretation of Hall-mobility data is difficult because of the complexity of the theory of the Hall effect[34] when only localized states are present, or when both localized and mobile carriers are present.[8,35] Fang and Fowler[1] and Murphy *et al.*[2] found that the carrier concentration deduced from the Hall constant with the simple free-electron model was essentially equal to the carrier concentration determined from the gate voltage and the capacitance,[16] even when the Hall mobility had already reached rather small values. Further experimental and theoretical examination of this problem is required.

We conclude that the mobility edge model, already proposed explicitly for inversion layers,[7,13(a),25] can account for the experimental evidence we have described. This model, to the extent that it postulates the existence of localized or strongly scattered carriers in the low-energy states of the inversion layer, resembles the models previously used by most of the authors we have cited. An advantage of the mobility-edge model is that it provides a somewhat simpler framework in which to consider the data, with the work on 3D systems available as a conceptual guide. The inversion layer can in principle provide a better test of theories than most 3D systems because of the relative ease of changing carrier concentration, and possibly even potential fluctuations, in a single sample.

ACKNOWLEDGMENTS

I am indebted to F. F. Fang, A. B. Fowler, S. Kirkpatrick, M. I. Nathan and T. H. Ning for valuable discussions and comments.

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