Magnetic Josephson junctions with noncentrosymmetric superconductors

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We study the dc Josephson effect in a clean noncentrosymmetric superconductor/ferromagnet/noncentrosymmetric superconductor junction within the quasiclassical theory of superconductivity. By considering charge and spin currents, we show that in such junctions an exotic Josephson effect can take place, depending on the superconducting pairing state and spin polarization direction. We focus on the importance of spin-triplet/spin-singlet gaps ratio in such systems showing that its value is related to the existence of even and odd high-order harmonics in the charge and spin current-phase relations, and to the possibility of 0-π transitions.

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I. INTRODUCTION

The discovery of superconductivity in the compound $CePt_3Si$ [1], characterized by a peculiar crystal structure missing an inversion center, has opened up new prospects in the yet vast field of unconventional superconductivity. Soon other compounds were identified as noncentrosymmetric superconductors (NCSs), and new members of this class of materials are constantly discovered nowadays. Relevant examples are $CeRhSi_3$ [2], $CeIrSi_3$ [3], $CeCoGe_3$ [4], $CeIrGe_3$ [5], UIr [6], $Li_2Pd_xPt_{3-x}B$ [7–9], Mo_3Al_2C [10,11], Y_2C_3 [12], YPtBi [13], BiPd [14], LaNiC [15], and LuPtBi [16]. While the physical properties of different NCSs may vary greatly [17], the existence of an antisymmetric spin-orbit coupling (SOC) and the possibility of parity mixing are common traits. Indeed electrons moving in a crystal structure with a missing inversion center are subjected to an electric field, which in the rest frame of electrons appears as a magnetic field proportional to their orbital angular momenta, resulting in an effective antisymmetric Rashba SOC [18] $\mathbf{g}_\mathbf{k} = -\mathbf{g}_{-\mathbf{k}}$. Moreover the absence of inversion symmetry is also responsible for the fact that parity is not a good quantum number. As a consequence a mixed-parity superconducting state with the coexistence of even-parity/spin-singlet and odd-parity/spin-triplet Cooper pairs is possible [19,20].

NCSs are currently widely studied since several theoretical works have pointed out the possibility of nontrivial topological phase developments [21–31] and their ability to host spin supercurrents [32–35]. However, despite many theoretical and experimental efforts, the actual superconducting pairing mechanism and pairing state symmetry realized in NCSs are still unclear. There are hints pointing towards both conventional and unconventional superconductivity, singlet and triplet pairing states, time-reversal invariance and breaking, unitary and nonunitary spin-triplet states [36–42], depending on the particular material.

We focus here on the cerium-based NCSs, the most studied family of superconductors. They admit a common description since they share the same generating point group C_{4v} lacking a mirror plane normal to the c axis. They also have similar T-P phase diagrams and a superconducting phase developing close to a magnetic instability [5,43–46]. Strong electronic correlations play a major role in $CePt_3Si$ [47,48], the compound with the largest Sommerfeld coefficient and the only one with a superconducting phase accessible at ambient pressure. A comprehensive list of cerium-based NCSs properties can be found in Ref. [17] and references therein. The superconducting pairing state in these compounds is yet to be determined. Several experiments point towards an unconventional pairing state [41,42,49–52]. A fundamental question is if spin-triplet superconductivity is, at least partially, realized in such materials. This is a controversial matter since critical magnetic field H_{c2} measurements report values larger than the condensation energies pointing towards a spin-triplet state [53], while the fact that the superconducting phase develops close to an antiferromagnetic phase seems to suggest the realization of a spin-singlet state. Indeed the strong resistance of the superconducting phase against large magnetic fields could also be explained considering strong electronic correlations and SOC [20,54], without invoking a spin-triplet pairing state. Important hints are predicted to come from Raman scattering [55] but such experiments are hard to perform on superconductors with such a low T_c. The actual realization of spin-triplet pairing in NCSs is considered to be likely but a definite answer is still lacking. Also the order relation between the spin-triplet and spin-singlet gaps is predicted to be extremely important and a very different physics is expected in NCSs with spin-triplet pair potential larger than the spin-singlet one and vice versa. One possibility to identify spin-triplet pairing is to exploit the unique interplay of spin-triplet superconductivity and magnetism that would take place in such a case. While Cooper pairs in a spin-singlet superconductor are destroyed by a large enough magnetic field of any orientation, in a spin-triplet superconductor the direction is important since Cooper pairs would be sensitive only to magnetic field components not parallel to their equal-spin-pairing direction. A constant (decreasing) magnetic susceptibility across the superconducting transition with a magnetic field parallel (not parallel) to the equal-spin-pairing direction is expected to be strong evidence for spin-triplet superconductivity [56]. However in NCSs the strong electronic correlations and SOC complicate the situation. In $CePt_3Si$ a constant susceptibility for any field direction is found [54]. Moreover theoretical calculations show a helical vortex phase stabilized in NCSs under a magnetic field [57]. A possible way to overcome these complications is to analyze hybrid structures of NCSs and ferromagnets since even if magnetism and superconductivity are separated in space the consequences of their interplay directly affect the physics of the system. Several theoretical studies exist on proximity contact, tunnel junctions, and Josephson junctions with NCSs [28,30,33,58–71], mainly employing quasiclassical theories to describe the system. More generally, Josephson junctions with spin-triplet superconductors and ferromagnets have been extensively studied [72–83], revealing several novel features compared to conventional Josephson junctions. However the interplay of magnetism and the exotic superconductivity of NCSs in a Josephson junction has never been analyzed. The study of such a system promises to be interesting to look for the unconventional Josephson effect and for signatures of spin-triplet pairing and parity mixing, if any, that could be ascertained by standard transport measurement.

In this paper we study the dc Josephson effect between two NCSs connected by a ferromagnetic link by means of the quasiclassical theory of superconductivity. We analyze charge, spin, and critical currents as a function of several materials/junction parameters, pointing out their direct connections to the superconducting state realized in NCSs. We have considered NCSs with different superconducting pair potentials, but we limit ourselves to show results only for the most common type and briefly discuss the other possibilities. The paper is organized as follows. In Sec. II the system and the theoretical formalism employed are introduced. In Sec. III the results for charge, spin and also critical currents are presented and commented in detail. We summarize our findings in Sec. IV.

II. MODEL AND FORMALISM

We consider an effective two-dimensional (2D) noncentrosymmetric superconductor/ferromagnet/noncentrosymmetric superconductor Josephson junction (NCS/F/NCS). The junction is assumed to be in the clean limit. While typical NCSs can safely be assumed to be in the clean limit (see Ref. [17] and references therein), ferromagnetic metals are known to have a varying degree of impurities concentration resulting in mean free paths ranging from a few to tens of angstroms [84]. This fact limits from above the values that the thickness of the ferromagnetic bridge can assume in our model in order for the clean limit to be satisfied. In the presence of impurities the spin-triplet pair potential and, more generally, any anisotropic pair potential, would be suppressed according to the Anderson theorem [85]. However if the impurities were confined only to the ferromagnetic bridge, the spin-triplet correlations generated in F by the proximity effect from the spin-triplet pairing in NCSs could avoid impurities suppression depending on the relative orientation between the spin polarization in F and equal-spin-pairing direction in the NCSs [63,64].

A sketch of the junction under study is depicted in Fig. 1. The ferromagnetic layer has a thickness d and a homogenous exchange field $\mathbf{h}$ lying in the y-z plane and α is its angle from z axis. The x axis lies in the direction normal to the interfaces. Interfaces are assumed to be transparent and

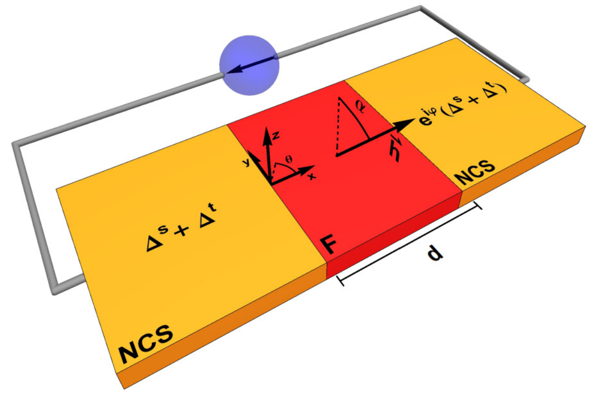

FIG. 1. (Color online) A sketch of the Josephson junction under examination with notation and coordinate system chosen. The NCSs are assumed to be semi-infinite while the ferromagnetic layer F has a finite width d. Its exchange field lies in the y-z plane and α is the angle it forms with the z axis. The junction is assumed to be in the clean limit.

magnetically inactive. The tunneling limit has been partly treated in Ref. [66]. We consider a current biased junction and study the dc Josephson effect. The system is described within the quasiclassical theory of superconductivity [86], a widely employed method to describe superconducting hybrids [87,88]. Quasiclassical theories have been able to successfully describe conventional and unconventional superconducting hybrids, and are usually applied by experimental groups to interpret/analyze their data. Within this formalism, all the physical information is coded in the position-, momentum-, and energy-dependent quasiclassical Green's function (GF) $\check{g}(\mathbf{r},\mathbf{k},\varepsilon)$. In the quasiclassical formalism momentum is fixed on the Fermi surface and only its direction is important, i.e., $\mathbf{k} = (\cos\theta, \sin\theta, 0)$, where θ is the azimuthal angle in the x-y plane (see Fig. 1). Throughout the paper $(\check{\bullet})$ will denote a 4×4 matrix in spin $\otimes$ Nambu space and $(\hat{\bullet})$ will denote a 2×2 matrix in spin space. We also use units such that $\hbar = k_B = 1$.

The quasiclassical GF for the system under examination can be obtained by solving the Eilenberger equation [89]. It reads (impurity scattering terms are omitted since we consider a clean junction)

$$\mathbf{v}_F \nabla \check{g} + [\varepsilon_m \check{\sigma}_3 + i(\check{\Delta} + \check{h}), \check{g}] = \check{0}, \tag{1}$$

with normalization condition $\check{g}\check{g} = \check{1}$. Here, $\varepsilon_m = \pi T(2m + 1)$ are discrete Matsubara energies, T is the temperature, $\mathbf{v}_F$ is the Fermi velocity and $\check{\sigma}_i = \hat{\tau}_i \otimes \hat{I}$ with $\hat{\tau}_i$ the Pauli matrices in particle-hole space. $\hat{\sigma}_j (j = 1,2,3)$ denote Pauli matrices in spin space. $\check{\Delta}$ and $\check{h}$ are the superconducting pair potential and exchange field matrices. The former (latter) is nonzero only in NCS (F). The exchange energy matrix explicitly reads

$$\check{h} = \Theta(x)\Theta(d - x) \begin{pmatrix} \mathbf{h} \cdot \hat{\boldsymbol{\sigma}} & \hat{0} \\ \hat{0} & (\mathbf{h} \cdot \hat{\boldsymbol{\sigma}})^{\dagger} \end{pmatrix}, \tag{2}$$

where

$$\mathbf{h} = (0, h \sin\alpha, h \cos\alpha), \tag{3}$$

and $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_1,\hat{\sigma}_2,\hat{\sigma}_3)$ is the vector of Pauli matrices. The superconducting pair potential matrix reads

$$\check{\Delta} = \Theta(-x)\check{\Delta}_L + \Theta(x-d)\check{\Delta}_R, \tag{4}$$

with

$$\check{\Delta}_{L(R)} = \begin{pmatrix} \hat{0} & \hat{\Delta}_{L(R)} \\ \hat{\Delta}^{\dagger}_{L(R)} & \hat{0} \end{pmatrix}, \tag{5}$$

In NCSs the superconducting pair potential in spin space has the general form

$$\hat{\Delta} = \Delta^s f^s(\mathbf{k}) i\hat{\sigma}_2 + \Delta^t f^t(\mathbf{k})\mathbf{d}_{\mathbf{k}} \cdot \hat{\boldsymbol{\sigma}} i\hat{\sigma}_2, \tag{6}$$

where $\Delta^{(s,t)}$ is the singlet (triplet) component amplitude, $f^{(s,t)}(\mathbf{k})$ are structure factors, and the well-known d vector $\mathbf{d}_{\mathbf{k}}$ [90] is used to parametrize the triplet pair potential components. For the sake of simplicity we normalize Eq. (6) introducing the total gap amplitude $\Delta_0 \equiv \max_{\mathbf{k}} \mathrm{Tr}(\hat{\Delta}^{\dagger}\hat{\Delta})/2$ and the triplet/singlet gap amplitudes ratio $r \equiv \Delta^t/\Delta^s \in [0,\infty)$, resulting in

$$\hat{\Delta} = \Delta_0\left(\frac{f^s(\mathbf{k})}{\sqrt{1+r^2}} i\hat{\sigma}_2 + \frac{r f^t(\mathbf{k})}{\sqrt{1+r^2}}\mathbf{d}_{\mathbf{k}} \cdot \hat{\boldsymbol{\sigma}} i\hat{\sigma}_2\right). \tag{7}$$

With this parametrization the relative weight of triplet and singlet gaps amplitudes r can be changed without affecting the total gap amplitude $\Delta_0 = \sqrt{\Delta^{s\,2} + \Delta^{t\,2}}$, and the limiting cases of pure singlet and triplet superconductors can be obtained with the substitutions $r = 0$ and $r \to \infty$, respectively. The value of r effectively realized in NCSs is extremely important since Andreev bound states can develop at the surface if $r > 1$ ($\Delta^t > \Delta^s$), depending on the orbital symmetry of pair potentials, i.e., the form of structure factors. It is usually assumed that they are the minimal spherical harmonics compatible with the spin structure, i.e., s wave for singlet and p wave for triplet pair potentials. The $s+p$ case is obtained by choosing $f^s(\mathbf{k}) = f^t(\mathbf{k}) = 1$. Theoretical studies [67–71] predict that higher spherical harmonics pair potentials are possible such as $d+p$, i.e., $f^s(\mathbf{k}) = f^t(\mathbf{k}) = 2k_xk_y$, and $d+f$, i.e., $f^s(\mathbf{k}) = f^t(\mathbf{k}) = k_x^2 - k_y^2$. We have studied all three cases but we explicitly report results only on $s+p$ pair potentials, since the other cases differ only quantitatively, rather than qualitatively. The d vector in Eq. (7) is closely linked to the SOC $\mathbf{g}_{\mathbf{k}}$ and the condition $\mathbf{d}_{\mathbf{k}} \parallel \mathbf{g}_{\mathbf{k}}$ holds [20]. The proper choice for our case, i.e., NCSs with C_{4v} generating point group, is $\mathbf{d}_{\mathbf{k}} = (-k_y,k_x,0)$ [91]. This corresponds to equal-spin-pairing direction along z, i.e., out of the plane of the junction (see Fig. 1). Both $\hat{\Delta}_L$ and $\hat{\Delta}_R$ in Eq. (5) take the form specified in Eq. (7). We also consider the possibility for the total gap amplitudes in the left and right superconductors to differ. In a real NCS/F/NCS this could be due by different materials, doping, or a temperature gradient across the junction.

We have not explicitly included a SOC term in Eq. (1) in order to simplify the problem and obtain analytical results in limiting cases. We believe this to be a legitimate assumption for several reasons. First of all it has been shown that the bulk quasiclassical GF in NCSs does not depend on SOC even if it this term is included in the Eilenberger equation [92]. This of course does not imply that the position-dependent GF also does not. However we expect the inclusion of SOC to induce only small quantitative changes in our results. Other works based on quasiclassical methods, e.g., Bogoliubov–de-Gennes equations in the Andreev approximation, have shown that the behavior of the tunneling conductance of NCSs is mainly dominated by the r value and results with a fixed r value associated to zero or finite SOC are almost identical with only small quantitative changes occurring [28,30]. It has also been shown that tunneling conductance spectra of NCS/F junctions are only weakly affected by the SOC and almost identical to the one of conventional S/F junctions in the pure singlet limit [60]. Studies beyond the quasiclassical approximation have shown that SOC and r are not independent parameters as always assumed in quasiclassical studies, which include both. Full self-consistent calculations based on extended Hubbard models [67] show clearly that there is a one to one correspondence (almost a linear dependence) between the calculated ratio of the triplet/singlet gaps nucleating in NCSs and the assumed SOC strength. The inclusion of a SOC term in Eq. (1) acting only on the half spaces occupied by the NCSs would give a small quantitative change in the calculated GFs in these half spaces. However the physics of the junctions is mostly dependent by the type (triplet, singlet, or both) of superconducting correlations in the system (information contained in the gap function form) and their interplay with the exchange field in the bridge, rather than by how these correlations were generated in the first place (the SOC in NCSs). Consequently we believe that our model or, in general, any quasiclassical model neglecting SOC, can be seen as toy models offering a sufficient description of NCSs since they are able to reproduce the effect of SOC through r only, in a way consistent with studies beyond the quasiclassical approximation which have shown the two to be not really independent but proportional. Consequently every subsequent result referring to a particular r value can be alternatively seen as results referring to a particular SOC value.

Equation (1) has to be solved with proper boundary conditions, i.e., GF has to be continuous at the interfaces and has to converge towards the bulk superconducting GFs far from the interfaces for all quasiparticle trajectories. We employ the following parametrization for the quasiclassical GF:

$$\check{g} = \begin{pmatrix} g_1\hat{1} + \mathbf{g}_1\cdot\hat{\boldsymbol{\sigma}} & (g_2\hat{1} + \mathbf{g}_2\cdot\hat{\boldsymbol{\sigma}})i\hat{\sigma}_2 \\ i\hat{\sigma}_2(g_3\hat{1} + \mathbf{g}_3\cdot\hat{\boldsymbol{\sigma}}) & g_4\hat{1} - \hat{\sigma}_2(\mathbf{g}_4\cdot\hat{\boldsymbol{\sigma}})\hat{\sigma}_2 \end{pmatrix}. \tag{8}$$

In terms of it, the bulk solutions in the left ($x \to -\infty$) and right ($x \to +\infty$) NCSs read [92]

$$\begin{aligned} g_1(\mp\infty) &= \frac{\varepsilon_m}{\Omega^-_{L,R}} + \frac{\varepsilon_m}{\Omega^+_{L,R}}, \\ g_2(\mp\infty) &= \frac{\Delta_{L,R}(1-r)}{\Omega^-_{L,R}\sqrt{1+r^2}} + \frac{\Delta_{L,R}(1+r)}{\Omega^+_{L,R}\sqrt{1+r^2}}, \\ \mathbf{g}_1(\mp\infty) &= \mathbf{d}_{\mathbf{k}}\left(\frac{\varepsilon_m}{\Omega^-_{L,R}} - \frac{\varepsilon_m}{\Omega^+_{L,R}}\right), \\ \mathbf{g}_2(\mp\infty) &= \mathbf{d}_{\mathbf{k}}\left(\frac{\Delta_{L,R}(1-r)}{\Omega^-_{L,R}\sqrt{1+r^2}} - \frac{\Delta_{L,R}(1+r)}{\Omega^+_{L,R}\sqrt{1+r^2}}\right), \end{aligned} \tag{9}$$

with

$$\Omega^{\pm}_{L,R} = \sqrt{\varepsilon_m^2 + \frac{|\Delta_{L,R}|^2(r\pm 1)^2}{1+r^2}}$$
$$\Delta_R = \Delta_L \exp(i\varphi), \tag{10}$$

where φ is the external phase difference between the superconducting order parameters, and $\Delta_{L,R}$ are the total gap amplitudes on the left and right sides of the junction. We assume that their temperature dependence is BCS-like, i.e., $\Delta_{L,R}(T) = \Delta_{L,R}(0)\tanh(1.74\sqrt{Tc/T-1})$. Not all GF terms of Eq. (8) have been included in Eq. (9) since it is well known that quasiclassical GFs contain twice the same physical information [93]. The lower blocks can be calculated by the upper blocks, or vice versa, with the aid of the symmetry relations

$$\breve{g}(\mathbf{k},\varepsilon_m)^\dagger = \breve{\sigma}_3\breve{g}(\mathbf{k},-\varepsilon_m)\breve{\sigma}_3, \tag{11}$$

$$\breve{g}(\mathbf{k},\varepsilon_m)^T = \breve{\sigma}_2\breve{g}(-\mathbf{k},-\varepsilon_m)\breve{\sigma}_2. \tag{12}$$

The Eilenberger equation should be also supplemented with a self-consistent equation to obtain the spatial variation of pair potential in the superconductors. However a very good agreement between non-self-consistent and self-consistent calculations [94–96], and experimental results [97–100] for unconventional Josephson junctions has been found. Though we are not performing full self-consistent calculations, GFs in the superconducting banks are not considered to be the bulk ones, and their position dependence is taken into account.

An analytical solution of the Eilenberger equation for a NCS/F/NCS is obtainable. However, the excessive length and intricacy of GFs causes it to be of little practical applicability. Simple solutions can be obtained, however, in limiting cases. When there is no exchange field in F, i.e., the ferromagnet is a normal metal, GFs have a compact form. An example is

$$g_1 = \frac{\eta\varepsilon_m^2\left(\Delta_R^2-\gamma^2\Delta_L^2\right)+\varepsilon_m\left[\gamma^2\Delta_L^2(\Omega_R^+ + \Omega_R^-)+\Delta_R^2(\Omega_L^+ + \Omega_L^-)\right]+\eta\left(\Delta_R^2\Omega_L^+\Omega_L^- - \gamma^2\Delta_L^2\Omega_R^+\Omega_R^-\right)}{[\varepsilon_m(\gamma\Delta_L-\Delta_R)-\eta(\gamma\Delta_L\Omega_R^+ + \Delta_R\Omega_L^+)][\varepsilon_m(\gamma\Delta_L-\Delta_R)-\eta(\gamma\Delta_L\Omega_R^- + \Delta_R\Omega_L^-)]}, \tag{13}$$

where $\eta = \mathrm{sign}(v_x)$ and $\gamma = \exp(-2\varepsilon_m d/v_x)$. Similar expressions hold for other GF elements.

Once the GFs are obtained analytically or evaluated numerically the charge and spin Josephson currents densities are calculated according to

$$\mathbf{J} = 2i\pi eTN(0)\sum_m\langle \mathbf{v}_F g_1(\hat{\mathbf{v}}_F,\mathbf{r},\varepsilon_m)\rangle, \tag{14}$$

$$\mathbf{J}_{S_i}(\mathbf{r}) = i\pi TN(0)\sum_m\langle \mathbf{v}_F[\hat{\mathbf{r}}_i\cdot\mathbf{g}_1(\hat{\mathbf{v}}_F,\mathbf{r},\varepsilon_m)]\rangle, \tag{15}$$

where $\langle\bullet\rangle$ is the angular average over incident trajectories and $N(0)$ is the density of states in the normal state at the Fermi energy [82,83,93].

III. RESULTS AND DISCUSSION

In this section we show results for charge and spin currents in NCS/F/NCS. We discuss dependence of these currents on magnitude and orientation of the exchange field in F and on its width. The effect of spin-singlet and spin-triplet degree of mixing in NCSs will be addressed. The orbital symmetry considered is $s+p$, which is a mixture of s-wave spin-singlet and chiral p-wave spin-triplet pairings. We have also considered two other types of pairing ($d+p$ and $d+f$), but the results are qualitatively similar and will not be shown. Moreover we restrict ourselves to the case of identical NCSs, i.e., the gap amplitude Δ_0 and mixing ratio r are the same in the left and right side of the junction. The gap Δ_0 is used as unit of energy and the junction length d is normalized to the superconducting coherence length $\xi = v_F/(\pi\Delta_0)$. Even if we are working with normalized units it is important to discuss the range of significant parameter values in order to compare our theoretical results with experiments. Cerium based NCSs are low-temperature superconductors ($T_c \approx 1$ K) with a small gap ($\Delta_0 \approx 10^{-1}-10^{-2}$ meV) and a large coherence length ($\xi \approx 10$ nm). We will show results for a normal bridge, i.e., $h=0$, and a weak ferromagnetic bridge with $h = 10-50\Delta_0$. Results for stronger ferromagnets are qualitatively similar. Indeed it is well known that in ferromagnetic Josephson junctions, currents and other relevant quantities such as free energy are oscillating functions of h to a good approximation [101–113]. However our results cannot be immediately extended to very strong ferromagnets such as cobalt or half metals such as CrO_2 since in these materials the exchange and Fermi energies are comparable and the quasiclassical theory cannot be applied in principle. It is also important to comment on the relevant length scales for our junction. In clean systems the decay length of superconducting correlations leaking in the nonsuperconducting side of the junction is the thermal coherence length $\xi_T = v_F/(2\pi T)$ whether the nonsuperconducting side is a normal metal or a ferromagnet. In moderately disordered systems the penetration length is the smallest between the thermal coherence length and the mean-free path [114,115]. This is in sharp contrast with the case of dirty systems, where this length differs in systems with normal metal and ferromagnets, with the magnetic coherence length much shorter than the normal one [87], except when equal-spin-pairing correlations are present [88]. However the magnetic coherence length in clean systems $\xi_F = v_F/(2h)$ does play a role, governing the oscillation length of superconducting correlations. In our model $\xi_T \approx 10^3$nm and $\xi_F \approx 5$A for $h = 50\Delta_0$. The maximum length of the ferromagnetic bridge used is $d = 1.2$nm, large enough to see one oscillation in the critical current but still compatible with the assumption of ballistic junction.

A. Charge current

We start by analyzing the φ dependence of the charge current, i.e., the Josephson current-phase relation [116]. Even if critical current measurements are much more common, several reliable experimental techniques have been created to probe the current-phase relation of Josephson junctions with conventional and unconventional superconductors. The sensitivity of these techniques is large enough to ascertain the existence and amplitude of harmonics of higher order than the fundamental one $\sin(\varphi)$ [117–121]. In Fig. 2, the charge current for different exchange field amplitudes (h) and orientations (α), and for several values of the relative weight of triplet and singlet gaps (r) is shown. In each panel the exchange field direction and amplitude values are fixed as reported. $T = 0.1T_c$ and $d = 0.08\xi$ everywhere. The current is normalized to $J_0 = \pi e N(0) v_F T_c$. Each curve is associated with a $r = \Delta^t/\Delta^s$ value. In the left panels $\Delta^s \geq \Delta^t$ with $r = 0,0.2,0.4,\ldots,1$ from red/solid (pure singlet NCSs) to blue/dotted (maximally mixed NCSs with $\Delta^s = \Delta^t$). In the right panels $\Delta^s \leq \Delta^t$ with $r = 1,1/0.8,1/0.6,\ldots,\infty$ from blue/dotted (maximally mixed NCSs with $\Delta^s = \Delta^t$) to green/solid (pure triplet NCSs).

In Figs. 2(a), 2(b) the exchange field is set to zero, i.e., they depict the current-phase relation of a NCS/N/NCS junction. In this case the current mainly obeys the Josephson relation $J \propto \sin(\varphi)$ with only small contributions from higher-order harmonics $\sin(n\varphi)$ [66]. The amplitude of the current clearly depends on the degree of mixing r: it is maximum for no mixing (pure singlet and pure triplet limits) and minimum for maximum mixing $r = 1$. In NCS/F/NCS, Figs. 2(c)–2(p), the current-phase relation becomes more complicated and contributions from higher-order harmonics can become more important. This is evident in Fig. 2(c) showing the current for $h = 12.5\Delta_0$ and with no misalignment with the equal-spin-pairing direction in the superconductors ($\alpha = 0$), for a NCS/F/NCS junction with $\Delta^s \geq \Delta^t$. The difference from the Josephson relation is larger for smaller contribution of the spin-triplet gap. This appears manifest in Fig. 2(d) showing the current in the same junction with $\Delta^s \leq \Delta^t$. In this case the current is again mostly sinusoidal, and numerically converge to the one in NCS/N/NCS for larger triplet to singlet ratios. The behavior of NCS/F/NCS with no misalignment ($\alpha = 0$) is a consequence of the fact that in such a configuration the exchange field in F is pair breaking only for the spin-singlet Cooper pairs in NCS. When $\alpha \neq 0,\pi$ the exchange field is pair breaking even for spin-triplet Cooper pairs since spin-flip scattering processes become possible. In this case strong modifications in the current-phase relation can appear even for large r values and in the pure spin-triplet limit, e.g., Figs. 2(n), 2(p). Also when there is misalignment $\cos(n\varphi)$ harmonics are present and current is no more zero at $\varphi = 0$, see Figs. 2(e)–2(h) and 2(m)–2(p). These contributions are largest in the maximally mixed $r = 1$ case and disappear in the pure singlet $r = 0$ and pure triplet $r \to \infty$ limits. The existence of these $\cos(n\varphi)$ components and spontaneous charge currents are related to some sort of frustration in the system. Indeed in the configurations where they appear, also a spin current is flowing (see Sec. III B). The latter is expected to exert a torque on the spin polarization of the ferromagnetic bridge [78], triggering a precessional motion. When spin relaxation

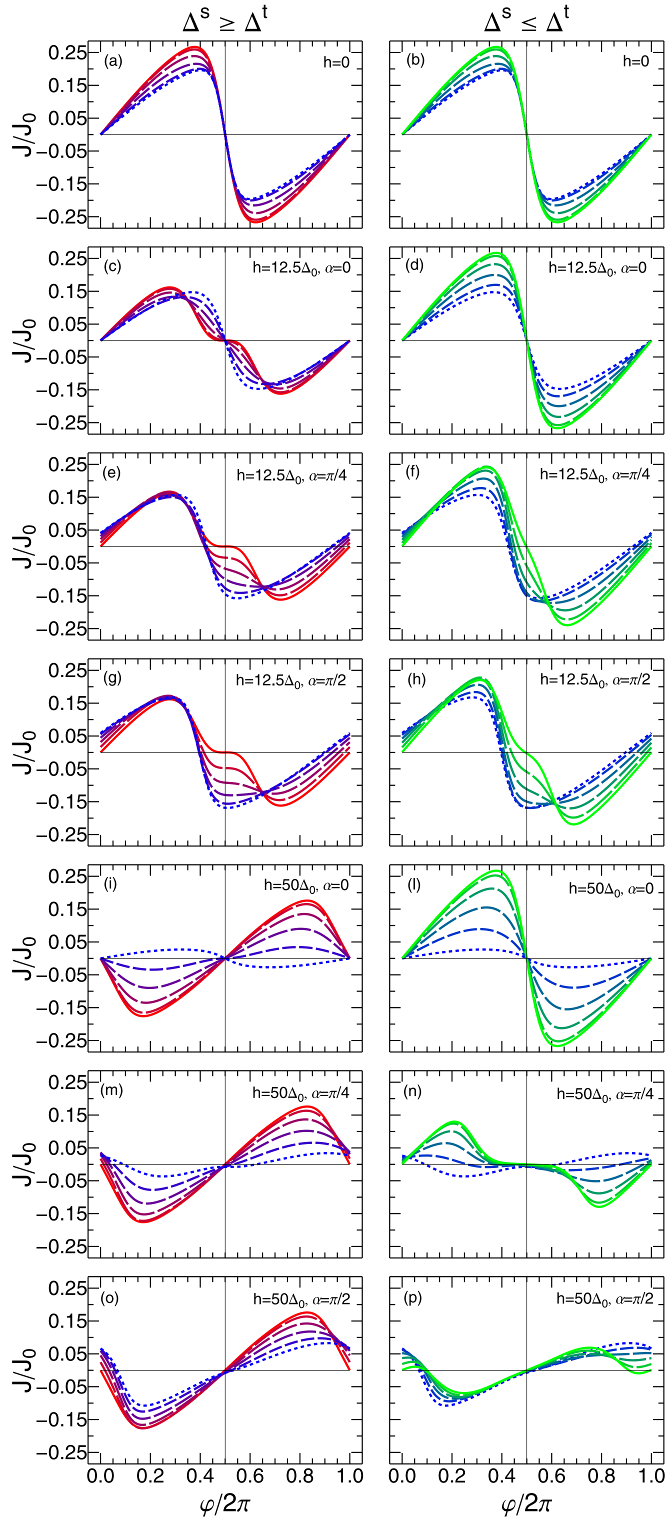

FIG. 2. (Color online) Josephson current in NCS/N/NCS and NCS/F/NCS. In each panel the exchange field direction and amplitude values are fixed. Each curve is associated to a $r = \Delta^t/\Delta^s$ value. In the left panels $\Delta^s \geq \Delta^t$ with $r = 0,0.2,0.4,\ldots,1$ from red/solid (pure singlet NCSs) to blue/dotted (NCSs with $\Delta^s = \Delta^t$). In the right panels $\Delta^s \leq \Delta^t$ with $r = 1,1/0.8,1/0.6,\ldots,\infty$ from blue/dotted (NCSs with $\Delta^s = \Delta^t$) to green/solid (pure triplet NCSs). $T = 0.1T_c$ and $d = 0.08\xi$ everywhere. Currents are given in units of $J_0 = \pi e N(0) v_F T_c$.

is considered, the precession is damped and the system reaches a relaxed configuration without any misalignments, spin-flip scattering processes, and spontaneous current [122]. We have reserved to study this problem in future works.

Like in a conventional magnetic Josephson junction with s-wave superconductors (S/F/S) 0-π transitions are possible in NCS/F/NCS as a function of h or d (when $h \neq 0$) [101–113]. By comparing Figs. 2(a) and 2(c) with Fig. 2(i), we see that h-driven 0-π transitions are possible in NCS/F/NCS when $\Delta^s \geq \Delta^t$. We also see that for a fixed h value the Josephson ground-state phase depends on triplet/singlet gaps ratio. The π-0 transition for $h = 50\Delta_0$ in Fig. 2(i) takes place at $r \approx 0.92$. For different T, d, h, and α values the discussed features of Josephson current do not change, e.g., the conditions triggering the harmonics other than the fundamental ones and the possibility of 0-π transitions, while quantitative differences exist, e.g., the harmonics amplitudes and ground-state phase depends on the particular values assumed by these parameters.

B. Spin current

In the Josephson junction with NCSs a Josephson spin current also can flow. While charge current does not depend on the position in the junction when it is evaluated, the spin current does [72–83,123]. In our setup, the only nonzero spin current component is the x-polarized one. We evaluate it at the left interface (it differs from the one at the right interface simply by a -1 factor). In Fig. 3 relevant examples of spin current are plotted with and without misalignment for several r values at $T = 0.1T_c$ and $d = 0.08\xi$. The current is normalized to $J_{0,S} = \pi N(0) v_F T_c$. The notation used is the same as the one in Fig. 2. For $\alpha = 0$ the magnetic barrier acts as a spin filter and the spin current generated is mostly sinusoidal [see Figs. 3(a), 3(b)]. In this case the current is maximum for maximally mixed NCSs with $r = 1$ and minimum in the pure singlet and triplet limits, at odds with the charge current. In these limits there is no spin current at all. For finite α [see Figs. 3(c), 3(d)] the spin current is generated mainly by spin-flip processes. Its magnitude is zero in the pure singlet limit and maximum in pure triplet limit. In this limit the spin current is an even function of phase difference and only $\cos(n\varphi)$ harmonics exist [72–83]. For finite r values the spin current has both $\sin(n\varphi)$ and $\cos(n\varphi)$ components. It is also worth noticing that the spin current generated by spin-flip scattering processes [Figs. 3(c), 3(d)] is much larger than the one generated by spin filtering [Figs. 3(a), 3(b)]. The general features discussed do not depend on the particular T, h, and d values chosen in Fig. 3 and only quantitative changes occur for different sets of parameters values.

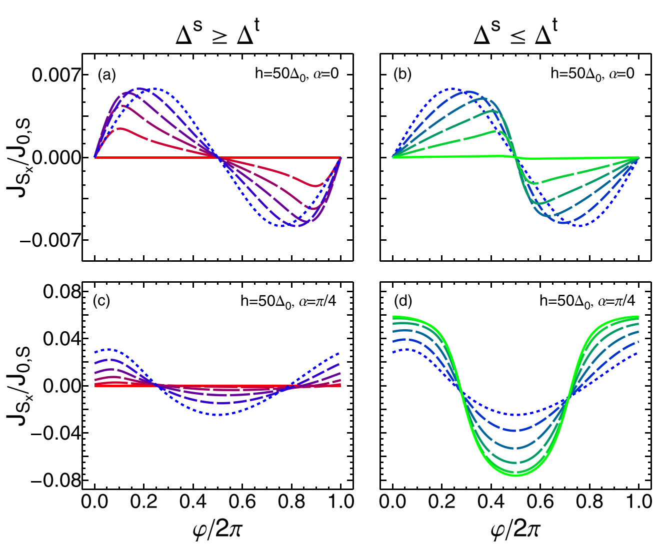

FIG. 3. (Color online) Josephson spin current in NCS/F/NCS. In each panel the exchange field direction and amplitude values are fixed. Each curve is associated to a $r = \Delta^t/\Delta^s$ value with the same pattern of Fig. 2. $T = 0.1T_c$ and $d = 0.08\xi$ everywhere. Currents are calculated in units of $J_{0,S} = \pi N(0) v_F T_c$.

The conditions to obtain a finite spin current can be stated by analyzing what type of superconducting correlations, i.e., anomalous Green's functions, are generated by proximity effect in the system. In a quasiclassical linearized treatment, where the limit of weak proximity effect is taken, one can show that the spin current is a sum of two terms products between the equal-spin- and opposite-spin-pairing anomalous Green's functions (or their derivatives). Consequently in order for the spin current to be finite both types of correlations have to be finite in the system. This is why in Josephson junctions with triplet superconductors a finite spin current is not always allowed. In particular one needs d vectors misalignment or a ferromagnetic barrier with misaligned spin polarization to observe it [72–83]. Both conditions trigger spin-flip scattering processes and it can be shown that when they are allowed a part of the equal-spin-pairing correlations transforms in opposite-spin-pairing correlations. This case is represented by the green curve (pure triplet NCSs) in Fig. 3(d). In Josephson junctions with NCSs spin-flip scattering process are not strictly necessary since equal-spin- and opposite-spin-pairing correlations are yet both in the system given the singlet/triplet mixed pair potential typical of NCSs. This case is represented by every curve in Figs. 3(a) and 3(b), i.e., the configuration without misalignment and spin-flip processes, with the exception of red and green curves (pure singlet and pure triplet limits, with only opposite-spin- and equal-spin-pairing correlations, respectively).

C. Critical current

The critical current is defined as the modulus of the maximum current (with respect to phase difference φ) allowed to flow in a Josephson junction. It can be measured easily in experiments as a function of parameters such as T, h, d. Excluding the former, this usually requires performing measurements on different samples differing in barrier width and magnetization strength. The sign of the critical current (positive or negative) determines the ground-state phase of the junction (0 or π) in conventional S/F/S junction. In more complex systems like the one under examination the free energy should be minimized as a function of φ to ascertain the phase value reached in the ground state. In NCS/F/NCS we find that the critical current is a simple decaying function of temperature with no sign changes (not shown) consistently with Ref. [66]. The critical current as a function of F width d is more interesting since 0-π transitions can take place. This

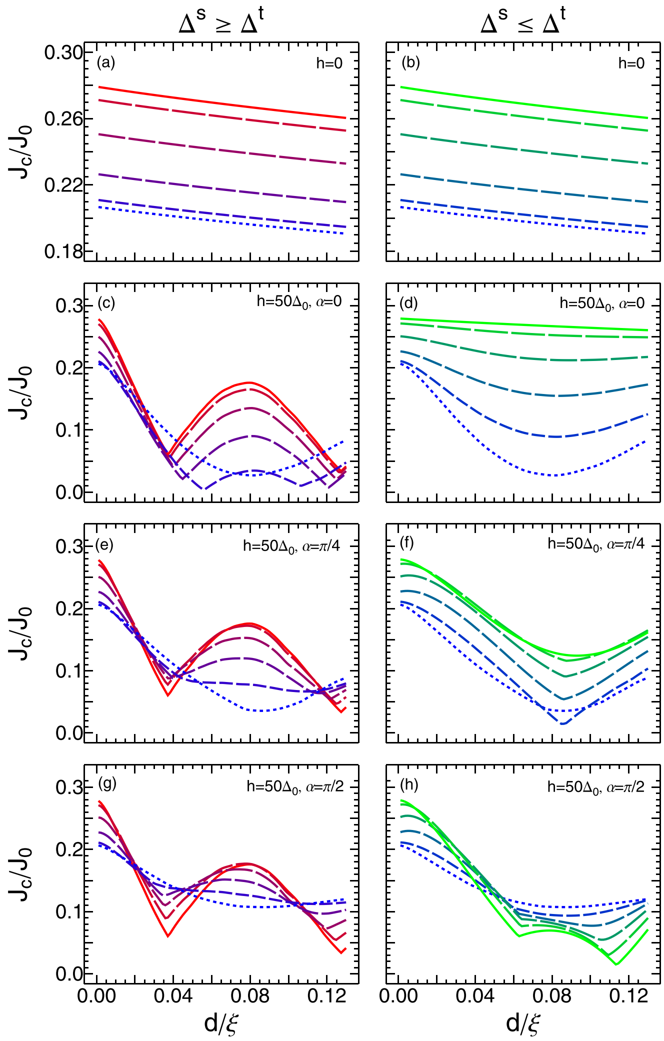

FIG. 4. (Color online) Critical Josephson current in NCS/N/NCS and NCS/F/NCS as a function of the barrier width d. In each panel the exchange field direction and amplitude values are fixed. Each curve is associated to a $r = \Delta^t/\Delta^s$ value with the same pattern of Fig. 2. $T = 0.1T_c$ everywhere.

case is shown in Fig. 4 where the same notation of Fig. 2 is again employed. In Figs. 4(a) and 4(b) the critical current for NCS/N/NCS is shown to be a simple decaying function of d. Its magnitude dependence on the triplet/singlet ratio r is the same as the one for the Josephson current. Critical current in NCS/N/NCS appears linear when plotted on the short length scale typical of magnetic oscillations, i.e., the magnetic coherence length $\xi_F = v_F/(2h)$, but it is indeed exponential on the larger length scale of nonmagnetic junction, i.e., the thermal coherence length $\xi_T = v_F/(2\pi T)$. We used the short length scale in the figure for a direct comparison with NCS/F/NCS [Figs. 4(c)–4(h)]. In this case the critical current can be an oscillating and decaying function of d. When there is no misalignment [Figs. 4(c), 4(d)] there are sharp oscillations and 0-π transitions for $\Delta^s > \Delta^t$. When $\Delta^s \leq \Delta^t$ the oscillations disappear approaching the pure triplet limit but also they are not accompanied by 0-π transitions as we have verified by evaluating the free energy. This does not depend on the particular h value chosen in the plot. When $\alpha \neq 0$ [Figs. 4(e)–4(h)] there are always sharp oscillations and transitions for $\Delta^s > \Delta^t$ while when $\Delta^s < \Delta^t$ this is the case only for orthogonal equal-spin-pairing direction and exchange field [the case $\alpha = \pi/2$ in Fig. 4(h)]. When $\alpha \neq 0,\pi/2$ and $\Delta^s < \Delta^t$, e.g., Fig. 4(f), the weak oscillation are not accompanied by 0-π transitions except if the pure triplet limit is considered. Again these general features do not depend on T and h values used in Fig. 4. Similar considerations hold for the analysis of the critical current as a function of h (not shown). The only difference is that without misalignment the decaying behavior of the current for $\Delta^s < \Delta^t$ is suppressed and disappear in pure triplet limit since for $\alpha = 0$ the exchange field is not pair breaking for spin-triplet Cooper pairs.

IV. CONCLUSIONS

We have theoretically studied the dc Josephson effect in a clean noncentrosymmetric superconductor/ferromagnet/noncentrosymmetric superconductor junction showing that its features are closely related to the interplay of superconductivity and magnetism in the system. We have shown that the transport properties of the junction are strictly connected to the type of superconductivity, namely if a spin-triplet pairing exists, and the degree of parity mixing. When this is the case, the transport properties strongly depends on exchange field direction in the ferromagnet.

We have shown the charge current behavior as a function of the spin-triplet/spin-singlet mixing. It reaches its maximum in the pure singlet or pure triplet limit, and its minimum when the two gaps are equal in magnitude, i.e., at maximum mixing. We have shown that the conventional Josephson current-phase relation $J \propto \sin(\varphi)$ approximately holds only in the limit of weak exchange field. The current can contain higher-order harmonics of appreciable magnitude depending on the exchange field amplitude and direction. Also $\cos(n\varphi)$ harmonics can be generated when the exchange field in the ferromagnet and the equal-spin-pairing direction in the superconductors are misaligned. These harmonics' existence and amplitudes are closely related to the degree of mixing, i.e., they disappear in the pure singlet or triplet limits and are maximum for maximum mixing. We have pointed out that 0-π transitions as a function only of the degree of mixing can exist, depending on the (fixed) junction parameters. We have shown that a Josephson spin current can flow depending on the junction properties. Its magnitude and Fourier components are related to exchange field amplitude and direction. In particular when there is no misalignment between exchange field and equal-spin-pairing direction, the ferromagnet works as a spin filter barrier generating a spin current with only $\sin(n\varphi)$ terms. In this case the spin current, at odds with the charge current, is maximum for maximum mixing and disappears in the pure singlet and pure triplet limits. In the case of misaligned exchange field, i.e., when spin flips are included, the spin current in general has both even and odd components. Its magnitude is zero in the pure singlet limit, and maximum in the pure triplet limit, where only $\cos(n\varphi)$ harmonics exist.

We also showed results for the critical current as a function of barrier width. The considerations about Josephson current-phase relation amplitude are still valid for the critical current. An oscillating and decaying behavior is found except in the limit of zero exchange field, where the current is a monotonically decaying function of the barrier width. When this is not the case there are always sharp oscillations and 0-π transitions for $\Delta^s > \Delta^t$ while in the $\Delta^s < \Delta^t$ case the oscillations, if any, are weak and generally not accompanied by transitions unless the equal-spin-pairing direction in the superconductors and the exchange field direction in the ferromagnet are orthogonal or the superconductors are considered to be in the pure triplet limit. Similar considerations hold for the critical current as a function of exchange field amplitude. All these results have been explicitly shown only for $s + p$ noncentrosymmetric superconductors but remain valid also for $d + p$ and $d + f$ noncentrosymmetric superconductors.

We believe our results can be useful in the field of spin-polarized transport in superconductors and for experimental probes of noncentrosymmetric superconductors. Also singlet/triplet mixing possibility in superconductors is not exclusive to noncentrosymmetric materials. Indeed several theoretical studies [124–131] have pointed out that in heterostructures with singlet (triplet) superconductors triplet (singlet) superconducting correlations can be generated such that an effective singlet/triplet coexistence takes place. Experimental signatures of this effect have been found, e.g., in measurement of supercurrents in a magnetic Josephson junction with singlet superconductors too wide to be compatible with the short propagation of singlet superconducting correlations in ferromagnets [132–135].

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