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# Measurement of the probability distribution of thermally excited fluxoid quantum transitions in a superconducting ring closed by a Josephson junction

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The thermally induced distribution of transitions between fluxoid quantum states of a superconducting ring closed by a Josephson junction has been measured. The observed dependence of its width $\sigma$ on temperature, ring inductance, and weak-link critical current, as well as its shape, confirms the recent theoretical results of Kurkijärvi, $\sigma \propto L(i_c)^{1/3}T^{2/3}$.

We report here the first diagnostic measurements of the effect of thermodynamic fluctuations on the transitions between the fluxoid quantum states of a superconducting ring closed with a Josephson junction. Recent studies have characterized fluctuations in the Josephson junction itself.[1–5] However, the system consisting of a superconducting ring shunted by the junction has resisted experimental study because the intrinsic fluctuations in the internal flux $\phi$ are much smaller than the noise of the best available measuring instruments, superconducting flux detectors. We have, however, devised experiments which overcome this difficulty and allow study of the intrinsic fluctuations through their effects on the transitions between fluxoid quantum states induced by an applied flux.

In the absence of thermal fluctuations, external flux applied to a superconducting ring induces just enough supercurrent to preserve fluxoid quantization until a potential barrier preventing transition vanishes. Then the supercurrent in the ring decays abruptly and a fluxoid is admitted. With fluctuations these transitions are distributed stochastically over a range of values of applied flux $\phi_x$ which is reduced from the discrete value $\phi_{xc}$ expected in the absence of fluctuations. We report here measurements of the probability distribution $P(\phi_x)$ for induced transitions, with accuracy better than $0.002\phi_0$, where $\phi_0$ is the flux quantum. Recently, Kurkijärvi,[6] here referred to as (K), has analyzed the dynamics of the transitions between fluxoid quantum states and has calculated the functional form of the transition distribution. Our measurements confirm his results including the explicit termperature dependence of the distribution width $\sigma \propto T^{2/3}$ under the conditions where the energy spacing between fluxoid quantum states is much larger than $k_B T$ such as is common in various quantum superconducting devices. We have also measured the dependence of $\sigma$ on ring inductance $L$ and junction critical current $i_c$. Kurkijärvi and Webb[7] have applied (K)'s theory to fluctuations in the SQUID where these fluxoid transitions take place at radio frequencies.

The system treated in (K)'s calculation of the effect of thermal fluctuations assumes a superconducting ring of inductance L closed by an ideal Josephson junction shunted by small capacitance $C$ and resistance $R$. Fluxoid quantization implies that the phase difference across the weak link is $-2\pi\phi/\phi_0$.[8] Thus, the total current through the ring is $i=-i_c \times\sin(2\pi\phi/\phi_0)+V/R+\dot{V}C$, where $V=-\dot{\phi}$ is the voltage across the line. The flux $\phi$ threading the ring is then related to the applied flux $\phi_x$ by

$$\phi=\phi_x-Li_c\sin(2\pi\phi/\phi_0)-LC\ddot{\phi}-L\dot{\phi}/R \quad . \qquad (1)$$

Equation (1) is homologous with the equation of motion of a classical particle of mass $C$ and damping constant $\eta=1/RC$ in an effective potential

$$U(\phi,\phi_x)=(\phi-\phi_x)^2/2L-(i_c\phi_0/2\pi)\cos(2\pi\phi/\phi_0). \qquad (2)$$

The behavior of the system in the limit of large $\eta$ can be understood by considering its trajectory in $U(\phi,\phi_x)$; see Fig. 1. The essential feature is that thermal fluctuations effect transitions from the potential trough associated with the initial fluxoid quantum state over a potential barrier to a lower energy fluxoid state. As $\phi_x$ is increased the potential barrier preventing transitions $\Delta U(\phi_x)$ decreases, finally vanishing at a critical value $\phi_{xc}$, but thermally induced transitions occur over a range $\phi_x<\phi_{xc}$ with the distribution $P(\phi_x)$ rather than occurring only at $\phi_{xc}$. Expressing $\phi_x$ in terms of $\Delta\phi_x=\phi_{xc}-\phi_x$, (K) finds that when $Li_c \gg \Delta\phi_x$, $\Delta U(\Delta\phi_x) =(2\sqrt{2}/3\pi)\phi_0(\Delta\phi_x/L)^{3/2}i_c^{-1/2}$. (K) also finds that if $\dot{\phi}_x$ is constant, $P(\phi_x)$ has the width

$$\sigma=\sigma_u Li_c(2\pi k_B T/\phi_0 i_c)^{2/3}(3/4\sqrt{2})^{2/3} \qquad (3)$$

in the relevant limit, where $(\phi_0/2\pi Li_c)^2 \ll 1$. The

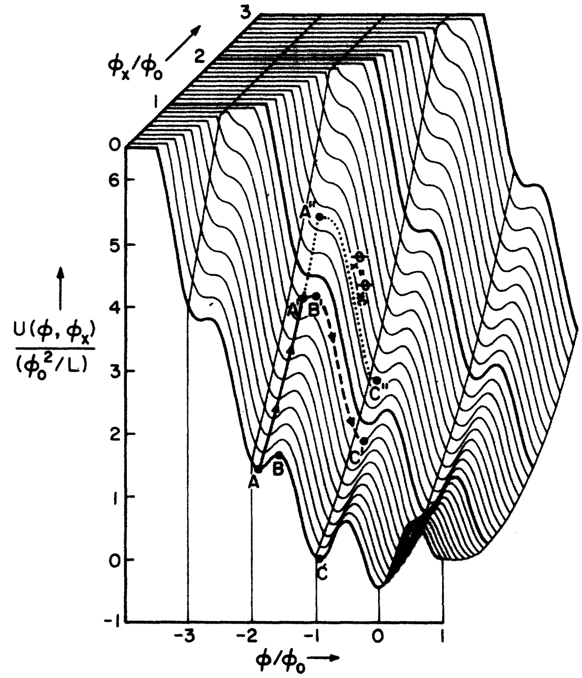


FIG. 1. Potential $U(\phi, \phi_x)$ surface for $\phi_x = 0$ to $3\phi_0$ and $\phi = \phi_0$ to $-4\phi_0$ in the case where $Li_c = 3\phi_0$. When $\phi_x = 0$ the system will be trapped around a minimum such as point $A$ in the potential well associated with a fluxoid quantum state. It is constrained by a potential barrier at $B$. Since $\Delta U(\phi_x = 0) = U(\phi_B) - U(\phi_A)$ is much greater than $k_BT$, it is unlikely that a fluctuation will be able to drive $\phi$ over $B$ to a point of lower potential near $C$. As $\phi_x$ is slowly increased ($\dot{\phi}_x/\phi_0 \ll R/L$) the potential energy increases along the valley $A$-$A''$ and the system can be carried to a point near $A'$ where the barrier at $B'$ is smaller and a fluctuation induced transition to $C'$ is more likely. In the absence of fluctuations the system would follow the dotted path with a transition from point $A'$, where $\phi_x = \phi_{xc}$ and $\Delta U = 0$, to point $C''$; thus fluctuations displace the transition to lower $\phi_x$.

coefficient $\sigma_u$ depends only very weakly on $\dot{\phi}_x$, $L$, $R$, $i_c$, and $T$ and is $\cong 0.3$ for our experiments. Therefore, $\sigma \propto i_c^{1/3}T^{2/3}L$. For rings with $L \sim 10^{-9}$ H and $i_c \sim 10^{-5}$ A, $\sigma$ is about $\frac{1}{30}\phi_0$. (K) predicts $\langle\Delta\phi_x\rangle = \langle\phi_{xc} - \phi_x\rangle \approx \frac{3}{4}\phi_0$ for our slow sweep rates ($\dot{\phi}_x \sim$ a few $\phi_0$/sec).

We measured the distribution $P(\phi_x)$ by applying a periodic ramped flux of sufficient amplitude to induce several fluxoid transitions in the sample ring. The flux inside the ring was monitored by an rf-biased SQUID weakly coupled to the sample through an rf shield using a dc flux transformer. The distribution $P(\phi_x)$ was established either by recording in a multichannel analyzer the values of $\phi_x$ at which the transitions occurred for $\sim 10^4$ cycles of $\phi_x$, or alternatively by summing the SQUID output with a signal averager for successive cycles so that the resultant accumulated signal was proportional to $\int_0^{\phi_x} P(\phi_x')d\phi_x'$.

To exclude external noise the entire system was elaborately shielded from rf and magnetic fields, reducing the ambient field to $< 10^{-4}$ G. The weak coupling ($\kappa < 0.02$) between the sample and the monitoring SQUID ensured that virtually all the noise in the monitoring circuitry was excluded from the sample except a small portion of the SQUID's audio modulation which introduced demonstrably negligible effects. Variation of $\kappa$ verified the adequacy of isolation. Temperatures were regulated to $\sim 10$ $\mu$K. Many of the design details have previously been described.[9,10]

For these measurements double rings connected in parallel were machined from a single block of superconducting niobium and were closed by a point contact (see Fig. 2). For this system $L$ is an effective inductance (essentially the parallel combination of the inductance of the separate rings) and $\phi_x$ is an effective applied flux depending on geometry. Both of these effective parameters were determined by direct measurements made under the same conditions as the measurements of $P(\phi_x)$, thus automatically correcting for any effects of the measurement circuits such as the diamagnetism of the dc transformer. The behavior of these point-contact weak links is consistent with that of weakly coupled low-capacitance Josephson junctions. The current-phase relation of point contacts in our samples was measured directly for critical currents as large as $0.5\times10^{-6}$ A.[11] We found the higher-order terms in $i = \sum_n i_n \sin 2n\pi\phi/\phi_0$ to be less than 5% of $i_1$. Henkels measured $I$-$V$ curves of identical point contacts with critical currents up to $4\times10^{-6}$ A and found

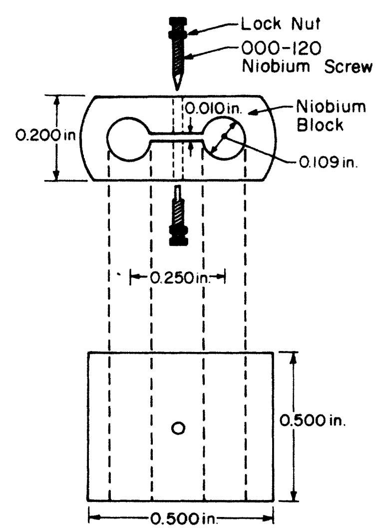


FIG. 2. Superconducting double ring. The weak link is provided by contact between the pointed niobium screw and the oxidized niobium flat screw.

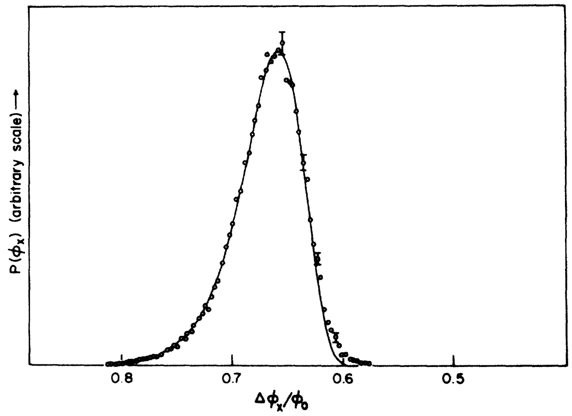


FIG. 3. Measured transition distribution $P(\phi_x)$ vs $\Delta\phi_x$ for $T=2.8$ K and $Li_c=3\phi_0$, $\Delta\phi_x=\phi_{xc}-\phi_x$. The solid line is the theoretical curve fitted to the data with $R=1\ \Omega$ and $L=5\times10^{-10}$ H. Representative statistical errors are shown for four points.

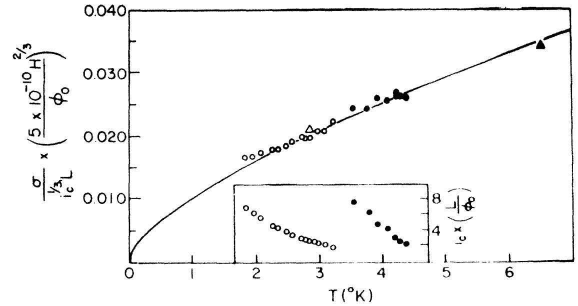


FIG. 4. Temperature dependence of $(\sigma/i_c^{1/3}L)\times(5\times10^{-10}\ \text{H}/\phi_0)^{2/3}$ in mks units. The data for two contacts (open and filled circles) across one ring are plotted using $L=5\times10^{-10}$ H. Measured temperature-dependent critical current for these contacts is shown as $i_c(L/\phi_0)$ in the insert. Data for two other rings (open and filled triangles) are plotted using values of $L$ measured with 5% accuracy. The solid curve shows (K)'s theoretical result.

good agreement with calculations assuming the dc and ac Josephson equations.[12]

The measured absolute widths of the distributions agree with the theory within the accuracy of the measurement of the ring inductance (usually 20%, but in one set of experiments, 5%). One typical distribution (out of about 50 cases measured) is shown in Fig. 3. We have fitted the theoretical curve to the data by assuming $R=1\ \Omega$ and then adjusting the value of the inductance to $L=5\times10^{-10}$H; an independent measurement yields $L=(4.8\pm1.0)\times10^{-10}$ H. Although direct measurement of $R$ was not possible in a closed ring, our previous experience as well as published results of others suggests $1\ \Omega$ for a reasonable estimate. The only effect of a change of a factor of 10 in $R$ is a change of less than 6% in the best fit value of $L$. The slight broadening (~1%) of the observed distribution and smearing of the sharp corner due to the known instrumental noise in the detection system is noticeable near $\Delta\phi_x/\phi_0 \sim 0.6$.

Figure 4 shows the temperature dependence of the parameter $\sigma/(i_c^{1/3}L)$ for several rings with $2\phi_0 \lesssim Li_c \lesssim 7\phi_0$. The data for two contacts (open and filled circles) across one ring were fitted using $L=5\times10^{-10}$H as the single (slightly) adjustable parameter as described above. The measured temperature-dependent critical current for these contacts is shown in the insert. Data for two other rings (open and filled triangles) are plotted using more precise values of $L$ experimentally determined with 5% accuracy. The solid curve shows (K)'s theoretical result without adjustable parameters. The agreement with experiment is evident. The explicit $T$ dependence of $\sigma$ was also checked by a least-squares fit of the form $\sigma\propto T^\alpha/i_c^{1/3}$ to the data yielding $\alpha=0.64\pm0.08$ in accord with the theoretical value of $\frac{2}{3}$. Similarly, a fit of $\sigma\propto i_c^\delta/T^{2/3}$ yielded $\delta=0.37\pm0.06$ in accord with the theoretical $\delta=\frac{1}{3}$. We have also directly measured the inductance dependence of $\sigma$ by filling some of a ring's enclosed area with a niobium plug to change the inductance by about a factor of 2. The relative values of $L$ were measured to better than 5% and the corresponding reductions in the transition width agreed with theory within this accuracy.

We conclude that the experimental results are in good agreement with the theory. The observed magnitudes and functional dependence of $P(\phi_x)$ on temperature, critical current and inductance, as well as the distribution shape, confirm Kurkijärvi's calculations and thus his model for the thermal fluctuations in this system. The particular explicit temperature dependence of $\sigma$ arises simply from the functional form of the energy barrier $\Delta U(\phi_x)$. The remarkable power of the effect is demonstrated by noting that in our conditions the fluctuations reduce the value of the external flux $\phi_x$ at which transitions occur by $\sim\frac{3}{4}\phi_0$ to a point where the energy barrier $\Delta U(\phi_x)$ is still $\sim 20k_BT$. Finally, we note that the sensitive methods developed for these measurements may be useful in the study of the various weak link structures currently of interest.[13,14]

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