PHYSICAL REVIEW B VOLUME 9, NUMBER 6 15 MARCH 1974

Magnetoflicker noise in Na and K†

A. W. Overhauser
Department of Physics, Purdue University, West Lafayette, Indiana 47907
(Received 25 October 1973)

It is shown that a physical model put forward to explain the induced-torque anomalies observed by Schaefer and Marcus in single-crystal spheres of Na and K requires the existence of a new type of $1/f$ flicker noise in large magnetic fields. A quantitative theory of this effect is developed. The mean-square noise voltage is proportional to $I^2 R_0^2 B^3 \Delta f/f$, where I is the current, R_0 is the zero-field resistance, B is the magnetic field, and Δf is the bandwidth. For a convenient experiment the noise could be as large as $(1\times10^{-6}\ \text{V})^2$, 13 orders of magnitude greater than Johnson noise. Experimental study is recommended to illuminate further the perplexing magnetoconductivity properties of Na and K.

I. BACKGROUND

The failure of the magnetoresistance of Na and K to saturate at high fields has been a challenging puzzle for many years.[1] Fear that this anomalous behavior might be an artifice caused by electrode geometry has been eliminated by the advent of inductive techniques.[2] Data obtained by these newer methods substantiate those obtained from well-executed four-terminal measurements.[3]

The most spectacular results obtained so far are the torque measurements of Schaefer and Marcus[4] on single-crystal spheres. A torque about the vertical axis arises when a sample experiences a magnetic field rotating in the horizontal plane.[5] The torque is proportional to the induced current, which depends on the magnetoconductivity tensor of the sample. The theoretical treatment[6] for a spherical sample with an isotropic conductivity (and quite generally[7] for tensors of arbitrary symmetry) requires that the high-field torque saturate if the magnetoresistance does. Not only do the observed torques increase linearly at high field, but they are highly anisotropic (sometimes by factors exceeding 5). Data from oriented single crystals[4] indicate that the anisotropy is in conflict with the presumed cubic symmetry of Na and K.

Lass[8] has calculated the torque pattern for a sample having a nonspherical shape, and has shown that it is possible to predict curves similar to those of Schaefer and Marcus. There are, however, serious difficulties with this explanation. A 10% deviation from spherical shape is required to explain the observations. Experimentally, the deviations from sphericity were about 2% or less.[9] This was determined by torque measurements (e.g., at 80 °K) when $\omega_c\tau \ll 1$. Under these conditions a 1% shape anisotropy gives rise to a 1% torque anisotropy. On the other hand the high-field torque anisotropy is proportional to the square of the shape deviation.[8] Consequently, Lass's explanation appears inadequate by about a factor of 25.

Another problem is that the high-field anisotropies were correlated with [110] crystal axes, whereas shape anisotropies would likely have random orientation. Furthermore, if the degree of shape anisotropy had a reasonable statistical distribution (~70 samples were studied) a large fraction should have had quite small high-field anisotropies in view of the quadratic dependence mentioned above. This was not the case.[4] A further difficulty is that an intrinsic high-field linear magnetoresistance with a Kohler slope $S\sim0.025$ had also to be assumed. This value is an order of magnitude larger than that generally observed in single crystals.[2,3] In this connection, however, a recent study[10] suggests that the longitudinal magnetoresistance may be significantly larger than the transverse. This latter work was carried out on spheres that were accurately round to better than $\frac{1}{2}$%. Nevertheless anisotropies up to a factor 2.4 were observed for $\omega_c\tau=150$.

It is remotely conceivable that a spherical sample, electrically isotropic at 80 °K (where phonon scattering dominates the resistivity), could be electrically anisotropic at 4 °K from an unusual impurity concentration at one end of the specimen. However, the large diffusion coefficients obtained[11] for solutes in Na and K should provide effective homogenization during the time a sample awaits its destined application.

We shall now turn to the other proposed[12] explanation of the Schaefer-Marcus experiments. This model is based on the hypothesis that the electronic ground state of Na or K has a (almost) static charge-density-wave (CDW) structure.[13] This model had been invoked to explain other alkali-metal anomalies.[14] It is noteworthy that no new embellishments of the CDW model were required to provide a detailed account of all the Schaefer-Marcus results. Our purpose is to observe that this success requires the existence of a large magnetoflicker noise in K wires. Experimental test of this prediction is crucial, since it may show that success can be a measure of ingenuity without being a measure of truth.

II. CDW STRUCTURE

The possibility for a simple metal to have a CDW ground state rests on a firm theoretical foundation. It has been shown[13] that exchange and correlation act constructively to cause such an instability. The only uncertain contribution is whether the positive-ion background is sufficiently deformable to allow neutralization of the electronic charge density

$$\rho(\vec{r}) = \rho_0[1 - p\cos(\vec{Q}\cdot\vec{r}+\varphi)] \quad . \qquad (1)$$

$\vec{Q}$ and p are the wave vector and amplitude of the CDW. Neutralization will be optimum if the ion displacements are

$$\vec{u}(\vec{L}) = (p\vec{Q}/Q^2)\sin(\vec{Q}\cdot\vec{L}+\varphi) \quad , \qquad (2)$$

relative to their ideal lattice sites $\vec{L}$. Na and K are prime candidates for such a phenomenon since their direct ion-ion interaction is very weak.[15]

The magnitude of $\vec{Q}$ is very nearly the diameter $2k_F$ of the Fermi surface, since that choice optimizes both the exchange[16] and correlation[13] energy. Q is larger than the radius of the Brillouin zone. Accordingly, the wave vector $\vec{Q}'$ of the static phonon equivalent to Eq. (2) is

$$\vec{Q}' = (2\pi/a)(1, 1, 0) - \vec{Q} \quad . \qquad (3)$$

We have concluded[14] that the lowest-energy direction for $\vec{Q}$ is along the [110] direction of the bcc lattice (having lattice constant a) because that minimizes the magnitude of $\vec{Q}'$. This is desirable because any deformation energy associated with the neutralization of Eq. (1) by (2) should have some proportional relationship to the energy of a phonon of wave vector $\vec{Q}'$. Since $Q \approx 1.33(2\pi/a)$,[17]

$$Q' \approx (2\pi/a)(\sqrt{2} - 1.33) \approx 0.08(2\pi/a) \quad . \qquad (4)$$

The periodicity $\vec{Q}'$ gives rise to two small energy gaps (heterodyne gaps[18]) which cut through the Fermi surface very near $k=0$. Although these energy gaps are quite small[19] ($E_g \sim 0.015$ eV) cyclotron orbits which intersect them will on the average not suffer magnetic breakdown except in fields exceeding 100 kG. The geometric relationship of the heterodyne gaps to the Fermi surface is shown in Fig. 1 for $\vec{Q}$ parallel to [110] and for $\vec{Q}$ having a 2° angular deviation from [110].

The effect of the heterodyne gaps is to divide the Fermi surface into two parts (not three). Two "hemispherical" ends can be pieced together to form one simply connected surface that is almost spherical. (It is somewhat distorted near the poles and the equator.) The second piece is the cylindrical slice, which we shall call the "wedding band." It will give rise to open orbits whenever $\vec{Q}'$ is perpendicular to $\vec{B}$. This allows one to explain the anomalous high-field magnetoresistance, as we show in Sec. IV, and to account[12] for the bizarre

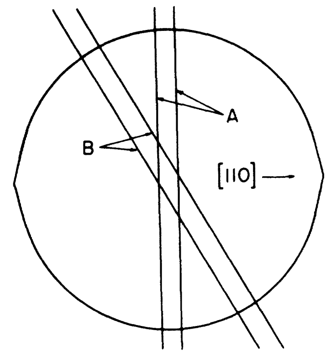

FIG. 1. Brillouin zone and Fermi surface of K, assuming a CDW ground state. The heterodyne gaps A occur when $\vec{Q}$ is parallel to [110]. The heterodyne gaps B occur when $\vec{Q}$ is tilted 2° from [110].

torque patterns of Shaefer and Marcus without a separate assumption.[20] The predicted correlation of the torque patterns with one of the [110] crystal axes was verified in the 15 oriented samples that were studied.[4] The fluctuation (in time) of the axis $\vec{Q}'$ of the wedding band plays a crucial role in all of these phenomena. For example, the angular width of the high-field torque peaks is directly related to the range of the fluctuation. In Sec. V we show that these fluctuations must give rise to a flicker noise in the resistivity.

The major physical difficulty of the CDW model is a reconciliation with the isotropy[21] of the de Haas–van Alphen effect. Prior to the work of Schaefer and Marcus, one could postulate[14] that $\vec{Q}$ aligns parallel to a strong magnetic field, thereby preventing observation of a CDW anisotropy in the Fermi surface. The torque anomalies "show" that $\vec{Q}$ is confined (within a few degrees) to a [110] axis. An alternative reconciliation is necessary, and this is currently under study. It should be mentioned that the conduction-electron-spin-resonance (CESR) splitting[22] provides indirect evidence of a Fermi-surface distortion (~7%) that is in quantitative agreement[23] with the CDW model. The splitting arises when a sample has two or more $\vec{Q}$ domains, since[23] the CESR g factor depends on the angle between $\vec{B}$ and $\vec{Q}$. Recent experiments[24] at 40 kG show that the splitting is proportional to B, and that the CESR can have as many as five well-resolved components.

III. FLUCTUATIONS OF $\vec{Q}$ AND $\vec{Q}'$

A CDW state is an ordered phase with a four-dimensional order parameter. The four components are the amplitude p [Eq. (1)], the magnitude of $\vec{Q}$, and the two angles needed to specify the directional deviation of $\vec{Q}$ from a [110] axis. Since the charge density amplitude is large[13] ($p \sim 0.17$), we shall neglect its variation and consider only the three remaining components. A dynamical theory of these (fluctuations in $\vec{Q}$) has been given.[14] It was prompted by a necessity to explain why diffraction satellites associated with the new periodicity $\vec{Q}$ could not be observed in a feasible experiment. (Analogous fluctuations in a one-dimensional metal have recently become of interest.[25])

Long-wavelength fluctuations in $\vec{Q}$ can be most easily described by letting the phase angle φ in Eqs. (1) and (2) be a slowly varying function of position and time[14]:

$$\varphi(\vec{L}, t) = \sum_{\vec{q}} \varphi_{\vec{q}} \sin(\vec{q} \cdot \vec{L} - \omega_{\vec{q}} t) \quad , \qquad (5)$$

where $\varphi_{\vec{q}}$ is the amplitude of an excitation having wave vector $\vec{q}$ and frequency $\omega_{\vec{q}}$. We have shown that $\omega_{\vec{q}}$ has a linear dispersion relation near $q = 0$. These new modes are, of course, part of the vibrational spectrum of the crystal. In view of their description as a phase modulation of the CDW we have called them phason modes. Relative to the ordinary $\vec{k}$ space of the Brillouin zone the phason $q = 0$ is located a distance Q' from $k = 0$ along one of the [110] directions. The fact that this mode can have $\omega = 0$ (in addition to the three acoustic phonon modes for $k = 0$) depends on $\vec{Q}$ being incommensurate with the reciprocal lattice.

Let q_1, q_2, q_3 be the three components of $\vec{q}$, corresponding to the [110] ($\vec{Q}$ direction), [$\bar{1}$10], and [001] directions. For small q,

$$\omega_{\vec{q}} = [u_1^2 q_1^2 + u_2^2 q_2^2 + u_3^2 q_3^2]^{1/2} \quad , \qquad (6)$$

where u_1, u_2, u_3 are the three principle components of the phason velocity tensor. There has been no attempt to calculate the magnitude of these velocities. However, they are probably smaller than phonon velocities since the condensation energy of a CDW state is only $\sim 2\times10^{-4}$ eV per atom.[14] A phason (q_1, 0, 0) describes a periodic variation of the magnitude of $\vec{Q}$, whereas (0, q_2, q_3) describes a periodic variation of the direction of $\vec{Q}$. These attributes are based on the assumption that phason modes are underdamped. Damping caused by electron-phason interactions has not been studied.[26]

The fact that Q' [Eq. (4)] is 16 times smaller than Q, together with the geometric constraint on these vectors given by Eq. (3), means that a 1° rotation of $\vec{Q}$ causes a 16° rotation of $\vec{Q}'$. It is clear that zero-point and thermal excitation of phasons will cause profound time-dependent fluctuations in the Fermi surface and all physical properties which depend on it. We now turn to its influence on the magnetoconductivity tensor. The dominant effect arises from fluctuations of the axis of the wedding band.

Consider a coordinate system u, v, w so that the axis $\vec{Q}'$ of the wedding band is in the w direction. The angular relationship of u, v, w to an x, y, z coordinate system is shown in Fig. 2. The con-

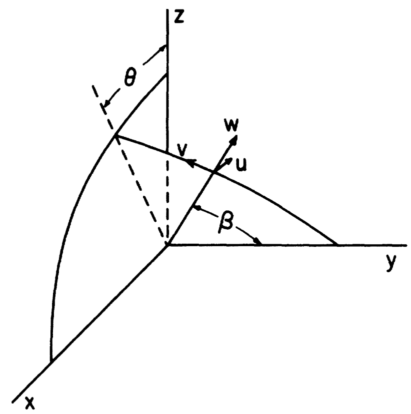

FIG. 2. Relative orientation of the uvw axes and the xyz axes. The axis $\vec{Q}'$ of the wedding band is parallel to w. The magnetic field $\vec{B}$ is parallel to z.

ductivity tensor of the wedding band in the u, v, w frame is[12]

$$\sigma_{\mathrm{wb}} = \frac{\eta\rho_0^{-1}}{1+\delta^2}\begin{pmatrix} 1 & -\delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \qquad (7)$$

where $\delta \equiv \omega_c\tau \sin\beta\cos\theta$, ρ_0 is the $B = 0$ resistivity of the metal, and η is the fraction of the conduction electrons between the heterodyne gaps:

$$\eta \approx \pi k_F^2 Q' / (\tfrac{4}{3}\pi) k_F^3 \sim 0.10 \quad . \qquad (8)$$

When $\vec{Q}'$ is tilted the fraction η is somewhat larger. We have taken the field $\vec{B}$ in the z direction.

The quasispherical Fermi surface, which contains the remaining fraction, $1-\eta$, of electrons will have a conductivity tensor in the xyz frame given approximately by

$$\sigma_{\mathrm{qs}} \approx \frac{(1-\eta)\rho_0^{-1}}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & 1+(\omega_c\tau)^2 \end{pmatrix} \quad . \qquad (9)$$

We have here neglected any anisotropy in σ_{qs} for $B = 0$ since it is not germane to a calculation of the magnetophason noise. (It was relevant to the low-field torque patterns.[12]) The total conductivity in the xyz frame is obtained by summing (7) and (9) after transforming (7) into the xyz frame.

$$\sigma = \tilde{S}\sigma_{\mathrm{wb}}S + \sigma_{\mathrm{qs}} \quad . \qquad (10)$$

The (orthogonal) transformation matrix is[12]

$$S \equiv \begin{pmatrix} -\cos\theta & 0 & \sin\theta \\ \sin\theta\cos\beta & -\sin\beta & \cos\theta\cos\beta \\ \sin\theta\sin\beta & \cos\beta & \cos\theta\sin\beta \end{pmatrix} \quad . \qquad (11)$$

It is of course the phason-induced fluctuations of the first term of Eq. (10) that generate a high-field linear magnetoresistance. This was already shown by the theory of the torque anomalies,[12] since the high-field torques are a direct measure of the magnetoresistance.[6]

In the following section we shall compute the transverse magnetoresistance in a long, thin wire using Eq. (10). Even if the wire were a single crystal, it will have many $\vec{Q}$ domains. A $\vec{Q}$ domain is a region within a single crystal where $\vec{Q}$ is aligned parallel to just one of the six [110] axes. There will be $\vec{Q}$-domain boundaries across which the direction of $\vec{Q}$ changes abruptly to another [110]-type axis. In a macroscopic sample the size and distribution of $\vec{Q}$ domains will play an important role in determining electrical transport. One would have to embark on studies of even greater difficulty than those considered by Herring.[27] Very little can be inferred about $\vec{Q}$-domain structure from experiment. If the CDW hypothesis is correct, the variability of $\vec{Q}$-domain structure would certainly be the explanation of the nonrepeatability of experiments from sample to sample, or from run to run on the same sample, which has baffled and frustrated all workers who have studied alkali metals. Slight deformation, thermal or mechanical, may cause nucleation and growth of new $\vec{Q}$ domains and change the transport coefficients by large factors.

The 200 experimental runs reported by Schaefer and Marcus[4] are perhaps the set with the least overall variability to date. Their samples were small, 2–7-mm-diam spheres. It is reasonable to regard the qualitative uniformity of their results, together with the fact that a single $\vec{Q}$-domain model could explain them, as indicating that $\vec{Q}$ domains can be comparable in size to their specimens. There is also indirect support. We have shown[28] by neutron diffraction studies of primary extinction that the mosaic blocks in single crystal K are about 1 mm in size.

Extreme variability appears to be an intrinsic property of Na and K. We now turn to one effect of $\vec{Q}$-domain structure and phason fluctuations which account, at least in part, for that.

IV. MAGNETORESISTANCE OF A THIN WIRE

Consider a wire of length L, diameter d, and let it be oriented along the y axis. We shall assume that d is sufficiently small so each $\vec{Q}$ domain extends across the cross section of the wire. The approximation we are attempting to justify is one for which the total resistance is the sum of the resistances of each $\vec{Q}$ domain. With $\vec{B}$ in the z direction the magnetoresistivity of a $\vec{Q}$ domain will be ρ_{yy}, where the matrix ρ is the inverse of σ [Eq. (10)]. Calculating this inverse is a strenuous algebraic exercise leading to expressions of great complexity. Our interest is only in the leading term for the high-field limit, $\omega_c\tau \gg 1$:

$$\rho_{yy} \approx \rho_0 + \eta\rho_0 \frac{(\omega_c\tau)^2\cos^2\beta\sin^2\theta}{1+(\omega_c\tau)^2\sin^2\beta\cos^2\theta} \quad . \tag{12}$$

We have neglected terms of order η^2 and all terms important only near $\omega_c\tau\sim 1$. Accordingly Eq. (12) is correct only for the low-field and high-field limits. Furthermore it is valid in the latter case only when the numerator of the second term is large compared to the denominator. From Fig. 2 one can see that $\epsilon \equiv \sin\beta\cos\theta$ is the angular deviation of $\vec{Q}'$ away from the xy plane, which is perpendicular to $\vec{B}$. The variation of $\Delta\rho_{yy}$ with ϵ is shown in Fig. 3. It has a Lorentzian shape with a height proportional to $(\omega_c\tau)^2$ and a half-width equal to $2/\omega_c\tau$. The area under this curve is proportional to $\omega_c\tau$.

We now assume that the wire has many $\vec{Q}$ domains of random orientation and that the phason fluctuations of $\vec{Q}'$ cause the directional distribution of $\vec{Q}'$ to be continuous and isotropic. The mean high-field resistivity increase is then

$$\left\langle\frac{\Delta\rho_{yy}}{\rho_0}\right\rangle \approx \frac{\eta(\omega_c\tau)^2}{4\pi}\int_0^\pi \cos^2\beta\sin\beta\,d\beta \times\int_0^{2\pi}\frac{d\theta}{1+(\omega_c\tau)^2\sin^2\beta\cos^2\theta} \quad . \tag{13}$$

We have set $\sin\theta=1$ when inserting the second term of Eq. (12) in the integrand of Eq. (13), since the important contribution to the integral arises only when $\theta\approx\frac{1}{2}\pi$. The θ integration yields $2\pi[1+(\omega_c\tau)^2$

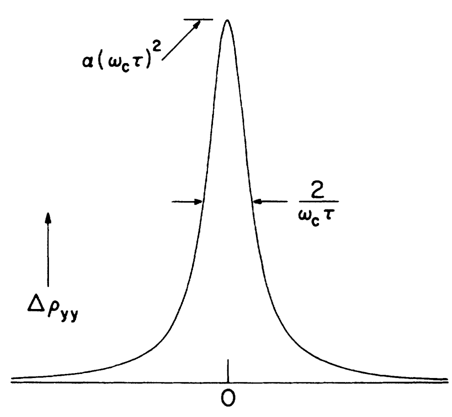

FIG. 3. Variation of ρ_{yy} with ϵ, the angle between the axis $\vec{Q}'$ of the wedding band and the xy plane. This curve also depicts the voltage pulse that a $\vec{Q}$ domain will contribute if $\epsilon = vt$.

$\sin^2\beta]^{-1/2}$, which for $\omega_c\tau \gg 1$ is just $2\pi/\omega_c\tau\sin\beta$. The β integration is then trivial and contributes a factor $\frac{1}{2}\pi$. We obtain, finally,

$$\langle \Delta\rho_{yy}/\rho_0\rangle \approx \tfrac{1}{4}\pi\eta\omega_c\tau \quad , \tag{14}$$

which is the (desired) high-field linear magnetoresistivity. The Kohler slope S is Eq. (14) divided by $\omega_c\tau$. From Eq. (8) the expected value of S is 0.08. Observed values of S vary about this value by two orders of magnitude. We therefore define a factor F so that

$$S = \tfrac{1}{4}\pi\eta F \quad . \tag{15}$$

This factor is intended to account for the sample variation of $\vec{Q}$-domain distribution, as well as one additional effect discussed below.

It is clear from the derivation that the high-field resistance results from resistance pulses $(\omega_c\tau)^2$ in magnitude and lasting a fraction $(\omega_c\tau)^{-1}$ of the time. Suppose a $\vec{Q}$ domain does not occupy the entire cross section of the wire. Its resistance pulses may be "shorted out" by the surrounding material. This is a likely occurrence in large specimens, as is frequently the case with single crystals, which generally show values ~0.003 for S. A very interesting theoretical question is the extent to which inductive effects may inhibit this shorting process during the very brief time interval of a resistance pulse.

One can perhaps predict in advance whether mechanical deformation will cause S to increase or decrease. The shorting effect would be enhanced if the $\vec{Q}$ domains become smaller in size. Deformation (which should initially decrease the size) can initiate a strain-anneal process that may then allow $\vec{Q}$ domains to grow larger. Deformation of single-crystal and polycrystal specimens[2] has been found to give an increase in S; but these data were obtained on samples strained at 77 °K, well above 15 °K where K begins its mechanical recovery.[29] In contrast, polycrystalline wires strained at 4 °K show a decrease in S with strain.[30] If the strain temperature is the difference, (and this should be given further study) the contrasting behavior can be understood in terms of the anticipated change in $\vec{Q}$-domain size outlined above.

V. MAGNETOPHASON NOISE

The proposed origin of the high-field linear magnetoresistance given previously[12] and in the foregoing section must necessarily cause a large flicker noise, the sum total of all pulses from every $\vec{Q}$ domain delivered randomly in time. We calculate here the noise spectrum and its absolute magnitude without attempting to be exact at every step.

Suppose that the angle ϵ by which $\vec{Q}'$ deviates from the xy plane varies with time as follows:

$$\epsilon \sim \zeta \sin 2\pi\nu t \quad , \tag{16}$$

where ζ is a characteristic amplitude of the $\vec{Q}'$ oscillation angle, and ν is a characteristic frequency. We expect that $\zeta \sim 0.4$ rad, since 2ζ must equal the width of the Schaefer-Marcus torque peaks (~45°). From Fig. 3 one can see that the resistivity pulse is large only for $|\epsilon| \lesssim 1/\omega_c\tau$. So we may approximate the time dependence of ϵ during one such pulse by $\epsilon = vt$, where v is an angular velocity $\sim 2\pi\nu\zeta$. If r_0 is the zero-field resistance of the Q domain, then from (12),

$$\Delta r \approx \eta r_0(\omega_c\tau)^2\cos^2\beta/[1+(\omega_c\tau v t)^2] \quad . \tag{17}$$

This resistance pulse has the shape depicted in Fig. 3. The Fourier transform of (17) is

$$\Delta r_f = \int_{-\infty}^{\infty} \Delta r\, e^{-2\pi i f t}\, dt \quad , \tag{18}$$

where f is the frequency. The integration is elementary. The one-sided spectral density for one such pulse per second is obtained by squaring Eq. (18) and multiplying by 2:

$$(\Delta r_f)^2 \approx 2(\pi\eta v^{-1} r_0\omega_c\tau\cos^2\beta)^2 e^{-4\pi f/v\omega_c\tau} \quad . \tag{19}$$

The factor 2 is inserted so that Eq. (19) includes contributions from f and $-f$.

We must next average (19) over a suitable angular velocity distribution $P(v)$. The maximum velocity associated with a perpendicular traversal of $\vec{Q}'$ through the xy plane is $v_m \sim 2\pi\nu\zeta$. The characteristic frequency ν is unknown since the phason velocities [Eq. (6)] are unknown. A reasonable guess for an upper limit might be u/d, where d is the wire diameter (and $\vec{Q}$-domain size.) For $u \sim 10^5$ cm/sec and $d \sim 0.1$ cm, $\nu < 10^6$ Hz.

The traversal of $\vec{Q}'$ through the xy plane may occur, however, at any angle, say α. Equation (16) corresponds to $\alpha = \frac{1}{2}\pi$; so in general the angular velocity will be

$$v = v_m \sin\alpha \quad . \tag{20}$$

The probability distribution $P(v)$ is related to that for α,

$$P(v) = \left(\frac{dv}{d\alpha}\right)^{-1} P(\alpha) \quad , \tag{21}$$

where $P(\alpha) = 2/\pi$, $0 \le \alpha \le \frac{1}{2}\pi$. One calculates the needed derivative from Eq. (20), whereupon

$$P(v) = 2/\pi(v_m^2 - v^2)^{1/2} \quad . \tag{22}$$

We must integrate the v dependent terms of Eq. (19) with this distribution. The factor of interest is

$$\int_0^{v_m} P(v)\, e^{-4\pi f/v\omega_c\tau}\, v^{-2}\, dv \quad . \tag{23}$$

This integral can be approximated very well by observing that most of the contribution comes from small v as long as $f \ll \omega_c\tau\nu$. For small v, Eq. (22)

is just $2/\pi v_m$, and the integration becomes elementary. Accordingly expression (23) has the approximate value:

$$\omega_c\tau/2\pi^2 f v_m = \omega_c\tau/4\pi^3\zeta\nu f \quad . \tag{24}$$

At this point we see that the spectral density has a $1/f$ frequency dependence. Magnetophason noise is a $1/f$ noise, in common with many other types of flicker noise. We combine (24) with the remaining terms of (19), multiply by 2ν (a mean number of plane crossings per second for an active $\vec{Q}$ domain), and obtain the spectral density per active $\vec{Q}$ domain:

$$(\Delta r_f)^2 \approx \eta^2 r_0^2(\omega_c\tau)^3 \cos^4\beta/\pi\zeta f \quad . \tag{25}$$

We note that the characteristic fluctuation frequency ν has dropped out. Our multiplication of (19) by 2ν to obtain (25) is justified only if pulses from the same $\vec{Q}$ domain are uncorrelated in time. Of course this cannot be completely true. The continuous distribution of phason frequencies between 0 and ν prevents long-time correlations. Nevertheless this step must be regarded as an approximation.

The resistance pulses from different $\vec{Q}$ domains will be uncorrelated. We need only count how many $\vec{Q}$ domains in the sample (of length L and diameter d) will participate. If the average length of a $\vec{Q}$ domain is γd, there will be $L/\gamma d$ domains in the sample. (By definition γ is a shape parameter for the typical $\vec{Q}$ domain, and should be near unity.) Not all of the domains can contribute to the flicker noise, since some will have $\vec{Q}$ oriented too far from the xy plane to allow $\vec{Q}'$ much chance of fluctuating into the plane. Only those domains having $\vec{Q}$ within an equatorial strip 2ζ wide will contribute fully to the noise. The fractional solid angle of this strip is ζ. We must modify the total expected number by the same factor F that corrects the linear magnetoresistance [Eq. (15)]. Consequently, the total sample spectral density will be (25) multiplied by the factor

$$\zeta FL/\gamma d \quad . \tag{26}$$

Within the effective (equatorial) strip, $\langle\cos^4\beta\rangle \cong \frac{3}{8}$. The mean-square noise voltage is the spectral density times the bandwidth Δf, times the square of the measuring current I. We combine all these factors and obtain for the noise voltage V,

$$\langle V^2\rangle = \frac{3\eta^2 r_0^2(\omega_c\tau)^3 LFI^2}{8\pi\gamma d}\left(\frac{\Delta f}{f}\right) \quad . \tag{27}$$

The sample resistivity for $B=0$ is $R_0 = Lr_0/\gamma d$. We can therefore eliminate r_0 from (27), and also F with the help of Eq. (15). Our final result is

$$\langle V^2\rangle = \frac{3\gamma\eta d S I^2 R_0^2(\omega_c\tau)^3}{2\pi^2 L}\left(\frac{\Delta f}{f}\right) \quad . \tag{28}$$

It is noteworthy that except for γ (the $\vec{Q}$-domain shape parameter which should be close to unity), none of the factors in (28) are adjustable. The fractional number η of electrons enclosed by the heterodyne gaps is given by Eq. (8). The theoretical uncertainty in η cannot exceed about 50%. (For Na, $\eta \sim 0.07$.) The $B=0$ resistance and the Kohler slope S must be measured for each sample. Similarly the residual resistance ratio determines $\omega_c\tau$ (which is proportional to B). Equation (28) predicts a spectacularly large magnetoflicker noise in Na and K. Such a phenomenon is an inescapable conclusion from the premises that Na and K have CDW ground states and that the high-field torque patterns of Schaefer and Marcus are correctly explained by the model. This latter premise is needed so one can assume that $\vec{Q}$ domains are large enough in size to span the cross section of a thin wire.

VI. CONCLUSION

Let us evaluate the noise voltage [Eq. (28)] for a typical experiment. The residual resistivity of K is $\sim 1\times10^{-9}$ Ω cm. Consequently a specimen, $d=0.1$ cm, $L=50$ cm, will have a 4 °K resistance of 6×10^{-6} Ω. $\omega_c\tau \sim 10^2$ for $B=40$ kG. Take $I=1$ A, $\Delta f=100$ Hz, $f=10^3$ Hz, $S=0.01$, $\eta=0.1$, and $\gamma=1$. The noise power is proportional to

$$\langle V^2\rangle \sim (1\times10^{-6}\text{ V})^2 \quad . \tag{29}$$

Let us compare this with Johnson noise under the same conditions:

$$\langle V^2\rangle_J = 4k_B T R_0 \Delta f \quad . \tag{30}$$

Accordingly, the Johnson noise power will be proportional to

$$\langle V^2\rangle_J \sim (4\times10^{-13}\text{ V})^2 \quad . \tag{31}$$

Although Eq. (29) is 13 orders of magnitude greater than Eq. (31), carrying out such a measurement is not trivial. There will be serious microphonic noise caused by vibration of the sample and leads in the magnetic field. This can be minimized, of course, by a compensating winding in series opposition. Microphonic noise can be distinguished from magnetophason noise since the former is proportional to B^2 only, whereas the latter is proportional to I^2B^3.

The predicted flicker noise is so unambiguous in magnitude that an experimental measurement will provide important clarification of the mechanisms contributing to numerous anomalies in alkali-metal behavior. One hopes that such a study will be forthcoming soon.

Note added in proof. R. S. Hockett and D. Lazarus [Bull. Am. Phys. Soc. (to be published)].

have carried out the experiment proposed above. They found no noise greater than 4 nV. This result shows that the direction of $\vec{Q}'$ cannot be assumed constant within a $\vec{Q}$ domain. A modified theory of the four-peaked torque pattern is therefore required and takes into account the finite coherence length a for the direction of $\vec{Q}'$. (This will be submitted later.) The magnitude of the magnetoflicker noise is then given by

$$\langle V^2\rangle \approx \frac{a^3\omega_c \tau S I^2 R_0^2}{d^2 L}\,\frac{\Delta f}{f} \quad . \tag{32}$$

This result is much smaller than that given by Eq. (28) and leads to values $\sim(10^{-9}\text{ V})^2$. The coherence length a is unknown, but it must be $\gtrsim 2\times10^{-4}$ cm, the cyclotron radius at 30 kG. Otherwise the de Haas-van Alphen effect would be incompatible with the CDW model.

---

†Work supported by the National Science Foundation, Material Research Laboratory Program No. GH 33574.

[1]D. Gugan, Nature Phys. Sci. 235, 61 (1972).

[2]P. A. Penz and R. Bowers, Phys. Rev. 172, 991 (1968).

[3]H. Taub, R. L. Schmidt, B. W. Maxfield, and R. Bowers, Phys. Rev. B 4, 1134 (1971).

[4]J. A. Schaefer and J. A. Marcus, Phys. Rev. Lett. 27, 935 (1971).

[5]W. R. Datars and J. R. Cook, Phys. Rev. 187, 769 (1969).

[6]J. S. Lass and A. B. Pippard, J. Phys. E 3, 137 (1970).

[7]P. B. Visscher and L. M. Falicov, Phys. Rev. B 2, 1518 (1970).

[8]J. S. Lass, Phys. Lett. A 39, 343 (1972).

[9]J. A. Schaefer (private communication).

[10]A. M. Simpson, J. Phys. F 3, 1471 (1973).

[11]L. W. Barr, J. N. Mundy, and F. A. Smith, Philos. Mag. 16, 1139 (1967).

[12]A. W. Overhauser, Phys. Rev. Lett. 27, 938 (1971).

[13]A. W. Overhauser, Phys. Rev. 167, 691 (1968).

[14]A. W. Overhauser, Phys. Rev. B 3, 3173 (1971).

[15]D. C. Wallace, Phys. Rev. 176, 832 (1968).

[16]A. W. Overhauser, Phys. Rev. 128, 1437 (1962).

[17]A. W. Overhauser, Phys. Rev. Lett. 13, 190 (1964).

[18]J. R. Reitz and A. W. Overhauser, Phys. Rev. 171, 749 (1968).

[19]A. W. Overhauser, Bull. Am. Phys. Soc. 17, 40 (1972).

[20]The magnetic-breakdown model for the magnetoresistance (Ref. 18) became inapplicable once it was realized (Ref. 14) that $\vec{Q}$ would align along a [110] direction. (The breakdown model requires hole orbits, which can occur only if $\vec{Q}$ is along a [100] or [111] direction.)

[21]D. Shoenberg and P. J. Stiles, Proc. R. Soc. A 281, 62 (1964).

[22]W. M. Walsh, Jr., L. W. Rupp, Jr., and P. H. Schmidt, Phys. Rev. 142, 414 (1966).

[23]A. W. Overhauser and A. M. de Graaf, Phys. Rev. 168, 763 (1968).

[24]G. L. Dunifer and T. G. Phillips (private communication).

[25]P. A. Lee, T. M. Rice, and P. W. Anderson, Phys. Rev. Lett. 31, 462 (1973).

[26]Another detail in need of study is the possibility that the minimum energy for $\vec{Q}$ might occur when $\vec{Q}$ has a small angular tilt ($\sim 1°$ or $2°$) from a [110] direction. Equation (3) would then correspond to a static phonon having a transverse component. It is well known that the [1$\bar{1}$0] transverse mode of a [110] phonon is much softer than the longitudinal mode. Such a tilt might reduce slightly the charge-neutralization energy.

[27]C. Herring, J. Appl. Phys. 31, 1939 (1960).

[28]A. W. Overhauser (unpublished).

[29]W. S. C. Gurney and D. Gugan, Philos. Mag. 24, 857 (1971).

[30]B. K. Jones, Phys. Rev. 179, 637 (1969).