Effect of nonparabolicity on the damping of helicons in semiconductors in the extreme quantum limit*

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The effect of nonparabolicity of the energy bands on the damping of helicons in semiconductors has been investigated in the extreme quantum limit of the magnetic fields so that all conduction electrons are in the ground oscillator state. Analytical expressions (corresponding to different dominant types of scattering) for the dissipative conductivity σ_{xx} (whose ratio to the Hall conductivity σ_{yx} determines helicon damping) have been obtained for both degenerate and nondegenerate energy distributions. Numerical results have been presented for the typical case of n-InSb and compared with the results for the parabolic model in the form of graphs. The nonparabolicity, in general, leads to a stronger damping of the helicons as compared to the case of parabolic bands.

I. INTRODUCTION

It is well known that low-frequency electromagnetic waves can, under certain circumstances, propagate in conducting solids with relatively little attenuation in the presence of a large magnetic field and are referred to as helicon waves. This possibility was first reported by Aigrain[1] and independently by Konstantinov and Parel.[2] Since then a great deal of work has been reported in the literature on the various aspects of helicon propagation in solids.[3] Such studies have a double interest, viz., as a measuring tool to yield important information about the electronic properties of solids and as a technique to study plasmas in solids.

Recently, Meilikhov[4] has considered the effect of a quantizing magnetic field on the damping of helicons in a semiconductor. Quantization of the orbital electrons was found to lead to a qualitative change in helicon damping from that in classical magnetic fields, owing to the corresponding change in the dissipative conductivity σ_{xx}. However, Meilikhov's[4] analysis was based on the formulation of the conductivity tensor by Kubo *et al.*,[5] which is valid for the parabolic energy bands and, as such, cannot be applied to most of the III-V compounds such as InSb, InAs, etc., which are known to have a markedly nonparabolic band structure. The effect of nonparabolicity in the band structure is to introduce an energy, and therefore, a magnetic-field-dependent effective mass for the electrons in the extreme quantum limit[6] (EQL). This dependence of the electron effective mass on the magnetic field is found to lead to a qualitative change in various galvanomagnetic and magnetoacoustic phenomena. For example, a magnetic-field-dependent effective mass alone can lead to a nonzero longitudinal magnetoresistance[7] as well as a large change in the longitudinal magnetoacoustic absorption[6] in the EQL as compared to a parabolic-band model.

In this paper, we have studied the effect of nonparabolicity of the energy bands on the damping of helicons in semiconductors in the EQL, the nonparabolicity being accounted for by Kane's[8] model. Analytical results have been obtained for both degenerate and nondegenerate samples and for various mechanisms of scattering, viz., ionized impurity, acoustic or piezoelectric. Our treatment has as its purpose to illustrate the dependence of the helicon damping on quantum effects in nonparabolic semiconductors subjected to an extremely strong magnetic field ($\hbar\omega_c \gg k_0 T$). In general, nonparabolicity is seen to give rise to a stronger damping of the helicons as compared to the case of parabolic bands.

In Sec. II the theory of the dissipative conductivity σ_{xx} for nonparabolic energy bands has been presented in the limit when all the carriers are in the ground oscillator state EQL. Numerical results are presented in Sec. III for the typical case of an n-type-InSb sample and compared with the results of the parabolic model by means of graphs along with a brief discussion.

II. THEORY

In this section the general formulation of the conductivity tensor in the extremely strong magnetic field EQL as described by Kubo *et al.*[5] will be used to obtain analytical expressions for the transverse conductivity σ_{xx} for nonparabolic energy bands.

The general expression for the conductivity σ_{xx} is given by[5]

$$\sigma_{xx}=\frac{\pi e^2}{\hbar V}\int_{-\infty}^{+\infty} de\,\frac{df}{dE}\,\langle \mathrm{Tr}\{\delta(E-\mathcal{H})\,[U,x]\times\delta(E-\mathcal{H})\,[U,x]\}\rangle_s \quad , \qquad (1)$$

where $\mathcal{H}$ is the Hamiltonian, V is the normalization

volume, U is the scatterer potential, x is the position coordinate of the center of the Landau oscillator, and the other symbols have their usual meaning. The two δ functions correspond to level densities at the initial and final states for transitions by collisions, $\langle \ldots \rangle_s$ denoting average with respect to scatterers' variables, and $f(E)$ denotes the Fermi function

$$f(E) = 1/(1 + e^{(E-\zeta)/k_0T}) \quad , \tag{2}$$

where ζ is the Fermi energy in the presence of a magnetic field. The energy E is, in general, a function of the electron wave vector $\vec{k}$ and is given for an electron in a spherical nonparabolic energy band of a sample subjected to a magnetic field along the z direction by[6]

$$E_{kN} = -\tfrac{1}{2}E_g + \tfrac{1}{2}E_g a_N + \hbar^2 k_z^2/2m_c a_N \quad , \tag{3}$$

where E_g is the band gap, m_c and ω_c are, respectively, the effective mass and cyclotron frequency of the electron at the band edge, and a_N is given by

$$a_N = [1 + (4\hbar\omega_c/E_g)(N + \tfrac{1}{2})]^{1/2} \quad . \tag{4}$$

The quantum number N levels the Landau oscillators or the magnetic subbands. Equation (3) shows that the effective mass of the electron depends on the magnetic field strength through the relation

$$m^* = m_c a_N \quad . \tag{5}$$

Using Eqs. (3)–(5) the general expression of the density of states of electron in a nonparabolic energy band in the presence of a magnetic field can be shown to be given by

$$\begin{aligned} N(E)dE &= (1/V)\,\mathrm{Tr}\,\delta(E - \mathcal{H}_e) \\ &= 2\left(\frac{(2m^*)^{1/2}}{2\pi\hbar}\right)(2\pi l^2)^{-1} \\ &\quad \times \sum_{N=0}^{N_{\max}} [E - \tfrac{1}{2}(a_N - 1)E_g]^{-1/2} \quad , \end{aligned} \tag{6}$$

where $l = (c\hbar/eH)^{1/2}$ is the quantum Larmor radius. $N_{\max}$ is the maximum value of N that makes $[E - (a_N - 1)E_g/2]$ non-negative. It can be easily shown that Eq. (6) reduces to the Kubo formula[5] of the level density in the limit $\hbar\omega_c \ll E_g$ (which corresponds to the case of parabolic energy bands).

The subsequent analysis closely follows the steps of Kubo *et al.*[5] and therefore, we refrain from reproducing the steps in detail and give only the relevant intermediate results. Thus, using Eqs. (1) and (6), σ_{xx} for elastic scattering can be shown, in general, to be of the form

$$\sigma_{xx} \sim \int dE \sum_{NN'} \frac{G_{NN'}(E)\,df/dE}{(E' - \tfrac{1}{2}E_g a_N)^{1/2}(E' - \tfrac{1}{2}E_g a_{N'})^{1/2}} \quad , \tag{7}$$

where $G_{NN'}(E)$ is a smooth function of E and $E' = E + \tfrac{1}{2}E_g$. The denominators in Eq. (7) arise from the density of initial and final states. However, in extremely strong magnetic fields EQL, only the lowest Landau subband would be occupied by the electrons, and transitions between different widely separated Landau levels may be neglected so that $N = N' = 0$. But this makes the integral in Eq. (7) logarithmically divergent, owing to the overlapping of two level densities in elastic scattering, as was noted by Davydov and Pomeranchuk[9] for the case of parabolic bands. The divergence can be removed by incorporating the inelasticity in the collisions (e.g., electron-phonon interaction via acoustic or piezoelectric scattering) or, in the case of elastic scattering by impurities, the divergence is cut off either by the collision broadening of the Landau levels or by the interference of electron waves occurring at a scattering. Thus, following Kubo *et al.*[5] and Roth and Argyres,[10] we obtain the following expressions of the dissipative conductivity σ_{xx} corresponding to nonparabolic energy bands when different scattering mechanisms are individually dominant.

A. Acoustic scattering

Case I: High temperatures ($k_0T \gg \hbar v_s/l$), where v_s is the sound velocity in the material

When the electrons are degenerate ($\hbar\omega_c \gg \zeta - \tfrac{1}{2}E_g a_0 + \tfrac{1}{2}E_g \gg k_0T$),

$$\sigma_{xx} = \frac{D^2 e^2 m_c^2 a_0 \omega_c}{8\pi^3\hbar^4\rho v_s^2} \frac{k_0T}{\zeta - \tfrac{1}{2}E_g a_0 + \tfrac{1}{2}E_g} \tag{8}$$

$$= \frac{D^2 e^2 a_0 n_0 (k_0T)^{3/2}}{\pi\hbar^2\rho v_s^2 (2\zeta_0)^{3/2}(1 + \zeta_0/E_g)^{3/2}} \times \frac{3}{8}\left(\frac{m_c}{k_0T}\right)^{1/2} \frac{\hbar\omega_c}{\zeta - \tfrac{1}{2}E_g a_0 + \tfrac{1}{2}E_g} \quad , \tag{9}$$

where D is the deformation potential constant, ρ is the material density, and n_0 is the number density of electrons in the absence of a magnetic field and is determined from the condition

$$n_0 = \int_0^\infty N(E) f(E)\,dE = \frac{(2m_c\zeta_0)^{3/2}}{3\pi^2\hbar^3}\left(1 + \frac{\zeta_0}{E_g}\right)^{3/2} \quad , \tag{10}$$

where ζ_0 is the Fermi energy in the absence of a magnetic field. n_0 is related to the number density n in the presence of a magnetic field by

$$\frac{n}{n_0} = 3\,\frac{\hbar\omega_c}{2\zeta_0}\left(\frac{\zeta a_0 - \tfrac{1}{2}E_g a_0^2 + \tfrac{1}{2}E_g a_0}{\zeta_0}\right)^{1/2}\left(1 + \frac{\zeta_0}{E_g}\right)^{-3/2} \tag{11}$$

in the present approximation. However, the dependence of electron concentration on the magnetic field is, in general, very weak except for the case of magnetic "freeze out" of impurities,[11] and therefore n is assumed to be independent of H. Therefore, from Eq. (11) one obtains

$$\zeta = \tfrac{1}{2}E_g a_0 - \tfrac{1}{2}E_g + \frac{4\zeta_0^3(1 + \zeta_0/E_g)^3}{9a_0\hbar^2\omega_c^2} \quad . \tag{12}$$

When the electrons are nondegenerate ($\hbar\omega_c \gg k_0T \gg \zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g$),

$$\sigma_{xx} = \frac{D^2e^2m_c^2a_0\omega_c}{8\pi^3\hbar^4\rho v_s^2} \exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}{k_0T}\right) \ln\left(\frac{4}{(2e\gamma)^{1/2}} \frac{k_0T}{\hbar v_s/l}\right) \tag{13}$$

$$= \frac{D^2e^2n}{16\pi\hbar^2\rho v_s^2} \left(\frac{m_c a_0}{2\pi k_0T}\right)^{1/2} \ln\left(\frac{4}{(2e\gamma)^{1/2}} \frac{k_0T}{\hbar v_s/l}\right) , \tag{14}$$

where γ is the Euler constant, and the electron number density n in the presence of a magnetic field is given by

$$n = \frac{(2m_c)^{3/2}}{2\pi^{3/2}\hbar^3} \frac{1}{2} \hbar\omega_c (a_0k_0T)^{1/2} \exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}{k_0T}\right) = n_0 \frac{\hbar\omega_c}{k_0T} \frac{1}{2} \frac{(\pi a_0)^{1/2}}{I(x)} \exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g - \zeta_0}{k_0T}\right) . \tag{15}$$

$I(x)$ stands for the integral

$$I(x) = \int_0^\infty x^{1/2}(1 + x/x_g)^{1/2}(1 + 2x/x_g)\, e^{-x} dx , \tag{16}$$

where $x = E/k_0T$, $x_g = E_g/k_0T$ and n_0 is the number density of electrons in the absence of a magnetic field and is given by

$$n_0 = \frac{(2m_c k_0T)^{3/2} I(x)}{2\pi^2\hbar^3} e^{\zeta_0/k_0T} . \tag{17}$$

Case II: Low temperature ($k_0T \ll \hbar v_s/l$)

For degenerate electrons $\{[8m^* v_s^2(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g)]^{1/2} \ll k_0T \ll \zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g\}$,

$$\sigma_{xx} = \frac{D^2e^2a_0(k_0T)^4}{16\pi^3\hbar^6\rho v_s^6\omega_c} \frac{k_0T}{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g} \left[I_5(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_7(\xi)\right] \tag{18}$$

$$= \frac{3}{16} \frac{D^2e^2a_0n_0(k_0T)^4}{\pi\hbar^3\rho v_s^6\omega_c(2m_c\zeta_0)^{3/2}} \frac{k_0T}{(1 + \zeta_0/E_g)^{3/2}(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g)} \left[I_5(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_7(\xi)\right] , \tag{19}$$

where n_0 is given by Eq. (10) and $I_n(\xi)$ stands for the integral

$$I_n(\xi) = \int_0^{\Theta/T} \frac{\xi^n d\xi}{(e^\xi - 1)(e^{-\xi} - 1)} , \tag{20}$$

where Θ is the Debye temperature.

For nondegenerate electrons ($k_0T \gg \zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g$),

$$\sigma_{xx} = \frac{D^2e^2a_0(k_0T)^4\pi^{1/2}}{16\pi^3\hbar^6\rho v_s^6\omega_c} \exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}{k_0T}\right) \left[I_{7/2}(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_{11/2}(\xi)\right] \tag{21}$$

$$= \frac{D^2e^2n(k_0T)^4}{8\pi^{3/2}\hbar^4\rho v_s^6\omega_c^2} \left(\frac{a_0\pi}{2k_0T}\right)^{1/2} \left[I_{7/2}(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_{11/2}(\xi)\right] , \tag{22}$$

where n is given by Eq. (15) and the integral $I_n(\xi)$

$$I_n(\xi) = \int_0^{\Theta/T} \frac{\xi^n d\xi}{(e^\xi - 1)} . \tag{23}$$

B. Piezoelectric scattering

Case I: High temperature ($k_0T \gg \hbar v_s/l$)

For degenerate electrons ($\hbar\omega_c \gg \zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g \gg k_0T$),

$$\sigma_{xx} = \frac{P^2e^2m_c a_0}{16\pi^3\hbar^3\rho v_s^2} \frac{k_0T}{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g} \tag{24}$$

$$= \frac{3}{16} \frac{P^2e^2m_c a_0 n_0}{\pi\rho v_s^2(2m_c\zeta_0)^{3/2}} \times \frac{k_0T}{(1 + \zeta_0/E_g)^{3/2}(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g)} . \tag{25}$$

Here P is the coupling constant $e^2\beta^2/\chi$, where e is the electronic charge, β is the piezoelectric modulus, and χ is the static dielectric constant. n_0 is given by Eq. (10).

For the nondegenerate case ($\hbar\omega_c \gg k_0T \gg \zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g$),

$$\sigma_{xx} = \frac{P^2e^2m_c a_0}{16\pi^3\hbar^3\rho v_s^2} \exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}{k_0T}\right)$$

$$\times\ln\left(\frac{4}{(2e\gamma)^{1/2}}\,\frac{k_0T}{\hbar v_s/l}\right) \tag{26}$$
$$=\frac{P^2e^2n}{8\pi^{3/2}\rho v_s^2\hbar\omega_c}\left(\frac{a_0}{2m_ck_0T}\right)^{1/2}$$

$$\times\ln\left(\frac{4}{(2e\gamma)^{1/2}}\,\frac{k_0T}{\hbar v_s/l}\right)\ , \tag{27}$$

where n is given by Eq. (15).

Case II: Low temperature ($k_0T \ll \hbar v_s/l$)

For the degenerate case $\{(m^*v_s^2\hbar\omega_c)^{1/2} \gg k_0T \gg [m^*v_s^2(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g)]\}^{1/2}$,

$$\sigma_{xx}=\frac{P^2e^2a_0(k_0T)^2}{16\pi^3\hbar^4\rho v_s^4\omega_c}\,\frac{k_0T}{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}\left[I_3(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_5(\xi)\right] \tag{28}$$

$$=\frac{P^2e^2a_0(k_0T)^2n_0}{\pi\rho v_s^4\hbar\omega_c}\,\frac{3}{16}\left(\frac{1}{2m_c\zeta_0}\right)^{3/2}\frac{k_0T}{(1+\zeta_0/E_g)^{3/2}(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g)}\left[I_3(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_5(\xi)\right]\ , \tag{29}$$

where the integrals $I_n(\xi)$ are defined by Eq. (20) and n_0 by Eq. (10).

For the nondegenerate case $\{(m^*v_s^2\hbar\omega_c)^{1/2} \gg k_0T \gg m^*v_s^2\}$,

$$\sigma_{xx}=\frac{P^2e^2a_0(k_0T)^2\pi^{1/2}}{16\pi^3\hbar^4\rho v_s^4\omega_c}\exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}{k_0T}\right)\left[I_{3/2}(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_{7/2}(\xi)\right] \tag{30}$$

$$=\frac{P^2e^2a_0^2n}{4\pi\rho v_s^4\hbar^2\omega_c^2}\left(\frac{k_0T}{2m_ca_0}\right)^{3/2}\left[I_{3/2}(\xi) - \frac{1}{2}\left(\frac{k_0T}{\hbar v_s/l}\right)^2 I_{7/2}(\xi)\right]\ , \tag{31}$$

where the integrals $I_n(\xi)$ are defined by Eq. (23) and n is given by Eq. (15).

C. Screened ionized-impurity scattering

For degenerate electrons (with $q_s^{-1} \gg l$, where q_s is the Fermi-Thomas screening wave number),

$$\sigma_{xx}=\left(\frac{Ze^2}{\chi}\right)^2\frac{N_ie^2a_0}{16\pi\hbar^2\omega_c(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g)}$$
$$\times\ln\frac{(\hbar\omega_c)^2}{[4(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g) + E_s]E_s a_0^2} \tag{32}$$
$$=\left(\frac{Ze^2}{4\chi}\right)^2\frac{3\pi n_0N_ie^2a_0}{2m_c\omega_c(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g)(\zeta_0 + \zeta_0^2/E_g)^{3/2}}$$
$$\times\left(\frac{\hbar^2}{2m_c}\right)^{1/2}\ln\frac{(\hbar\omega_c)^2}{[4(\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g) + E_s]E_s a_0^2}\ , \tag{33}$$

where Ze is the charge on the ionized impurity, N_i is the impurity concentration, and

$$E_s=\frac{\hbar^2q_s^2}{2m_c}=\frac{2\pi e^2\hbar^2}{m_c\chi}\,\frac{dn_0}{d\zeta_0}\ , \tag{34}$$

$dn_0/d\zeta_0$ being the density of states at the Fermi level; n_0 is given by Eq. (10).

For nondegenerate electrons (with $q_s^{-1} \gg l$),

$$\sigma_{xx}=\left(\frac{Ze^2}{\chi}\right)^2\frac{N_ie^2a_0}{2\pi\hbar^2\omega_ck_0T}\exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}{k_0T}\right)$$
$$\times\ln(4k_0T + E_s)E_s\ln\left(\frac{k_0T}{E_c}\right) \tag{35}$$
$$=\left(\frac{Ze^2}{\chi}\right)^2\frac{n_0N_ie^2a_0\pi}{2m_c\omega_c(k_0T)^{5/2}I(x)}\left(\frac{\hbar^2}{2m_c}\right)^{1/2}$$
$$\times\exp\left(\frac{\zeta - \frac{1}{2}E_g a_0 + \frac{1}{2}E_g}{k_0T}\right)$$
$$\times\ln(4k_0T + E_s)E_s\ln(k_0T/E_c)\ , \tag{36}$$

with

$$E_s=\frac{\hbar^2q_s^2}{2m_c}=\frac{2\pi e^2\hbar^2n_0}{\chi m_ck_0T}\ . \tag{37}$$

$I(x)$ and n_0 are given by Eqs. (16) and (17), respectively. The factor $\ln(k_0T/E_c)$ arises from the cutoff of the logarithmic divergence with E_c as the proper physical cutoff energy and involves the assumption[10] that $E_c \ll k_0T$. In the case of cutoff by collision broadening, E_c is given by

$$E_c=\left(\frac{\hbar}{2\tau_i}\,\frac{\frac{1}{2}\hbar\omega_c}{a_0(k_0T)^{1/2}}\right)^{2/3}\ , \tag{38}$$

where

$$\tau_i^{-1}=\left(\frac{\pi e^2}{\chi}\right)^2\frac{N_iF(3k_0T)\gamma'}{8\gamma^{3/2}(2\pi m_c)^{1/2}} \tag{39}$$

is the collision frequency for screened ionized-impurity scattering with

$$\gamma \simeq E(1 + E/E_g)\ , \tag{40}$$

$$\gamma' \simeq 1 + 2E/E_g\ , \tag{41}$$

and

$$F(E)=\ln(1 + 4\gamma/E_s) - (1 + E_s/4\gamma)^{-1}\ . \tag{42}$$

TABLE I. Material parameters.[a]

Effective mass m_c/m_0	0.013
Band gap E_g (eV)	0.265
Static dielectric constant χ	18.7
Material density ρ (g/cm^3)	5.82
Debye temperature Θ (°K)	278
Sound velocity v_s (km/sec)	3.8
Temperature T (°K)	4.2

[a]Reference 13.

On the other hand, cutoff energy due to interference of electron waves (non-Born scattering) following Skobov[12] is given by

$$E_c \sim \hbar^2 f^2 / 2 m_c a_0^4 \quad , \tag{43}$$

where f, the scattering amplitude, can be shown to be given by

$$f^2 = \left(\frac{e^2}{8\chi}\right)^2 \frac{\pi^{1/2} F(3k_0T)}{(k_0T)^{1/2}} \frac{\gamma'}{\gamma^{3/2}} \quad . \tag{44}$$

III. NUMERICAL RESULTS AND DISCUSSIONS

In order to have a quantitative estimate of the effect of nonparabolic energy bands on the helicon damping, some numerical results have been presented in the form of graphs for the typical case of n-InSb, assuming screened ionized-impurity scattering to be the dominant scattering mechanism at low temperatures. However, with a judicious combination of the various scattering mechanisms as warranted by the experimental conditions, the results can be applied to any semiconductor characterized by spherical nonparabolic energy bands. An additional advantage with n-InSb is that it is known to possess a very low electron effective mass, which allows one to realize the EQL at relatively low magnetic fields. Numerical parameters used in the calculation are listed in Table I. The helicon damping is governed by the ratio of the dissipative conductivity to the Hall conductivity,[14] i. e., σ_{xx}/σ_{yx} and is weak if $(\sigma_{xx}/\sigma_{yx}) < 1$. But it is well known from dc transport theory that σ_{yx} $(= nec/H)$ remains independent of the quantum effects, even for the condition $\hbar\omega_c \gg k_0T$ as long as the carrier density is conserved[15] [i. e., $n \neq n(H)$]. Therefore, any change in helicon damping due to the nonparabolic energy bands would be attributed to the corresponding change in magnitude and the magnetic field dependence of σ_{xx} as compared to its value in the parabolic-band approximation. It can be readily seen that all the expressions for σ_{xx} reduce to the corresponding expressions of Kubo *et al.*[5] if $\hbar\omega_c \ll E_g$, which is the case for a parabolic-band model.

Figure 1 shows the variation of σ_{xx} as a function of H in a degenerate sample $(n_0 \sim 10^{17}/\text{cm}^3)$ when ionized-impurity scattering is the dominant scattering mechanism. The whole curve (marked I) corresponds to our results appropriate for nonparabolic energy bands, while the dashed curve (marked II) refers to the corresponding results for parabolic bands as described by Kubo *et al.*[5] and Roth and Argyres.[10] It is apparent from the figure that the inclusion of nonparabolicity leads to stronger damping of the helicons due to increase in σ_{xx} for the same value of the magnetic field. The plot of Hall conductivity σ_{xx} as a function of H is also shown on the same figure (curve marked III). The intersection of the curves $\sigma_{xx}(H)$ and $\sigma_{yx}(H)$ determines the so-called critical field H_{cr}, which in turn determines the boundary between helicon transparancy and absorption regions.[4] It is seen from the figure that the critical field corresponding to nonparabolic energy bands lies at a lower value (~360 kG) of the magnetic field as compared to parabolic bands (~550 kG). Consequently, the wave would get totally absorbed at a lower value of the magnetic field in the nonparabolic case. Figure 2 shows the corresponding variation of σ_{xx} (curves marked I and II and σ_{yx} (curve III) both as a function of H for nondegenerate electrons $(n_0 \sim 5.0 \times 10^{13}\,\text{cm}^{-3})$. The dependence of σ_{xx} on H in this case is qualitatively different from that in the case of degenerate electrons. σ_{xx} decreases with H while it increases with H for degenerate electrons. It is worthwhile to point out that since the mechanism with the largest cutoff

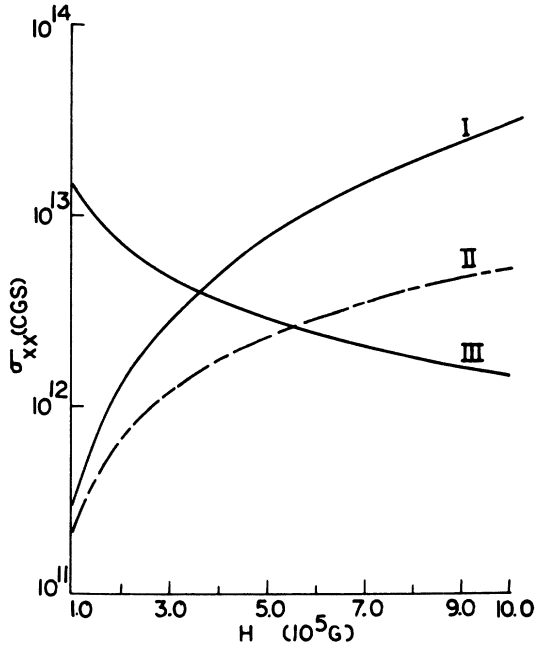

FIG. 1. Magnetic field (H) dependence of σ_{xx} and σ_{yx} for degenerate electrons $(n_0 \sim 10^{17}\,\text{cm}^{-3})$; Curve I (solid curve), $\sigma_{xx}(H)$ for nonparabolic energy bands; curve II (dashed curve), $\sigma_{xx}(H)$ for parabolic energy bands; curve III, Hall conductivity σ_{yx} as a function of H.

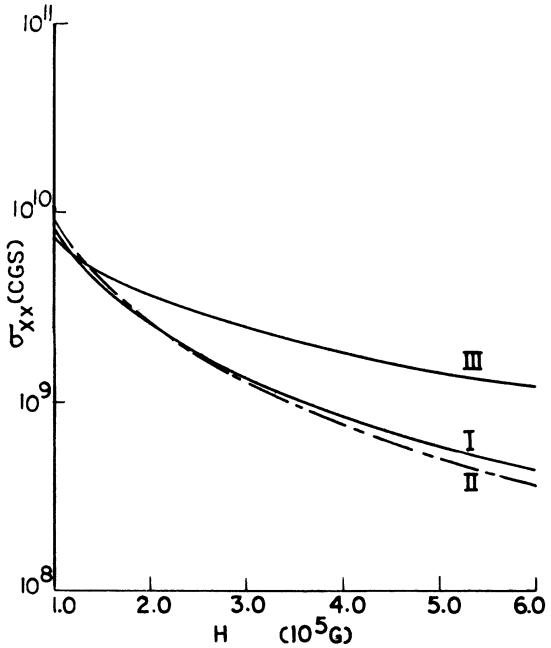

FIG. 2. Magnetic field (H) dependence of σ_{xx} and σ_{yx} for nondegenerate electrons ($n_0 \sim 5.0 \times 10^{13}$ cm^{-3}); Curve I (solid curve), $\sigma_{xx}(H)$ for nonparabolic energy bands; curve II (dashed curve), $\sigma_{xx}(H)$ for parabolic energy bands; curve III, Hall conductivity σ_{yx} as a function of H.

energy by nature cuts off the logarithmic divergence first,[5,10] we have calculated the cutoff energy E_c from both collision broadening and non-Born scattering as a function of H in order to determine their relative importance. It is found that the mechanism of non-Born scattering is more effective in cutting off the divergence than collision broadening for the parameters chosen in the present calculation. Figure 2 also shows that the critical field in this case lies at a much lower value as compared to degenerate electrons. Moreover, the critical field corresponding to parabolic and nonparabolic energy bands do not differ much, unlike the previous case of degenerate electrons. For the parabolic model it is ~130 kG while the inclusion of nonparabolicity makes it ~110 kG. This is to be expected because the effect of nonparabolicity of the energy bands would manifest only at very strong magnetic fields. Physically, this is evident from Eq. (5) because m^* increases with H through the factor a_0. The numerical results should find application in the interpretation of EQL helicon-damping experiments performed for contactless investigation of n-type-InSb samples. However, an explicit comparison of the theoretical results would require the experiments to be carried out in very strong magnetic fields such that $\hbar\omega_c \gg k_0 T$. Such strong magnetic fields are experimentally feasible with the present technology, e.g., experimental magnetoresistance measurements in InSb have been reported[16] in the literature in pulsed magnetic fields up to 800 kG.

In summary, we have provided a more realistic model appropriate for the determination of helicon damping in narrow band-gap semiconductors like n-InSb in the EQL by including the nonparabolic nature of the energy bands in the calculation. Our calculations show that the effect of nonparabolicity in the band structure on the helicon damping is important in quantum magnetic fields. In strong magnetic fields such that $\hbar\omega_c \simeq E_g$, the nonparabolicity of the energy bands, in general, leads to a stronger damping of the helicons as compared to the results of the parabolic model.

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