# **Ground state and magnetic phase transitions of orthoferrite DyFeO3**

Z. Y. Zhao,<sup>1</sup> X. Zhao,<sup>2,\*</sup> H. D. Zhou,<sup>3,4</sup> F. B. Zhang,<sup>1</sup> Q. J. Li,<sup>1,5</sup> C. Fan,<sup>1</sup> X. F. Sun,<sup>1,†</sup> and X. G. Li<sup>1,6</sup>

<sup>1</sup>*Hefei National Laboratory for Physical Sciences at Microscale, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*

<sup>2</sup>*School of Physical Sciences, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China* <sup>3</sup>*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996-1200, USA*

<sup>4</sup>*National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32306-4005, USA* <sup>5</sup>*School of Physics and Material Science, Anhui University, Hefei, Anhui 230039, People's Republic of China*

<sup>6</sup>*Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*

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Low-temperature thermal conductivity (*κ*), as well as magnetization (*M*) and electric polarization (*P*), of multiferroic orthoferrite DyFeO<sub>3</sub> single crystals are studied with  $H \parallel c$ . When the crystal is cooled in zero field, *M*, *P*, and *κ* all consistently exhibit irreversible magnetic-field dependencies. In particular, with 500 mK  $\lt T \leq$ 2 K, all these properties show two transitions at the first run of increasing field but only the higher-field transition is present in the subsequent field sweepings. Moreover, the ultra-low-*T* (*T <* 500 mK) *κ*(*H*) shows a different irreversibility and there is only one transition when the field is swept both up and down. All the results indicate a complex low-*T H*-*T* phase diagram involving successive magnetic phase transitions of the Fe<sup>3+</sup> spins. In particular, the ground state, obtained with cooling to sub-Kelvin temperatures in zero field, is found to be an unexplored phase.

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the  $Dy^{3+}$  spins develop a long-range antiferromagnetic (AF) order below  $T_{\rm N}^{\rm Dy} = 4.2$  K [\[8\]](#page-6-0). In the AF state,  $\rm Dy^{3+}$  spins are confined in the *ab* plane and the spin configuration can

## **I. INTRODUCTION**

Magnetic phase transition induced by magnetic field is an outstanding phenomenon in the strongly-correlated electron systems and is associated with many physical interests, such as the unconventional superconductivity  $[1,2]$ , the non-Fermi-liquid behaviors [\[3\]](#page-6-0), and the multiferroicity [\[4\]](#page-6-0), etc. Multiferroicity induced by spin order has attracted much attention due to its large magnetoelectric (ME) coupling. The spin-current model or the inverse Dzyaloshinsky-Moriya (DM) interaction [\[5\]](#page-6-0) can explain well the production of electric polarization (*P*) in the noncollinear spin systems, such as the perovskite  $R MnO<sub>3</sub>$  ( $R =$  rare earth) [\[4,6,7\]](#page-6-0). When the spins are aligned collinearly, *P* can also be formed through the exchange striction mechanism, such as in  $RFeO_3$  [\[8–10\]](#page-6-0), Ca<sub>3</sub>CoMnO<sub>6</sub> [\[11\]](#page-6-0), and  $RMn_2O_5$  [\[12–14\]](#page-6-0). In these materials, the spin structures are playing a key role in the ME coupling and the formation of the spontaneous electric polarization. The rare-earth-based orthoferrites *RFeO*<sub>3</sub> have received a lot of research interests in last several decades, particularly in the manifestations of the spin structures and spin reorientations [\[15–19\]](#page-6-0).

In DyFeO<sub>3</sub>, the Fe<sup>3+</sup> moments exhibit  $G_xA_yF_z$  (Fe<sub>I</sub>) spin configuration in Bertaut's notation [\[20\]](#page-6-0) at room temperature, that is, the main component of the magnetic moment is along the *a* axis, accompanying with weak ferromagnetism (WFM) along the *c* axis [\[21,22\]](#page-6-0). Upon lowering temperature, the Fe<sup>3+</sup> spins undergo a Morin transition [\[23\]](#page-6-0) at  $T_M = 50$  K. At this transition, the spin configuration changes to  $A_x G_y C_z$ (Fe<sub>II</sub>) [\[22\]](#page-6-0). Moreover, with an applied magnetic field,  $H >$  $H_{\rm r}^{\rm Fe}$ , along the *c* axis the Fe<sup>3+</sup> spin configuration could change back to Fe<sub>I</sub> [\[8,24\]](#page-6-0). With further lowering temperature,

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be expressed as  $G_xA_y$  with the Ising axis deviating about 33 $\degree$  from the *b* axis [\[25,26\]](#page-6-0). The spin-induced multiferroicity was observed only at  $T < T_{\text{N}}^{\text{Dy}}$  and when the spin flop of  $Fe<sup>3+</sup>$  moments is introduced by a *c*-axis field [\[8\]](#page-6-0). Due to the interaction between  $Dy^{3+}$  and  $Fe^{3+}$  spins, the  $Dy^{3+}$  spins shift towards the layers of  $Fe<sup>3+</sup>$  with opposite spin directions and far away from those with the same spin directions, resulting in a collective displacement of the  $Dy^{3+}$  ions and producing a spontaneous *P*. Therefore both the  $Dy^{3+}$  and  $Fe^{3+}$  spin structures are crucial for the ME phenomenon. However, the spin structure of  $Fe<sup>3+</sup>$  at low temperatures actually has not been determined. An obvious inconsistency in the early studies is that the Mossbauer spectroscopy suggested a  $G_xG_y$ structure [\[27\]](#page-6-0), while the electric polarization results suggested a  $A_x G_y C_z$  structure [\[8\]](#page-6-0). Moreover, the ground state and the low-*T* magnetic phase transitions are actually not known since all the previous works had not studied the physics of  $DyFeO<sub>3</sub>$ at temperatures lower than 1.5 K.

Heat transport has been proved to be a useful probe for the low-*T* magnetic transitions  $[28-35]$ . In this work, we study the low-*T* thermal conductivity  $(\kappa)$ , as well as the magnetization  $(M)$  and electric polarization, of  $DyFeO<sub>3</sub>$  single crystals with  $H \parallel c$  after a zero field cooling (ZFC) process. In particular, the *P* measurements can be done at temperatures below 1 K, while the  $\kappa$  is measured at temperatures down to several tens of millidegrees of Kelvin. It is found that  $M(H)$ ,  $P(H)$ , and  $\kappa(H)$  all consistently exhibit peculiar low-field irreversible behaviors at *T >* 500 mK with the irreversibility field smaller than  $H_r^{\text{Fe}}$ . At  $T < 500 \text{ mK}$ ,  $\kappa(H)$ shows a different irreversibility with larger  $\kappa$  in the field-up process, which is in contrast with the case above 500 mK. The results suggest a complex low-*T H*-*T* phase diagram involving

<sup>\*</sup>xiazhao@ustc.edu.cn

<sup>†</sup> xfsun@ustc.edu.cn

<span id="page-1-0"></span>successive field-induced magnetic phase transitions of  $Fe<sup>3+</sup>$ spins.

### **II. EXPERIMENTS**

High-quality  $DyFeO<sub>3</sub>$  single crystals were grown by a floating-zone technique in flowing oxygen-argon mixture with the ratio of 1:4. The crystals were cut precisely along the crystallographic axes after orientation by using back-reflection x-ray Laue photographs. The sample for magnetization measurement is rodlike and the dimension is  $1.70 \times 0.75$  $\times$  0.55 mm<sup>3</sup> with the length, width and thickness along the *b*, *c*, and *a* axis, respectively. The sample for electric polarization measurement is platelike and the wide face is perpendicular to the *c* axis with dimension of  $2.06 \times 2.14$  $\times$  0.13 mm<sup>3</sup>. The dimension of the sample for thermal conductivity measurement is 3.86  $\times$  0.63  $\times$  0.14 mm<sup>3</sup> and the longest dimension is parallel to the *c* axis.

Magnetization was measured by a SQUID-VSM (Quantum Design). Electric polarization was obtained by integrating the displacement current measured by an electrometer (model 6517B, Keithley) in a <sup>3</sup>He refrigerator and a 14 T magnet.  $P(T)$ was measured at a rate of about 2 K/min from 300 mK to 5 K. In order to stabilize temperature in the  $P(H)$  measurements, the sweeping-field rate must be slower for lower temperature, which is 0.25, 0.2, 0.15, and 0.1 T/min at 2, 1.4, 1, and 0.7 K, respectively. Thermal conductivity was measured by using a "one heater, two thermometers" technique and three different cryostats  $[30-36]$ : (i) in a <sup>3</sup>He-<sup>4</sup>He dilution refrigerator at temperature regime of 70 mK–1 K; (ii) in a  ${}^{3}$ He refrigerator at 0.3–8 K, and (iii) in a pulse-tube refrigerator for zero-field data at  $T > 5$  K. In all these measurements, the magnetic fields were applied along the *c* axis.

#### **III. RESULTS AND DISCUSSION**

#### **A. Magnetization**

The basic magnetic properties of  $DyFeO<sub>3</sub>$  are characterized by the  $M(T)$  and  $M(H)$  measurements, of which the representative data are shown in Fig. 1. These results are in good consistency with the earlier works  $[8,22]$ . The temperature dependence of *M* along the *c* axis measured in  $H = 500$  Oe, shown in Fig.  $1(a)$ , has two transitions at 50 and 4.2 K. The AF order of  $Fe<sup>3+</sup>$  spins is known to be formed at a high temperature of  $T_N^{\text{Fe}} \sim 645 \text{ K}$ , with a  $G_x A_y F_z$  (Fe<sub>I</sub>) spin configuration at room temperature. The transition at 50 K  $(T_M)$  is a Morin transition, where the Fe<sup>3+</sup> structure changes to  $A_x G_y C_z$  (Fe<sub>II</sub>) [\[22\]](#page-6-0). Another transition at 4.2 K ( $T_N^{\text{Dy}}$ ) corresponds to the AF ordering of  $Dy^{3+}$  moments in the  $G_xA_y$ configuration. The low-*T*  $M(H)$  curves shown in Fig. 1(b) are consistent with these spin structures.

When the field is applied along the *c* axis, the transition from  $Fe<sub>I</sub>$  to  $Fe<sub>II</sub>$  shifts to lower temperature rapidly with increasing field, but the AF order of  $Dy^{3+}$  is robust against the field, as seen in Fig. 1(c). It is known that  $Dy^{3+}$  moments have strong anisotropy and are confined in the *ab* plane; therefore the *c*-axis field can hardly to change either the Néel transition or the  $Dy^{3+}$  spin orientation. Thus the irreversible  $M(H)$ ,  $P(H)$ , and  $\kappa(H)$  behaviors at low temperatures shown in the



FIG. 1. (Color online) Magnetic properties of  $DyFeO<sub>3</sub>$  single crystal. (a) Magnetic susceptibility in *H* = 500 Oe along the *c* axis. (Inset) Zoom in of the low-*T* data. The transitions at about 50 and 4.2 K correspond to the Morin transition of  $Fe<sup>3+</sup>$  and the Néel transition of  $Dy^{3+}$ , respectively. The schematics illustrate the spin structures of Fe<sup>3+</sup> (bottom, black) and  $Dy^{3+}$  (top, blue). (b) Anisotropic magnetization at  $T = 2$  K. (c) Representative data of the low-*T* magnetic susceptibility in different magnetic fields along the *c* axis. (Inset) Field dependencies of the transition temperatures of the  $Dy^{3+}$  Néel order and the Fe<sup>3+</sup> spin reorientation, determined from the  $M(T)$  curves.

following sections are unambiguously related to the successive magnetic transitions of  $Fe<sup>3+</sup>$  spins.

Figure [2](#page-2-0) shows the magnetization of  $DyFeO<sub>3</sub>$  single crystal with *H*  $\parallel$  *c* after ZFC. With  $T > T_{\text{N}}^{\text{Dy}}$ , *M* exhibits an abrupt increase at  $H_r^{\text{Fe}}$ , which corresponds to the spin-flop transition of  $Fe^{3+}$  moments from  $Fe_{II}$  to  $Fe_{I}$  [\[8,24\]](#page-6-0). With decreasing temperature, this transition gradually moves to higher field but becomes weaker, and finally evolves into a change of the slope [Fig. [2\(a\)\]](#page-2-0). With  $T < T_N^{\text{Dy}}$ , there is a lower-field transition shown by a peak at  $H_m$  in the differential  $dM/dH$  curves, besides the peak at  $H_r^{\text{Fe}}$ , as shown in Figs. [2\(d\)](#page-2-0) and [2\(g\).](#page-2-0) Moreover, a peculiar irreversible *M*(*H*) behavior is observed at  $H < H_{\rm r}^{\rm Fe}$ , with the irreversibility field slightly larger than  $H<sub>m</sub>$ , as shown in Figs.  $2(b)$ –2(g). Note that this irreversibility is repeatable at  $T > 2$  K [Figs. [2\(c\)](#page-2-0) and [2\(d\)\]](#page-2-0), which means that the field-up  $M$  for the second time (solid symbol, label 5) is identical to that for the first field-up process (open symbol,

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FIG. 2. (Color online) Magnetization of  $DyFeO<sub>3</sub>$  single crystal with  $H \parallel c$  after ZFC. (a) shows the magnetization in the positive field-up process. Other panels show full scan of magnetization between 7 and −7 T, the magnetization curves in the positive fields, and the differential curves with  $T = 3$  (b)–(d) and 2 K (e)–(g). The numbers and arrows denote the sweeping-field sequences and directions. The open and solid symbols denote the magnetization in the first full scan and the field-up process for the second time.  $H_r^{\text{Fe}}$ and  $H<sub>m</sub>$  represent two transition fields, indicated by the peaks of the differential curves.

label 1). However, with  $T \le 2$  K, the irreversibility becomes unrepeatable [Figs.  $2(f)$  and  $2(g)$ ], that is, the field-up *M* for the second time is equal to that in the field-down process (open symbol, label 2). Therefore there are two transitions in the field-up process and only the higher-field one exists when the field is swept down.

## **B. Electric polarization**

The electric polarization along the  $c$  axis of  $DyFeO<sub>3</sub>$  single crystal is measured down to sub-Kelvin temperatures. Figure  $3(a)$  shows the temperature dependencies of *P* measured after ME cooling from 5 to 0.3 K with magnetic field and  $E = 3 \text{ kV/cm}$  along the *c* axis. *P* is temperature independent





FIG. 3. (Color online) (a) Temperature dependencies of the *c*axis electric polarization of  $DyFeO<sub>3</sub>$  single crystal measured after ME poling in magnetic field and  $E = 3$  kV/cm electric field, both along the *c* axis. (b)–(e) Magnetic field dependencies of *P* measured after ME poling in 7 T and  $E = 3$  kV/cm. The numbers and arrows denote the sweeping-field sequences and directions.

at very low temperatures and decreases to zero at around  $T_{\rm N}^{\rm Dy}$ , which indicates that the  $Dy^{3+}$  spin order contributes to this electric polarization. The magnitude of *P* is enhanced and the transition temperature is slightly suppressed with increasing field. Figures  $3(b)-3(e)$  shows the magnetic-field dependence of *P*, measured after ME poling from 5 K in 7 T magnetic field and  $E = 3$  kV/cm along the *c* axis. At low fields,  $P(H)$ shows a linear ME behavior.With increasing field, the spin-flop transition of Fe<sup>3+</sup> from Fe<sub>II</sub> to Fe<sub>I</sub> causes a change of the slope of  $P(H)$  at  $H = H_r^{\text{Fe}}$ , and the multiferroicity shows up. These results are also in good agreement with the earlier work [\[8\]](#page-6-0).

However, *P*(*H*) obtained with the ZFC process (cooling in zero magnetic field and electric field) shows very different behavior [Figs.  $4(b) - 4(d)$ ]. First, the magnitude of *P* after ZFC is one or two orders smaller than that after ME poling, which is mainly due to the electric domains. Second and more importantly, an irreversible  $P(H)$  is observed in the linear ME phase. At  $T = 2$  K [Fig. [4\(b\)\]](#page-3-0), for example, *P* is zero in low fields and exhibits a steplike increase at  $H_m \approx 2.5$  T for the first field-up process. In higher-field,  $P(H)$  is reversible and linear with field. Note that  $H<sub>m</sub>$  increases with decreasing

<span id="page-3-0"></span>

FIG. 4. (Color online) Magnetic-field dependencies of the electric polarization of  $DyFeO<sub>3</sub>$  single crystal measured at (a) 2 K after ME poling and (b)–(d) at 2–0.7 K after ZFC in  $H \parallel c$ . The numbers and arrows denote the sweeping-field sequences and directions. In (a) and (b),  $H_{\rm m}$  and  $H_{\rm r}^{\rm Fe}$  indicates two transition fields and are the same as those from the magnetization.

temperature and approaches  $H_r^{\text{Fe}}$  at 0.7 K [Figs. 4(b)–4(d)]. When  $H > H_{\rm r}^{\rm Fe}$  (≈3.5 T), the system enters the multiferroic phase due to the spin-flop transition of  $Fe^{3+}$  sublattice [\[8\]](#page-6-0). The values of  $H_{\text{m}}$  and  $H_{\text{r}}^{\text{Fe}}$  observed from  $P(H)$  are essentially consistent with those of  $M(H)$ . This consistency suggests a common origin for the observed irreversibility of *M*(*H*) and *P*(*H*).

#### **C. Thermal conductivity**

The thermal conductivity of  $DyFeO<sub>3</sub>$  single crystal is studied for probing the magnetic transitions at even lower temperatures that our  $M(H)$  and  $P(H)$  measurements cannot be done. Before presenting the heat transport results of DyFeO<sub>3</sub> crystal, we show in Fig.  $5(a)$  the low-T specific heat data. A large and sharp peak is observed at 4.1 K, which is related to the Néel transition of  $Dy^{3+}$  spins. It can be seen that the magnetic contributions to the specific heat are important only at very low temperatures and are likely to be negligible above ∼12 K, where the specific heat data show a minimum. Therefore the phonon specific heat can be estimated from the high-*T* data in Fig.  $5(a)$ . It is known that in the temperature range  $0.02 < T/\theta_D < 0.1$  ( $\theta_D$  is the Debye temperature), the phonon specific heat follows a low-frequency expansion of the Debye function,  $C = \beta T^3 + \beta_5 T^5 + \beta_7 T^7 + \ldots$ , where  $β$ ,  $β_5$  and  $β_7$  are temperature-independent coefficients [\[39\]](#page-7-0). It is found that this formula fits well to the experimental data at  $T > 15$  K, as shown in Fig.  $5(a)$ , with the fitting parameters  $\beta = 1.28 \times 10^{-4}$  J/K<sup>4</sup>mol,  $\beta_5 = 2.49 \times 10^{-7}$  J/K<sup>6</sup>mol, and  $\beta_7 = -1.75 \times 10^{-10}$  J/K<sup>8</sup>mol.

Figure  $5(b)$  shows the temperature dependencies of the thermal conductivity down to 70 mK in zero field, with the heat



FIG. 5. (Color online) (a) The specific heat of  $DyFeO<sub>3</sub>$  single crystal below 30 K, plotted in  $C/T$  vs  $T^2$ . The inset displays all the data at  $T \leq 30$  K, which show a sharp peak at 4.1 K. The dashed line shows the fitting to the high-*T* data by using the formula of phonon specific heat, that is,  $C = \beta T^3 + \beta_5 T^5 + \beta_7 T^7$ . (b) Zero-field thermal conductivity of  $DyFeO<sub>3</sub>$  with heat current along the  $c$  axis. For comparison, the data of  $YFeO<sub>3</sub>$  is also presented. The sample sizes are 3.86  $\times$  0.63  $\times$  0.14 mm<sup>3</sup> and 4.40  $\times$  0.66  $\times$  0.16 mm<sup>3</sup> for DyFeO<sub>3</sub> and YFeO<sub>3</sub>, respectively. The dash-dot line indicates a  $T^{2.7}$  temperature dependence. The inset shows the temperature dependence of the phonon mean free path *l* divided by the averaged sample width *W*.

current  $J_H$  applied along the  $c$  axis. For comparison, the data for YFeO<sub>3</sub> single crystal are also taken in the same temperature regime. Note that the  $Y^{3+}$  ions are nonmagnetic and there is only AF order of Fe<sup>3+</sup> ions. YFeO<sub>3</sub> actually shows a simple and pure phonon heat transport phenomenon at low temperatures: (i) the  $\kappa(T)$  curve exhibits a very large peak at about 20 K, with the peak value of 520 W*/*Km, indicating a very high quality of the single crystal; (ii) the temperature dependence of  $\kappa$  is roughly  $T^{2.7}$  at sub-Kelvin temperatures, which is close to the  $T^3$  boundary scattering limit of phonons [\[38\]](#page-7-0). The DyFe $O_3$  data are rather comparable to those of YFe $O_3$ , <span id="page-4-0"></span>except for two notable differences. At first, the  $\kappa$  of DyFeO<sub>3</sub> is smaller at high temperatures, although a phonon peak of 330 W*/*Km is also exceptionally larger for transition-metal oxides. More remarkably, there is a clear concave structure in the  $\kappa(T)$  curve in the temperature regime of 0.3–3 K. A similar result has been found in another orthoferrite GdFe $O<sub>3</sub>$  [\[32\]](#page-7-0). It is clear that at  $T < T_{\rm N}^{\rm Dy}$ , the magnon excitations from the  $Dy^{3+}$  spin system can have a significant scattering on phonons, which results in a downward deviation from the  $T^{2.7}$ behavior. However, with lowering temperature further, the *κ* recovers to the  $T^{2.7}$  dependence at  $T < 300$  mK. This means that the magnon scattering effect is gradually smeared out. Apparently, the magnon spectra has a finite energy gap (for example, originated from the spin anisotropy), which prevents the low-energy magnons from being thermally excited at very low temperatures.

It is possible to estimate the mean free path of phonons at low temperatures and to judge whether the phonons are free from microscopic scattering at sub-Kelvin temperatures. The phononic thermal conductivity can be expressed by the kinetic formula  $\kappa_{ph} = \frac{1}{3}Cv_p l$  [\[38\]](#page-7-0), where  $C = \beta T^3$  is the phonon specific heat at low temperatures,  $v_p$  is the average velocity, and *l* is the mean free path of phonons. Using the *β* value obtained from the above specific-heat data, the phonon velocity can be calculated and then the mean free path is obtained from the  $\kappa$  [\[32,33\]](#page-7-0). The inset to Fig. [5\(b\)](#page-3-0) shows the ratio between *l* and the averaged sample width  $W = 2\sqrt{A/\pi} = 0.335$  mm [\[32,33,38\]](#page-7-0), where *A* is the area of cross section. It can be seen that *l/W* increases with lowering temperature and becomes larger than one at lowest temperatures, which indicates that all the microscopic phonon scatterings (including magnon scattering) are negligible and the boundary scattering limit is established [\[38\]](#page-7-0).

The field dependence of thermal conductivity after ZFC is measured down to 92 mK and the results are presented in Fig. 6. With decreasing temperature, *κ* exhibits complicated field dependence: (i) with  $T > 2$  K,  $\kappa(H)$  is reversible and shows a sharp decrease at  $H_r^{\text{Fe}}$ ; (ii) with 0.7 K  $\leq T \leq 2$  K,  $\kappa$ (*H*) shows an analogous irreversible behaviors to those of  $M(H)$  and  $P(H)$ . There are two maximums in the field-up process and only the high-field one is kept when decreasing field, resulting in a larger  $\kappa$  in the field-down process at low fields; (iii) when the temperature is further decreased down to below 500 mK, *κ*(*H*) exhibits a different irreversible behavior, that is, there is only one dip in the sweeping-field run with a higher location in the field-up process. One notable feature is that the  $\kappa$  in the field-up process becomes larger than that in the field-down process, which is opposite to the case above 500 mK.

It has been known from many previous studies on the antiferromagnetic materials that the thermal conductivity could exhibit drastic changes at the magnetic phase transitions or the spin reorientations [\[28–35\]](#page-7-0). The mechanisms are mostly due to the sudden changes in the population of the magnetic excitations, which can effectively scatter phonons [\[31–35\]](#page-7-0). In some rare case, the magnons well populated at the spin-flop transition can also contribute to the heat transport by acting as heat carriers [\[33\]](#page-7-0). In present work, the anomalies of the  $\kappa$ (*H*) curves can be apparently attributed to the transitions of magnetic structures of  $DyFeO<sub>3</sub>$ , which is also a common phenomenon to GdFeO<sub>3</sub> [\[32\]](#page-7-0). However, the hysteresis of  $\kappa(H)$ are rather different between these two materials.

#### **D. Phase diagram and magnetic transitions**

Considering the above experimental results, one can find a common feature for the ZFC  $M(H)$ ,  $P(H)$ , and  $\kappa(H)$  data, that is, an irreversible behavior is observed only in the first fieldup process, and it disappears in the following sweeping-field process. One possible reason for the observed irreversibility is the multidomain effect in the multiferroic materials [\[9\]](#page-6-0). Since the crystal is cooled in zero field without ME poling, at zero and low fields, the macroscopic polarization might be zero due to the compensation among different electric domains. With increasing field, the sample gradually changes into a fewerdomain state and the finite magnitude of *P* appears, which could be accompanied with some change in the  $M(H)$  curve. However, from our results at  $T \le 2$  K, the  $P(H)$  and  $M(H)$ curves are reversible in the subsequent sweeping-field process except for the first one. This is obviously different from the case



FIG. 6. (Color online) Magnetic-field dependencies of the *c*-axis thermal conductivity of DyFeO<sub>3</sub> single crystal in  $H \parallel c$  after ZFC. As indicated by the colored arrows, the data shown with solid symbols are measured in the field-up process, while the open symbols show the data in the field-down process. The black upright arrows indicate the characteristic fields in  $\kappa(H)$  isotherms.

of the  $P(H)/M(H)$  loop in usual ferroelectric/ferromagnetic materials, in which the loop always exists due to the presence of domains. On the other hand, if the domain effect is the major reason for the observed irreversibility, the phonon scattering by the domain walls should be effective in the whole low-*T* range. In the case of  $GdFeO<sub>3</sub>$  [\[9\]](#page-6-0), when the crystal is cooled in zero field, the field-up *P* is larger than that in the field-down run, which indicates that there are less ferroelectric domains with increasing field. The larger  $\kappa(H)$  in the field-up process can be attributed to the weak phonon scattering by the domain walls [\[32\]](#page-7-0). Similar to the case of GdFeO<sub>3</sub>, the  $\kappa(H)$  at  $T >$ 500 mK behaves consistently with the *M*(*H*) and *P*(*H*) curves in  $DyFeO<sub>3</sub>$ , that is, the ME domain walls are likely playing a role in scattering phonons. However, upon further cooling  $(\leq 300 \text{ mK})$ , the *κ* in the field-up process becomes larger than that in the field-down process, which can not be explained by the domain wall scattering on phonons. In a word, although the multidomain effect could have influence on the magnitudes of  $M(H)$ ,  $P(H)$ , and  $\kappa(H)$ , when the crystal is cooled without ME poling, it seems to be not the dominant factor for the observed irreversible behaviors.

The second possible reason for the steplike increase of *P* and the consistent low-field irreversibility shown by  $M(H)$ , *P*(*H*), and  $\kappa$ (*H*) is the presence of an unknown zero-field magnetic structure below  $T_{\rm N}^{\rm Dy}$  after ZFC, which is metastable in the field and disappears when the field is swept down. Actually, it is a rather common feature of multiferroic materials that a metastable spin configuration is formed in ZFC process while another spin structure is stabilized after ME cooling. A recent example is the field-induced metastable phase in hexaferrites, observed by a neutron scattering [\[40\]](#page-7-0). This scenario can easily explain the  $P(H)$  and  $M(H)$  behaviors. Due to the strong out-of-plane anisotropy of  $Dy^{3+}$  spins, the applied field along the *c* axis could not change their spin arrangements directly, as the magnetization in Fig.  $1(c)$ indicated. Most likely, only the change of  $Fe<sup>3+</sup>$  spin structure is involved. Therefore the zero-field structure of Fe<sup>3+</sup> at  $T < T_{\text{N}}^{\text{Dy}}$ should be different from the known Fe<sub>II</sub> at  $T > T_{\text{N}}^{\text{Dy}}$  and is hereafter named as Fe $_{III}$ . It is also notable that the repeatable irreversibility begins to appear with  $T < T_{\text{N}}^{\text{Dy}}$  and becomes unrepeatable with  $T \le 2$  K, the transition from Fe<sub>II</sub> to Fe<sub>III</sub> is therefore a gradual process between  $T_N^{\rm Dy}$  and 2 K. Another notable feature is that the transition from Fe<sub>II</sub> to Fe<sub>I</sub> at  $T>T_{\rm N}^{\rm Dy}$ could cause a sharp increase of *M*. Due to the presence of irreversibility, this feature gradually vanishes at  $T < T_{\rm N}^{\rm Dy}$ , instead, two successive magnetic transitions are present, and the higher-field one is from  $Fe<sub>II</sub>$  to  $Fe<sub>I</sub>$ , which is the same as the case at high temperatures.

Based on the above discussions and experimental data of  $M(H)$ ,  $P(H)$ , and  $\kappa(H)$ , a low temperature  $H$ -*T* phase diagram with  $H \parallel c$  and in the case of ZFC is constructed, as shown in Fig. 7. We propose that besides the known  $Fe<sub>I</sub>$ and  $Fe<sub>II</sub>$ , the phase diagram involves another magnetic phase, Fe<sub>III</sub>. The most important feature is that the  $Fe^{3+}$  spins undergo successive transitions:  $Fe_{III} \rightarrow Fe_{II} \rightarrow Fe_{I}$  with increasing field and at 500 mK  $T < T_{\text{N}}^{\text{Dy}}$ , and only the transition from  $Fe<sub>I</sub> \rightarrow Fe<sub>II</sub>$  is present when the field is decreased, which is consistent with the FC results. It is known that there are four  $Fe^{3+}$  spin configurations that could produce linear



FIG. 7. (Color online) ZFC  $H - T$  phase diagram of DyFeO<sub>3</sub> obtained from the  $M(H)$ ,  $P(H)$ , and  $\kappa(H)$  data. The transition from Fe<sub>II</sub> to Fe<sub>III</sub> is actually a gradual change, as a continuous change of color indicates. Above 500 mK,  $Fe_{III}$  is a metastable state, that is, it exits when sweeping-up field to the boundary indicated by the dashed line but does not appear when sweeping-down field. Below  $500$  mK, Fe $_{III}$  becomes stable in sweeping field. The crossed red area represents an irreversible region due to the first-order transition from  $Fe<sub>III</sub>$  to Fe<sub>I</sub>. The transition temperatures of the Néel order and the spin reorientation for Dy<sup>3+</sup> and Fe<sup>3+</sup>, determined from the  $M(T)$  curves [Fig. [1\(c\)\]](#page-1-0), are also shown.

ME response, i.e.,  $G_xA_yF_z$  (Fe<sub>I</sub>),  $A_xG_yC_z$  (Fe<sub>II</sub>),  $F_xC_yG_z$ , and  $C_xF_yA_z$  [\[37\]](#page-7-0). Since the crystal is cooled without ME poling, it is, in principal, difficult to judge from the electric polarization whether there is spontaneous *P* or not in zero field, and accordingly to judge the definite magnetic structure of Fe $_{III}$ . This is because that the sample after ZFC could be in a multidomain phase with spontaneous polarization, even though the measured *P* is zero. However, since the low-*T P*(*H*) loops actually behave rather differently from the usual *P*(*H*) hysteresis curves, some kind of magnetic-structure transition other than the multidomain effect may play a more important role. Nevertheless, we have to leave the definition of the Fe<sub>III</sub> magnetic structure as an open question, and further investigations like neutron scattering studies on single crystals are called for.

This phase diagram could explain the behavior of  $\kappa(H)$ down to the lowest temperature. (i) With  $T > T_{N}^{Dy}$ , when the field is applied, the  $Fe^{3+}$  spin structure changes from  $Fe_{II}$  to Fe<sub>I</sub>. Accordingly, a sharp decrease of  $\kappa$  is observed, due to the phonon scattering by the magnetic excitations [\[31,32,34\]](#page-7-0). (ii) With  $T \leq 2$  K, the applied field could induce successive transitions from Fe $_{III}$  to Fe $_{II}$  and then to Fe $_{I}$  in the field-up process, and  $\kappa$ (*H*) shows two local maximums due to the weakened scattering of phonons across each transition. The transition from Fe $_{II}$  to Fe $_{III}$  is absent when the field is decreased, resulting in a larger *κ* below *H*m. (iii) With further cooling, *H*<sup>m</sup> becomes larger and there is only a first-order transition from Fe $_{III}$  to Fe<sub>I</sub> directly with  $T < 500$  mK, which <span id="page-6-0"></span>results in a different field dependence of *κ*. It should be noted that below 500 mK the  $Fe_{III}$  state is always present in the sweeping-field process. This indicates that the  $Fe_{III}$  state, which is metastable in sweeping magnetic field above 500 mK, becomes stable at *T <* 500 mK.

In passing, it should be pointed out that, besides the magnon excitations, the paramagnetic spins could also have influence on the phonon thermal conductivity [\[38,41,42\]](#page-7-0). Due to the paramagnetic scattering effect,  $\kappa(H)$  usually shows a diplike behavior. From Figs.  $6(e)$ – $6(j)$ , the dip around 1 T may be ascribed to the paramagnetic scattering on phonons. The complicated field dependencies at low fields between 0.97 K and 300 mK could be regarded as a superposition of magnon scattering at  $H_m$  on the recovery of  $\kappa$  due to the paramagnetic scattering, as indicated by the shoulderlike feature around 2 T for both field-up and field-down processes.

### **IV. SUMMARY**

Our detailed results of magnetization, electric polarization, and thermal conductivity point to a complex magnetic phase diagram of  $DyFeO<sub>3</sub>$  involving successive field-induced magnetic phase transitions of  $Fe<sup>3+</sup>$  spins. First, the unknown ground state at ultra-low temperatures ( $Fe_{III}$ ) is likely to have no linear ME effect. Apparently, this ground state is determined by the interaction between  $Dy^{3+}$  and  $Fe^{3+}$  spins. In particular,

when the Dy<sup>3+</sup> sublattice orders antiferromagnetically at  $T_N^{\text{Dy}}$ , the  $Fe<sup>3+</sup>$  spins have to change their orientation. Second, with 500 mK  $\lt T \leq 2$  K, a low-field irreversibility of the magnetic transitions is observed when the sample is cooled in zero field, which indicates that  $Fe_{III}$  structure is metastable with sweeping magnetic field. A similar irreversible behavior is probably the one found in another orthoferrite  $TbFeO<sub>3</sub>$ , probed by the capacitance measurement [\[43\]](#page-7-0), in which an unknown low-field phase also disappeared with sweeping-down field. It seems that only at very low temperatures (*<*500 mK), the interaction between  $Dy^{3+}$  and  $Fe^{3+}$  spins is strong enough to stabilize the  $Fe_{III}$  structure. The further studies of these materials and probably other members of orthoferrite *RFeO*3 could help to explore their complicated low-*T* magnetic structures and related interesting ME effect.

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