Strong-coupling effects in the temperature dependence of $\kappa_2(T)$*

D. Rainer†
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850

K. D. Usadel
Institut für Theoretische Physik, det Universität Göttingen, Göttingen, Germany
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A calculation of $\kappa_2(T)$ in dirty strong-coupling superconductors is presented. It is shown that strong-coupling effects lead to an enhanced temperature dependence of $\kappa_2(T)$.

This addendum deals with a computer analysis of the effect of strong coupling on the Ginzburg-Landau parameter $\kappa_2(T)$ which is based on a theoretical investigation by Usadel.[1] The computer program has already been used for a discussion of strong-coupling anomalies in $H_{c2}(T)$ and $\kappa_1(T)$. Results can be found in Refs. 2 and 3.

Measurements of the Ginzburg-Landau parameter κ_2 in strong-coupling superconducting alloys have been published recently by Fearday and Rollins.[4] The authors find that κ_2 at T_c is in quantitative agreement with the theoretical results of Usadel. The experiments of Fearday and Rollins furthermore show that the temperature dependence of κ_2 deviates significantly from weak-coupling behavior. The ratio of κ_2 at zero temperature to that at T_c exceeds the corresponding weak-coupling ratio by about 20%.

We want to demonstrate in the present paper that this enhanced temperature dependence of $\kappa_2(T)$ is a strong-coupling effect which also follows from the theory of one of the authors.[1] For this purpose we have performed a numerical evaluation of the strong-coupling equations for $\kappa_2(T)$.

According to Ref. 1, $\kappa_2(T)$ can be obtained in the dirty limit from the solution of the linearized gap equation

$$\Delta(n) = 2\pi T \sum_m [\lambda(\omega_n - \omega_m) - \mu^*] \times \frac{1}{2|\omega_m Z(m)| + D\epsilon_0}\Delta(m) \quad , \tag{1}$$

$$\omega_n Z(n) = \omega_n + \pi T \sum_m \lambda(\omega_n - \omega_m)\,\mathrm{sgn}\,\omega_m \quad , \tag{2}$$

$$\omega_n = (2n-1)\pi T \quad ,$$

via the formula

$$\kappa_2^2(T) = \left[\sum_n \xi^3(n)\,\Delta(n) \Big/ 8\pi e^2 D^2 N(0) 4\pi T \times \left(\sum_n \xi^2(n)\right)^2\right][1 + F(T)] \quad , \tag{3}$$

where

$$F(T) = -2\pi T \sum_{n,m>0} \xi^2(n)[\lambda(\omega_n - \omega_m) - \lambda(\omega_n + \omega_m)]\,\xi^2(m) \Big/ \sum_{n>0} \xi^3(n)\,\Delta(n) \quad .$$

Here

$$\xi(n) = \frac{\Delta(n)}{2|\omega_n Z(n)| + D\epsilon_0} \quad ;$$

$N(0)$ and D are the bare density of states and the bare electronic diffusion constant, respectively; $\lambda(\omega_n - \omega_m)$ is the effective electron-electron attraction which is conventionally expressed in terms of a spectral function $\alpha^2 F(\omega)$;

$$\lambda(\omega_n - \omega_m) = 2\int d\omega \frac{\alpha^2 F(\omega)\omega}{\omega^2 + (\omega_n - \omega_m)^2} \quad ; \tag{4}$$

and μ^* is the Coulomb pseudopotential. We have solved Eqs. (1)–(4) numerically using the procedure described in Ref. 5.

As a crude measure for the temperature dependence of $\kappa_2(T)$ one can use the ratio $\kappa_2(0)/\kappa_2(T_c)$ which is 1.2 for a dirty *weak*-coupling superconductor. In *strong*-coupling superconductors this ratio is no larger universal, but, in general, depends on the coupling strength and on the particular shape of the spectral function $\alpha^2 F(\omega)$. In order to demonstrate qualitatively the dependence of $\kappa_2(0)/\kappa_2(T_c)$ on the coupling strength we have eval-

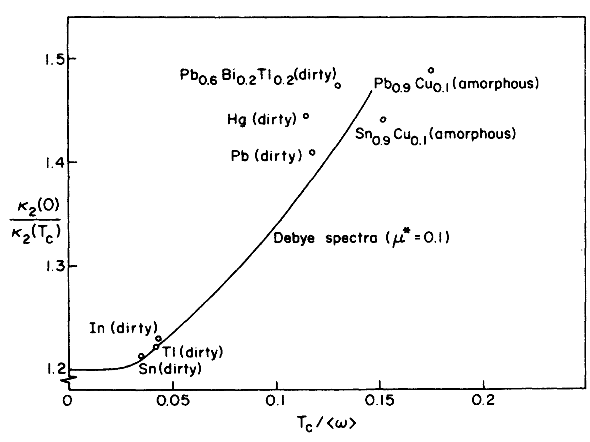

FIG. 1. Numerical results for $\kappa_2(0)/\kappa_2(T_c)$.

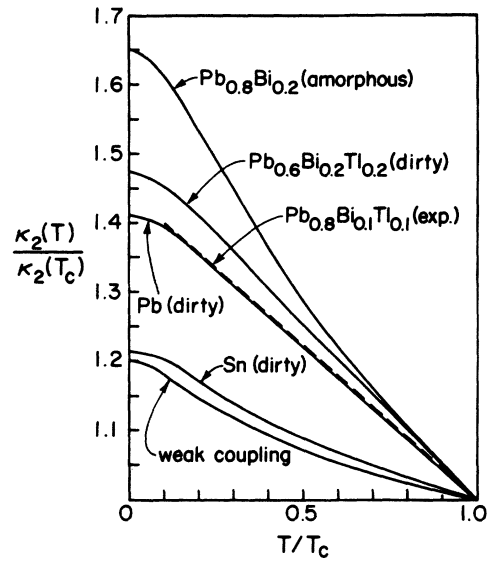

FIG. 2. Numerical results for $\kappa_2(T)$ in the dirty limit. The experimental result is taken from Ref. 4.

uated this ratio for a series of simple spectra (Debye spectra) with increasing coupling strength. The results are shown in Fig. 1 where, in addition, we have plotted the $\kappa_2(0)/\kappa_2(T_c)$ ratios calculated from several realistic spectral functions $\alpha^2F(\omega)$. It should be mentioned that we investigated $\kappa_2(t)$ in the *dirty* limit whereas the α^2F spectra of Pb, Hg, In, ... are taken from tunneling experiments on *pure* crystals. Pb_{dirty}, Hg_{dirty}, ... therefore refer to hypothetical dirty materials with α^2F spectra of pure Pb, Hg, ... The strong-coupling anomalies are not very sensitive to small variations of a given spectrum. The results for such hypothetical materials are therefore representative for realistic dirty materials with α^2F spectra of "Pb-type," ... .

Figure 1 demonstrates that the ratio $\kappa_2(0)/\kappa_2(T_c)$ *increases* with increasing coupling strength and that its magnitude can be roughly estimated from the ratio $T_c/\langle\omega\rangle$, where $\langle\omega\rangle$ is an average phonon frequency

$$\langle\omega\rangle = \int d\omega\,\omega\frac{\alpha^2F(\omega)}{\omega} \Big/ \int d\omega\frac{\alpha^2F(\omega)}{\omega} \quad .$$

In Fig. 2 the complete temperature dependence of $\kappa_2(t)$ is shown for several typical examples. The strong-coupling theory predicts that the anomalies are most pronounced for amorphous superconductors. One obtains a 40% deviation from weak-coupling behavior. This strong effect comes from the anomalously large weight in $\alpha^2F(\omega)$ at low frequencies which has been observed in these materials.[6] In Table I we summarize several characteristic results for lead-based alloys. For comparison we include in Table I the experimental results for $Pb_{0.8}Bi_{0.1}Tl_{0.1}$, which is nearest to the dirty limit of all the compounds analyzed by Fearday and Rollins. The agreement between strong-coupling theory and experiment is once more satisfactory. It should be mentioned that the only input parameters in our calculations are the measured α^2F spectra and μ^*'s. No further experimental information is necessary for our analysis, e.g., the ratios $(dH_c^{expt}/dT)/(dH_c^{BCS}/dT)$ or $\Delta^{expt}/\Delta^{BCS}$, which are used by Fearday and Rollins to make comparisons with the theoretical results. Table I indicates that κ at T_c is reduced below the weak coupling value. A more detailed investigation[3] shows that this reduction occurs only for strong-coupling compounds with $T_c/\langle\omega\rangle$ ratios larger than about 0.1. For intermediate-coupling superconductors κ at T_c slightly *exceeds* the weak-coupling value (by at most 5%). In all cases, however, the ratio $\kappa_2(0)/\kappa_2(T_c)$ is enhanced relative to the weak-coupling ratio of 1.2. In dirty materials such an enhancement can therefore be considered as an indication of strong-coupling effects.

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TABLE I. Numerical and experimental results for some lead-based alloys.

Alloy	State	$\alpha^2F(\omega)$ from Ref.	$\kappa_2(T_c)/\kappa^{BCS}$	$\kappa_2(0)/\kappa_2(T_c)$	$\kappa_2(0.1T_c)/\kappa_2(T_c)$
Weak-coupling theory (dirty)	...	...	1.0	1.2	1.18
Pb (dirty)	cryst.	7	0.95	1.41	1.39
$Pb_{0.6}Bi_{0.2}Tl_{0.2}$(dirty)	cryst.	8	0.92	1.47	1.45
$Pb_{0.8}Bi_{0.2}$(dirty)	amorphous	6	0.79	1.65	1.61
$Pb_{0.8}Bi_{0.1}Tl_{0.1}$ (experimental data[a])	cryst.	...	0.92	...	1.4

[a]Reference 4.

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